PHY1112: Assignment 11

> Random Notebooks

Assigned: March 26th, 2024

Due: April 2nd, 2024

Learning Objectives

1. Learn how to use Jupyter notebooks
2. Practice Monte-Carlo methods (integration by rejection)

Grade Breakdown

Part	1	Total
Points	21	21
Score		

Question 0: A Noteworthy Teaching Tool.

This assignment will be answered using a "Jupyter Notebook". These notebooks allow you to use Markdown and Python in the same document, as explained in more detail in your lab. They are relatively easy to use and are excellent teaching/explanatory tools.

Code that is run in a notebook will not necessarily be as fast or efficient as code run in a script of its own, however the ability to walk through each step, and include documentation throughout can help organize the script writer and/or allow you to walk someone else through the problem.

When making your Jupiter notebook for this assignment, *write it as if you are showing someone else how to solve the problem.* Include clear explanations of the steps you are taking, with any justifications necessary.

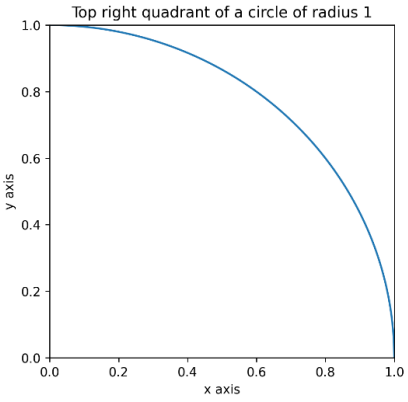
Question 1: Point and Shoot.

It is possible to make use of random numbers to do integration. In fact, for integrals over many dimensions, this is by far the most efficient way of doing it. In this problem, we will make use of Monte-Carlo integration by rejection to estimate the value of π.

Consider the top right quadrant of a circle of radius R, given by

$$y = \sqrt{1 - \frac{x^2}{R^2}}, \qquad 0 \leq y \leq R$$

We will use $R = 1$ to answer the question, making it a unit circle. This is plotted at right.



a) In a Jupyter notebook, create a Markdown cell and in it include the title of the assignment, as well as your name and student number, and any other information you would include at the top of an assignment.
(1 mark)

b) Continuing in the Jupyter notebook, create a Python cell and in it write a function that returns the y value as defined above, taking in as input the x value. Define the radius just before the function definition as a global variable, so it can be easily changed.
(1 mark)

c) Continuing in the Jupyter notebook, create a Markdown cell and in it explain how Monte-Carlo integration by rejection works in general. This method is also known as the "point

and shoot" and "hit and miss" methods. Discuss the algorithm and break down the steps. Make sure it flows in such a way that will relate well to the comments you will insert in your code.
(2 marks)

d) Continuing in the Juptyer notebook, create a Python cell and in it write a Monte-Carlo integration by rejection function that takes as input:

- A function handle for the function to be integrated
- The lower and upper bounds of *x* and *y*.
- The number of points you will use to do the integration (meaning, total number of hits+misses).

The output of the function will be the area. Use either `np.random.rand` or `np.random.uniform` for your random number generation. Use vectorization wherever possible in your functions.
(5 marks)

e) Continuing in your Jupyter notebook, create a Markdown cell and in it discuss the particular problem we are solving, that is, to integrate the top right quadrant of a circle to estimate the value of π.

Include the formula for the area of a quarter circle in terms of its radius. Rearrange the formula such that π is on the left-hand side, with all other factors (including the area A) on the right-hand side. You can write π as π (which is π in LaTex).
(2 marks)

f) Continuing in your Jupyter notebook, create a Python cell and in it write a script that uses the Monte-Carlo integration function you made in part d) to calculate the area of a quarter circle. Use this area, along with your formula from part e) to estimate the value of π.

Do this for the following values of *N*: 10, 100, 1000, 10 000, 100 000.
(5 marks)

g) Continuing in your Jupyter notebook, create a Python cell and in it plot your results from part f), that is, your estimates of π as a function of N. Use pyplot's `semilogx` function for the plotting (so that the x axis, that is, the N values, appear in a log scale), and use markers rather than a line. Include also a horizontal line placed at numpy's value of π, `np.pi`.
(3 marks)

h) Continuing in your Jupyter notebook, create a Markdown cell and in it comment on your plot from part g).
(1 mark)

(21 marks total, 1 for docstrings/file header/variable naming/comments)