

SCIENCE I - Assignment 5

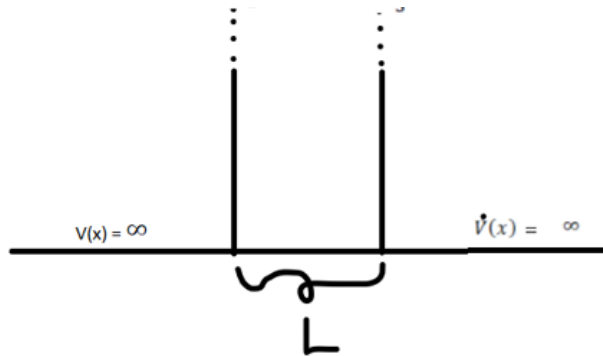
Quantum Mechanics

Total Marks: $3 * 4 + 3 * 4 = 24$

Problem I : The matrix representation of the three operators are given by:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Evaluate the commutation relations among the operators $[\sigma_i, \sigma_j]$
- Calculate eigenvalues and orthogonal eigen vectors of σ_1, σ_2 and σ_3
- Calculate the probability distribution of measuring σ_1, σ_3 in the one of the eigenstates of σ_2
- Evaluate the variances in measuring $\sigma_1(\Delta\sigma_1^2)$ and $\sigma_3(\Delta\sigma_3^2)$ and the corresponding uncertainty relation $\Delta\sigma_1^2 * \Delta\sigma_3^2$.



Problem II: An electron is moving inside an infinite 1D potential wall of length L. The wave function for the particle is given by (at $t=0$)

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$$

Where $|\phi_1\rangle$ and $|\phi_2\rangle$ are first two lowest energy eigenstates

- Calculate the average energy and variance in energy measurements at $t=0$ and later time t
- Calculate the probability density of finding the particle at $x = L/2$ at $t=0$
- Evaluate the average measuring X and variance of X at $t=0$ and later time t
- Calculate the average momentum in the state $|\psi\rangle$