Review of commutation and eigen-states

correspondence between classical and quantum mechanics

Time evolution of an observable in classical mechanics

$$\frac{dA}{dt} = \{A, H\} \leftarrow \text{poisson bracket}$$

in quantum mechanics

$$\frac{d\langle A\rangle}{dt} = \langle [A, H] \rangle$$

If $[A, H] = 0$ the observable is conserved

From classical mechanics

symmetry $\longrightarrow$ conservation $\rightarrow$ degeneracy of eigenstate

Under a central force potential

Rotational symmetry $\rightarrow$ angular momentum conservation

$\downarrow$

Energy eigenstates are degenerate

$$\Rightarrow \quad \hat{H}|\phi\rangle = E_n|\phi\rangle$$

$$L^2|\phi\rangle = l(l+1)\hbar^2|\phi\rangle$$

$$L_z|\phi\rangle = m\hbar|\phi\rangle$$

$$\phi \sim \phi_{nlm}(r, \theta, \phi)$$

$$E_n = \frac{e^2}{2a_0}\frac{1}{n^2} \qquad l = 0 \ldots (n-1)$$

$$m = -l \ldots +l$$

$$\text{degeneracy} = n^2 \quad \sum_{n=0}^{n-1} 2J+1 = n^2$$

Effective potential in central force

$$\frac{p_r^2}{2m} + \frac{l^2}{2mr^2} + V(r) = E$$

for

The Schrodenger equation for radial part

$$\left[\frac{p_r^2}{2m} + \frac{l(l+1)\hbar^2}{2mr^2} + V(r)\right]\phi(r) = E\phi(r)$$

$$\psi(r,\theta,\phi) = \phi_{nl}(r)\, Y_{lm}(\theta,\phi)$$

$$\phi = \frac{u(r)}{r}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial r^2} + \left(\frac{\hbar^2 l(l+1)}{2mr^2} - \frac{e^2}{r}\right)u = Eu$$

$$r = \frac{a_0}{2}x$$

$$\frac{2e^2}{a_0}\left(-\frac{d^2}{dx^2} + \frac{l(l+1)}{x^2} - \frac{1}{x}\right)u = Eu$$

$$\left(-\frac{d^2}{dx^2} + \frac{l(l+1)}{x^2} - \frac{1}{x}\right)u = -K^2 u$$

$= R(r)$

$$K^2 = -\frac{E}{2e^2/a_0}$$

$$\frac{l^2}{2mr^2} = 0$$

$x \to \infty \qquad u = e^{-kx}$

$$x \to 0 \qquad -\frac{d^2 u}{dx^2} + \frac{l(l+1)}{x^2} u = k^2 u$$

$$u \sim \cancel{\rho}\, x^{l} \qquad \rho = \cancel{k}\, x$$

$$\left(-\frac{d^2 \rho}{dx^2} + \frac{l(l+1)}{\rho^2} - \frac{1}{k\rho} \right) u = -u$$

$x \to 0 \qquad u = \rho^{l+1}$

$$u = \rho^{l+1} e^{-\rho} w(\rho) \qquad kr$$

Equation for $w(\rho)$

$$\rho \frac{d^2 w}{d\rho^2} + 2(l+1-\rho) \frac{dw}{d\rho} + \left[\frac{1}{k} - 2(l+1) \right] w = 0$$

$$\sum^{\infty} a\, \rho^{j}$$

$$w(\rho) = \sum_{j=0} a_j \rho^j$$

$$a_{j+1} = \frac{2(j+l+1) - \frac{1}{k}}{(k+1)(k+2+2l)}$$

$$2(n+l+1) = \frac{1}{k}$$

$$k^2 = \frac{1}{4(n+l+1)^2}$$

$$\frac{E}{e^2/2a_0} = \frac{1}{(n+l+1)^2}$$

$$E_{nl} = \frac{2e^2}{\cancel{4}a_0}\frac{1}{(n+l+1)^2}$$

$n+l+1 \rightarrow n$ principal quantum number

$n \rightarrow 0, 1 \ldots \ldots$

$l \rightarrow 0, 1 \ldots \ldots$

$$(\cancel{n}j+l+1) = n \qquad l = j-1$$

$$\phi_n(\rho) = \rho^{l+1} e^{-\rho} \underbrace{\nu_{nl}(\rho)}_{\text{degree of polynomial } (n+l+1)}$$

Laplacian in 3d

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\psi = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$\frac{\partial^2\Phi}{\partial\phi^2} = m^2\,\Phi(\phi) \qquad \Phi(\phi) \sim e^{\pm im\phi}$$

The effective potential