Postulates of QM

1) Every state is described by a wave function $\Psi(t)$

2) Every observable is described by a Hermitian operator A

$$A = A^{\dagger}$$

3) The eigen values of the operator correspond to the ~~values~~ outcomes of experimental measurement.

4) Time evolution of the wave function is described by the Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

$$|\Psi(t)\rangle = e^{iHt}|\Psi(0)\rangle$$

Proof. 1) All Hermeetian matrix has real eigenvalues.

2) If two operators commute they have ~~set~~ same set of eigenvectors

3) The eigenvectors of a Hermitian operator form an complete orthonormal basis.

$$A|\phi_1\rangle = a_1|\phi_1\rangle$$

$$A|\phi_2\rangle = a_2|\phi_2\rangle$$

$$\Rightarrow \langle \phi_1 | \overbrace{A|\phi_2\rangle}^{a_2|\phi_2\rangle} = a_2\langle \phi_1|\phi_2\rangle$$

Similarly $\langle \phi_1|A = \langle \phi_1| a_1^* = \langle \phi_1| a_1$

$$\Rightarrow \langle \phi_1 | A \phi_2 \rangle = a_1 \langle \phi_1 | \phi_2 \rangle$$

$$\therefore (a_2 - a_1) \langle \phi_1 | \phi_2 \rangle = 0$$

Hen if $a_2 \neq a_1$ $\langle \phi_1 | \phi_2 \rangle = 0$

$\Rightarrow \langle \phi_1 | \phi_2 \rangle$ are ~~normal~~ orthogonal

$\langle \phi_n | \phi_m \rangle = \delta_{nm}$ orthonormal basis

$\int |\phi_n|^2 dx < \infty$ square integrable

Gram-Schmidt orthonormalization

4) ~~The~~ If the state ψ is expanded as a linear combination of the eigenvectors of A ϕ_n

$$\psi = \sum_n a_n \phi_n$$

prob of getting value $\lambda_n = |a_n|^2$

$$= \langle A \rangle = \sum_n |a_n|^2 \lambda_n$$

$$= \langle \Psi | A \Psi \rangle$$

Two level atom

$$\mathcal{H} = \mu M$$

Eigen values $\epsilon_1 = \mu M \quad \epsilon_2 = -\mu M$

$$\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{1}{\sqrt{2}} |\phi_2\rangle$$

$$\langle \Psi | H | \Psi | =$$

$$\langle H \rangle = \frac{1}{2} \mu M - \frac{1}{2} \mu M = 0$$

$$\langle H^2 \rangle = \frac{1}{2} \mu^2 M^2 + \frac{1}{2} \mu^2 M^2$$

$$= \mu \sim \mathcal{A}_{ij}$$

$$\langle \Delta H \rangle = \mu M$$

<u>Stationary state</u>

$$|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$$

If the basis vectors are eigenvectors of H

$$\psi(0) = \sum_n a_n |\phi_n\rangle$$

H is diagonal

$$|\psi(t)\rangle = \sum_n e^{-iE_n t} a_n |\phi_n\rangle$$

If $|\psi(0)\rangle = \phi_1$

$$|\Psi(t)\rangle = a_1 e^{-iE_1 t} |\phi_1\rangle$$

Hence the vector remains the same. Hence, the eigenstates of Hamiltonian are called stationary state, since the probability does not evolve with time..

Momentum operator

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{p}\,\Psi = p\,\Psi \Rightarrow \Psi = A e^{+ikx}$$

$$k = \frac{p}{\hbar}$$

Commutation with x

$$[\hat{x}, \hat{p}]\Psi = -x\, i\hbar \frac{\partial \Psi}{\partial x} + i\hbar \frac{\partial}{\partial x}(x\Psi)$$

$$= -x\, i\hbar \frac{\partial\ }{\partial x} + i\hbar + x\, i\hbar \frac{\ }{\partial x}$$

$$= i\hbar$$

Time evolution of an operator

$$\frac{d\langle A\rangle}{dt} = \frac{d}{dt}\langle \psi | A | \psi \rangle$$

$$= \left\langle \frac{1}{i\hbar}[A, H] \right\rangle$$

$$H = \frac{p^2}{2m} + kx \qquad \dot{p} = -\frac{\partial H}{\partial x} = -K$$

$$\frac{d\langle \hat{p}\rangle}{dt} = \left\langle \frac{1}{i\hbar}[p, x] \right\rangle = -K$$

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\frac{d\langle p\rangle}{dt} = \frac{1}{i\hbar}\langle [p, x^2] \rangle = -\frac{1}{i\hbar}\langle [x^2, p] \rangle$$

$$\frac{}{dt} = \frac{1}{i\hbar}\langle[\ \cdot\ ,\ \cdot\ ]\rangle = \frac{1}{i\hbar}\langle[\ \cdot\ ,\ \cdot\ ]\rangle$$

$$[x^2, p] = 2i\hbar x$$

$$\Rightarrow \frac{d\langle p\rangle}{dt} = -K(x) \Rightarrow$$

correspondence Quantum to classical

average values follow the classical equation of motion

$$\frac{d\langle x\rangle}{dt} = \left\langle \frac{1}{i\hbar}[x, H]\right\rangle = \left\langle \frac{1}{i\hbar}\left[x, \frac{p^2}{2m}\right]\right\rangle$$

$$= \frac{1}{i\hbar}\frac{1}{2m}\left\langle [x, p^2]\right\rangle$$

$$[x, p^2] = xp^2 - p^2x = xp^2 - pxp + pxp - p^2x$$

$$= [xp - px]p + p[xp - px]$$

$$= [x, p]\, p + p\, [x, p]$$

$$= 2 i \hbar\, p$$

$$\Rightarrow \quad \frac{d\langle x \rangle}{dt} = \frac{\langle p \rangle}{m}$$