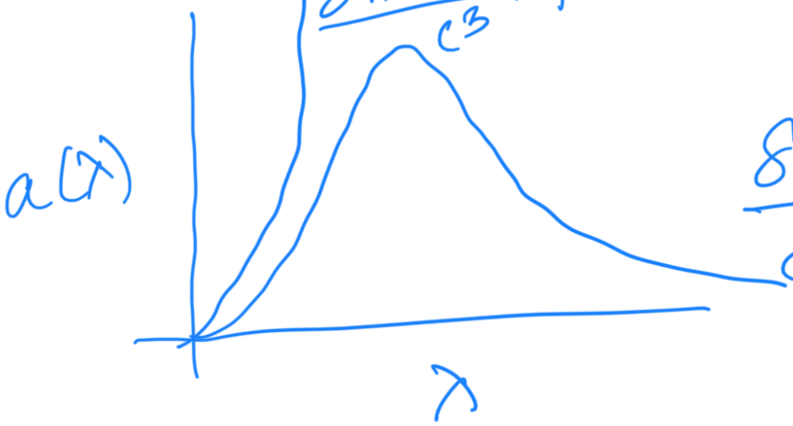
Quantum mechanics

1) Ultraviolet catastrophy

Max planck black body radiation

$$\frac{8\pi\nu^2}{c^3}kT$$



$$\frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_BT}-1}$$

2) Wave particle duality

Double slit experiment

3) photo electric effect

4) stability of atoms

Heisenberg uncertainty principle

$\hbar$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Fundamental postulates of QM

1) The state of a particle is described by a wave function $\psi(x,t)$

2) Every measurable observable there is an operator

3) The operator is hermitean as the eigen values must be real

Hermitean operator

If eigenvalues are real the matrix must be hermitean

$$H^{\dagger} = (H^{T})^{*}$$

$$H x = \lambda x$$

$$x^{\dagger} H^{\dagger} = \lambda x^{\dagger}$$

$$x^{\dagger} H x = \lambda x^{\dagger} x$$

$$x^{\dagger} H^{\dagger} x = \lambda x^{\dagger} x$$

$$x^{\dagger}(H - H^{\dagger}) x = 0 \Rightarrow H = H^{\dagger}$$

Commutativity and eigenfunctions

$$A \phi = a \phi$$

$$B \phi = b \phi$$

$$A B \psi = A b \psi$$

$$= b A \psi = b a \psi$$

$$B A \psi = B a \psi$$

$$= a b \psi$$

$$(AB - BA)\Psi = 0$$

$$AB = BA$$

If two operators commute they have same set of complete eigen vectors.

linear superposition and vector space

~~base~~

$$|\Psi\rangle = \sum_n a_n |\phi_n\rangle$$

Any vector can be written as linear combination of basis vectors $|\phi_n\rangle$

$a_n =$

Scalar product

$$\langle \Psi | \phi \rangle = \int \Psi^*(x,t)\, \phi(x,t)\, dx.$$

Orthogonality and normality of the basis sets of eigenvectors

$$\langle \phi_i | \phi_j \rangle = \delta_{ij}$$

If ϕ_n's are set of eigen vectors of an operator A

Any state vector $|\psi\rangle$ can be written as linear combination of ϕ_n's

$$|\psi\rangle = \sum_n a_n |\phi_n\rangle$$

$$\Rightarrow a_n = \langle \phi_n | \psi \rangle$$

If λ_n is the eigenvalue for $|\phi_n\rangle$

prob of measurement outcome λ_n

$$= |a_n|^2 = |\langle \phi_n | \psi \rangle|^2$$

$\Rightarrow$ The average value of the operator $\hat{A}$ in state ψ

$$\langle A \rangle = \sum_n |a_n|^2 \lambda_n = \langle \psi | \hat{A} | \psi \rangle$$

$$\langle A^2 \rangle = \sum_n |a_n|^2 \lambda_n^2 = \langle \psi | \hat{A}^2 | \psi \rangle$$

Position operator.

$$x \, \psi(x,t) = a \, \psi(x,t)$$

$$\psi(x,t) = \delta(x-a)$$

Momentum operator

$$\tilde{p} = -i\hbar \frac{\partial}{\partial x}$$

$$p\Psi = p\Psi$$

$$-i\hbar \frac{\partial \Psi}{\partial x} = p\Psi$$

$$\Psi(x) = A e^{-ikx} \qquad k = \frac{p}{\hbar}$$

Hamiltonian operator

$$H = \frac{p^2}{2m} + V$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V$$

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$ Schrodinger equation