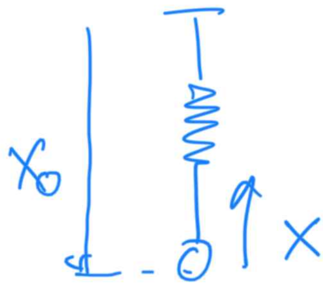
Solution Q1



1. Kinetic energy $= \frac{1}{2} m\dot{x}^2$

Extension of the spring $= (x_0 - x)$

potential energy $= \frac{1}{2} k (x_0 - x)^2 + mgx$

II. Lagrangian ϕ

$$L = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} k (x_0 - x)^2 - mgx$$

Hamiltonian $H = p_x \dot{x} - L$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \dot{x} = \frac{p_x}{m}$$

$$H = \frac{p_x^2}{2m} + \frac{1}{2} k (x_0 - x)^2 + mgx$$

III. $\frac{\partial L}{\partial \dot{x}} = m\dot{x}$ $\quad \frac{\partial L}{\partial x} = k(x_0 - x) + mg$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

$$\Rightarrow m\ddot{x} = K(x_0 - x) + mg$$

IV At equilibrium $\ddot{x} = 0$ $\quad x = 0$

$$\Rightarrow x_0 = -\frac{mg}{K}$$

V The equation of motion becomes

$$m\ddot{x} = -K(x - x_0) - mg$$

$$m\ddot{x} = -Kx \qquad x_0 = +\frac{mg}{K}$$

$\therefore$ frequency of oscillation $= \sqrt{\frac{K}{m}}$

For amplitude, at position x_m

$$\dot{x} = 0$$

$$\Rightarrow \quad \mathcal{H} = E$$

$$\frac{1}{2} K (x_c - x_m)^2 + mg\, x_m = E$$

$$\frac{1}{2} K x_0^2 + \frac{1}{2} K x_m^2 - K x_0 x_m + mg x_m = E$$

$$\because \quad x_c = + \frac{mg}{K}$$

$$\frac{1}{2} K x_m^2 = E - \frac{1}{2} K x_0^2$$

amplitude $$x_m = \sqrt{\frac{2E}{K} - x_0^2}$$

$$\omega = \sqrt{K/m}$$

Q2.

I. $$L = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + \frac{K}{r}$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m \dot{r}$$

$$p_\theta = \frac{\partial L}{\partial \dot\theta} = M r^2 \dot\theta$$

$$\Rightarrow \dot r = \frac{p_r}{M} \qquad \dot\theta = \frac{p_\theta}{M r^2}$$

II Hamiltonian

$$H = \frac{p_r^2}{2M} + \frac{p_\theta^2}{2Mr^2} - \frac{K}{r}$$

$$\dot p_\theta = -\frac{\partial H}{\partial \theta} = 0 \Rightarrow p_\theta = A$$

Total energy

$$E = \frac{p_r^2}{2M} + \frac{A^2}{2Mr^2} - \frac{K}{r}$$

III

$$A = M r_0 V_0$$

$$E = \frac{M^2 R_0^2 V_0^2}{2 M R_0^2} - \frac{K}{R_0}$$

$$= \frac{1}{2} M V_0^2 - \frac{K}{R_0}$$