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NEW METHODS FOR TRANSACTIONS IN BLOCKCHAIN SYSTEMS

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Preface

A preface is not mandatory. It would typically be written by some other person (eg your thesis director).
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Lausanne, 12 Mars 2011 T. D.

Abstract

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Keywords: keyword1, keyword2, keyword3

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1 Introduction

Bitcoin is a decentralized peer-to-peer currency that allow users to to pay for things electronically, if both parties are willing. Their exist thousands of other crypt-currencies, but only some of them are really interesting in a political, economical or technical point of view. We can also mention Ethereum, ZCash, or Monero for expemple. Bitcoin was created by a pseudonymous software developer going by the name of Satoshi Nakamoto in 2008, as an electronic payment system based on mathematical proof. The idea was to produce a means of exchange, independent of any central authority, that could be transferred electronically in a secure, verifiable and immutable way. The blockchain is the output of this secure, verifiable and immutable mathematical proof.

Yet, the largest challenge in Bitcoin for the coming years is scalability. Currently, Bitcoin enforces a block-size limit which is equivalent to only some transactions per second on the network. This is not sufficient in comparison to big payment infrastructures, which allows tens of thousands of transactions per second and even more in peak times such as Christmas. To address this, some proposals modifying the transaction structure (like SegWit), some proposals also modifying the block-size limit (such as SegWit2x) and others creating a second layer based on top of the Bitcoin protocol (like the Lightning Network) exist. In the same idea of the Lightning Network, an new implementation of a one-way payment channel is proposed for special contexts. A one-way payment channel allows two parties to transact over the blockchain while minimizing the number of transactions needed on the blockchain in a secure and trustless way. This kind of channel needs multi-signature addresses which might be improved with a threshold scheme. Finding a threshold scheme that fulfill the requirements for the payment channel is not trivial. A threshold scheme is analyze in this thesis and its implementation is intergrated into the standard Bitcoin crypto-library.

2 Bitcoin, a peer-to-peer payment network

The Bitcoin ecosystem is composed of multiple actors. Users of the network access information via wallets in their laptop or mobile phone. These users can see the amount present in their addresses. An address is the representation of a public key, herself being the representation of a private key. An address is owned by a user if this user has in his possession the associated private key. Users can transfer funds from some of their addresses to other addresses owned by other users or theirself. When funds are transfered a transaction is created and send to the network. The network is composed by nodes and these nodes take care of its proper functioning. Some of these nodes are called miners, they listen to new transactions and try to include them into the blockchain. This blockchain is the output, the intrinsec result, of the Bitcoin protocol and can be compared to a distributed public ledger. Nodes are softwares running all over the world, these softwares are maintained and improved by a group of developers present all over the world and for Bitcoin, the original and reference implementation is Bitcoin-core. The software allows to interact with the blockchain. It is possible to retreive information such as current unconfirmed transactions, information present in the blockchain, amount available for an address, etc. An unconfirmed transaction is a transaction that has not been yet included into the blockchain.

In the following some building blocks needed to figure out how payment channels works and how we can improve them with some cryptography are traveled. If you are a master of Bitcoin and you already know how blocks are created, how transactions are structured, how fees are calculated and how segregated witness works, this chapter will be just a reminder. For further explaination the best ressource today is the book "Mastering Bitcoin" by Andreas Antonopoulos [1].

Contents

2.1 The blockchain

The blockchain, as indicated by his name, is a chain of blocks. Blocks are created by the miners in a race to find the next valid block called the mining process. A block is considered valid if its identifier, i.e. the double hash of its header, is lower to the current difficulty target. It is worth noting that the validity of a block is based on multiple other criterion who are not exposed here, for further information please refer to the book "Mastering Bitcoin". The header of a block is composed of a version number, a creation timestamp, a nonce, or the difficulty target used as boundary.

The difficulty target is adjusted so a valid block is found in the network every ten minutes in average. Mining can be modelized as a poisson process, i.e. the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate is independent of the time since the last event. A miner will create a candidate block and compute its identifier, if this identifier is lower than the current difficulty target then the block is valid and the miner notice the network that he found the next block. Then the process start again. If the block identifier is not valid the miner can change the nonce value in the header and check with the new identifier. Finding the next valid block is so a computation that require an enormous amount of power. All the network, round the clock, keeps searching for the next valid block and the power of computation increase day after days.

2.1.1 A chain of blocks

As mentioned before, the blockchain is a chain who must be secure, verifiable, and immutable. To achieve immutability, modification of previous blocks must invalidate the chain. The block identifier is affected by information like the creation timestamp or the nonce used to adapt the modifier but also from the previous block identifier in the chain. That means that if the previous block identifier is changed for exemple, because its content changed, the child block into the chain will become invalid as well as its child, and so on.

Modifying the blockchain without invalidate the chain require to recompute all the block identifiers after the changed block. This require a quantity of power that can be estimated and for which the costs represent a certain safety threshold. It is established that a transaction included in a block can be considered as safe after six child blocks. The amount of power needed to erase this transaction became statistically too high to be probable, but it does not means that it will never happen due to the fact that there is the same probability to find a valid block with the first nonce than with the thousandth.

2.1.2 A list of transactions

To be useful, a block needs a content. In Bitcoin the content of a block is composed of transactions. As mentioned before, a transaction is called *confirmed* when she is included in a block. The number of confirmation is related to the number of blocks mined after that the transaction has been included.

To keep track of all the transactions included into a block, a Merkle tree is created. A Merkle tree or hash tree is a is a tree in which every leaf node is labelled with the hash of a data block and every non-leaf node is labelled with the cryptographic hash of the labels of its child nodes.

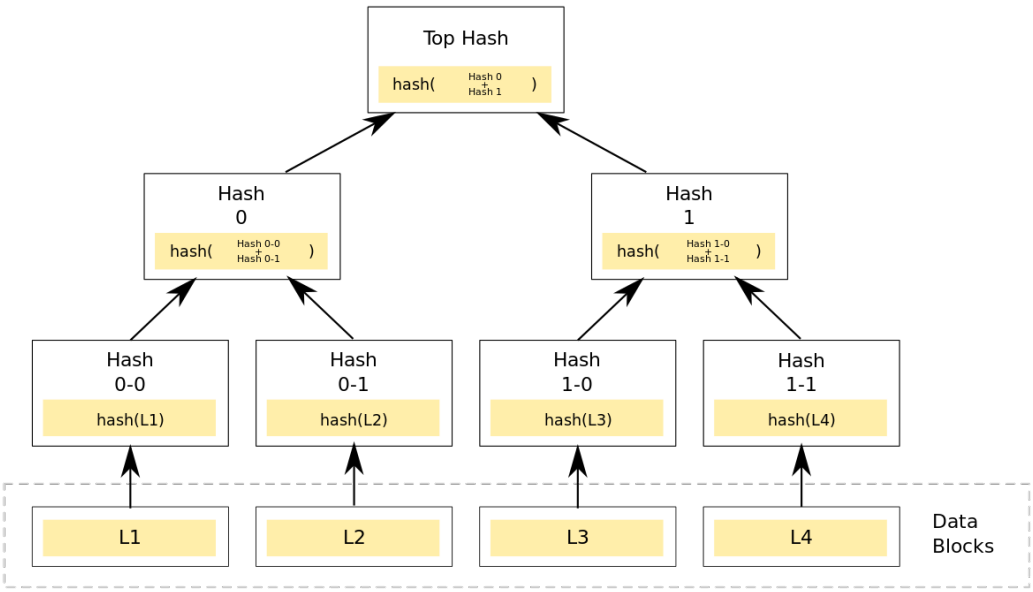

Figure 2.1 *Merkle tree construction*
Source: *https://en.wikipedia.org/wiki/Merkle_tree*

Given the top hash or Merkle root and a leaf, it is possible to prove the membership by given the path for each complementary hashes. E.g given the Merkle root and `L1`, the proof is `Hash 0-1` and `Hash 1`. The verifier can then compute the hash of `L1`, the result of this hash with `Hash 0-1`, and then with `Hash 1`. If the result is the same as the Merkle root, then `L1` is a part of the tree.

In a block, a Merkle tree of all included transaction identifiers is created. The obtained Merkle root is put into the header of the block. To validate if a transaction is included in a block the path must be given, then the resulting hash is compared to the Merkle root registered in the block's header.

2.2 Transactions

Transactions allow users to move Bitcoins from an address to another and they create the content of the Bitcoin's blockchain. In Bitcoin, the blockchain does not store a balance for each user, the blockchain keeps only the history of all transactions that have been made since the begining.

2.2.1 A list of inputs & outputs

A transaction is composed of a list of inputs and a list of outputs. In other words, where the Bitcoins come from and where they go. An input refers to an address where the funds will be spend, and an output refers to an address where the funds will go. An input points to another transaction output who has not been used already. Inputs and outputs are links, to spend funds the user need to control addresses where unspend outputs are present. These unspend outputs are called `UTXOs` and represent the total amount own by a user.

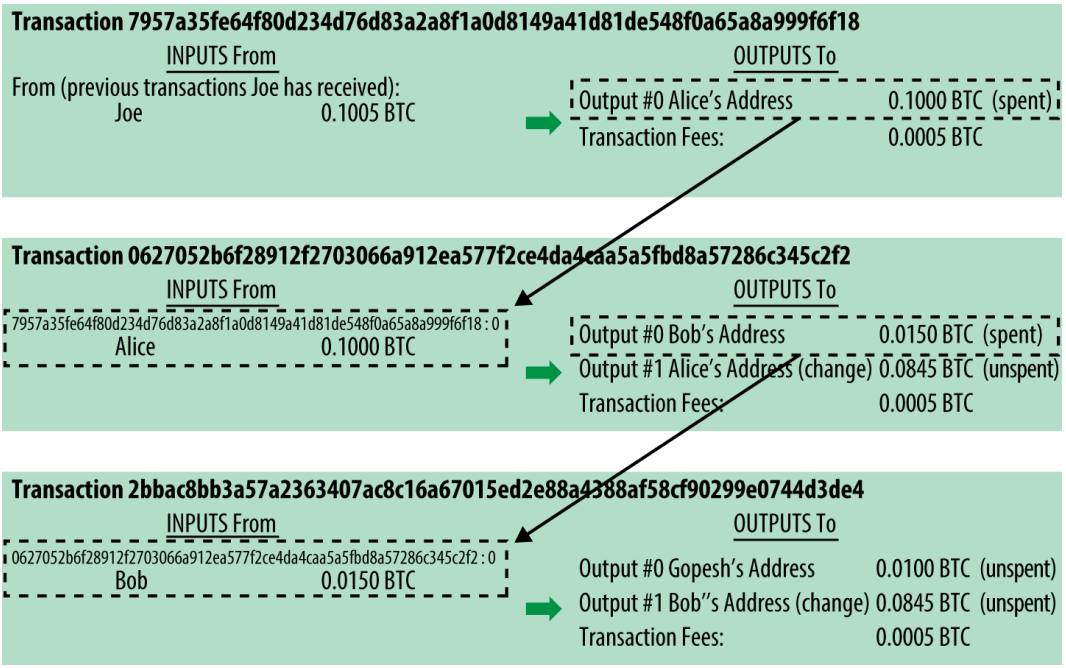

Figure 2.2 *A chain of transactions where inputs and outputs are linked*
Source: *https://github.com/bitcoinbook/bitcoinbook/blob/second_edition/ch02.asciidoc*

Each input is attached to a value (the value specified in the output to which the input points). The sum of all inputs in a transaction must correspond to the sum of all the values specified in the outputs, as in a dual entry accounting. So an input must be spend entirely.

So, the most simple transaction is composed of one input and one output with the same amount of money in and out. However, the most standard transaction is composed of one input, refering from where the funds come from, and two outputs. Indeed, it is very rare to have the right amount available in one `UTXO`. So, the first output is the user who will receive the funds, with the amount transfered, and the second output is an other address owned by the sender to gather the change, i.e. the remaining amount.

As blocks, transactions have identifier. These identifiers are created in the same way as blocks, by taking the double hash of the data, i.e. the signed transaction. That means that, in the original design, a transaction does not have its final transaction identifier (TXID) before she is completely signed (every inputs).

2.2.2 Transaction fees

The sum of all inputs in a transaction must correspond to the sum of all the values specified in the outputs, yes but if the sum of all outputs is lower than the sum of the inputs, the difference is implicitly considered as a fee (as shown in Figure 2.2.) At the beginning no fee was required, but today a transaction will not be included in a block without paying fees. A miner, when he find a block, can create a special transaction—the first transaction in the block—without inputs, where a regulated amount of new coins is created plus the total amount of fees collected in all the included transactions. A miner will therefore select the transactions that pay the more

fees in relation to the amount of work needed to validate them. Fees are calculated in relation to the size of the transaction in bytes, an ratio of fee per byte is then selected to find the fee for a transaction.

2.2.3 Scripting language

As described before, `UTXO`s, so outputs, are related to addresses and a proof of ownership is required to spend an `UTXO`. To achieve this, the Bitcoin protocol use digital signatures. To spend an `UTXO`, a valid signature for to the address, and so the public key, is required and to sign a transaction the private key is required. Thus, while signing a transaction corresponding to the right address it is possible to prove that the user own the address. But the protocol does not just require signatures and public keys, conditions to *unlock* an `UTXO` are structured in scripts. Bitcoin has a stack based script language called "Bitcoin Script".

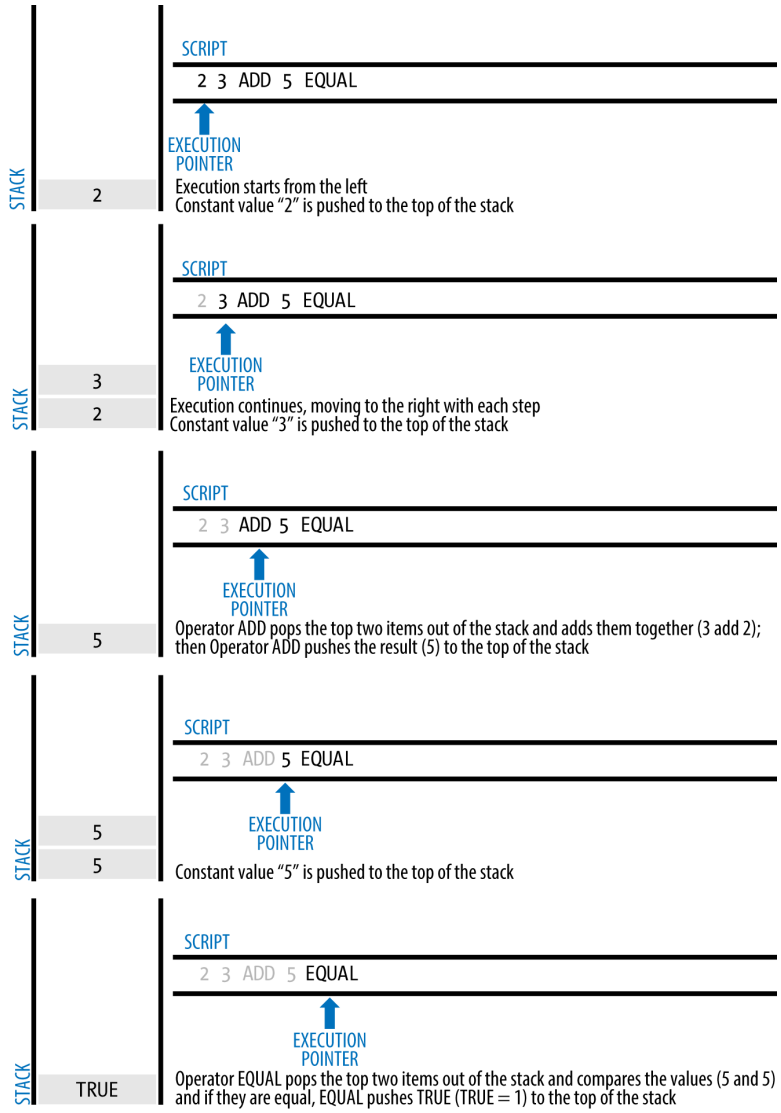

Figure 2.3 Example of simple Bitcoin script program execution
Source: *https://github.com/bitcoinbook/bitcoinbook/blob/second_edition/ch06.asciidoc*

The list of all the `OP_CODES` available in the Bitcoin script language is in the documentation. Among them, `OP_CHECKSIG` to verify a signature given a signature and a public key onto the stack, `OP_IF, OP_ELSE, OP_ENDIF` to create execution branch given a boolean onto the stack, `OP_DUP` to duplicate the value onto the stack, or `OP_HASH160, OP_SHA256` to compute hashes of values onto the stack.

Each input and output have a script. For an output the script correspond to the requirement to be fulfilled to be allowed to spend it. An address is the result of the public key hashed with a `SHA256` and then hashed with a `RIPEMD160` encoded with a checksum in a more human readable format. When an address is given, a user can decode the human readable format to retreive the hash data and create an output script called Pay To Public Key Hash (P2PKH). With the script, the address can be retreived, and given the address and the script, only the user who own the private key corresponding will be able to sign the transaction and spend the funds. The user owning the address can create a transaction such as an input points to the `UTXO`. To unlock the funds, the user needs to sign the transaction and give the signature with the public key in the input's unlocking script. A miner, before including a transaction in a candidate block, validate all the inputs. He needs to check if the pointed output is realy an `UTXO` and execute the locking script with the unlocking script. To do so, both scripts are concatenated with the unlocking script first and executed (as shown if Figure 2.4).

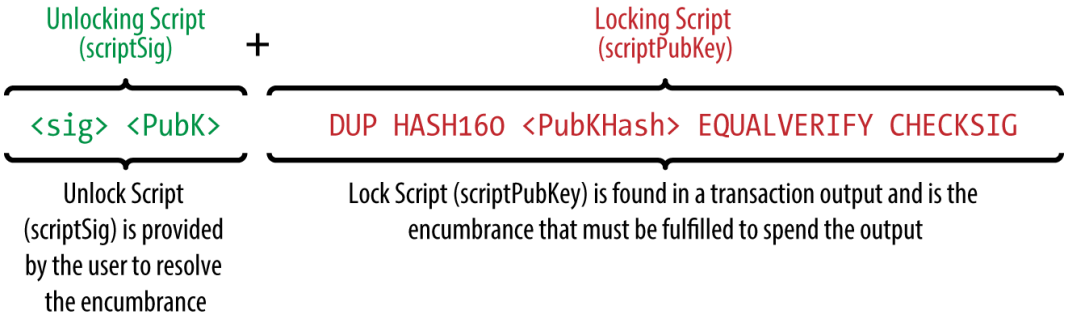

Figure 2.4 *Example of pay to public key hash script*
Source: *https://github.com/bitcoinbook/bitcoinbook/blob/second_edition/ch06.asciidoc*

This script in Figure 2.4, when executed, will put the signature onto the stack, then the public key, the public key will be duplicate and the public key ontop of the stack will be hashed, the public key hash present in the locking script is put ontop of the stack and the two first element ontop of the stack will be compare. If the comparison failed, the script will failed and the transaction will be rejected as invalid. If the test pass, the signature will be check with the two remaining parameter onto the stack, the public key and the signature. If the signature is valid, the value `True` is put onto the stack, otherwise the value `False` is put onto the stack. If the value `True` is present onto the stack at the end of the script the transaction is valid, otherwise the transaction is invalid.

2.2.4 Segregated witness

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2.3.2 Layer-two applications

3 Payment channels, a micro-transaction network

Concept

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3.1 Types of payment channel

3.2 Our one-way channel

3.3 Optimizing channels

4 ECDSA asymmetric threshold scheme

Threshold cryptography has been discussed since a long time already, many cryptographic scheme like RSA or Paillier [2, 3] exists but sometines it is difficult to use them in real case application. Since Bitcoin become popular, people lose funds because they lost keys or their keys have been hacked. And since then researches have been done to secure Bitcoin wallets [4, 5]. But the biggest problem today in Bitcoin that slow down the adoption of threshold cryptosystem is the complexity of creating an efficient and flexible scheme for Elliptic Curve Digital Signature Algorithm (ECDSA). Most recent researches are focused on finding more efficient and more generic scheme, but fortunately a scheme fulfilling perfectly the needs described into the previous chapter required to improve payment channel in Bitcoin already exists. Nevertheless this scheme describe how to perform a Digital Signature Algorithm (DSA) threshold signature and so needs to be adapted.

The scheme analysed, transformed, and implemented in the following has been presented by MacKenzie and Reiter in their paper "Two-Party Generation of DSA Signatures" [6]. This scheme is also the basis of several other papers cited before. Their construction of a threshold signature scheme with ECDSA is based on a simple multiplicative shared secret and homomorphic encryption to keep the private values unknown by the other player. The homomorphic encryption used as exemple in the paper and choosed for the implement is the Paillier cryptosystem [7]. The following chapter describe how to adapt the scheme form DSA to ECDSA and introduce some fundamental building blocks needed for a real case scenario like hierarchical deterministic threshold wallet or deterministic signatures.

Contents

4.1 Reminder

Before introducing the threshold scheme, a reminder of basic components used after in the scheme is presented. The reminder is composed of Elliptic Curves (EC) basic mathematics, Paillier homomorphic encryption scheme, and digital signature—in particular DSA and ECDSA.

4.1.1 Elliptic curves

Bitcoin use EC cryptography for securing his transaction. ECDSA—based on the DSA proposal by the National Institute of Standards and Technology (NIST)—over the curve secp256k1—proposed by the Standards for Efficient Cryptography Group (SECG)—is used.

Secp256k1 curve

The curve secp256k1 is define over the finite field $\mathbb{F}_p$ of 2^{256} bits with a Koblitz curve $y^2 = x^3 + ax + b$ where $a = 0$ and $b = 7$.

$$y^2 = x^3 + 7$$

$$\begin{aligned} p = &\ \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFFC2F} \\ G = &\ (\text{79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798}, \\ &\ \text{483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8}) \\ n = &\ \text{FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141} \end{aligned}$$

The curve order n define the number of element (points) generated by the generator G on the curve. An exponentiation of the generator $g^a \mod p$ become a point multiplication with the generator $a \cdot G$.

Points addition

With two distinct point P and Q on the curve $\mathcal{E}$, geometrically the resulting point of the addition is the inverse point, $(x, -y)$ of the intersection point with a straight line between P and Q. An infinity point $\mathcal{O}$ represent the identity element in the group. Algebraically the resulting point is obtained with:

$$\begin{aligned} P + Q = Q + P = P + Q + \mathcal{O} &= R \\ (x_p, y_p) + (x_q, y_q) &= (x_r, y_r) \\ x_r &\equiv \lambda^2 - x_p - x_q \pmod p \\ y_r &\equiv \lambda(x_p - x_r) - y_p \pmod p \\ \lambda &= \frac{y_q - y_p}{x_q - x_p} \\ &\equiv (y_q - y_p)(x_q - x_p)^{-1} \pmod p \end{aligned} \tag{4.1}$$

Point doubling

For P and Q equal, the formula is similar, the tangent to the curve $\mathcal{E}$ at point P determine R.

$$\begin{aligned} P + P &= 2P = R \\ (x_p, y_p) + (x_p, y_p) &= (x_r, y_r) \\ x_r &\equiv \lambda^2 - 2x_p \pmod{p} \\ y_r &\equiv \lambda(x_p - x_r) - y_p \pmod{p} \\ \lambda &= \frac{3x_p^2 + a}{2y_p} \\ &\equiv (3x_p^2 + a)(2y_p)^{-1} \pmod{p} \end{aligned} \tag{4.2}$$

Point multiplication

A point P can be multiply by a scalar d. The straightforward way of computing a point multiplication is through repeated addition where $dP = P_1 + P_2 + \cdots + P_d$.

Lemma 4.1.1 (Elliptic Curve Discrete Logarithm Problem) *Given a multiple Q of P where $Q = nP$ it is infeasible to find n if n is large.*

Lemma 4.1.2 (Point Order) *A point P has order 2 if $P + P = \mathcal{O}$, and therefore $P = -P$. A point Q has order 3 if $Q + Q + Q = \mathcal{O}$, and therefore $Q + Q = -Q$.*

4.1.2 Paillier cryptosystem

The Paillier cryptosystem, invented by and named after Pascal Paillier in 1999, is a probabilistic asymmetric algorithm for public key cryptography. The problem of computing n-th residue classes is believed to be computationally difficult. The decisional composite residuosity assumption is the intractability hypothesis upon which this cryptosystem is based.

Encryption

With a public key (n, g) and a message $m < n$, select a random $r < n$ and compute the ciphertext $c = g^m \cdot r^n \mod n^2$ to encrypt the plaintext.

Decryption

With a private key (n, g, λ, μ) and a ciphertext $c \in \mathbb{Z}_{n^2}^*$ compute the plaintext as $m = L(c^\lambda \mod n^2) \cdot \mu \mod n$ where $L(x) = \frac{x-1}{n}$.

4.1.3 Signature schemes

Digital Signature Algorithm

The Digital Signature Algorithm (DSA) is a Federal Information Processing Standard for digital signatures. In August 1991 the National Institute of Standards and Technology (NIST) proposed DSA for use in their Digital Signature Standard (DSS) and adopted it as FIPS 186 in 1993.

Signing With public parameters (p, q, g), `hash` the hashing function, m the message, and $x \in \mathbb{Z}_q$ the private key.

- Generate a random $k \in \mathbb{Z}_q$
- Calculate $r \equiv (g^k \pmod p) \pmod q : r \neq 0$
- Calculate $s \equiv k^{-1}(\texttt{hash}(m) + xr) \pmod q : s \neq 0$
- The signature is (r, s)

Verifying With public parameters (p, q, g), `hash` the hashing function, m the message, (r, s) the signature, and $y = g^x \mod p$ the public key.

- Reject the signature if $r, s \notin \mathbb{Z}_q$
- Calculate $w \equiv s^{-1} \pmod q$
- Calculate $u_1 \equiv \texttt{hash}(m) \cdot w \pmod q$
- Calculate $u_2 \equiv rw \pmod q$
- Calculate $v \equiv (g^{u_1} y^{u_2} \pmod p) \pmod q$
- The signature is valide iff $v = r$

Elliptic Curve Digital Signature Algorithm

ECDSA is a variant of DSA which uses elliptic curve cryptography and require a different set of parameters and smaller keys.

Signing With public parameters $(\mathcal{E}, G, n)$, `hash` the hashing function, m the message, and $x \in \mathbb{Z}_n$ the private key.

- Generate a random $k \in \mathbb{Z}_n$
- Calculate $(x_1, y_1) = k \cdot G$
- Calculate $r \equiv x_1 \pmod n : r \neq 0$
- Calculate $s \equiv k^{-1}(\texttt{hash}(m) + xr) \pmod n : s \neq 0$
- The signature is (r, s)

Verifying With public parameters $(\mathcal{E}, G, n)$, `hash` the hashing function, m the message, (r, s) the signature, and $Q = x \cdot G$ the public key.

- Reject the signature if $r, s \notin \mathbb{Z}_n$
- Calculate $w \equiv s^{-1} \pmod{n}$
- Calculate $u_1 \equiv \texttt{hash}(m) \cdot w \pmod{n}$
- Calculate $u_2 \equiv rw \pmod{n}$
- Calculate the curve point $(x_1, y_1) = u_1 \cdot G + u_2 \cdot Q$ if $(x_1, y_1) = \mathcal{O}$ then the signature is invalid
- The signature is valide iff $r \equiv x_1 \pmod{n}$

Schemes' analysis

In (r, s) the part s remain the same in each signature scheme, the only difference for s is the modulus applied. In DSA the modulus q, i.e. the order of the generator g modulo p, is took while in ECDSA the modulus n, i.e. the order of the generator G on the curve $\mathcal{E}$, is took.

The biggest adaptation is on how to calculate the part r from the private random k. In DSA the generator g is used with, at first, modulo p and then modulo q while in ECDSA the curve is used. A point is calculated and the coordinate x_1 is used modulo n.

Postulate 4.1.3 *This statement* $a \equiv g^b \pmod{p}$ *is equivalent in term of security to* $a = b \cdot G$ *and* $a \equiv (g^b \pmod{p}) \pmod{q}$ *is equivalent to* $a \equiv x \pmod{n} : (x, y) = b \cdot G$.

The previous postulate is used to adapt zero-knowledge proofs from DSA to ECDSA hereafter. This postulate has not been further researched by lack of time.

4.2 Threshold scheme

The "Two-party generation of DSA signatures" scheme presented by MacKenzie and Reiter, as mentionned before, is an asymmetric scheme, i.e. at the end of the protocol only the initiator can retreive the full signature. The scheme proposed is a cryptographic (1,2)-threshold, i.e. one player can be corrupted on the two players and the scheme remain safe. It is worth noting that this is qualified as an optimal (t, n)-threshold scheme, i.e. $t = n - 1$, because if only one honest player remain the safety is guarantee.

As also mentionned before, the scheme correspond to the same requirement of a Bitcoin 2-out-of-2 multi-signatures script. This means that it is possible to use it to improve the payment channels. However, it is necessary to adapt the scheme and particulary the zero-knowledge proofs construction to ECDSA. The approach is explained in the following.

The presented scheme use a multiplicative shared secret and a multiplicative shared private random value. The secret x is shared between Alice and Bob, so that Alice holds the secret value $x_1 \in \mathbb{Z}_q$ and Bob $x_2 \in \mathbb{Z}_q$ such that $x \equiv x_1 x_2 \pmod{q}$.

Along with the public key y, $y_1 \equiv g^{x_1} \pmod{p}$ and $y_2 \equiv g^{x_2} \pmod{p}$ are public. Alice holds a private key, hereinafter mentionned as sk, corresponding to a public encryption key pk. Alice also knows a public encryption key pk' for which she does not know the private key sk'. Here the Paillier homomorphic cryptosystem is used as the encryption scheme, but it is worth noting that others homomorphic encryption systems can be use to implement the scheme. Alice and Bob know a set of parameters used for both zero-knowledge proofs.

Hereinafter, the initialisation is not taken into account. The choice was made to decrease the amount of work and because the implementation is not part of the cryptographic `C` library. This part can be a further research for an other thesis.

4.2.1 Adapting the scheme

Except for the zero-knowledge proofs, the adaptation is trivial and require just the same adaptation from DSA signature scheme and ECDSA signature scheme, i.e. compute the r value with the curve. The following figure describe the adapted scheme. Messages stay the same and are not repeated. The postulate 4.1.3 is used to perform the adaptation.

The secret remain shared multiplicatively so that so that Alice holds the secret value $x_1 \in \mathbb{Z}_n$ and Bob $x_2 \in \mathbb{Z}_n$ such that $x \equiv x_1 x_2 \pmod{n}$. Alice holds her public key $Q_1 = x_1 \cdot G$ and Bob $Q_2 = x_2 \cdot G$ such that $Q = x_1 \cdot Q_2$ for Alice or $Q = x_2 \cdot Q_1$ for Bob. The notation $\cdot$ is used to denote the point multiplication over EC.

All the random values k are choose in $\mathbb{Z}_n$ instead of $\mathbb{Z}_q$, also in the case of deterministic signature. All the computation modulo q is replaced by modulo n, as shown in the foregoing digital signature recap. Values R_2 and R become points. Verifications of values R_2 and R become point verifications on the curve and r' is calculated by Alice and Bob as shown in the foregoing remainder. The value cq added to the homomorphic encrypted signature is transformed into cn to hide values z_2 and $x_2 z_2$.

A error of notation is noticed in the original paper, on the second line Alice compute $z_1 \equiv (k_1)^{-1} \pmod{n}$ and the original paper use the random value selection $\overset{R}{\leftarrow}$ instead of a standard assignation $\leftarrow$, this error has been corrected in the following version of the protocol.

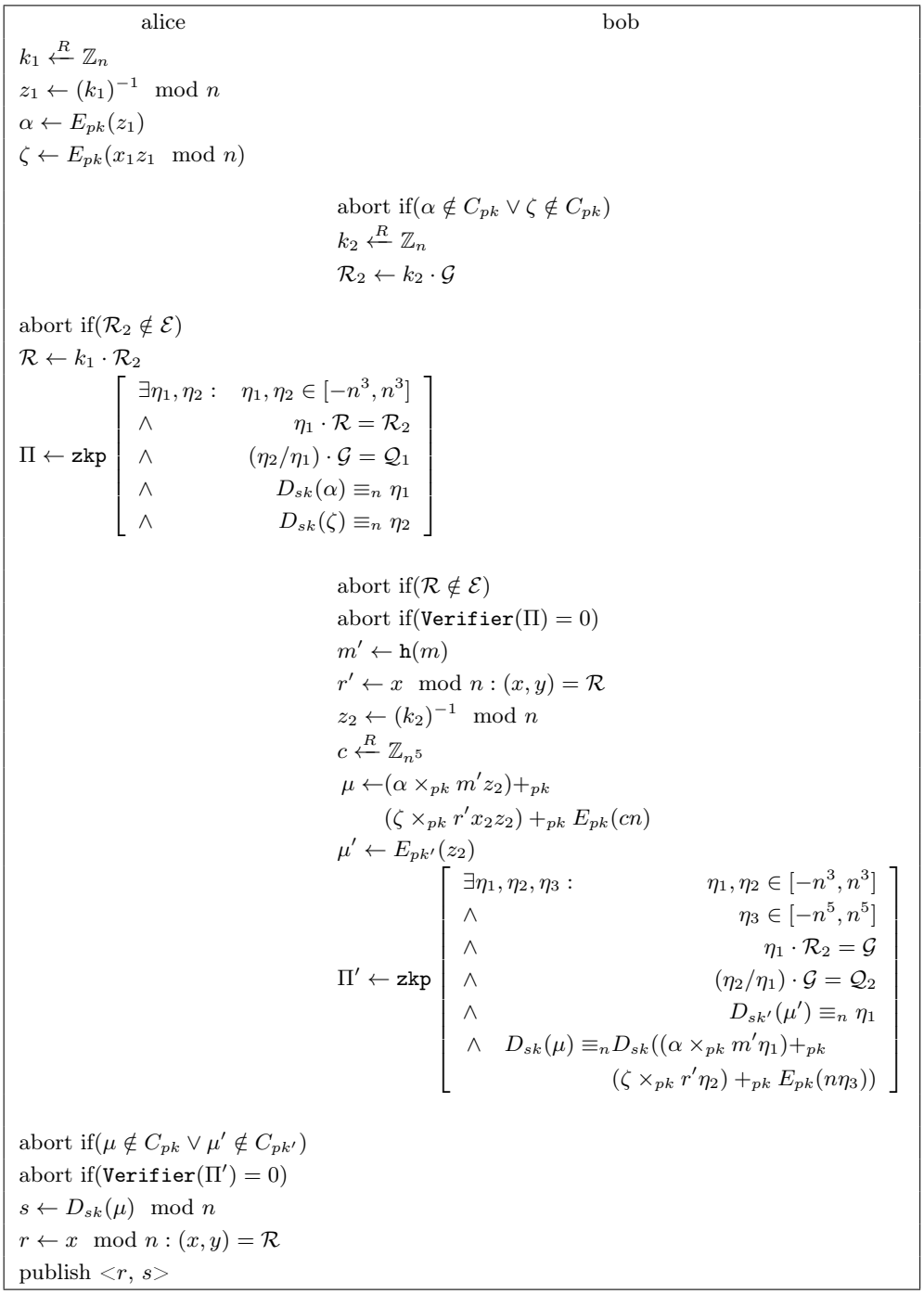

Figure 4.1 Adapted protocol for ECDSA

4.2.2 Adapting zero-knowledge proofs

Initialy the proofs have been designed for the DSA architecture, so the values tested in the proofs are values in $\mathbb{Z}_q$. These values are used to create a challenge e with two hash function (a different one per proof.) For ECDSA some of these values become points and some equations need to be adapted. Points are serialized in the

long form, 65 bytes starting with `0x04` and two 32 bytes number for the coordinates (x, y). As mentionned is the original paper, the variables names are not consistent with the first part of the paper. Hereinafter the variable names follow the same notation as the original paper and are therefore no longer consistent with the previous pages.

Zero-knowledge proof Π

The first zero-knowledge proof Π is created by Alice to prove to Bob that she act correctly and have encrypted coherent data with Paillier encryption, proving the ownership and the validity of the two encrypted values in relation to the public address Q_1 with $(x_1 z_1 / z_1) \cdot G = Q_1$. The proof states that the encrypted value α is related to R and R_2 such that $(k_1)^{-1} \cdot R = ((k_1)^{-1} k_1 k_2) \cdot G = k_2 \cdot G = R_2$.

$$\Pi \leftarrow \texttt{zkp} \left[\begin{array}{ll} \exists x_1, x_2 : & x_1, x_2 \in [-n^3, n^3] \\ \wedge & x_1 \cdot \mathcal{C} = \mathcal{W}_1 \\ \wedge & (x_2 / x_1) \cdot \mathcal{D} = \mathcal{W}_2 \\ \wedge & D_{sk}(m_1) \equiv_n x_1 \\ \wedge & D_{sk}(m_2) \equiv_n x_2 \end{array} \right]$$

Figure 4.2 The proof Π

$x_1 = z_1$	$\mathcal{C} = \mathcal{R}$
$x_2 = x_1 z_1 \mod n$	$\mathcal{D} = \mathcal{G}$
$m_1 = \alpha$	$\mathcal{W}_1 = \mathcal{R}_2$
$m_2 = \zeta$	$\mathcal{W}_2 = \mathcal{Q}_1$

Table 4.1 Mapping between the protocol's variable names and the ZKP Π

$$\langle z_1, z_2, \mathcal{Y}, e, s_1, s_2, s_3, t_1, t_2, t_3, t_4 \rangle \leftarrow \Pi$$

Verify $s_1, t_1 \in \mathbb{Z}_{n^3}$	$\mathcal{V}_1 \leftarrow (t_1 + t_2) \cdot \mathcal{D} + (-e) \cdot \mathcal{Y}$
$\mathcal{U}_1 \leftarrow s_1 \cdot \mathcal{C} + (-e) \cdot \mathcal{W}_1$	$\mathcal{V}_2 \leftarrow s_1 \cdot \mathcal{W}_2 + t_2 \cdot \mathcal{D} + (-e) \cdot \mathcal{Y}$
$u_2 \leftarrow g^{s_1} (s_2)^N (m_1)^{-e} \mod N^2$	$v_3 \leftarrow g^{t_1} (t_3)^N (m_2)^{-e} \mod N^2$
$u_3 \leftarrow (h_1)^{s_1} (h_2)^{s_3} (z_1)^{-e} \mod \tilde{N}$	$v_4 \leftarrow (h_1)^{t_1} (h_2)^{t_4} (z_2)^{-e} \mod \tilde{N}$

$$\text{Verify } e = \texttt{hash}(\mathcal{C}, \mathcal{W}_1, \mathcal{D}, \mathcal{W}_2, m_1, m_2, z_1, \mathcal{U}_1, u_2, u_3, z_2, \mathcal{Y}, \mathcal{V}_1, \mathcal{V}_2, v_3, v_4)$$

Figure 4.3 Adaptation of Π*'s verification in ECDSA*

$\alpha \xleftarrow{R} \mathbb{Z}_{n^3}$	$\delta \xleftarrow{R} \mathbb{Z}_{n^3}$
$\beta \xleftarrow{R} \mathbb{Z}_N^*$	$\mu \xleftarrow{R} \mathbb{Z}_N^*$
$\gamma \xleftarrow{R} \mathbb{Z}_{n^3\tilde{N}}$	$\nu \xleftarrow{R} \mathbb{Z}_{n^3\tilde{N}}$
$\rho_1 \xleftarrow{R} \mathbb{Z}_{n\tilde{N}}$	$\rho_2 \xleftarrow{R} \mathbb{Z}_{n\tilde{N}}$
	$\rho_3 \xleftarrow{R} \mathbb{Z}_n$
	$\epsilon \xleftarrow{R} \mathbb{Z}_n$
$z_1 \leftarrow (h_1)^{x_1}(h_2)^{\rho_1} \mod \tilde{N}$	$z_2 \leftarrow (h_1)^{x_2}(h_2)^{\rho_2} \mod \tilde{N}$
$\mathcal{U}_1 \leftarrow \alpha \cdot \mathcal{C}$	$\mathcal{Y} \leftarrow (x_2 + \rho_3) \cdot \mathcal{D}$
$u_2 \leftarrow g^\alpha \beta^N \mod N^2$	$\mathcal{V}_1 \leftarrow (\delta + \epsilon) \cdot \mathcal{D}$
$u_3 \leftarrow (h_1)^\alpha (h_2)^\gamma \mod \tilde{N}$	$\mathcal{V}_2 \leftarrow \alpha \cdot \mathcal{W}_2 + \epsilon \cdot \mathcal{D}$
	$v_3 \leftarrow g^\delta \mu^N \mod N^2$
	$v_4 \leftarrow (h_1)^\delta (h_2)^\nu \mod \tilde{N}$
$e \leftarrow \texttt{hash}(\mathcal{C}, \mathcal{W}_1, \mathcal{D}, \mathcal{W}_2, m_1, m_2, z_1, \mathcal{U}_1, u_2, u_3, z_2, \mathcal{Y}, \mathcal{V}_1, \mathcal{V}_2, v_3, v_4)$	
$s_1 \leftarrow ex_1 + \alpha$	$t_1 \leftarrow ex_2 + \delta$
$s_2 \leftarrow (r_1)^e \beta \mod N$	$t_2 \leftarrow e\rho_3 + \epsilon \mod n$
$s_3 \leftarrow e\rho_1 + \gamma$	$t_3 \leftarrow (r_2)^e \mu \mod N^2$
	$t_4 \leftarrow e\rho_2 + \nu$
$\Pi \leftarrow \langle z_1, z_2, \mathcal{Y}, e, s_1, s_2, s_3, t_1, t_2, t_3, t_4 \rangle$	

Figure 4.4 Adaptation of Π*'s construction in ECDSA*

Zero-knowledge proof Π'

The second zero-knowledge proof is created by Bob to prove to Alice that he acted honestly according to the protocol. The proof states that the point $\mathcal{R}_2$ is generated accordingly to the value z_2 and then to the value k_2, the public key $\mathcal{Q}_2$ is related to the values z_2 and $x_2 z_2$, and the encrypted values μ and μ' are correctly composed.

$x_1 = z_2$	$\mathcal{C} = \mathcal{R}_2$
$x_2 = x_2 z_2 \mod n$	$\mathcal{D} = \mathcal{G}$
$x_3 = c \mod n$	$\mathcal{W}_1 = \mathcal{G}$
$m_1 = \mu'$	$\mathcal{W}_2 = \mathcal{Q}_2$
$m_2 = \mu$	$m_3 = \alpha$
$m_4 = \zeta$	

Table 4.2 Mapping between the protocol's variable names and the ZKP Π'

$$\Pi' \leftarrow \texttt{zkp}\left[\begin{array}{ll} \exists x_1, x_2, x_3 : & x_1, x_2 \in [-n^3, n^3] \\ \wedge & x_3 \in [-n^5, n^5] \\ \wedge & x_1 \cdot \mathcal{C} = \mathcal{W}_1 \\ \wedge & (x_2/x_1) \cdot \mathcal{D} = \mathcal{W}_2 \\ \wedge & D_{sk'}(m_1) \equiv_n x_1 \\ \wedge & D_{sk}(m_2) \equiv_n D_{sk}((m_3 \times_{pk} m' x_1) +_{pk} \\ & \qquad (m_4 \times_{pk} r' x_2) +_{pk} E_{pk}(n x_3)) \end{array}\right]$$

Figure 4.5 The proof Π'

Correcting the verification of Π' If $x_1 = z_2$, $x_2 = x_2 z_2$, $x_3 = c$, and $m_2 = \mu$ such that $\mu = (\alpha)^{m' x_1} (\zeta)^{r' x_2} g^{n x_3} (r_2)^N$, then the equation v_3 in the validation proccess does not correspond to construction of v_3 in the original paper. The result in the verification proccess Π' need to match $v_3 \leftarrow (m_3)^{\alpha}(m_4)^{\delta} g^{n\sigma} \mu^N \mod N^2$. The original equation proposed $v_3 \leftarrow (m_3)^{s_1}(m_4)^{t_1} g^{n t_5} (t_3)^N (m_2)^{-e} \mod N^2$ does not include m' and r' present in μ, so m_2 cannot be used correctly as showed next.

$$\begin{aligned} v_3 &\equiv (m_3)^{s_1}(m_4)^{t_1} g^{n t_5} (t_3)^N (m_2)^{-e} \pmod{N^2} \\ &\equiv (m_3)^{e x_1 + \alpha}(m_4)^{e x_2 + \beta} g^{n(e x_3 + \sigma)} \big((r_2)^e \mu\big)^N \big((m_3)^{m' x_1}(m_4)^{r' x_2} g^{n x_3} (r_2)^N\big)^{-e} \\ &\equiv (m_3)^{e x_1 + \alpha}(m_4)^{e x_2 + \beta} g^{n(e x_3 + \sigma)} (r_2)^{eN} \mu^N (m_3)^{-e m' x_1}(m_4)^{-e r' x_2} g^{-e n x_3} (r_2)^{-eN} \\ &\equiv (m_3)^{e x_1 + \alpha - e m' x_1}(m_4)^{e x_2 + \beta - e r' x_2} g^{e n x_3 + n\sigma - e n x_3} (r_2)^{eN - eN} \mu^N \\ &\equiv (m_3)^{e x_1 + \alpha - e m' x_1}(m_4)^{e x_2 + \beta - e r' x_2} g^{n\sigma} \mu^N \end{aligned} \tag{4.3}$$

The equation v_3 needs to be adapted to include $x_4 = m'$ and $x_5 = r'$ (m' and r' cannot be include directly in x_1 and x_2 without breaking equations u_1, u_2, u_3, v_2.) Two new parameters $s_4 \leftarrow e x_1 x_4 + \alpha$ and $t_7 \leftarrow e x_2 x_5 + \delta$ are added into the proof to correct the equation.

$$\begin{aligned} v_3 &\equiv (m_3)^{s_4}(m_4)^{t_7} g^{n t_5} (t_3)^N (m_2)^{-e} \pmod{N^2} \\ &\equiv (m_3)^{e x_1 x_4 + \alpha}(m_4)^{e x_2 x_5 + \beta} g^{n(e x_3 + \sigma)} \big((r_2)^e \mu\big)^N \big((m_3)^{x_1 x_4}(m_4)^{x_2 x_5} g^{n x_3} (r_2)^N\big)^{-e} \\ &\equiv (m_3)^{e x_1 x_4 + \alpha}(m_4)^{e x_2 x_5 + \beta} g^{n(e x_3 + \sigma)} (r_2)^{eN} \mu^N (m_3)^{-e x_1 x_4}(m_4)^{-e x_2 x_5} g^{-e n x_3} (r_2)^{-eN} \\ &\equiv (m_3)^{e x_1 x_4 + \alpha - e x_1 x_4}(m_4)^{e x_2 x_5 + \beta - e x_2 x_5} g^{e n x_3 + n\sigma - e n x_3} (r_2)^{eN - eN} \mu^N \\ &\equiv (m_3)^{\alpha}(m_4)^{\beta} g^{n\sigma} \mu^N \end{aligned} \tag{4.4}$$

$\alpha \xleftarrow{R} \mathbb{Z}_{n^3}$	$\delta \xleftarrow{R} \mathbb{Z}_{n^3}$
$\beta \xleftarrow{R} \mathbb{Z}^*_{N'}$	$\mu \xleftarrow{R} \mathbb{Z}^*_N$
$\gamma \xleftarrow{R} \mathbb{Z}_{n^3\tilde{N}}$	$\nu \xleftarrow{R} \mathbb{Z}_{n^3\tilde{N}}$
$\rho_1 \xleftarrow{R} \mathbb{Z}_{n\tilde{N}}$	$\rho_2 \xleftarrow{R} \mathbb{Z}_{n\tilde{N}}$
	$\rho_3 \xleftarrow{R} \mathbb{Z}_n$
	$\rho_4 \xleftarrow{R} \mathbb{Z}_{n^5\tilde{N}}$
	$\epsilon \xleftarrow{R} \mathbb{Z}_n$
	$\sigma \xleftarrow{R} \mathbb{Z}_{n^7}$
	$\tau \xleftarrow{R} \mathbb{Z}_{n^7\tilde{N}}$
$z_1 \leftarrow (h_1)^{x_1}(h_2)^{\rho_1} \mod \tilde{N}$	$z_2 \leftarrow (h_1)^{x_2}(h_2)^{\rho_2} \mod \tilde{N}$
$\mathcal{U}_1 \leftarrow \alpha \cdot \mathcal{C}$	$\mathcal{Y} \leftarrow (x_2 + \rho_3) \cdot \mathcal{D}$
$u_2 \leftarrow (g')^{\alpha}\beta^{N'} \mod (N')^2$	$\mathcal{V}_1 \leftarrow (\delta + \epsilon) \cdot \mathcal{D}$
$u_3 \leftarrow (h_1)^{\alpha}(h_2)^{\gamma} \mod \tilde{N}$	$\mathcal{V}_2 \leftarrow \alpha \cdot \mathcal{W}_2 + \epsilon \cdot \mathcal{D}$
	$v_3 \leftarrow (m_3)^{\alpha}(m_4)^{\delta} g^{n\sigma}\mu^N \mod N^2$
	$v_4 \leftarrow (h_1)^{\delta}(h_2)^{\nu} \mod \tilde{N}$
	$z_3 \leftarrow (h_1)^{x_3}(h_2)^{\rho_4} \mod \tilde{N}$
	$v_5 \leftarrow (h_1)^{\sigma}(h_2)^{\tau} \mod \tilde{N}$
$e \leftarrow \texttt{hash'}(\mathcal{C}, \mathcal{W}_1, \mathcal{D}, \mathcal{W}_2, m_1, m_2, z_1, \mathcal{U}_1, u_2, u_3, z_2, z_3, \mathcal{Y}, \mathcal{V}_1, \mathcal{V}_2, v_3, v_4, v_5)$	
$s_1 \leftarrow ex_1 + \alpha$	$t_1 \leftarrow ex_2 + \delta$
$s_2 \leftarrow (r_1)^e\beta \mod N'$	$t_2 \leftarrow e\rho_3 + \epsilon \mod n$
$s_3 \leftarrow e\rho_1 + \gamma$	$t_3 \leftarrow (r_2)^e\mu \mod N$
$s_4 \leftarrow ex_1x_4 + \alpha$	$t_4 \leftarrow e\rho_2 + \nu$
	$t_5 \leftarrow ex_3 + \sigma$
	$t_6 \leftarrow e\rho_4 + \tau$
	$t_7 \leftarrow ex_2x_5 + \delta$
$\Pi' \leftarrow \langle z_1, z_2, z_3, \mathcal{Y}, e, s_1, s_2, s_3, s_4, t_1, t_2, t_3, t_4, t_5, t_6, t_7 \rangle$	

Figure 4.6 Adaptation of Π''s construction in ECDSA

$$\langle z_1, z_2, z_3, \mathcal{Y}, e, s_1, s_2, s_3, s_4, t_1, t_2, t_3, t_4, t_5, t_6, t_7 \rangle \leftarrow \Pi'$$

Verify $s_1, t_1 \in \mathbb{Z}_{n^3}$	$\mathcal{V}_1 \leftarrow (t_1 + t_2) \cdot \mathcal{D} + (-e) \cdot \mathcal{Y}$
Verify $t_5 \in \mathbb{Z}_{n^7}$	$\mathcal{V}_2 \leftarrow s_1 \cdot \mathcal{W}_2 + t_2 \cdot \mathcal{D} + (-e) \cdot \mathcal{Y}$
$\mathcal{U}_1 \leftarrow s_1 \cdot \mathcal{C} + (-e) \cdot \mathcal{W}_1$	$v_3 \leftarrow (m_3)^{s_4} (m_4)^{t_7} g^{n t_5} (t_3)^N (m_2)^{-e} \mod N^2$
$u_2 \leftarrow (g')^{s_1} (s_2)^{N'} (m_1)^{-e} \mod (N')^2$	$v_4 \leftarrow (h_1)^{t_1} (h_2)^{t_4} (z_2)^{-e} \mod \tilde{N}$
$u_3 \leftarrow (h_1)^{s_1} (h_2)^{s_3} (z_1)^{-e} \mod \tilde{N}$	$v_5 \leftarrow (h_1)^{t_5} (h_2)^{t_6} (z_3)^{-e} \mod \tilde{N}$

$$\text{Verify } e = \texttt{hash'}(\mathcal{C}, \mathcal{W}_1, \mathcal{D}, \mathcal{W}_2, m_1, m_2, z_1, \mathcal{U}_1, u_2, u_3, z_2, z_3, \mathcal{Y}, \mathcal{V}_1, \mathcal{V}_2, v_3, v_4, v_5)$$

Figure 4.7 Adaptation of Π' verification to ECDSA

4.3 Threshold Hierarchical Determinitic Wallets

Hierarchical deterministic wallets are sophisticated wallets in wich fresh keys can be generated from a previous key. Adapting hierarchical deterministic wallets with a threshold scheme can be achieve by sharing the private key additively:

$$\begin{aligned} pk_i &= sk_i \cdot G \\ sk_{mas} &= \sum_{i=1}^{s} sk_i \bmod n \\ pk_{mas} &= \left[\sum_{i=1}^{s} sk_i \bmod n\right] \cdot G \\ &= \sum_{i=1}^{s}(sk_i \cdot G) = \sum_{i=1}^{s} pk_i \end{aligned}$$

or multiplicatively:

$$\begin{aligned} sk_{mas} &= \prod_{i=1}^{s} sk_i \bmod n \\ pk_{mas} &= \left[\prod_{i=1}^{s} sk_i \bmod n\right] \cdot G \\ &= (((G \cdot sk_1) \cdot sk_2) \dots) \cdot sk_i \end{aligned}$$

In the additive case, the master public key pk_{mas} is also the sum of all the public points pk_i, which means that if each player publish his own public share point, every one can compute the master public key. The multiplicative sharing is more communication applicant because the computation of the public key is sequential instead of parallel.

A extended private key share is a tuple of (sk_i, c) with sk_i the normal private key and c the chain code, such that c is the same for each player. In the following it is asume that the private key is shared multiplicatively.

4.3.1 Private parent key to private child key

The function `CKDpriv` compute a child extended private key from the parent extended private key. The derivation can be *hardened*. This proposal differ from the `BIP32` [8] standard in the chain derivation proccess. The `ser` function and `point` function are the same as described in `BIP32`.

$$f(l) = \begin{cases} \texttt{HMAC-SHA256}(c_{par}, \texttt{0x00} \mid\mid \texttt{ser}_{256}(sk_i^{par}) \mid\mid \texttt{ser}_{32}(k)) & \text{if } k \geq 2^{31} \\ \texttt{HMAC-SHA256}(c_{par}, \texttt{ser}_p(\texttt{point}(sk_{mas}^{par})) \mid\mid \texttt{ser}_{32}(k)) & \text{if } k < 2^{31} \end{cases}$$

$$sk_i \equiv l \cdot sk_i^{par} \pmod{n}$$

The function $f(l)$ compute the partial share l at index k, such that multiplied with the parent private key share sk_i^{par} for the player i the result is sk_i.

4.3.2 Public parent key to public child key

The function `CKDpub` compute a child extend public key from the parent extended public key. It is worth noting than it is not possible to compute an *hardened* derivation without the parent private key. It is worth noting that every player update the master public key for the threshold, not the public key share.

$$f(l) = \begin{cases} \texttt{failure} & \text{if } k \geq 2^{31} \\ \texttt{HMAC-SHA256}(c_{par}, \texttt{ser}_p(pk_{mas}^{par}) \mid\mid \texttt{ser}_{32}(k)) & \text{if } k < 2^{31} \end{cases}$$

$$\begin{aligned} pk_{mas} &= l \cdot pk_{mas}^{par} \\ &= l \cdot (sk_{mas}^{par} \cdot G) \\ &= (l \cdot sk_{mas}^{par} \bmod n) \cdot G \end{aligned}$$

4.3.3 Child key share derivation

It is asume that one of the players P_i is designated as the leader L. The function `CKSD` compute a threshold child extended key share from the threshold parent extended key share for the derivation index k. It is worth noting that only the leader L use `CKDpriv` and if the derivation is *hardened*, i.e. if $k \geq 2^{31}$, a special case occured and a round of communication is needed. Let's define `CKSD` for $k < 2^{31}$:

$$\forall i \in P_i : f(t) = \begin{cases} \texttt{CKDpriv}(k) & \text{if } i = L \\ \texttt{CKDpub}(k) & \text{if } i \neq L \end{cases} \tag{4.5}$$

such that:

$$\begin{aligned} sk_{i=L} &= sk_i^{par} \cdot t \\ sk_{i\neq L} &= sk_i^{par} \\ sk_{mas} &= \left[\prod_{j=1}^{i} sk_j^{par} \right] \cdot t \\ &= sk_{mas}^{par} \cdot t \end{aligned} \tag{4.6}$$

and then $\forall i \in P_i$:

$$\begin{aligned} pk_{mas} &= pk_{mas}^{par} \cdot t \\ &= (sk_{mas}^{par} \cdot G) \cdot t \\ &= \left[\prod_{j=1}^{i} sk_j^{par} \right] \cdot G \end{aligned} \tag{4.7}$$

The chain code is updated for each player at each derivation index. The derivation does not depend on the secret key because the chain code must remain deterministic and have same value for each player, without requiring communication round.

$$c_i = \texttt{HMAC-SHA256}(c_i^{par}, \texttt{ser}_{32}(k)) \tag{4.8}$$

If the index $k \geq 2^{31}$ the new master public key, only calculable by the master player L, must be revealed to other players. A round of communication is then needed to continue de derivation.

In this threshold HD scheme only one private share change at each derivation. In other workds, the master private share is derived either with public information or with private information, i.e. *hardened* derivation. If the derivation is private, then a communication round between the players is necessary, more specificly we asume that a secure broadcast channel is open from the master player to other players.

This scheme is sufficient for the payment channels, a threshold key used for the $\texttt{Multisig}_i$ address with a root derivation path `m/44'/0'/a'/0'` is negociated at the opening of the channel (variable `a` is related to the channel account number between the client and the provider as shown in the paper). Then the index `i` in the paper is used to derive each addresses without require any communication. It is worth noting that the root derivation path can also be simplify at `m/a'` or even `m/` because the compatibility with a standard wallet is not anymore a requirement. Noted that the version `m/a'` is more flexible and allows multiple channels between a client and a provider with only one threshold key.

4.3.4 Proof-of-concept implementation

A proof-of-concept implemented in Python has been made. A share can be tagged as master share as described previously. The result of the script is presented thereafter, three share are created, and the first one is tagged as the master share. The root threshold public key `m/` is computed and display, then individual shares' addresses are displayed. The share s_1 is derived with and without *hardened* path, as expected the resulted address is different. The master public key resulting of each share derivations for the path `m/44/0/1` is the same as computing the private key with all individual secret shares and getting the associated address, as expected. To note that only the master individual address for `m/` and `m/44/0/1` has changed.

```
=== Threshold addresses ===
Master root public key m/   : 1BF5ZpQMCg3eGDEm51rkiwcKR12UnFu

*** Individual addresses m/ ***
s1: 1tRFxbAfKKowtqrSC3bVUi491hTXqg1
s2: 16uCytSc9oAJyi5FbxmH6NyTJuYkCLj
s3: 1TcYLZUZYd86AFaT58tzFGBW1BVVw7K

*** Hardened derivation for one share ***
s1 m/44/0/1  : 128PvDGSbZuNpz1zG1Mh1fjJFN3eNaTb
s1 m/44/0/1' : 12883vUsA2gyCAcSNogGUMFuCJsrj58

*** Master public key m/44/0/1 ***
s1: 128PvDGSbZuNpz1zG1Mh1fjJFN3eNaTb
s2: 128PvDGSbZuNpz1zG1Mh1fjJFN3eNaTb
s3: 128PvDGSbZuNpz1zG1Mh1fjJFN3eNaTb

Master public key m/44/0/1 : 128PvDGSbZuNpz1zG1Mh1fjJFN3eNaTb

*** Individual addresses m/44/0/1 ***
s1: 1nNL1gozCk4J1agV667kJFmsyu4RvF5
s2: 16uCytSc9oAJyi5FbxmH6NyTJuYkCLj
s3: 1TcYLZUZYd86AFaT58tzFGBW1BVVw7K
```

Listing 4.1 Result of using threshold HD wallet

A share is composed of four main information: (i) the secret share, (ii) the chain code, (iii) the tag for the master share, and (iv) the threshold public key. The threshold public key address can be set after computation. The derive function `d`

```
252 if __name__ == "__main__":
253     print("=== Threshold addresses ===")
254 
255     chain = ecdsa.gen_priv()
256     # Shares
257     s1 = Share(chain, True, ecdsa.gen_priv())
258     s2 = Share(chain, False, ecdsa.gen_priv())
259     s3 = Share(chain, False, ecdsa.gen_priv())
260 
261     sec = (s1.secret * s2.secret * s3.secret) % ecdsa.n
262     pub = ecdsa.get_pub(sec)
263     add = get(pub)
264     print "Master root public key m/   :", add
265 
266     s1.set_master_pub(pub)
267     s2.set_master_pub(pub)
268     s3.set_master_pub(pub)
269 
270     print "\n*** Individual addresses m/ ***"
271     print "s1:", s1.address()
272     print "s2:", s2.address()
273     print "s3:", s3.address()
274 
275     print "\n*** Hardened derivation for one share ***"
276     print "s1 m/44/0/1  :", get(s1.derive("m/44/0/1").master_pub)
277     print "s1 m/44/0/1' :", get(s1.derive("m/44/0/1'").master_pub)
278 
279     print "\n*** Master public key m/44/0/1 ***"
280     s1 = s1.derive("m/44/0/1")
281     s2 = s2.derive("m/44/0/1")
282     s3 = s3.derive("m/44/0/1")
283     print "s1:", get(s1.master_pub)
284     print "s2:", get(s2.master_pub)
285     print "s3:", get(s3.master_pub)
286 
287     sec = (s1.secret * s2.secret * s3.secret) % ecdsa.n
288     pub = ecdsa.get_pub(sec)
289     add = get(pub)
290     print "\nMaster public key m/44/0/1 :", add
291 
292     print "\n*** Individual addresses m/44/0/1 ***"
293     print "s1:", s1.address()
294     print "s2:", s2.address()
295     print "s3:", s3.address()
```

Listing 4.2 Demonstration of using threshold HD wallet

derives with `CKDpub` or `CKDpriv` depending on the master tag and return a new share for a given index. The path derivation function `derive` take a path and generate the chain of shares. In this Implementation, if a share not tagged as master try to derive a path with an `hardened` index, an exception is raised and the proccess stops. But in real world case, a communication process must take place to complete the derivation.

```
163 class Share(object):
164     def __init__(self, chain, master, secret=ecdsa.gen_priv()):
165         super(Share, self).__init__()
166         self.chain = chain
167         self.master = master
168         self.secret = secret
169         self.master_pub = None
170 
171     def pub(self):
172         return ecdsa.get_pub(self.secret)
173 
174     def address(self):
175         return get(self.pub())
176 
177     def set_master_pub(self, pub):
178         self.master_pub = pub
179 
180     def d_pub(self, i):
181         if i >= pow(2, 31): # Only not hardened
182             raise Exception("Impossible to hardened")
183         k = "%x" % self.chain
184         data = "00%s%08x" % (ecdsa.expand_pub(self.master_pub), i)
185         hmac = hashlib.pbkdf2_hmac('sha256', k, data, 100)
186         point = ecdsa.point_mult(self.master_pub, long(binascii.hexlify(hmac), 16))
187         data = "%08x" % (i)
188         hmac = hashlib.pbkdf2_hmac('sha256', k, data, 100)
189         c = long(binascii.hexlify(hmac), 16)
190         share = Share(c, self.master, self.secret)
191         share.set_master_pub(point)
192         return share
193 
194     def d_priv(self, i):
195         k = "%x" % self.chain
196         data = "%08x" % (i)
197         hmac = hashlib.pbkdf2_hmac('sha256', k, data, 100)
198         c = long(binascii.hexlify(hmac), 16)
199         if i >= pow(2, 31): # Hardened
200             data = "00%32x%08x" % (self.secret, i)
201         else: # Not hardened
202             data = "00%s%08x" % (ecdsa.expand_pub(self.master_pub), i)
203         hmac = hashlib.pbkdf2_hmac('sha256', k, data, 100)
204         key = long(binascii.hexlify(hmac), 16) * self.secret
205         point = ecdsa.point_mult(self.master_pub, long(binascii.hexlify(hmac), 16))
206         share = Share(c, self.master, key)
207         share.set_master_pub(point)
208         return share
209 
210     def d(self, index):
211         if self.master:
212             return self.d_priv(index)
213         else:
214             return self.d_pub(index)
215 
216     def derive(self, path):
217         path = string.split(path, "/")
218         if path[0] == "m":
219             path = path[1:]
220             share = self
221             for derivation in path:
222                 if "'" in derivation:
223                     i = int(derivation.replace("'", "")) + pow(2, 31)
224                     share = share.d(i)
225                 else:
226                     i = int(derivation)
227                     share = share.d(i)
228             return share
229         else:
230             return False
```

Listing 4.3 Construction of a share for a threshold HD wallet

4.4 Threshold deterministic signatures

One of the simplest way to compromise the private key in ECDSA is to select a weak pseudo random number generator for k or even worst, select a static value for k. This problem already append to Sony in December 2010 when a group of hacker calling itself *fail0verflow* announced recovery of the ECDSA private key used to sign software for the PlayStation 3.

Given two signatures (r, s) and (r, s') employing the same unknown k for different messages m and m'. Let's define x as the private key, z as the hash of m and z' of m', an attacker can calculate:

$$\begin{aligned}
s &\equiv k^{-1}(z + rx) \pmod{n} \\
s' &\equiv k^{-1}(z' + rx) \pmod{n} \\
s - s' &\equiv k^{-1}(z + rx) - k^{-1}(z' + rx) \pmod{n} \\
&\equiv k^{-1}(z - z') \pmod{n} \\
k &\equiv \frac{z - z'}{s - s'} \pmod{n} \\
x &\equiv \frac{sk - z}{r} \pmod{n}
\end{aligned}$$

But this issue can be prevented by deterministic generation of k, as described by the RFC 6979 [9]. The random value k can be generated deterministicly by using an `HMAC` function such that the parameters are the private key and the message to sign.

The other positive point is that signatures for the same key paire and the same message are deterministic, i.e. if we sign multiple times the same message, the signature remain the same. This is aslo a big advantage in Bitcoin to help preventing transaction malleability. The deterministic signature construction can also be applied to the threshold scheme with the same properties.

$$\begin{aligned}
k_1 &= \texttt{HMAC}(m, x_1) \\
k_2 &= \texttt{HMAC}(m, x_2) \\
k &= k_1 k_2 \mod n
\end{aligned}$$

The values k_1 and k_2 remain secret as well as the value x_1 and x_2 but the signature will always be the same for the given message and the threshold key.

5 Implementation in Bitcoin-core secp256k1

As mentionned before, Bitcoin use eliptic curve cryptography (ECC) for signing transactions. When the first release of Bitcoin core appeared in the early 2009, the cryptographic computations was performed with the OpenSSL library. Some years after a project started with the goal of replacing OpenSSL and creating a custom and minimalistic `C` library for cryptography over the curve secp256k1. This library is now available on GitHub at `bitcoin-core/secp256k1` project and it is one of the most optimized, if not the most optimized, library for the curve secp256k1. It is worth noting that this library is also used by other major crypto-currencies like Ethereum, so extending the capabilities of this library is a good choice to attract other cryptographer to have a look and increase the amount of reviews for this thesis.

The implementation is spread into four main components: (i) a `DER` parser-serializer, (ii) a textbook implementation of Paillier homomorphic cryptosystem, (iii) an implementation of the Zero-Knowledge Proofs adaptation, and (iv) the threshold public API. It is worth noting that the current implementation is NOT production ready and NOT side-channel attack resistant. Paillier and ZKP are not constant time computation and use `libgmp` for all arithmetic computations, even when secret values are used. This implementation is a textbook implementation of the scheme and need to be reviewed and more tested before been used in production. It is also worth noting that this library doesn't implement the functions needed to initialize the setup. Only the functions needed to parse existing keys and compute a distributed signature are implemented.

This chapter refers to the implementation available on GitHub at `https://github.com/GuggerJoel/secp256k1/tree/threshold` at the time when this lines are wrote. Note that the sources can evolve after that this report is written, to be sure to read the latest version of the code check out the sources directly on GitHub.

Contents

5.1 Configuration

The library use `autotools` to manage the compilation, installation and uninstallation. A system of module is already present in the structure with an ECDH experimental module for shared secret computation and a recovery module for recover ECDSA public key. A module can be flag as experimental, then, at the configuration time, an explicit parameter enabling experimental modules must be passed and a warning is shown to warn that the build contains experimental code.

5.1.1 Add new experimental module

In this structure, the threshold extension is all indicated to be an experimental module also. A new variable `$enable_module_recovery` is declared with a m4 macro defined by autoconf in the `configure.ac` file with the argument `--enable-module-threshold`. The default value is set to `no`.

```
137 AC_ARG_ENABLE(module_threshold,
138     AS_HELP_STRING([--enable-module-threshold],[enable Threshold ECDSA computation with
        ↪  Paillier homomorphic encryption system and zero-knowledge proofs (experimental)]),
139     [enable_module_threshold=$enableval],
140     [enable_module_threshold=no])
```

Listing 5.1 Add argument into `configure.ac` to enable the module

If the variable `$enable_module_recovery` is set to yes into `configure.ac` (lines 443 to 445) a compiler constant is declared, again with a m4 marco defined by autoconf, and set to 1 in `libsecp256k1-config.h` (lines 20 and 21.) This header file is generated when `./configure` script is run and is included in the library.

```
443 if test x"$enable_module_threshold" = x"yes"; then
444   AC_DEFINE(ENABLE_MODULE_THRESHOLD, 1, [Define this symbol to enable the threshold module])
445 fi
```

```
20 /* Define this symbol to enable the threshold module */
21 #define ENABLE_MODULE_THRESHOLD 1
```

Listing 5.2 Define constant `ENABLE_MODULE_THRESHOLD` if module enable

The main file `secp256k1.c` (lines 586 to 590) and the tests file `tests.c` include headers based on the compiler constant definition.

```
586 #ifdef ENABLE_MODULE_THRESHOLD
587 # include "modules/threshold/paillier_impl.h"
588 # include "modules/threshold/eczkp_impl.h"
589 # include "modules/threshold/threshold_impl.h"
590 #endif
```

Listing 5.3 Include implementation headers if `ENABLE_MODULE_THRESHOLD` is defined

The module is set to experimental to avoid enabling it without explicitly agree to build experimental code. If experimental is set to yes a warning is display during the configuration process, if experimental is not set and any experimental module is enable an error message is display and the process failed.

```
465 if test x"$enable_experimental" = x"yes"; then
466   AC_MSG_NOTICE([******])
467   AC_MSG_NOTICE([WARNING: experimental build])
468   AC_MSG_NOTICE([Experimental features do not have stable APIs or properties, and may not be
      ↪  safe for production use.])
469   AC_MSG_NOTICE([Building ECDH module: $enable_module_ecdh])
470   AC_MSG_NOTICE([Building Threshold module: $enable_module_threshold])
471   AC_MSG_NOTICE([******])
472 else
473   if test x"$enable_module_ecdh" = x"yes"; then
474     AC_MSG_ERROR([ECDH module is experimental. Use --enable-experimental to allow.])
475   fi
476   if test x"$enable_module_threshold" = x"yes"; then
477     AC_MSG_ERROR([Threshold module is experimental. Use --enable-experimental to allow.])
478   fi
479   if test x"$set_asm" = x"arm"; then
480     AC_MSG_ERROR([ARM assembly optimization is experimental. Use --enable-experimental to
      ↪  allow.])
481   fi
482 fi
```

Listing 5.4 Set threshold module to experimental into `configure.ac`

5.1.2 Configure compilation

A module is composed of one or many `include/` headers that contain the public API with a small description of each functions, these headers are copied in the right folders when `sudo make install` command is run. The file `Makefile.am` define which headers need to be installed, which not and how to compile the project. This file is parsed by autoconf to generate the final `Makefile` with all the fonctionalities expected.

Each module has its own `Makefile.am.include` which describe what to do with all the files present into the module folder. This file is included in the main `Makefile.am` (lines 179 to 181) if the module is enable.

```
179 if ENABLE_MODULE_THRESHOLD
180 include src/modules/threshold/Makefile.am.include
181 endif
```

Listing 5.5 Include specialized Makefile if threshold module is enable

The specialized `Makefile.am.include` declare the header requisite to be include and declare the list of all the headers that must not be installed on the system when `sudo make install` command is run.

```
1  include_HEADERS += include/secp256k1_threshold.h
2  noinst_HEADERS += src/modules/threshold/der_impl.h
3  noinst_HEADERS += src/modules/threshold/paillier.h
4  noinst_HEADERS += src/modules/threshold/paillier_impl.h
5  noinst_HEADERS += src/modules/threshold/paillier_tests.h
6  noinst_HEADERS += src/modules/threshold/eczkp.h
7  noinst_HEADERS += src/modules/threshold/eczkp_impl.h
8  noinst_HEADERS += src/modules/threshold/eczkp_tests.h
9  noinst_HEADERS += src/modules/threshold/threshold_impl.h
10 noinst_HEADERS += src/modules/threshold/threshold_tests.h
```

Listing 5.6 Specialized Makefile for threshold module

It is possible to build the library and enable the threshold module with the command below.

```
./configure --enable-module-threshold --enable-experimental
```

5.2 DER parser-serializer

Transmit messages and retreive keys are an important part of the scheme. Because between all steps a communication on the network is necessary, a way to export and import data is required. Bitcoin private key are simple structures because of the fixed curve and their intrinsic nature, a single 2^{256} bits value. Threshold private key are composed of multiple parts like: (i) the private share, (ii) a Paillier private key, (iii) a Paillier public key, and (iv) Zero-Knowledge Proof parameters. To serialize these complex structures the DER standard has been choosed. Three simple data types are implemented in the library: (i) sequence, (ii) integer, and (iii) octet string.

5.2.1 Sequence

The sequence data structure holds a sequence of integers and/or octet strings. The sequence start with the constant 0x30 and is followed by the content lenght and the content itself. A lenght could be in the short form or the long form. If the content number of bytes is shorter to 0x80 the lenght byte indicate the lenght, if the content is equal or longer than 0x80 the seven lower bits 0 to 6 where byte $= \{b_7, \ldots, b_1, b_0\}$ indicate the number of followed bytes which are used for the lenght.

```
10 void secp256k1_der_parse_len(const unsigned char *data, unsigned long *pos, unsigned long
    ↪  *lenght, unsigned long *offset) {
11     unsigned long op, i;
12     op = data[*pos] & 0x7F;
13     if ((data[*pos] & 0x80) == 0x80) {
14         for (i = 0; i < op; i++) {
15             *lenght += data[*pos+1+i]<<8*(op-i-1);
16         }
17         *offset = op + 1;
18     } else {
19         *lenght = op;
20         *offset = 1;
21     }
22     *pos += *offset;
23 }
```

Listing 5.7 Implementation of a DER lenght parser

The sequence parser check the first byte with the constant 0x30 and extract the content lenght. Position in the input array are holds in the *pos variable, extracted lenght is stored in *lenght, and the offset holds how many bytes in the data are used for the header and the lenght. A coherence check is performed to ensure that the current offset and the retreived lenght result to the same amount of bytes passed in argument.

When a sequence holds other sequence, retreive their total lenght (including header and content lenght bytes) is needed to recursivly parse them. A specific function is created to retreive the total lenght of a struct given a pointer to its first byte.

The serialization of a sequence is implemented as a serialization of an octet string with the sequence header 0x30 without integrity check of the content. The content lenght is serialized first, then the header is added.

```
25  int secp256k1_der_parse_struct(const unsigned char *data, size_t datalen, unsigned long *pos,
    ↪  unsigned long *lenght, unsigned long *offset) {
26      unsigned long loffset;
27      if (data[*pos] == 0x30) {
28          *pos += 1;
29          secp256k1_der_parse_len(data, pos, lenght, &loffset);
30          *offset = 1 + loffset;
31          if (*lenght + *offset != datalen) { return 0; }
32          else { return 1; }
33      }
34      return 0;
35  }
```

Listing 5.8 Implementation of a DER sequence parser

The result of a content lenght serialization can be ≥ 1 byte-s. If the content is shorter than `0x80`, then one byte is enough to store the lenght. Else multiple bytes (≥ 2) are used. Because the number of byte is undefined before the computation a memory allocation is necessary and a pointer is returned with the lenght of the array.

```
155 unsigned char* secp256k1_der_serialize_sequence(size_t *outlen, const unsigned char *op,
    ↪  const size_t datalen) {
156     unsigned char *data = NULL, *len = NULL;
157     size_t lensize = 0;
158     len = secp256k1_der_serialize_len(&lensize, datalen);
159     *outlen = 1 + lensize + datalen;
160     data = malloc(*outlen * sizeof(unsigned char));
161     data[0] = 0x30;
162     memcpy(&data[1], len, lensize);
163     memcpy(&data[1 + lensize], op, datalen);
164     free(len);
165     return data;
166 }
```

Listing 5.9 Implementation of a DER sequence serializer

If the content lenght is longer than `0x80`, then `mpz` is used to serialize the lenght into a bytes array in big endian most significant byte first. The lenght of this serialization is stored into `longsize` and is used to create the first byte with the most significant bit set to 1 (line 93).

5.2.2 Integer

Integers are used to store the most values in the keys and Zero-Knowledge Proofs. An integer can be positive, negative or zero and are represented in the second complement form. The header start with `0x02`, followed by the lenght of the data. Parsing and serializing integer are already implemented in `libgmp`, functions are juste wrapper to extract information from the header and start the `mpz` importation at the right offset.

5.2.3 Octet string

Octet strings are used to holds serialized data like points/public keys. An octet string is an arbitrary array of bytes. The header start with `0x04` followed by the size of the content. The serialization implementation retreive the lenght of the content,

```
81  unsigned char* secp256k1_der_serialize_len(size_t *datalen, size_t lenght) {
82      unsigned char *data = NULL; void *serialize; size_t longsize; mpz_t len;
83      if (lenght >= 0x80) {
84          mpz_init_set_ui(len, lenght);
85          serialize = mpz_export(NULL, &longsize, 1, sizeof(unsigned char), 1, 0, len);
86          mpz_clear(len);
87          *datalen = longsize + 1;
88      } else {
89          *datalen = 1;
90      }
91      data = malloc(*datalen * sizeof(unsigned char));
92      if (lenght >= 0x80) {
93          data[0] = (uint8_t)longsize | 0x80;
94          memcpy(&data[1], serialize, longsize);
95          free(serialize);
96      } else {
97          data[0] = (uint8_t)lenght;
98      }
99      return data;
100 }
```

Listing 5.10 Implementation of a DER lenght serializer

copy the header and the octet string into a new memory space, and return the pointer with the total lenght. The parser implementation copy the content and set the conent lenght, the position index, and the offset.

5.3 Paillier cryptosystem

Homomorphic encryption is required in the scheme and Paillier is proposed in the white paper. Paillier homomorphic encryption is simple to implement in a textbook way, this implementation is functional but not optimized and need to be reviewed.

5.3.1 Data structures

Encrypted message, public and private keys are transmited. As mentionned before, the `DER` standard format is used to parse and serialize data. `DER` schema for all data structures are defined to ensure portability over different implementations.

Public keys

The public key is composed of a public modulus and a generator. The implementation data structure add a big modulus corresponding to the square of the modulus. A version number is added for future compatibility purposes.

```
HEPublicKey ::= SEQUENCE {
    version           INTEGER,
    modulus           INTEGER,  -- p * q
    generator         INTEGER
}
```

Listing 5.11 `DER` schema of a Paillier public key

`libgmp` is used for all the arithmetic in Paillier implementation, all numbers are stored in `mpz_t` type. The parser take in input an array of bytes with a lenght and the public key to fill.

```
typedef struct {
    mpz_t modulus;
    mpz_t generator;
    mpz_t bigModulus;
} secp256k1_paillier_pubkey;

int secp256k1_paillier_pubkey_parse(
    secp256k1_paillier_pubkey *pubkey,
    const unsigned char *input,
    size_t inputlen
);
```

Listing 5.12 `DER` parser of a Paillier public key

Private keys

The private key is composed of a public modulus, two primes, a generator, a private exponant $\lambda = \varphi(n) = (p-1)(q-1)$, and a private coefficient $\mu = \varphi(n)^{-1} \mod n$. Again, a version number is added for future compatibility purposes.

The parser take in input an array of bytes with a lenght and the private key to fill. The big modulus is computed after the parsing to accelerate encryption and decryption.

```
HEPrivateKey ::= SEQUENCE {
    version           INTEGER,
    modulus           INTEGER,  -- p * q
    prime1            INTEGER,  -- p
    prime2            INTEGER,  -- q
    generator         INTEGER,
    privateExponent   INTEGER,  -- (p - 1) * (q - 1)
    coefficient       INTEGER   -- (inverse of privateExponent) mod (p * q)
}
```

Listing 5.13 *DER schema of a Paillier private key*

```
typedef struct {
    mpz_t modulus;
    mpz_t prime1;
    mpz_t prime2;
    mpz_t generator;
    mpz_t bigModulus;
    mpz_t privateExponent;
    mpz_t coefficient;
} secp256k1_paillier_privkey;

int secp256k1_paillier_privkey_parse(
    secp256k1_paillier_privkey *privkey,
    secp256k1_paillier_pubkey *pubkey,
    const unsigned char *input,
    size_t inputlen
);
```

Listing 5.14 *DER parser of a Paillier private key*

Encrypted messages

An encrypted message with Paillier cryptosystem is a big number $c \in \mathbb{Z}_{n^2}^*$. No version number is added in this case. The implementation structure contain a nonce value that could be set to 0 to stores the nonce used during encryption.

```
HEEncryptedMessage ::= SEQUENCE {
    message           INTEGER
}
```

Listing 5.15 *DER schema of an encrypted message with Paillier cryptosystem*

An encrypted message can be serialized and parsed and they are used in messages exchange during the signing protocol by both parties.

5.3.2 Encrypt and decrypt

Like all other encryption schemes in public key cryptography, the public key is used to encrypt and the private key to decrypt. To encrypt the message `mpz_t m` where $m < n$, a random value r where $r < n$ is selected with the fonction pointer `noncefp` and set into the nonce value `res->nonce`. This nonce is stored because his value is needed to create Zero-Knowledge Proofs. Then, the cipher $c = g^m \cdot r^n \mod n^2$ is putted into `res->message` to complete the encryption process. All intermediray states are wipe out before returning the result.

```
int secp256k1_paillier_encrypt_mpz(secp256k1_paillier_encrypted_message *res, const mpz_t m,
 ↪  const secp256k1_paillier_pubkey *pubkey, const secp256k1_paillier_nonce_function
 ↪  noncefp) {
    mpz_t l1, l2, l3;
    int ret = noncefp(res->nonce, pubkey->modulus);
    if (ret) {
        mpz_inits(l1, l2, l3, NULL);
        mpz_powm(l1, pubkey->generator, m, pubkey->bigModulus);
        mpz_powm(l2, res->nonce, pubkey->modulus, pubkey->bigModulus);
        mpz_mul(l3, l1, l2);
        mpz_mod(res->message, l3, pubkey->bigModulus);
        mpz_clears(l1, l2, l3, NULL);
    }
    return ret;
}
```

Listing 5.16 Implementation of encryption with Paillier cryptosystem

If the random value selection process failed the encryption fail also. The random function of type `secp256k1_paillier_nonce_function` must use a good CPRNG and his implementation is not part of the library.

```
typedef int (*secp256k1_paillier_nonce_function)(
    mpz_t nonce,
    const mpz_t max
);
```

Listing 5.17 Function signature for Paillier nonces generation

To decrypt the cipher $c \in \mathbb{Z}^*_{n^2}$ with the private key, the function compute $m = L(c^\lambda \mod n^2) \cdot \mu \mod n$ where $L(x) = (x-1)/n$. The cipher is raised to the lambda $c^\lambda \mod n^2$ in line 4 and the result is putted to an intermediray state variable. Then the $L(x)$ function is applied on the intermediray state in lines 5-6. Finally, the multiplication with μ and the modulo of n are taken (lines 7-8) to lead to the result. It is worth noting that, in line 6, only the quotient of the division is recovered.

```
1  void secp256k1_paillier_decrypt(mpz_t res, const secp256k1_paillier_encrypted_message *c,
    ↪  const secp256k1_paillier_privkey *privkey) {
2      mpz_t l1, l2;
3      mpz_inits(l1, l2, NULL);
4      mpz_powm(l1, c->message, privkey->privateExponent, privkey->bigModulus);
5      mpz_sub_ui(l2, l1, 1);
6      mpz_cdiv_q(l1, l2, privkey->modulus);
7      mpz_mul(l2, l1, privkey->coefficient);
8      mpz_mod(res, l2, privkey->modulus);
9      mpz_clears(l1, l2, NULL);
10 }
```

Listing 5.18 Implementation of decryption with Paillier cryptosystem

5.3.3 Homomorphism

The choice of this scheme is not hazardous, homomorphic addition and multiplication are used to construct the signature composant $s = D_{sk}(\mu) \mod q : \mu = (\alpha \times_{pk} m'z_2) +_{pk} (\zeta \times_{pk} r'x_2z_2) +_{pk} E_{pk}(cn)$ where $+_{pk}$ denotes homomorphic addition over the ciphertexts and $\times_{pk}$ denotes homomorphic multiplication over the ciphertexts.

Addition

Addition $+_{pk}$ over ciphertexts is computed with $D_{sk}(E_{pk}(m_1, r_1) \cdot E_{pk}(m_2, r_2) \mod n^2) = m_1 + m_2 \mod n$ or $D_{sk}(E_{pk}(m_1, r_1) \cdot g^{m_2} \mod n^2) = m_1 + m_2 \mod n$ where D_{sk} denotes descryption with private key sk and E_{pk} denotes encryption with public key pk. Only the first variant is implemented, where two ciphertexts are added together to result in a third ciphertext.

```
void secp256k1_paillier_add(secp256k1_paillier_encrypted_message *res, const
 ↪  secp256k1_paillier_encrypted_message *op1, const secp256k1_paillier_encrypted_message
 ↪  *op2, const secp256k1_paillier_pubkey *pubkey) {
    mpz_t l1;
    mpz_init(l1);
    mpz_mul(l1, op1->message, op2->message);
    mpz_mod(res->message, l1, pubkey->bigModulus);
    mpz_clear(l1);
}
```

Listing 5.19 Implementation of homomorphic addition with Paillier cryptosystem

Multiplication

Multiplication $\times_{pk}$ over ciphertexts can be performed with $D_{sk}(E_{pk}(m_1, r_1)^{m_2} \mod n^2) = m_1 m_2 \mod n$, the implementation is straight forward in this case. The nonce value from the ciphertext is copied in the resulted encrypted message for not lose information after opperations.

```
void secp256k1_paillier_mult(secp256k1_paillier_encrypted_message *res, const
 ↪  secp256k1_paillier_encrypted_message *c, const mpz_t s, const secp256k1_paillier_pubkey
 ↪  *pubkey) {
    mpz_powm(res->message, c->message, s, pubkey->bigModulus);
    mpz_set(res->nonce, c->nonce);
}
```

Listing 5.20 Implementation of homomorphic multiplication with Paillier cryptosystem

5.4 Zero-knowledge proofs

Two Zero-Knowledge Proofs are used in the scheme, each party generate a proof and validates the other one. A proof is generated and verified under some ZKP parameters, these parameters are fixed at the initialization time and don't change over the time.

5.4.1 Data structures

Three data structures are created, one for each ZKP and one for storing the parameters. Zero-Knowledge Proofs are composed of big numbers and points and need to be serialized and parsed to be included in the messages exchange protocol.

Zero-Knowledge Parameters

Zero-Knowledge parameter is composed of three numeric values: (i) $\tilde{N}$ a public modulus, (ii) h_2 a value selected randomly $\in \mathbb{Z}^*_{\tilde{N}}$, and (iii) h_1 a value where $\exists x, \log_x(h_1) = h_2 \mod \tilde{N}$. One function is provided in the module to parse a `ZKPParameter DER` schema.

```
ZKPParameter ::= SEQUENCE {
    modulus             INTEGER,
    h1                  INTEGER,
    h2                  INTEGER
}
```

Listing 5.21 *`DER` schema of a Zero-Knowledge parameters sequence*

Zero-Knowledge Proof Π

Zero-Knowledge Proof Π is composed of numeric values and one point. The point is stored in a public key internal structure inside the implementation and is exported with the secp256k1 library as a 65 bytes uncompressed public key. The uncompressed public key is then stored as an octet string in the schema. A version number is added for future compatibility purposes. Two functions are provided in the module to parse and serialize a `ECZKPPi DER` schema.

```
ECZKPPi ::= SEQUENCE {
    version             INTEGER,
    z1                  INTEGER,
    z2                  INTEGER,
    y                   OCTET STRING,
    e                   INTEGER,
    s1                  INTEGER,
    s2                  INTEGER,
    s3                  INTEGER,
    t1                  INTEGER,
    t2                  INTEGER,
    t3                  INTEGER,
    t4                  INTEGER
}
```

Listing 5.22 *`DER` schema of a Zero-Knowledge* Π *sequence*

Zero-Knowledge Proof Π'

Zero-Knowledge Proof Π' is composed of the same named values as ZKP Π plus five new ones. The construction of the proof is based on Π but needs more than values to express all the proven statements. Again, the point y is a point serialized as an uncompressed public key in an octet string and a version number is added for future compatibility purposes. Two functions are provided in the module to parse and serialize a `ECZKPPiPrim DER` schema.

```
ECZKPPiPrim ::= SEQUENCE {
    version             INTEGER,
    z1                  INTEGER,
    z2                  INTEGER,
    z3                  INTEGER,
    y                   OCTET STRING,
    e                   INTEGER,
    s1                  INTEGER,
    s2                  INTEGER,
    s3                  INTEGER,
    s4                  INTEGER,
    t1                  INTEGER,
    t2                  INTEGER,
    t3                  INTEGER,
    t4                  INTEGER,
    t5                  INTEGER,
    t6                  INTEGER,
    t7                  INTEGER
}
```

Listing 5.23 `DER` *schema of a Zero-Knowledge* Π' *sequence*

5.4.2 Generate proofs

Proofs are generated in relation to a specific setup and a specific in progress signature. which makes them linked to a large number of values (points, encrypted messages, secrets, parameters, etc.) The complexity of these constructions is strongly felt in the code. Heavy mathematic computations are needed with two `hash` functions.

A CPRNG function is required to generate both proofs. This function generate random number in $\mathbb{Z}_{max}$ and $\mathbb{Z}^*_{max}$. The `flag` argument indicate which case is treated, `STD` or `INV`. If the function have not access to a good source of randomness or cannot generate a good random number a zero is returned, otherwise a one is returned.

```
typedef int (*secp256k1_eczkp_rdn_function)(
    mpz_t res,
    const mpz_t max,
    const int flag
);

#define SECP256K1_THRESHOLD_RND_INV 0x01
#define SECP256K1_THRESHOLD_RND_STD 0x00
```

Listing 5.24 *Function signature for ZKP CPRNG*

Zero-Knowledge Proof Π

As shown in figure 4.2, the proof states that: (i) it exists a known value by the proover that link $r \to r_2$, (ii) it exists a second known value by the proover that, related to the first one, link $G \to y_1$, (iii) the result of $D_{sk}(\alpha)$ is this first value, and (iv) the result of $D_{sk}(\zeta)$ is this second value.

To do computation on the curve a context object need to be passed in argument, then the ZKP object to fill, the ZKP parameters, the two encrypted messages α and ζ, scalar values sx_1 and sx_2 representing $z_1 = (k_1)^{-1} \mod n$ and $x_1 z_1$, then the point r, the point r_2, the partial public key y_1, the proover Paillier public key which has been used to encrypt α and ζ, and finally a pointer to a CPRNG function used to generate all needed random values.

```
int secp256k1_eczkp_pi_generate(
    const secp256k1_context *ctx,
    secp256k1_eczkp_pi *pi,
    const secp256k1_eczkp_parameter *zkp,
    const secp256k1_paillier_encrypted_message *m1,
    const secp256k1_paillier_encrypted_message *m2,
    const secp256k1_scalar *sx1,
    const secp256k1_scalar *sx2,
    const secp256k1_pubkey *c,
    const secp256k1_pubkey *w1,
    const secp256k1_pubkey *w2,
    const secp256k1_paillier_pubkey *pubkey,
    const secp256k1_eczkp_rdn_function rdnfp
);
```

Listing 5.25 Function signature to generate ZKP Π

The function implementation can be splited in four main parts: (i) generate all the needed random values v, (ii) compute the challenge values, (iii) compute the `hash` of these values v, and (iv) compute the ZKP values with $e = \texttt{hash}(v)$.

Zero-Knowledge Proof Π'

As shown in figure 4.5, the proof states that: (i) it exists a known value by the proover x_1 that link $r_2 \rightarrow G$, (ii) it exists a second known value by the proover that, related to the first one, link $G \rightarrow y_2$, (iii) the result of $D_{sk'}(\mu')$ is this first value, and (iv) it exists a third known value by the proover x_3 and the result of $D_{sk}(\mu)$ is the homomorphic operation of $(\alpha \times x_1) + (\zeta \times x_2) + x_3$.

```
int secp256k1_eczkp_pi2_generate(
    const secp256k1_context *ctx,
    secp256k1_eczkp_pi2 *pi2,
    const secp256k1_eczkp_parameter *zkp,
    const secp256k1_paillier_encrypted_message *m1,
    const secp256k1_paillier_encrypted_message *m2,
    const secp256k1_paillier_encrypted_message *m3,
    const secp256k1_paillier_encrypted_message *m4,
    const secp256k1_paillier_encrypted_message *r,
    const mpz_t x1,
    const mpz_t x2,
    const mpz_t x3,
    const mpz_t x4,
    const mpz_t x5,
    const secp256k1_pubkey *c,
    const secp256k1_pubkey *w2,
    const secp256k1_paillier_pubkey *pairedkey,
    const secp256k1_paillier_pubkey *pubkey,
    const secp256k1_eczkp_rdn_function rdnfp
);
```

Listing 5.26 Function signature to generate ZKP Π'

The function implementation can also be splited in four main parts: (i) generate all the needed random values v, (ii) compute the proof values, (iii) compute the `hash'` of these values v, and (iv) compute the ZKP values with $e = \texttt{hash'}(v)$.

It is worth noting that `hash` and `hash'` must be different hashing function to avoid reusing Π proofs, even not satisfying the predicate, to construct fraudulent Π' proofs.

5.4.3 Validate proofs

Validation of proofs Π and Π' can be done with: (i) the Paillier public keys, (ii) the ZKP parameters, and (iii) the exchanged messages. The process can be splited in three steps: compute the proof values, retreive the candidate value e', and compare if $e = e'$. If the values match the proof is valid.

```
int secp256k1_eczkp_pi_verify(
    const secp256k1_context *ctx,
    secp256k1_eczkp_pi *pi,
    const secp256k1_eczkp_parameter *zkp,
    const secp256k1_paillier_encrypted_message *m1,
    const secp256k1_paillier_encrypted_message *m2,
    const secp256k1_pubkey *c,
    const secp256k1_pubkey *w1,
    const secp256k1_pubkey *w2,
    const secp256k1_paillier_pubkey *pubkey
);

int secp256k1_eczkp_pi2_verify(
    const secp256k1_context *ctx,
    secp256k1_eczkp_pi2 *pi2,
    const secp256k1_eczkp_parameter *zkp,
    const secp256k1_paillier_encrypted_message *m1,
    const secp256k1_paillier_encrypted_message *m2,
    const secp256k1_paillier_encrypted_message *m3,
    const secp256k1_paillier_encrypted_message *m4,
    const secp256k1_pubkey *c,
    const secp256k1_pubkey *w2,
    const secp256k1_paillier_pubkey *pubkey,
    const secp256k1_paillier_pubkey *pairedkey
);
```

Listing 5.27 Function signature to validate ZKP Π and Π'

5.5 Threshold module

The threshold module exposes the public API usefull to create an application that wants to use the distributed signature protocol. The public API includes all the function needed to parse-serialize keys, messages, and signature parameters. Signature parameters holds the values k, $z = k^{-1}$, and $r = k \cdot G$, these values are—in a normal signature mode—computed, used, and destroy in one time. However, a mechanisme to save et restore these values is required in the distributed mode because the context can be destroy and re-created between each steps.

The public API also includes the five functions that implement the protocol. One function is one step in the protocol and between two functions, the generated message is serialized by the caller and parsed by the sender. The signature parameters could also be serialized and parsed during the response waiting time.

Nomenclature

A proposal for exchanged messages names and actions is done in this report. Players P_1 and P_2 represent the initiator and collaborator. Player P_1 initialize the communication and ask P_2 to collaborate on a signature, if P_2 collaborates and the protocol end successfully P_1 retreive the signature.

Four messages are necessary between the five steps. In order, the proposed name are: (i) call message, (ii) challenge message, (iii) response challenge, and (iv) terminate message. The functions are named after the corresponding action and message name.

5.5.1 Create call message

The `call_create` function, as indicated by his name, create the call message. Arguments are checked to be non-null, if one of them is the function will fail. The secret share is loaded in a 32 bytes array and the nonce (k) is retreived with the `noncefp` function pointer. It is worth noting that this function could be call multiple times until a nonce that is not zero and which doesn't overflow is found. However, this function as a limited number of calls and if the limit is reached the function will fail. The signatures parameters are then set and encrypted in the call message. The parameters k and z are set for P_1. The `noncefp` can point to an implementation of a deterministic signature mode or a random signature mode. If the deterministic mode is choosed, the counter indicates the number of round done by the function.

```
247 int secp256k1_threshold_call_create(const secp256k1_context *ctx,
    ↪  secp256k1_threshold_call_msg *callmsg, secp256k1_threshold_signature_params *params,
    ↪  const secp256k1_scalar *secshare, const secp256k1_paillier_pubkey *paillierkey, const
    ↪  unsigned char *msg32, const secp256k1_nonce_function noncefp, const
    ↪  secp256k1_paillier_nonce_function pnoncefp) {
248     int ret = 0;
249     int overflow = 0;
250     unsigned char nonce32[32];
251     unsigned char sec32[32];
252     unsigned int count = 0;
253     secp256k1_scalar privinv;
254
255     ARG_CHECK(ctx != NULL);
256     ARG_CHECK(callmsg != NULL);
257     ARG_CHECK(params != NULL);
258     ARG_CHECK(secshare != NULL);
259     ARG_CHECK(paillierkey != NULL);
260     ARG_CHECK(msg32 != NULL);
261     secp256k1_scalar_get_b32(sec32, secshare);
262     while (1) {
263         ret = noncefp(nonce32, msg32, sec32, NULL, NULL, count);
264         if (!ret) {
265             break;
266         }
267         secp256k1_scalar_set_b32(&params->k, nonce32, &overflow);
268         if (!overflow && !secp256k1_scalar_is_zero(&params->k)) {
269             secp256k1_scalar_inverse(&params->z, &params->k); /* z1 */
270             secp256k1_scalar_mul(&privinv, &params->z, secshare); /* x1z1 */
271             if (secp256k1_paillier_encrypt_scalar(callmsg->alpha, &params->z, paillierkey,
                ↪  pnoncefp)
272                 && secp256k1_paillier_encrypt_scalar(callmsg->zeta, &privinv, paillierkey,
                    ↪  pnoncefp)) {
273                 break;
274             }
275         }
276         count++;
277     }
278     memset(nonce32, 0, 32);
279     memset(sec32, 0, 32);
280     secp256k1_scalar_clear(&privinv);
281     return ret;
282 }
```

Listing 5.28 Implementation of `call_create` function

5.5.2 Receive call message

The `call_received` function set the parameter k and r of P_2 and prepare the challenge message with r. Again, the pointer can point to a deterministic implementation for generating the nonce.

```
284 int secp256k1_threshold_call_received(const secp256k1_context *ctx,
    ↪  secp256k1_threshold_challenge_msg *challengemsg, secp256k1_threshold_signature_params
    ↪  *params, const secp256k1_threshold_call_msg *callmsg, const secp256k1_scalar *secshare,
    ↪  const unsigned char *msg32, const secp256k1_nonce_function noncefp) {
285     int ret = 0;
286     int overflow = 0;
287     unsigned int count = 0;
288     unsigned char k32[32];
289     unsigned char sec32[32];
290
291     ARG_CHECK(ctx != NULL);
292     ARG_CHECK(challengemsg != NULL);
293     ARG_CHECK(params != NULL);
294     ARG_CHECK(callmsg != NULL);
295     ARG_CHECK(secshare != NULL);
296     ARG_CHECK(msg32 != NULL);
297     secp256k1_scalar_get_b32(sec32, secshare);
298     while (1) {
299         ret = noncefp(k32, msg32, sec32, NULL, NULL, count);
300         if (!ret) {
301             break;
302         }
303         secp256k1_scalar_set_b32(&params->k, k32, &overflow);
304         if (!overflow && !secp256k1_scalar_is_zero(&params->k)) {
305             if (secp256k1_ec_pubkey_create(ctx, &params->r, k32)) {
306                 memcpy(&challengemsg->r2, &params->r, sizeof(secp256k1_pubkey));
307                 break;
308             }
309         }
310         count++;
311     }
312     memset(k32, 0, 32);
313     memset(sec32, 0, 32);
314     return ret;
315 }
```

Listing 5.29 Implementation of `call_received` *function*

5.5.3 Receive challenge message

The `challenge_received` function is called by P_1 to compute the final public point r of the signature and create the first Zero-Knowledge Proof.

```
317 int secp256k1_threshold_challenge_received(const secp256k1_context *ctx,
    ↪  secp256k1_threshold_response_challenge_msg *respmsg,
    ↪  secp256k1_threshold_signature_params *params, const secp256k1_scalar *secshare, const
    ↪  secp256k1_threshold_challenge_msg *challengemsg, const secp256k1_threshold_call_msg
    ↪  *callmsg, const secp256k1_eczkp_parameter *zkp, const secp256k1_paillier_pubkey
    ↪  *paillierkey, const secp256k1_eczkp_rdn_function rdnfp) {
318     int ret = 0;
319     unsigned char k32[32];
320     secp256k1_pubkey y1;
321     secp256k1_scalar privinv;
322
323     ARG_CHECK(ctx != NULL);
324     ARG_CHECK(respmsg != NULL);
325     ARG_CHECK(params != NULL);
326     ARG_CHECK(challengemsg != NULL);
327     secp256k1_scalar_get_b32(k32, &params->k);
328     memcpy(&respmsg->r, &challengemsg->r2, sizeof(secp256k1_pubkey));
329     ret = secp256k1_ec_pubkey_tweak_mul(ctx, &respmsg->r, k32);
330     secp256k1_scalar_get_b32(k32, secshare);
331     if (ret && secp256k1_ec_pubkey_create(ctx, &y1, k32)) {
332         memcpy(&params->r, &respmsg->r, sizeof(secp256k1_pubkey));
333         secp256k1_scalar_mul(&privinv, &params->z, secshare);
334         VERIFY_CHECK(secp256k1_eczkp_pi_generate(
335             ctx,
336             respmsg->pi,
337             zkp,
338             callmsg->alpha,
339             callmsg->zeta,
340             &params->z,
341             &privinv,
342             &params->r,
343             &challengemsg->r2,
344             &y1,
345             paillierkey,
346             rdnfp
347         ) == 1);
348     }
349     memset(k32, 0, 32);
350     secp256k1_scalar_clear(&privinv);
351     return ret;
352 }
```

Listing 5.30 Implementation of `challenge_received` function

5.5.4 Receive response challenge message

The `response_challenge_received` function is called by P_2 and validates the first Zero-Knowledge Proof, Π. The final ciphertext which contain the s part of the distributed signature is computed and the second Zero-Knowledge Proof Π' is created.

The point r is normalized and the coordinate $r.x$ is get (modulo n). The `hash` is multiplied with z_2 and the coordinate $r.x$ is multiplied with $x_2 z_2$. A value x_3 where $n|x_3$ is added to the cipher to hide information about the secret share and the secret random. In ECDSA $s = k^{-1}(m + rx) \mod n$, so the ciphertext match the requirement as demonstrated below:

$$\begin{aligned} D_{sk}(\mu) &\equiv (\alpha \times mz_2) + (\zeta \times rx_2 z_2) + (x_3) \pmod{n} \\ &\equiv (z_1 \times mz_2) + (x_1 z_1 \times rx_2 z_2) \pmod{n} \\ &\equiv (z_1 z_2 m) + (x_1 z_1 r x_2 z_2) \pmod{n} \\ &\equiv z_1 z_2 (m + r x_1 x_2) \pmod{n} \\ &\equiv z(m + rx) \pmod{n} \\ &\equiv k^{-1}(m + rx) \pmod{n} \end{aligned}$$

5.5.5 Receive terminate message

The `terminate_received` function is called by P_1 and validates the second Zero-Knowledge Proof, Π'. After validation of the proof, the ciphertext is decrypted and the signature is composed. The signature is then tested and the protocol ends. Only P_1 can decrypt the ciphertext so the protocol is asymetric. If P_2 also needs the signature, P_1 must share it. There is now way for P_2 to know the signature without a cooperative P_1.

```
379     ret = secp256k1_eczkp_pi_verify(
380         ctx,
381         respmsg->pi,
382         zkp,
383         callmsg->alpha,
384         callmsg->zeta,
385         &respmsg->r,
386         &challengemsg->r2,
387         pairedshare,
388         pairedkey
389     );
390     if (ret) {
391         mpz_inits(m1, m2, c, n5, n, nc, m, z, rsig, inv, NULL);
392         secp256k1_scalar_inverse(&params->z, &params->k); /* z2 */
393         secp256k1_scalar_mul(&privinv, &params->z, secshare); /* x2z2 */
394         mpz_import(n, 32, 1, sizeof(n32[0]), 1, 0, n32);
395         secp256k1_scalar_set_b32(&msg, msg32, &overflow);
396         if (!overflow && !secp256k1_scalar_is_zero(&msg)) {
397             secp256k1_pubkey_load(ctx, &r, &respmsg->r);
398             secp256k1_fe_normalize(&r.x);
399             secp256k1_fe_normalize(&r.y);
400             secp256k1_fe_get_b32(b, &r.x);
401             secp256k1_scalar_set_b32(&sigr, b, &overflow);
402             /* These two conditions should be checked before calling */
403             VERIFY_CHECK(!secp256k1_scalar_is_zero(&sigr));
404             VERIFY_CHECK(overflow == 0);
405             mpz_import(rsig, 32, 1, sizeof(b[0]), 1, 0, b);
406             secp256k1_scalar_get_b32(b, &params->z);
407             mpz_import(z, 32, 1, sizeof(b[0]), 1, 0, b);
408             secp256k1_scalar_get_b32(b, &privinv);
409             mpz_import(inv, 32, 1, sizeof(b[0]), 1, 0, b);
410             secp256k1_scalar_get_b32(b, &msg);
411             mpz_import(m, 32, 1, sizeof(msg32[0]), 1, 0, msg32);
412             mpz_mul(m1, m, z); /* m'z2 */
413             mpz_mul(m2, rsig, inv); /* r'x2z2 */
414             mpz_pow_ui(n5, n, 5);
415             noncefp(c, n5);
416             mpz_mul(nc, c, n); /* cn */
417             secp256k1_paillier_mult(m3, callmsg->alpha, m1, pairedkey);
418             secp256k1_paillier_mult(m4, callmsg->zeta, m2, pairedkey);
419             secp256k1_paillier_add(m5, m3, m4, pairedkey);
420             ret = secp256k1_paillier_encrypt_mpz(enc, nc, pairedkey, noncefp);
421             secp256k1_scalar_get_b32(sec32, secshare);
422             if (ret && secp256k1_ec_pubkey_create(ctx, &y2, sec32)) {
423                 secp256k1_paillier_add(termsg->mu, m5, enc, pairedkey);
424                 ret = secp256k1_paillier_encrypt_mpz(termsg->mu2, z, p2, noncefp);
425                 VERIFY_CHECK(secp256k1_eczkp_pi2_generate(
426                     ctx,                    /* ctx */
427                     termsg->pi2,            /* pi2 */
428                     zkp,                    /* zkp */
429                     termsg->mu2,            /* m1 */
430                     termsg->mu,             /* m2 */
431                     callmsg->alpha,         /* m3 */
432                     callmsg->zeta,          /* m4 */
433                     enc,                    /* r */
434                     z,                      /* x1 */
435                     inv,                    /* x2 */
436                     c,                      /* x3 */
437                     m,                      /* x4 */
438                     rsig,                   /* x5 */
439                     &challengemsg->r2,      /* c */
440                     &y2,                    /* w2 */
441                     pairedkey,              /* pairedkey */
442                     p2,                     /* pubkey */
443                     rdnfp                   /* rdnfp */
444                 ) == 1);
445             }
446         }
```

Listing 5.31 *Core function of* `response_challenge_received`

```
460     unsigned char n32[32] = {
461         0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff, 0xff,
            ↪  0xff, 0xfe,
462         0xba, 0xae, 0xdc, 0xe6, 0xaf, 0x48, 0xa0, 0x3b, 0xbf, 0xd2, 0x5e, 0x8c, 0xd0, 0x36,
            ↪  0x41, 0x41
463     };
464     unsigned char b[32];
465     void *ser;
466     int ret = 0;
467     int overflow = 0;
468     size_t size;
469     mpz_t m, n, sigs;
470     secp256k1_ge sigr, pge;
471     secp256k1_paillier_pubkey *p1;
472     secp256k1_scalar r, s, mes;
473
474     ARG_CHECK(ctx != NULL);
475     ARG_CHECK(sig != NULL);
476     ARG_CHECK(termsg != NULL);
477     ARG_CHECK(params != NULL);
478     ARG_CHECK(p != NULL);
479     ARG_CHECK(pub != NULL);
480     ARG_CHECK(msg32 != NULL);
481     p1 = secp256k1_paillier_pubkey_get(p);
482     ret = secp256k1_eczkp_pi2_verify(
483         ctx,                    /* ctx */
484         termsg->pi2,            /* pi2 */
485         zkp,                    /* zkp */
486         termsg->mu2,            /* m1 */
487         termsg->mu,             /* m2 */
488         callmsg->alpha,         /* m3 */
489         callmsg->zeta,          /* m4 */
490         &challengemsg->r2,      /* c */
491         pairedpub,              /* w2 */
492         p1,                     /* pubkey */
493         pairedkey               /* pairedkey */
494     );
495     if (ret) {
496         secp256k1_scalar_set_b32(&mes, msg32, &overflow);
497         ret = !overflow && secp256k1_pubkey_load(ctx, &pge, pub);
498         if (ret) {
499             secp256k1_pubkey_load(ctx, &sigr, &params->r);
500             secp256k1_fe_normalize(&sigr.x);
501             secp256k1_fe_normalize(&sigr.y);
502             secp256k1_fe_get_b32(b, &sigr.x);
503             secp256k1_scalar_set_b32(&r, b, &overflow);
504             VERIFY_CHECK(!secp256k1_scalar_is_zero(&r));
505             VERIFY_CHECK(overflow == 0);
506             mpz_inits(m, n, sigs, NULL);
507             secp256k1_paillier_decrypt(m, termsg->mu, p);
508             mpz_import(n, 32, 1, sizeof(n32[0]), 1, 0, n32);
509             mpz_mod(sigs, m, n);
510             ser = mpz_export(NULL, &size, 1, sizeof(unsigned char), 1, 0, sigs);
511             secp256k1_scalar_set_b32(&s, ser, &overflow);
512             if (!overflow
513                 && !secp256k1_scalar_is_zero(&s)
514                 && secp256k1_ecdsa_sig_verify(&ctx->ecmult_ctx, &r, &s, &pge, &mes)) {
515                 secp256k1_ecdsa_signature_save(sig, &r, &s);
516             } else {
517                 memset(sig, 0, sizeof(*sig));
518             }
519         }
520         mpz_clears(m, n, sigs, NULL);
521         secp256k1_scalar_clear(&r);
522         secp256k1_scalar_clear(&s);
523         secp256k1_scalar_clear(&mes);
524     }
525     secp256k1_paillier_pubkey_destroy(p1);
526     return ret;
```

Listing 5.32 Core function of `terminate_received`

6 Further research

It is possible to list an enormous amount of idea or further research in a field like crypto-currencies or blockchain. But some of them more related to the work done in this paper are listed in the following. Some of them are improvements of the work already done but not yet ready for production, and some of them are completely exploratory.

6.1 Side-channel attack resistant implementation and improvements

The proposed implementation into the library `secp256k1` rely on `libgmp` for all complex mathematical calculus and `libgmp` is not strong against side channel attacks, and it is normal, the library has not been developed for that particular purpose. Therefore, a other implementation need to take the place and handle, in constant time and constant memory if possible, the mathematical calculus part. This is a big improvement that can be done, or must be done, before hoping to use the module is some real case scenario.

6.1.1 Second hash function

The current implementation use the hash function `SHA256` implemented into the library `secp256k1` for Π and Π'. This is not complient with the orignial paper requirements, a other hash function must be implemented and used for Π'.

6.1.2 Paillier cryptosystem

Two major improvements or modifications could be performed specificaly on the Paillier cryptosystem implementation. As shown in the original paper, the Chinese Remainder Theorem can be used to optimize the decryption. In the standard approach, with a private key (n, g, λ, μ) and a ciphertext $c \in \mathbb{Z}_{n^2}^*$ it is possible to compute the plaintext $m = L(c^\lambda \mod n^2) \cdot \mu \mod n$ where $L(x) = \frac{x-1}{n}$. With the CRT two function L_p and L_q are define by

$$L_p(x) = \frac{x-1}{p} \quad \text{and} \quad L_q(x) = \frac{x-1}{q}$$

Decryption can therefore be perform over mod p and mod q and recombining modular residues afterwards:

$$\begin{aligned} m_p &= L_p(c^{p-1} \mod p^2)\ h_p \mod p \\ m_q &= L_q(c^{q-1} \mod p^2)\ h_q \mod q \\ m &= \mathrm{CRT}(m_p, m_q) \mod pq \end{aligned}$$

with precomputations

$$\begin{aligned} h_p &= L_p(g^{p-1} \mod p^2)^{-1} \mod p \quad \text{and} \\ h_q &= L_q(g^{q-1} \mod p^2)^{-1} \mod q \end{aligned}$$

Paillier cryptosystem can be adapted to EC cryptography as shown in the paper "Trapdooring Discrete Logarithms on Elliptic Curves over Rings" by Pascal Paillier [10]. It is worth nothing however that the curve construction is different than the curve used to sign and so the code base cannot can not necessarily be reused.

6.1.3 Zero-knowledge proofs

Non-interactive zero-knowledge proofs are a big research field. The article "From Extractable Collision Resistance to Succinct Non-interactive Arguments of Knowledge, and Back Again" by Bitansky, Nir and Canetti, Ran and Chiesa, Alessandro and Tromer, and Eran [11] introduced the acronym zk-SNARK for zero-knowledge Succinct Non-interactive ARgument of Knowledge that are the backbone of the Zcash protocol [12]. In the recent paper "Bulletproofs: Efficient Range Proofs for Confidential Transactions" [13] a new non-interactive zero-knowledge proof protocol with very short proofs and without a trusted setup is proposed. Further research could be done to adapt the zero-knowledge proof construction and migrate to a more generic approach, to remember that the zero-knowledge proof construction proposed in the original paper dates from the early 2000s, progress has been made since.

6.2 Hardware wallets

Hardware wallet devices have become increasingly popular with people and society. They promise to keep the keys safe and, at least, expose less the keys thanks to a dedicated and controlled environment. Thus, keys can be stored safly and, in an organisation for exemple, multiple hardware wallets can be used to create a multi-signature and control the funds.

The development of this threshold library, even if it is just a 2-out-of-2 multi-signature script equivalent, can be used to create real threshold hardware wallet devices. Two hardware wallet devices can be setup together to create a multi-user setup, or an hardware wallet device can be couple with a phone to secure a lightweight wallet.

Usually, when a new Bitcoin wallet is created, a list of words called mnemonic is shown to the user as a backup of his wallet key. The mnemonics are between twelve and twenty-four and each word represent 11 bits of the primary seed [14], for a threshold key it is not possible to represent all the data in the same way given the size of the key (near 4.5 Kb). A other way to display and transmit these information is needed to increase usability. Further research could be done to find a better way to represent and display a threshold key.

The master tag is not included in the `DER` schema. Is the key itself responsible to store this information or this information is a part of the setup and can be stored outside, this question can be deepened.

6.3 More generic threshold scheme

As previously mentionned, research have been done to generalize and find an optimal (t, n) threshold in ECDSA [2, 3]. These papers base their work on the scheme chosen by the implementation, so a deeper analysis could be performed to assess the needed changes to adapt the current implementation to a generic threshold.

6.4 Schnorr signatures

In the paper "Efficient Identification and Signatures for Smart Cards" published in CRYPTO 1989, C.P. Schnorr propose the "Schnorr signature algorithm" [15]. The Schnorr signature is considered the simplest digital signature scheme to be provably secure in a random oracle model [16, 17]. Thus, Bitcoin developers and researchers

have a strong interest for this specific scheme since some years now. Schnorr signatures could greatly reduce the size of the signaure from 65 bytes (ECDSA in `DER` format) to 40 bytes.

With the arrival of SegWit, it is now possible to have script version, thus it is more easy to indroduce new `OP_CODE` and so introduce a new signature validation scheme. However, this will not invalidate the present work and researches because of the specific nature of its application.

Nevertheless, Schorr signatures are tipped to be the next scheme used in Bitcoin and maybe in other crypto-currencies. Further research could be done to find a protocol that fulfill the requirements defined for payment channels optimization.

7 Conclusions

Basics mechanisms of Bitcoin have been explain to introduce two major issues in Bitcoin today, i.e. the scalability problem and the latency problem. These issues already have existing drafted solutions like consensus changes or payment channel. Payment channels are not new to Bitcoin and multiple implementations of the Lightning Network specification already exist. A other scheme, with different capabilities, is proposed to handle a specific context explain in the white paper. This scheme can be improve with threshold cryptography by reducing the size of the transactions. This reduction is done by replacing the multi-signature script in Bitcoin by a single signature computed with threshold cryptography. The threshold scheme is analysed and adapted to ECDSA before been implemented inside the existing library used in Bitcoin-core implementation, the library `secp256k1`. Finally, further research about payment channels, Bitcoin, and threshold signature scheme are exposed.

The Bitcoin payment channel implementation will be released in open source soon and testing will begin in the next months. The threshold implementation will be part of a current project comprising the creation of a open BTM machine using payment channel to withdraw cash money and threshold signature to buy crypto-coins to avoid storing full private keys on the machine.

A | Experimental implementation in Python

[here the implementation in python] point also to a public GitHub repo

Optimal Trustless One-way Payment Channel for Bitcoin

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Abstract. The largest challenge in Bitcoin for the coming years is scalability. Currently, Bitcoin enforces a 1 Megabyte block-size limit which is equivalent to ~7 transactions per second on the network. This is not sufficient in comparison to big payment infrastructure such as credit card processors, which allows tens of thousands of transactions per second and even more in peaks like Christmas. To address this, some proposals modifying the transaction structure (like SegWit), some proposals modifying the block-size limit (such as SegWit2x) and others creating a second layer based on top of the Bitcoin protocol (like Lightning Network) exist. In the same idea of the Lightning Network, we propose a one-way payment channel that allows two parties to transact off-chain while minimizing the number of transactions needed in the blockchain in a secure and trustless way.

Keywords: Crypto-currencies, Bitcoin, Payment channels, State channels, Threshold ECDSA signatures

1 Introduction

Decentralized crypto-currencies such as Bitcoin [5] and its derivatives employs a special decentralized public append-only log based on proof-of-work called the *blockchain* to protect against equivocation in the form of *double-spending*, i.e., spending the same funds to different parties. In a decentralized crypto-currency, users transfer their funds by publishing digitally signed transactions. Transactions are confirmed only when they are included in the blockchain, which is generated by currency miners that solve proof-of-work puzzles. Although a malicious owner can sign over the same funds to multiple receivers through multiple transactions, eventually only one transaction will be approved and added to the publicly verifiable blockchain.

But the blockchain is slow and could be expensive in fees when comes the time to broadcast transactions. Scalability is one of the biggest challenge in blockchain systems these days, as mentionned before, some proposals are focused on the blockchain structure and modify the consensus, others like the Lightning Network [6] are focused on a second layer of transaction where transactions are

created off-chain and the blockchain itself is used as an conflict resolving system and a source of truth. These proposals are called payment channels and provide a wide number of advantages and possibilites.

1.1 Our contribution

Most of commecrial transactions are unidirectional and bidirectional channels are not necessary. Streaming payments in buying services context are mostly unidirectional and bidirectional channels implies both parties to policy the chain and listen the network, which greatly increase the complexity. In this paper we propose a simplified scheme aimed to be use in a context of a provider providing goods or services at many clients and this provider wants to receive the payments into payment channels for rapidity and convenience.

In our scheme the client is represented by Carol, she want to buy goods or services to the provider Bob. Bob is likely to belly serveral times goods or services to Carol and wants to receive payments into a channel to optimize his costs. For this scheme to be realistic, some requirements are assumed. The channel must stay open for undefined amount of time and the client must not be obliged to stay online and watch the blockchain to be safe, only the receiver, i.e., the provider, must stay online to be safe. The provider does not want to lock any fund for the clients, if he needs to send money to some clients, it is asumed that these transactions are regular on-chain transaction or via other channels. When clients send money into channels, the provider also must be able to manage when and how many funds are settle from which channels, so he must be able to settle a channel without closing it. A client, who has blocked funds specifically for the provider, must be able to, with the provider cooperation, withdraw an arbitrary amount out of the channel without closing it.

We propose two definitions to qualify a state channel and generalize the analysis of different implementation.

Definition 1. *A channel is trustless if the funds' safety for every players* $p_i \in \mathcal{P} = \{\mathcal{P}_0, \ldots, \mathcal{P}_n\}$ *at each steps* $\mathcal{S}$ *of the protocol does not depend on players'* $\Delta p = \mathcal{P} - p_i$ *behavior.*

Definition 2. *A channel is optimal if the number of transaction* $\mathcal{T}(\mathcal{C})$ *needed to claims the funds for a given constraint* $\mathcal{C}$ *is equal to the number of moves* $\mathcal{M}(\mathcal{C})$ *needed to satisfy the constraint at any time without breaking the first definition.*

For a containt $\mathcal{C}$ *in a channel* $\mathcal{P}_1 \rightarrow \mathcal{P}_2$*, refunding* $\mathcal{P}_1$ *where the amount* $M \geq 0$ *of* $\mathcal{P}_2$ *require* $\mathcal{M}(\mathcal{C}) \geq 2$ *because of the intermediary revokation's state. An optimal scheme require* $\mathcal{T}(\mathcal{C}) = \wedge\mathcal{M}(\mathcal{C}) = 2$*.*

The channel is trustless in the meaning of Def. 1 and is optimal in the meaning of Def. 2. This paper describe how to achive a trustless one-way payment channel for Bitcoin. Our work is inspired by the Lightning Network and "Yours Lightning Protocol" [1].

2 Building Blocks

In the following the concepts and sub-protocols used in this work are described in more detail.

2.1 Channel State

The channel state is expressed by two indexes i and n, hereinafter also $\texttt{Channel}_{i,n}$. Both indexes are independent and can only be positively incremented. Index i represents the offset of the multisig address where the channel's funds are locked. Index n represents the offset use to create the revocation secrets, this secret is used after in smart contracts.

A channel state always depends on an account a, this account is defined when the channel is created between the client and the server and never changes during his life. We need to share public hierachical deterministic addresses between the client and the server. Let's define the hierachical deterministic Bitcoin account path as:

$$\forall a \geq 2, \exists \texttt{xPriv}_a \mid \texttt{xPriv}_a = \texttt{m/44'/0'/a'}$$
$$\forall a \geq 2, \ \exists \texttt{xPub}_a \ \mid \texttt{xPub}_a = \texttt{m/44'/0'/a'}$$

For a given account a at $\texttt{Channel}_{i,n}$, the protocol and transactions depend on the private multi-signature node Π, the public multi-signature node π, the private revocation node Ω, the public revocation node ω, and the private secret node Θ. Let's define these nodes as:

$$\Pi_i = \texttt{xPriv}_a \texttt{ /0/i}$$
$$\pi_i \ = \texttt{xPub}_a \texttt{ /0/i}$$
$$\Omega_i = \texttt{xPriv}_a \texttt{ /1/i}$$
$$\omega_i \ = \texttt{xPub}_a \texttt{ /1/i}$$
$$\Theta_n = \texttt{xPriv}_a \texttt{ /2'/n'}$$

It is worth noting that Π_i, π_i, Ω_i, and ω_i aren't hardened derivations, instead of Θ_n. This because we need to be able to compute the public keys π_i and ω_i from the $\texttt{xPub}_a$.

Channel Dimensions The channel dimension, noted $|\texttt{Channel}|$, depends of the number of indexes present in the state. Let's define the channel dimension:

$$N = |\texttt{Channel}_{i,n}| = 2$$

Revocation Secret The revocation sercret $\Phi_{i,n}$ corresponds to the state $\texttt{Channel}_{i,n}$ and depends on the secret Θ_n and the revocation key Ω_i.

$$\Phi_{i,n} = \texttt{HMAC}(\Theta_n, \Omega_i)$$

The secret is the HMAC of Θ_n and Ω_i. Both indexes are used to protect Carol from the Old Settlement Attack With Weak Secret (see 4.3).

2.2 Smart Contracts

Two types of smart contract are used in the payment channel scheme. The fisrt one is a standard 2-out-of-2 multi-signature script and the second is a custom script used to prevent the client from broadcasting old transactions.

Multisig Contract The multi-signature contract at $\texttt{Channel}_{i,n}$, hereinafter $\texttt{Multisig}_i$, can be constructed with Carol's π_i key, and Bob's π_i key. Let's define the $\texttt{Multisig}_i$ script:

OP_2 <π_i^{carol}> <π_i^{bob}> OP_2 OP_CHECKMULTISIG

Revocable PubKey Contract Bob and Carol may wish make an output to Carol which Carol can spend after a timelock and Bob can revoke if it is an old state. The next contract, for a $\texttt{Channel}_{i,n}$, use Carol's ω_i key, Bob's ω_i key, and Carol's secret $\Phi_{i,n}$.

```
OP_IF
  <ω_i^carol> OP_CHECKSIG
  <timelock> OP_CHECKSEQUENCEVERIFY OP_DROP
OP_ELSE
  <ω_i^bob> OP_CHECKSIGVERIFY
  OP_HASH160 <Hash160(Φ_{i,n})> OP_EQUAL
OP_ENDIF
```

With this contract Carol can spend this output after the timelock with the script signature:

<Ω_i^{carol} signature> OP_TRUE

In the case if Carol broadcasts an older transaction Bob can revoke it with the script signature:

<Carol's $\Phi_{i,n}$> <Ω_i^{bob} signature> OP_FALSE

Bob has a head start during which, if he knows the secret $\Phi_{i,n}$ generated by Carol, he can spend the money while Carol cannot. This mechanism prevents Carol from broadcasting older transactions which do not match the current $\texttt{Channel}_{i,n}$.

2.3 Transactions

A transaction is noted $\texttt{Transaction}^{i,n}_{<>}$ to denote the name of the transaction, on which indexes this transaction depends—here on indexes i and n—and who has already signed this transaction—denoted by the <>. If a transaction is signed by Carol the transaction is noted $\texttt{Transaction}^{i,n}_{<carol>}$. Transactions that appear in blue on figures are only owned fully signed by Carol and red ones only by Bob, i.e., only the owner can broadcast the transaction.

Funding Transaction The funding transaction, hereinafter $\texttt{FundingTx}^{i}_{<>}$, is the transaction sendting funds to the first multisig address. This transaction depends only on the state index i used by the multisig contract and is fully signed as soon as Carol signs it.

A funding transaction is never broadcast by Carol before she owns the corresponding refund transaction that allows her to get her money back off the channel. This refund transaction has only one output that goes to the revocation contract. To be able to revoke this contract Bob has to know the secret $\Phi_{i,n}$, so if no transaction are made Bob cannot revoke the contract.

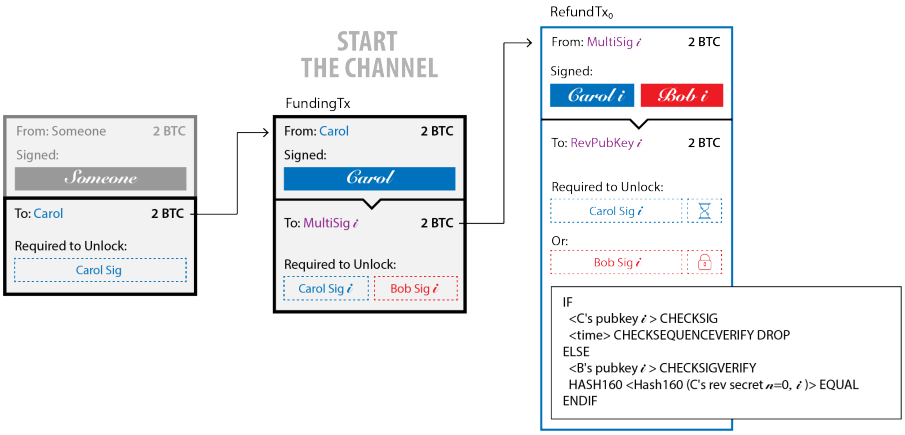

Fig. 1. Funding transaction that start the channel by sending money in the first multisig address with the first refund transaction that allows Carol to close the channel if no transaction is made.

Refund Transaction The refund transaction, hereinafter $\texttt{RefundTx}^{i,n}_{<>}$, is a transaction that keeps track the balances of Carol and Bob at $\texttt{Channel}_{i,n}$ and allows Carol to close the channel if Bob does not repsond or does not cooperate anymore. This transaction have one input or more—which come from the multisig address corresponding to the state index i—and two outputs. The first output represent the amount still owned by Carol, and the second—which can be non-present in the transaction if the balance is equal to zero—is the amount owned by Bob. This second output is not present when the channel is open and when Bob settle the channel.

The Carol balance is send to a revocation contract corresponding to the channel state. This prevent Carol to broadcast an old refund transaction such as $\texttt{RefundTx}^{i,n-1}_{<>}$. The amount owned by Bob is sent directly to Bob's address. The refund transaction is broadcasted by Carol so the fees are substracted to the first output, owned by Carol.

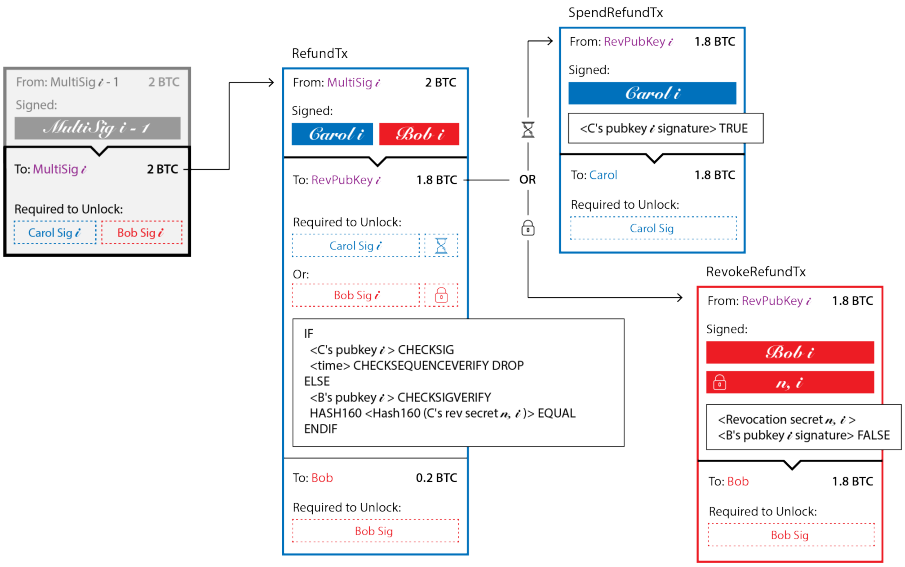

Fig. 2. Refund transaction based on the current multisig address with the associated spend and revoke transactions that allows Carol to get her money back and Bob to revoke the contract if he knows the secret.

Because a refund transaction spends funds from a multisig address, she must be signed by Carol and Bob to be considered fully signed. The revocation contract used in the output of Carol can be spent with a spend refund transaction after a timelock delay. She just needs to sign the output with her Ω_i key to unlock the funds. Bob can directly revoke the contract, without delay, if he knows the secret $\Phi_{i,n}$ and sign with his Ω_i key.

Settlement Transaction The settlement transaction, hereinafter also mentioned as $\texttt{SettlementTx}^{i,n}_{<>}$, is a transaction that keeps track the balances of Carol and Bob at $\texttt{Channel}_{i,n}$ and allows Bob to settle the channel without closing it. Because the settlement transaction spends the funds present into the multisig address, both Carol and Bob need to sign to consider the transaction as a fully signed transaction. Fees are substracted from Bob's owned output, because he is responsible to broadcast the transaction and settle the channel.

A settlement transaction has always one output that sends Bob's balance directly to Bob's address and one output that send the remaining funds to the next multisig address $\texttt{Channel}_{i+1,n}$. Because the funds are sends to the next multisig address a post settlement refund transaction is created—Carol needs a way to get her money back off the channel. This transaction has the same structure as the first refund transaction—one output to the next revocation contract—because the funds owned by Bob was already settle.

If Bob broadcast the fully signed settlement transaction, Carol has two choices (i) continue to transact on the channel with the new multisig address and (ii) close the channel with her post settlement transaction. It is worth noting that the secret for the revocation contract is $\Phi_{i+1,n}$, so in the case of a new transaction $\texttt{Channel}_{i+1,n}$ becomes $\texttt{Channel}_{i+1,n+1}$ and then, the secret for $\texttt{Channel}_{i,n-1}$ is shared:

$$\Phi_{i,n-1}(\texttt{Channel}_{i+1,n+1}) = \Phi_{i+1,n}$$

Post Settlement Refund Transaction The post settlement transaction aim to spend funds from the next multisig address directly to a revocation contract. As explained before, this contract is not revocable by Bob if no transaction is made after the settlement transaction, but when Carol sends an amount to Bob she shares the secret needed to revoke the contract, thus she cannot broadcast this transaction that is now attach to an old state.

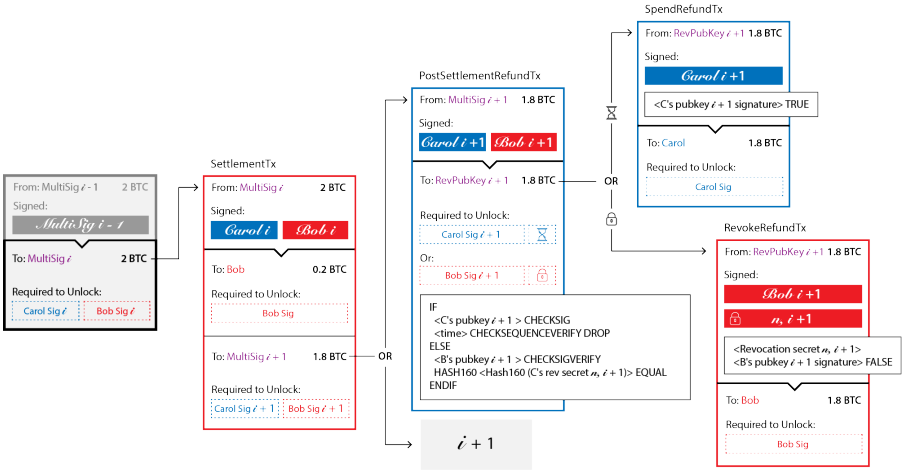

Fig. 3. Settlement transaction that allows Bob to settle the channel with moving the remaining funds to the next multisig address with the post settlement refund transaction that allows Carol to close the channel directly after the settlement.

Withdraw Transaction The withdraw transaction, hereinafter $\texttt{WithdrawTx}_i$, is a transaction that allows Carol to take an arbitrary amount of money out the channel. This amount is send to an arbitrary address specified by Carol. If Carol wants to withdraw the channel, she has to ask Bob for his cooperation. This transaction is not auto-generated when Carol send money to Bob, both have to be online to create this transaction.

This transaction automatically settle the channel with the amount owned by Bob at $\texttt{Channel}_{i,n}$ and changes the remaining amount available for Carol for the state $\texttt{Channel}_{i+1,n}$, thus, this remaining funds are moved to the next channel address.

Closed Channel Transaction The close channel transaction, hereinafter also mentioned as $\texttt{ClosedChannelTx}_i$, is also a cooperative transaction, that allows Carol or Bob to close the channel in the most effective way (less fee and quiker). This transaction has two output, one for Bob with the amount owned by Bob to his address and a second one for Carol with the remaining amount of money in the channel. When a $\texttt{ClosedChannelTx}_i$ is created, no more transaction must be created or accepted on the channel.

Pay To Channel Transaction The pay to channel transaction, hereinafter $\texttt{PayToChannelTx}_i$, allows Carol or Bob to send money directly to the current channel address. This transaction is usefull for Carol if there is not enough money onto the channel and she wants to send more money to Bob without opening an other payment channel. It is also usefull for Bob in the case he want to send money to Carol—he can send money directly to a Carol's address, but the payment can be related to a channel event or action—and allows her to reuse it into the channel, e.g fidelity points.

Before broadcasting the pay to channel transaction or before accepting that payment as part of the usable funds for Carol, an interim refund transaction needs to be created. This interim refund transaction is a safty garauntee for Carol until the merge occures.

For a state $\texttt{Channel}_{i,n}$ without any pay to channel transaction, the multisig address have, normaly, one unspend output. This unspend output is used as an input in each transactions and these transactions split it to track the balances of each parties. After a pay to channel transaction the multisig address have more than one unspend output. When Carol sends money to Bob they have to check if a pay to channel transaction occured and if it is the case they need to merge the interim refund and use all the unspend outputs. It is worth noting that the more pay to channel transactions occurred the more expensive the transaction become.

Interim Refund Transaction The interim refund transaction, hereinafter $\texttt{InterimRefundTx}_{i,n}$, is a temporary transaction used by Carol to get her money out of the channel. This transaction is created to protect Carol from Bob invalidating the current refund transaction.

Definition 3. *A channel merge occured each time a interim refund transaction is merged into the regular refund transaction and the regular settlement transaction.*

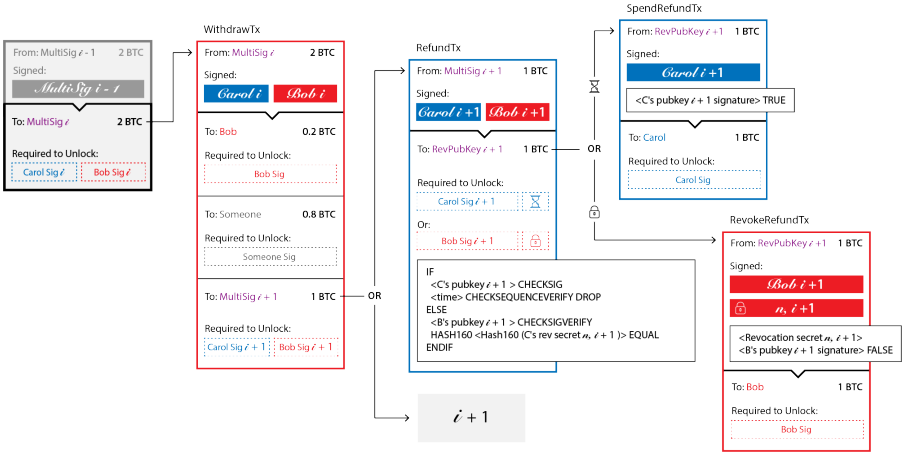

Fig. 4. Withdraw transaction that allows Carol to takes money out the channel, pay Bob, and move the remaining funds into the next multisig address. A refund transaction is created to allow Carol to recover her funds after the withdraw if no transaction is made. The refund transaction can be spent by Carol with a spend refund transaction and cannot be contested with the revoke refund transaction if no other transaction is made. If the state move to $\texttt{Channel}_{i+1,n+1}$, again, the secret for $\texttt{Channel}_{i,n-1}$ is shared, then Bob knows $\Phi_{i,n-1}(\texttt{Channel}_{i+1,n+1}) = \Phi_{i+1,n}$ and can revoke.

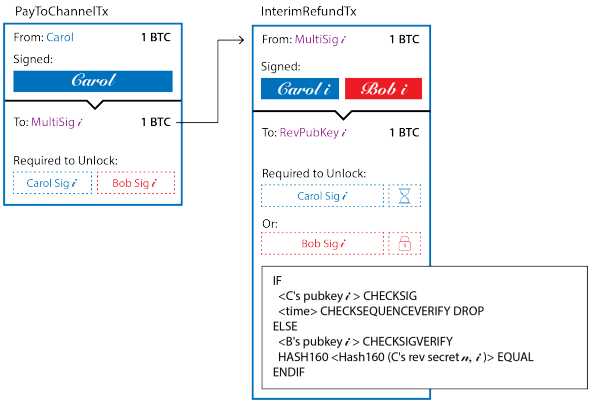

Fig. 5. Pay to channel transaction by Carol with the interim refund transaction. The interim refund transaction acts like a standard refund transaction but aims to be merged in the next round of transaction. The interim refund transaction has the same requirements to be spent than a standard refund transaction, if no transaction is made Carol can spend the interim refund, otherwise Bob can revoke the interim refund.

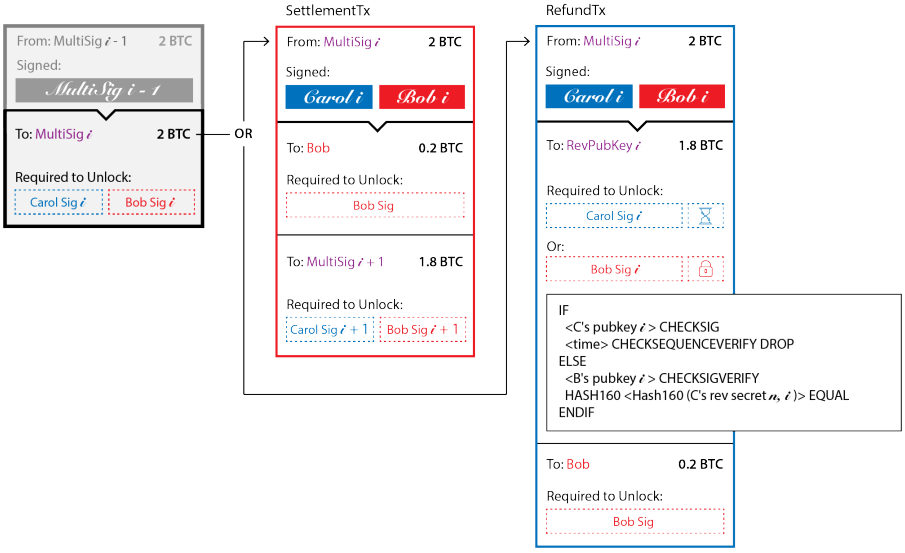

Fig. 6. Simplified view of possibilites for a standard state $\texttt{Channel}_{i,n}$ without second layer dependency transactions like spend and revoke. The content of a multisig can be settle by Bob (in red) or can be refund by Carol (in blue).

Definition 4. *A channel reduce occured each time a non-closing channel transaction is broadcast and included in the blockchain. The channel is then in reduced mode when one and only one* `UTXO` *is available in the current multisig address.*

3 Trustless One-way Payment Channel

3.1 Channel Setup

Before opening the channel Carol and Bob need to exchange keys for the channel account a and negotiate the relative timelock value.

1. Carol:
   (a) sends a request to open a channel with:
      i. the account a
      ii. the Carol's $\texttt{xPub}_a$
      iii. the relative timelock parameter
2. Bob:
   (a) if Bob agree with the request and timelock is whithin acceptable range, respond with:
      i. the Bob's $\texttt{xPub}_a$

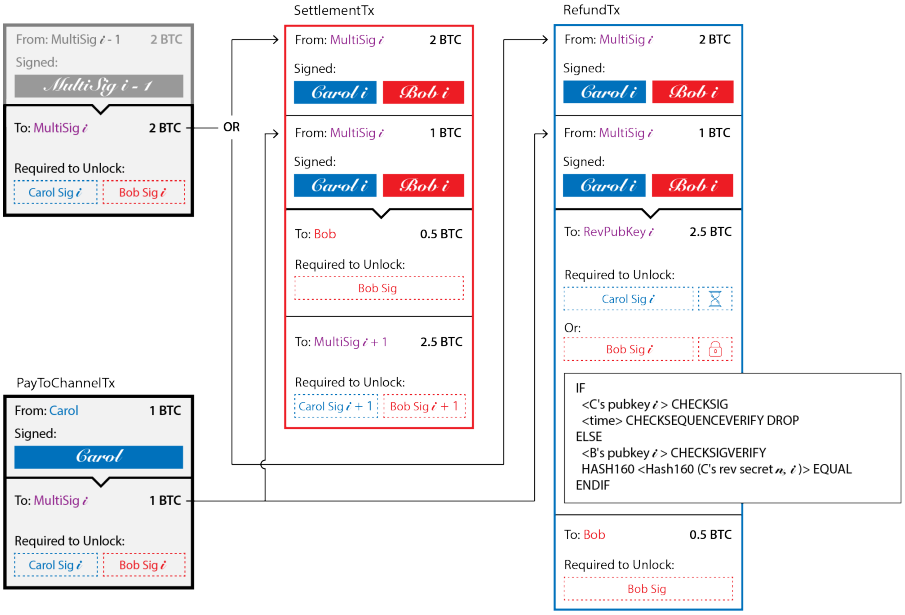

Fig. 7. Result of a merged pay to channel transaction after Carol sends 0.3 more Bitcoin to Bob. The $\texttt{Multisig}_i$ contains two `UTXOs`, (i) from the funding transaction or the last move from $\texttt{Multisig}_{i-1}$, and (ii) from the pay to channel transaction adding 1 BTC into the channel. Both settlement transaction and refund transaction contain the two `UTXOs` as input to sign and spend the totality with the adjusted balances.

3.2 Channel Opening

To open the channel, Carol and Bob have to cooperate to generate and fund a mutli-signatures address, hereinafter also mentioned as $\texttt{Multisig}_i$ address. This multi-signature address acts as the channel address and stores the totality of the channel's funds. This address holds normaly only one `UTXO`, but with pay to channel transactions, this could not be true.

1. Bob:
   (a) generates the $\texttt{Multisig}_i$ with Bob's Π_i and Carol's π_i and sends it
2. Carol:
   (a) create the $\texttt{FundingTx}^{i}_{<>}$ that funds the $\texttt{Multisig}_i$ address
   (b) generate $\Phi_{i,n}$
   (c) create $\texttt{RefundTx}^{i,n}_{<>}$ with $\texttt{Multisig}_i$ and $\Phi_{i,n}$ sending the full amount back to herself via the $\texttt{RevPubKey}_{i,n}$ contract
   (d) initiates the channel by sending:
      i. $\texttt{hash}(\Phi_{i,n})$

ii. $\texttt{RefundTx}^{i,n}_{<>}$

3. Bob:
   (a) receives the $\texttt{RefundTx}^{i,n}_{<>}$, signs it and returns $\texttt{RefundTx}^{i,n}_{<bob>}$
4. Carol:
   (a) broadcasts $\texttt{FundingTx}^{i}_{<carol>}$
5. Bob:
   (a) wait for transaction's confirmations
   (b) consider the channel as open

If Bob stops responding after the step 2, Carol has created transactions but has not use them. If Carol stops responding after step 3, Bob has signed a transaction who will maybe never been used. After a while, Bob must consider the channel opening as failed. If Bob stops responding after the setp 4, Carol can broadcast her refund transaction and she is safe. If Carol stops responding after opening the channel Bob do not lose anything.

3.3 Transact

Carol to Bob The Carol to Bob protocol allows Carol to send an arbitrary amount of money throught the channel. Carol desires to authorize a payment of M satoshis to Bob at $\texttt{Channel}_{i,n}$ state.

If there is a Bod to Carol transaction before and we need to use it because there is no more funds in the regular refund, we have to merge that refund in that new $\texttt{Channel}_{i,n+1}$ state. Bob can trust this unconfirmed output because it comes from himself.

1. Carol:
   (a) derives $\Phi_{i,n+1}$ and $\Phi_{i+1,n+1}$
   (b) generates the $\texttt{RefundTx}^{i,n+1}_{<>}$ with two outputs:
      i. Refund Output: Carol's new balance to $\texttt{RevPubKey}_{i,n+1}$ contract
      ii. Settlement Output: Bob's new balance to settlement address
   (c) sends a message to Bob containing:
      i. $\texttt{RefundTx}^{i,n+1}_{<>}$
      ii. $\texttt{hash}(\Phi_{i,n+1})$ and $\texttt{hash}(\Phi_{i+1,n+1})$
      iii. the amount of M satoshis being paid
2. Bob:
   (a) generates the $\texttt{SettlementTx}^{i}_{<>}$ with two outputs:
      i. Settlement Output: Bob's new balance
      ii. Change Output: Carol's new balance to $\texttt{Multisig}_{i+1}$ with Bob's Π_{i+1} and Carol's π_{i+1}
   (b) generates the $\texttt{PostSettlementRefundTx}^{i+1,n+1}_{<bob>}$ with:

i. Refund Output: sends Carol's funds to the associated $\texttt{RevPubKey}_{i+1,n+1}$ contract with the secret = $\texttt{hash}(\Phi_{i+1,n+1})$

(c) sends:

i. $\texttt{RefundTx}^{i,n+1}_{<bob>}$

ii. $\texttt{SettlementTx}^{i}_{<>}$

iii. $\texttt{PostSettlementRefundTx}^{i+1,n+1}_{<bob>}$

3. Carol:

(a) sends:

i. $\texttt{SettlementTx}^{i}_{<carol>}$

ii. the shared secret $\Phi_{i,n}$

4. Bob:

(a) upates state channel to $\texttt{Channel}_{i,n} \Rightarrow \texttt{Channel}_{i,n+1}$ and the payment can now be considered as final

If Bob stops responding after step 2, Carol can broadcast the refund transaction but she has no incentives to do that because she will go down her balance without counter-party. Because she has not yet share the secret $\Phi_{i,n}$, Bob cannot yet revoke the current $\texttt{Channel}_{i,n}$ state. If Carol does not respond at the step 3, Bob can settle the current $\texttt{Channel}_{i,n}$ state, but cannot settle the $\texttt{Channel}_{i,n+1}$ state in negotiation, Carol is safe. After step 3, Carol can refund herself and Bob can revoke the old $\texttt{Channel}_{i,n}$ state and settle the new $\texttt{Channel}_{i,n+1}$ state, the transaction is complete.

Channel Topop The channel topop protocol allows Bob or Carol to send an output directly to the current channel multisig address and allows Carol to include this output as part of usable funds immediatley. If the funds comes from Carol, they can be immediatley used for a withdraw transaction only.

To protect Carol, the refund for this additional amount is separate to the existing refund output to prevent Bob from invalidating Carol's refund transaction by sending an output which becomes invalid or not accepted by the network (lower fee, double spend, invalid script, etc.)

In subsequent transactions, once this output has confirmed, the refund should be merged into a single refund output as before, to be more efficient with refund transaction size.

1. Initiator:

(a) create the $\texttt{PayToChannelTx}^{i}_{<initiator>}$ that funds the $\texttt{Multisig}_i$ address

(b) create $\texttt{InterimRefundTx}^{i,n}_{<>}$ with $\texttt{Multisig}_i$ and $\texttt{hash}(\Phi_{i,n})$ sending the full amount back to Carol via the $\texttt{RevPubKey}_{i,n}$ contract

(c) sends:

i. $\texttt{InterimRefundTx}^{i,n}_{<initiator>}$

2. Receiver:
   (a) validate $\texttt{InterimRefundTx}^{i,n}_{<initiator>}$
   (b) sends if the payment is accepted or not
3. Initiator:
   (a) if the payment is accepted broadcast $\texttt{PayToChannelTx}^{i}_{<initiator>}$
4. Receiver:
   (a) wait for $\texttt{PayToChannelTx}^{i}_{<initiator>}$ transaction's confirmations

If the receiver does not validate the payment the initiator has no incentives to broadcast the transaction, if it is accepted, then the initiator can send money into the channel safely because of the interim refund transaction. Without negotiating a new state, Bob cannot revoke $\texttt{InterimRefundTx}^{i,n}_{<initiator>}$ and Carol can spend the refund. When a new $\texttt{Channel}_{i,n+1}$ state is negotiated, Bob can revoke the $\texttt{InterimRefundTx}^{i,n}_{<initiator>}$ if Carol try to broadcast it. At $\texttt{Channel}_{i,n+1}$, the refund transaction and the settlement transaction contain the merged refund transaction.

It is worth noting that the initiator does need to know the secret to create the pay to channel transaction and the interim refund transaction. An external player can ask the needed information to topop the channel knowing only public information.

Withdrawing The withdraw protocol allows Bob to authorize a withdrawal of M satoshis at Carol's request and with her cooperation for $\texttt{Channel}_{i,n}$ state. Bob needs to validate the withdrawal amount and can set up a set of rules internally to manage the channel economics. It is worth noting that when the withdraw takes action Bob automatically settle his funds without paying fee.

1. Carol:
   (a) derives $\Phi_{i+1,n}$
   (b) generates the $\texttt{Multisig}_{i+1}$ address with Bob's π_{i+1} and Carol's Π_{i+1}
   (c) generates the $\texttt{WithdrawTx}^{i}_{<>}$ with:
      i. Settlement Output: sends Bob's funds to the settlement address
      ii. Withdraw Output: withdrawal amount M to specified address
      iii. Change Output: new balance into $\texttt{Multisig}_{i+1}$
   (d) generates the $\texttt{RefundTx}^{i+1,n}_{<>}$ from address $\texttt{Multisig}_{i+1}$ with:
      i. Refund Output: Carol's new balance into $\texttt{RevPubKey}_{i+1,n}$ contract with the secret = $\texttt{hash}(\Phi_{i+1,n})$
   (e) sends:
      i. $\texttt{RefundTx}^{i+1,n}_{<>}$
      ii. $\texttt{WithdrawTx}^{i}_{<>}$
      iii. $\texttt{hash}(\Phi_{i+1,n})$

2. Bob:
   (a) verifies, signs and returns:
      i. $\texttt{RefundTx}^{i+1,n}_{<bob>}$
      ii. $\texttt{WithdrawTx}^{i}_{<bob>}$
3. Carol:
   (a) shares:
      i. $\Phi_{i,n}$ to invalidate the current state
      ii. $\texttt{WithdrawTx}^{i}_{<bob,carol>}$
4. Bob:
   (a) broadcast $\texttt{WithdrawTx}^{i}_{<bob,carol>}$
   (b) upates state channel to $\texttt{Channel}_{i,n} \Rightarrow \texttt{Channel}_{i+1,n}$ and validate exchange

If Bob does not respond in step 2, Carol has not disclosed any important informations. If Bob stops responding after step 2, Carol can withdraw the amount and safely refund her funds if no transaction is negotiated. If Carol does not respond after the step 2, Bob must wait a while and if the withdraw transaction is not broadcasted, he must broadcast the settlement transaction to force the transition to the next $\texttt{Channel}_{i+1,n}$ state.

Settling The settling protocol allows Bob to broadcast at $\texttt{Channel}_{i,n}$ state the $\texttt{SettlementTx}_{i,n}$ to get the settlement output and move the remaining funds into the next $\texttt{Multisig}_{i+1}$ address. In this case the channel stays open and Carol can create new transactions or close the channel.

If the $\texttt{SettlementTx}_{i,n}$ is broadcasted and Carol wants to close the channel, she can broadcast the $\texttt{PostSettlementRefundTx}_{i,n}$ and wait the timelock to get her money back. Carol has to query the network to know if the $\texttt{SettlementTx}_{i,n}$ is broadcast, she can only query the blockchain before each new transaction to be sure that the settlement transaction has not be broadcasted yet.

3.4 Channel Closing

Cooperative Closing cooperatively the channel allows Carol—or Bob if Carol is online—to ask if Bob agrees to close efficiently the channel, withdrawing the full remaining balance, at $\texttt{Channel}_{i,n}$ state. The following steps 3 and 4 can be merge and executed by the same player depending the implementation.

1. Carol:
   (a) generates the $\texttt{ClosedChannelTx}^{n+1}_{<carol>}$ with:
      i. Settlement Output: sends Bob's funds to Bob address
      ii. Change Output: sends Carol's funds to Carol address
   (b) sends $\texttt{ClosedChannelTx}^{n+1}_{<carol>}$

2. Bob:
   (a) verifies and signs $\texttt{ClosedChannelTx}^{n+1}_{<carol>}$
   (b) sends $\texttt{ClosedChannelTx}^{n+1}_{<carol,bob>}$
3. Carol:
   (a) broadcasts $\texttt{ClosedChannelTx}^{n+1}_{<carol,bob>}$

Contentious The contentious channel closing protocol allows Carol to close the channel alone, i.e., without Bob's cooperation or response, at $\texttt{Channel}_{i,n}$ state. Carol can broadcast her fully signed refund transaction sending her owned funds to $\texttt{RevPubKey}_{i,n}$ address. Carol would then need to spend from the revocation public key contract after the timelock delay with the spend refund transaction.

It is worth noting that only Carol can close the channel, but Bob can get his money by broacasting his settlement transaction at any time.

4 Evidence of Trustlessness

In the following, axioms, possible edge-cases, and discovered attacks, with an evidence of trustlessness for the channel protocol, are exposed. *Liveness* in the blockchain, i.e. transactions can be included in the next blocks, is asumed to guarantee the security model.

4.1 Axioms

Refund Transaction For $\texttt{Channel}_{i,n}$ state, if Carol broadcasts the current refund transaction, Bob cannot revoke it without knowing $\Phi_{i,n}$. After the timelock, Carol can generate and broadcast a spend refund transaction.

$$\forall i \wedge n, \exists \Phi_{i,n} : \quad \forall \Phi_{i,n}, \text{bob knows } \Phi_{i,n-1} \wedge \texttt{hash}(\Phi_{i,n})$$

The same rule is applied to interim refund transactions, if Carol broadcast the current interim refund transaction, Bob cannot revoke it without knowing $\Phi_{i,n}$, and Carol can spend the interim refund after the timelock.

Old Refund Transaction For $\texttt{Channel}_{i,n}$ state, if Carol broadcasts an old refund transaction, e.g. $n-1$, then Bob has a the time during the timelock to generate and broadcast the revoke transaction for the state $\texttt{Channel}_{i,n-1}$ with $\Phi_{i,n-1}$ secret.

$$\forall 0 \leq x < n, \exists \Phi_{i,x} : \quad \forall \Phi_{i,x}, \exists \texttt{RevokeTx}^{i,x}_{<bob>}$$

The same rule is applied to old interim refund transactions, if Carol broadcast an old interim refund transaction, e.g. $n-1$, Bob can revoke it with $\Phi_{i,n-1}$ secret.

Settlement Transaction For $\texttt{Channel}_{i,n}$ state, if Bob broadcasts his knowed $\texttt{SettlementTx}_{i,n}$ transaction, Carol has the choice to close the channel or transact on top of the new $\texttt{Multisig}_{i+1}$ address.

$$\forall \mathcal{C} = \texttt{Channel}_{i,n}, \exists \Phi_{i,n} \wedge \Phi_{i+1,n} : \quad \text{bob knows } \Phi_{i+1,n} \text{ iff } \mathcal{C} = \texttt{Channel}_{i+1,n+1}$$

Contentious Channel Closing By contentious it means that all players are not communicating anymore and/or do not agree on a valid state. Let's define the way for Carol to close the $\texttt{Channel}_{i,n}$ state.

$$\forall \texttt{Channel}_{i,n}, \exists \texttt{RefundTx}^{i,n}_{<carol,bob>} \text{only owned by Carol}$$

and

$$\forall \texttt{RefundTx}^{i,n}_{<carol,bob>}, \exists \texttt{SpendRefundTx}^{i,n}_{<carol>} \text{only owned by Carol}$$

so:

$$\forall \texttt{Channel}_{i,n}, \exists \texttt{SpendRefundTx}^{i,n}_{<carol>} \text{only owned by Carol}$$

4.2 Edge Cases

Someone does not broadcast Cooperative Transaction If one player does not share a fully signed cooperative transaction and the secret $\Phi_{i,n}$ attached to the current $\texttt{Channel}_{i,n}$ state, then the other player need to force after a while the transition into the new $\texttt{Channel}_{i+1,n}$ state with his own fully signed transaction, i.e $\texttt{RefundTx}_{i,n}$ or $\texttt{SettlementTx}_{i,n}$ transaction.

4.3 Attacks

In this section, attacks' vector discovered and fixes are discussed. Attacks exposed are no longer valid in the current scheme, but a deep analysis has been carry out to generalize the protocol construction and improve the scheme.

Old Settlement Attack With Weak Secret It is possible for Bob to lock the funds into the multisig or steal the money if the secret construction is too weak. For a channel at N dimensions the secret is considered weak if:

$$|\Phi| < N$$

Let's assume that the revocation secret Φ depends only on n and not on i for $\texttt{Channel}_{i,n}$. Thus, the secret can be expressed by:

$$|\texttt{Channel}_{i,n}| = N = 2 : |\Phi_n| = 1 \implies |\Phi_n| < N$$

Then, for $\texttt{Channel}_{i,n}$, if Bob broadcast an old settlement transaction, e.g. $n-1$, then Carol cannot use her post settlement refund transaction because

she previously shared the secret Φ_{n-1}. So the remaining funds are blocked in the $\texttt{Multisig}_{i+1}$ address. To be able to get her funds back, Carol would have to transact with Bob, if Bob does not cooperate, Carol has no way to recover her funds. If she try to refund the $\texttt{Multisig}_{i+1}$, then Bob can revoke with Φ_{n-1} secret.

If the secret dimension is equal to the channel dimension, i.e. $|\Phi_{i,n}| = |\texttt{Channel}_{i,n}|$, then the previous shared secret is $\Phi_{i,n-1}$ and the secret for refunding the $\texttt{Multisig}_{i+1}$ address at $\texttt{Channel}_{i,n-1}$ state is $\Phi_{i+1,n-1}$ and then:

$$\Phi_{i,n-1} \neq \Phi_{i+1,n-1}$$

Game theory is not sufficient to ensure the security of the channel if, when a player acts dishonestly, it exists an incentive to gain, even probabilistically, over the other player. In this case, the provider lose funds by broadcasting the $\texttt{Channel}_{i,n-1}$ state but can gain all funds if the client does not act correctly and does unlock his funds.

5 Further Improvements

Improvements can be done in two ways: (i) extending channel capabilities or (ii) optimizing the channel costs by reducing the transaction size or their number.

5.1 Threshold Signatures

The abilities of settle and withdraw the channel without closing has a downside, each time a transaction is broadcasted on chain and cost fees. Optimizing the channel transaction size or the number of transaction needed is an area of further research.

The principal cost of a transaction come from it inputs and their type. A channel transaction spend one or more `UTXOs` from the $\texttt{Multisig}_i$ address. These `UTXOs` are `P2SH` of a Bitcoin 2-out-of-2 multi-signature script that requires, obviously, two signatures. Knowing that a signature size is at least 64 bytes and an average transaction size (one simple input and one or two outputs) is a bit more than 200 bytes, it is easy to see that replacing the `P2SH` with a 2-out-of-2 multisig `UTXOs` by `P2PKH` `UTXOs` is more efficient in any cases.

To achive it, an ECDSA threshold signature scheme, with the same requirements as the 2-out-of-2 multisig, is required. This scheme exists and can be adapted from the paper "Two-Party Generation of DSA Signatures" by MacKenzie and Reiter [4].

5.2 Pre-authorized Payments

Pre-authorized payments are required in other real case scenario like provider acting as a payment processor. The client must be able to set a limit in which the provider can take the money during a time limit.

Further research can be done in this area to figure out the achievability and the most effective way to implement this feature into this scheme. May be a third layer, on top of layer's two, is necessary and achievable, may be the channel dimension can be increased.

6 Related Work

Simple micropayment channels were introduced by Hearn and Spilman [3]. The Lightning Network by Poon and Dryja [6], also creates a duplex micropayment channel. However it requires exchanging keying material for each update in the channels, which results in either massive storage or computational requirements in order to invalidate previous transactions. Finally, Decker and Wattenhofer introduced a payment network with duplex micropayment channels [2].

7 Conclusion

Trustless one-way payment channel for Bitcoin resolves many problems. Scalability is near infinite and costs of the channel decrease linearly with the number of transaction present into the channel. Delays to consider a transaction as valid are brought back to network delay and minimal check time. Clients do not need to be online to keep their funds safe and can withdraw arbitrary amount and refill the channel at any time. The provider does not need to lock funds to receive money and the cost of a channel setup is low.

8 Acknowledgement

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References

[1] Ryan X. Charles and Clemens Ley. *Yours Lightning Protocol*. 2016. URL: `https://github.com/yoursnetwork/yours-channels/blob/master/docs/yours-lightning.md`.

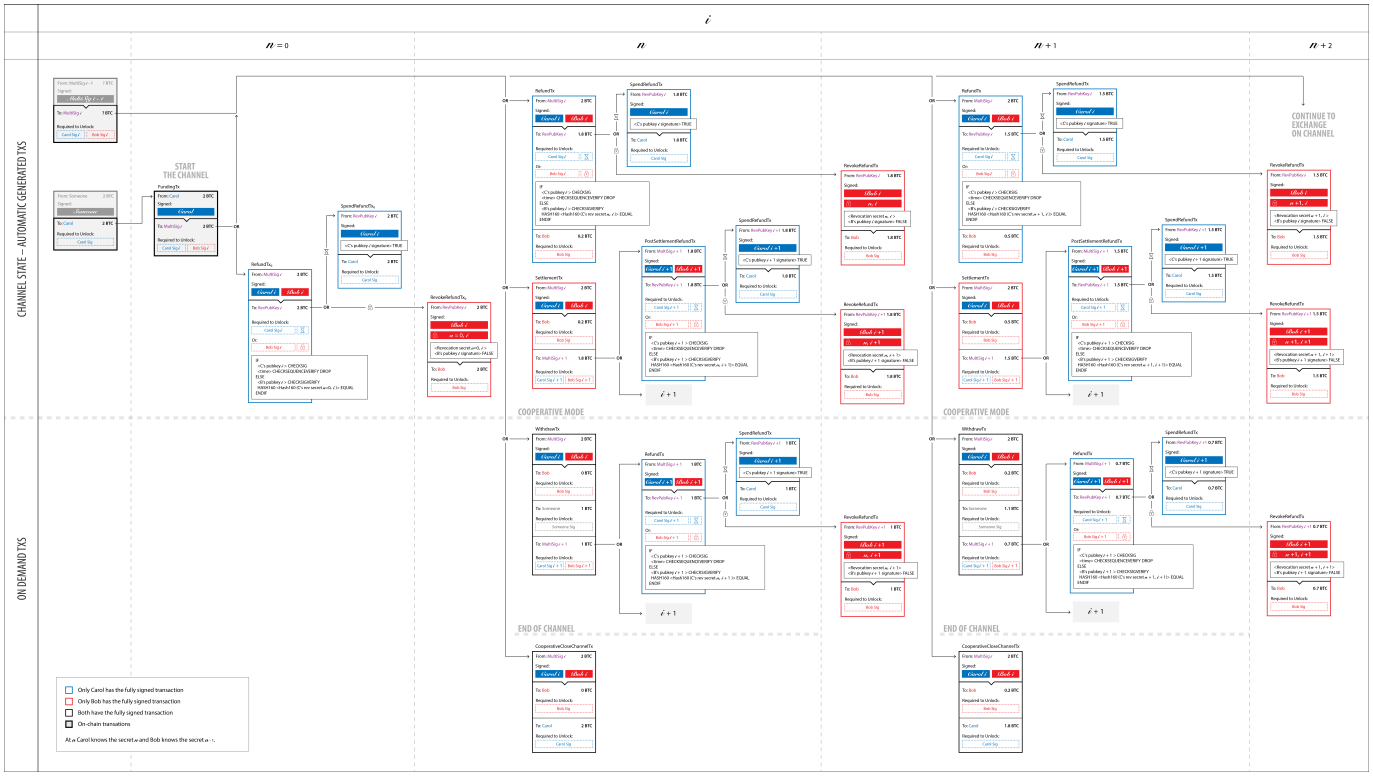
[2] Christian Decker and Roger Wattenhofer. "A Fast and Scalable Payment Network with Bitcoin Duplex Micropayment Channels". In: *Stabilization, Safety, and Security of Distributed Systems*. Ed. by Andrzej Pelc and Alexander A. Schwarzmann. Cham: Springer International Publishing, 2015, pp. 3–18. ISBN: 978-3-319-21741-3.

[3] Mike Hearn and Jeremy Spilman. *Bitcoin contracts*. URL: `https://en.bitcoin.it/wiki/Contracts`.

[4] Philip MacKenzie and Michael K. Reiter. "Two-Party Generation of DSA Signatures". In: *Advances in Cryptology — CRYPTO 2001*. Ed. by Joe Kilian. Berlin, Heidelberg: Springer Berlin Heidelberg, 2001, pp. 137–154. ISBN: 978-3-540-44647-7.

[5] Satoshi Nakamoto. *Bitcoin: A peer-to-peer electronic cash system.* 2009. URL: `http://bitcoin.org/bitcoin.pdf`.
[6] Joseph Poon and Thaddeus Dryja. *The bitcoin lightning network.*

A Transaction Dependency Graph



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Bibliography

[1] Andreas M. Antonopoulos. *Mastering Bitcoin: Unlocking Digital Crypto-Currencies*. 1st. O'Reilly Media, Inc., 2014. ISBN: 1449374042, 9781449374044.

[2] Dan Boneh and Matthew Franklin. "Efficient generation of shared RSA keys". In: *Advances in Cryptology — CRYPTO '97*. Ed. by Burton S. Kaliski. Berlin, Heidelberg: Springer Berlin Heidelberg, 1997, pp. 425–439. ISBN: 978-3-540-69528-8.

[3] Carmit Hazay et al. "Efficient RSA Key Generation and Threshold Paillier in the Two-Party Setting". In: *Topics in Cryptology – CT-RSA 2012*. Ed. by Orr Dunkelman. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012, pp. 313–331. ISBN: 978-3-642-27954-6.

[4] Steven Goldfeder et al. "Securing Bitcoin wallets via a new DSA/ECDSA threshold signature scheme". In: 2015.

[5] Rosario Gennaro, Steven Goldfeder, and Arvind Narayanan. "Threshold-Optimal DSA/ECDSA Signatures and an Application to Bitcoin Wallet Security". In: *Applied Cryptography and Network Security - 14th International Conference, ACNS 2016, Guildford, UK, June 19-22, 2016. Proceedings*. 2016, pp. 156–174. DOI: 10.1007/978-3-319-39555-5_9. URL: https://doi.org/10.1007/978-3-319-39555-5_9.

[6] Philip D. MacKenzie and Michael K. Reiter. "Two-Party Generation of DSA Signatures". In: *Advances in Cryptology - CRYPTO 2001, 21st Annual International Cryptology Conference, Santa Barbara, California, USA, August 19-23, 2001, Proceedings*. Vol. 2139. Lecture Notes in Computer Science. Springer, 2001, pp. 137–154. DOI: 10.1007/3-540-44647-8_8.

[7] Pascal Paillier. "Public-key Cryptosystems Based on Composite Degree Residuosity Classes". In: *Proceedings of the 17th International Conference on Theory and Application of Cryptographic Techniques*. EUROCRYPT'99. Prague, Czech Republic: Springer-Verlag, 1999, pp. 223–238. ISBN: 3-540-65889-0. URL: http://dl.acm.org/citation.cfm?id=1756123.1756146.

[8] Pieter Wuille. *Hierarchical Deterministic Wallets*. 2017. URL: https://github.com/bitcoin/bips/blob/master/bip-0032.mediawiki (visited on 02/01/2018).

[9] Thomas Pornin. *Deterministic Usage of the Digital Signature Algorithm (DSA) and Elliptic Curve Digital Signature Algorithm (ECDSA)*. RFC 6979. Aug. 2013. DOI: 10.17487/RFC6979. URL: https://rfc-editor.org/rfc/rfc6979.txt.

[10] Pascal Paillier. "Trapdooring Discrete Logarithms on Elliptic Curves over Rings". In: *Advances in Cryptology — ASIACRYPT 2000*. Ed. by Tatsuaki Okamoto. Berlin, Heidelberg: Springer Berlin Heidelberg, 2000, pp. 573–584. ISBN: 978-3-540-44448-0.

[11] Nir Bitansky et al. "From Extractable Collision Resistance to Succinct Non-interactive Arguments of Knowledge, and Back Again". In: *Proceedings of the 3rd Innovations in Theoretical Computer Science Conference*. ITCS '12. Cambridge, Massachusetts: ACM, 2012, pp. 326–349. ISBN: 978-1-4503-1115-1. DOI: 10.1145/2090236.2090263. URL: http://doi.acm.org/10.1145/2090236.2090263.

[12] Eli Ben-Sasson et al. *Zerocash: Decentralized Anonymous Payments from Bitcoin*. Cryptology ePrint Archive, Report 2014/349. `https://eprint.iacr.org/2014/349`. 2014.

[13] Benedikt Bünz et al. *Bulletproofs: Efficient Range Proofs for Confidential Transactions*. Cryptology ePrint Archive, Report 2017/1066. `https://eprint.iacr.org/2017/1066`. 2017.

[14] Marek Palatinus et al. *Mnemonic code for generating deterministic keys*. 2013. URL: `https://github.com/bitcoin/bips/blob/master/bip-0039.mediawiki` (visited on 02/01/2018).

[15] C. P. Schnorr. "Efficient Identification and Signatures for Smart Cards". In: *Advances in Cryptology — CRYPTO' 89 Proceedings*. Ed. by Gilles Brassard. New York, NY: Springer New York, 1990, pp. 239–252. ISBN: 978-0-387-34805-6.

[16] Mihir Bellare and Phillip Rogaway. "Random Oracles Are Practical: A Paradigm for Designing Efficient Protocols". In: *Proceedings of the 1st ACM Conference on Computer and Communications Security*. CCS '93. Fairfax, Virginia, USA: ACM, 1993, pp. 62–73. ISBN: 0-89791-629-8. DOI: `10.1145/168588.168596`. URL: `http://doi.acm.org/10.1145/168588.168596`.

[17] Yannick Seurin. "On the Exact Security of Schnorr-Type Signatures in the Random Oracle Model". In: *Advances in Cryptology – EUROCRYPT 2012*. Ed. by David Pointcheval and Thomas Johansson. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012, pp. 554–571. ISBN: 978-3-642-29011-4.

Glossary

DSA Digital Signature Algorithm. 13, 14, 16–18

EC Elliptic Curves. 14, 18, 53

ECDSA Elliptic Curve Digital Signature Algorithm. 13, 14, 16–18, 30, 54, 55, 57

National Institute of Standards and Technology (NIST) is a unit of the U.S. Commerce Department. Formerly known as the National Bureau of Standards, NIST promotes and maintains measurement standards.. 14

P2PKH Pay To Public Key Hash. 8

Standards for Efficient Cryptography Group (SECG) is an international consortium founded by Certicom in 1998. The group exists to develop commercial standards for efficient and interoperable cryptography based on elliptic curve cryptography (ECC). 14