ING175 Physics - Oblig 1

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Introduction

Task A has been developed in Python, and the corresponding script is available in the "Task_A.py" file. Upon execution, this script produces an output file named "coord_time.csv," which contains the resulting data. Task B then utilizes this CSV file to conduct the required analysis, building upon the output generated by Task A..

Results

Table 1. Result

Iteration	Internal time (s)	Coordinate time (s)
...	...	...
1440	86400	86399.9999614271

Excerpt from Coord_time.csv

(a) Table 1 illustrates the cumulative difference between coordinate time, (t), and physical time, τ. By iteration 1440, the physical time reaches $\tau = 86400$, corresponding to precisely one day, while the coordinate time is slightly less at $t = 86399.9999614271$.

$$discrepancy = \tau - t = 0.0000385729s = 38.5729\mu s$$

After a full day, the accumulated discrepancy is about 38.6 microseconds.

(b) Given that a time discrepancy of $1 \times 10^{-9}s$ results in a positional error of 0.3 meters, we can determine the expected location error over a one-day period using our findings from part (a). By dividing the time difference of $38.5729 \times 10^{-6}s$ by the known discrepancy ratio and then multiplying by the spatial error per unit time, we calculate the error as follows:

$$err = \frac{38.5729 \times 10^{-6}s}{1 \times 10^{-9}s} \times 0.3m = 11577.69m$$

This calculation reveals that after 24 hours, the accumulated location error exceeds 11.5 kilometers.

(c) To estimate the time required for the location error to reach 500 meters, we can adjust the proportionality defined in (b). First, we identify the time associated with a one-meter error and then scale this value proportionally to 500 meters.
Starting with the correspondence from (b):

$$1 \times 10^{-9}s = 0.3m$$

We normalize this to find the time for one-meter error:

$$\frac{1 \times 10^{-9}s}{0.3} = 1m$$

Scaling up to a 500-meter error:

$$\frac{500 \times 10^{-9}s}{0.3} = 500m$$

Simplifying, we get:

$$1666.\overline{6} \times 10^{-9}s = 500m$$

$$1.\overline{6} \times 10^{-6}s = 500m$$

$$500m = 1.\overline{6}\mu s$$

Hence, it would take approximately 1.7 microseconds for a satellite to accrue a location error of 500 meters.

(d) To explore the impact of using a longer time step on the accuracy of our approximation, we adapted the original script from "Task_A.py". The changes entail extending the time step to 3600 seconds and adjusting the script to only consider every 60th entry from the input data. The outcomes of these modifications are documented in Table 2.

Table 2. Result with longer time step

Iteration	Internal time (s)	Coordinate time (s)
...	...	...
24	86400	86399.9999614255

Excerpt from Coord_time2.csv

$$discrepancy = \tau - t = 0.0000385745s = 38.5745\mu s$$

$$38.5745\mu s - 38.5729\mu s = 1.6ns$$

Comparing the results with $\Delta\tau = 60s$ and $\Delta\tau = 3600s$ we see that the difference is minuscule, at 1.6ns.
This likely points to a flawed testing methodology as we expected a larger discrepancy when using a so much larger time step.