CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 6 Continuous Probability Distributions, Multivariate Probability Distributions

Questions

1. The weekly downtime x (in hours) for a certain industrial machine has approximately a gamma distribution with $\alpha = 4$ and $\beta = 1.5$. The loss (l) to the industrial operation as a result of this downtime is given by $l = 30x + 2$. **[4+1 points]**
   a. Find the probability that the down time is not more than 2 hours. *(Write Julia code)*
   b. Find the expected value and the variance of l
2. The weekly amount spent for maintenance and repairs in a certain company has an approximately normal distribution with a mean of \$600 and a standard deviation of \$40. **[2+3 points]**
   a. If \$700 is budgeted to cover repairs for next week, what is the probability that the actual costs will exceed the budgeted amount? *(Write Julia code)*
   b. How much should be budgeted weekly for maintenance and repairs to ensure that the probability that the budgeted amount will be exceeded in any given week is only 0.1? *(Write Julia code. cdf() is the cumulative distribution function in Julia.)*
3. The proportion of impurities per batch in a certain type of industrial chemical is a random variable x that has a probability density function **[3+2 points]**

$$f(x) = \begin{cases} 20x^3(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   a. Suppose that a batch with more than 30% impurities cannot be sold. What is the probability that a randomly selected batch cannot be sold for this reason? *(This is a Beta distribution. Identify the parameters and write Julia code)*
   b. Suppose that the dollar value of each batch is given by $v = 10 - 0.75x$. Find the epxected value and variance of v.
4. Fatigue life (in hundres of hours) for a certain type of bearing has a Weibull distribution with $\gamma = 2$ and $\theta = 4$. **[4+1 points]**
   a. What is the probability that a randomly selected bearing of this type will fail in less than 200 hours? *(Write Julia code)*
   b. What is the expected value of the fatigue life for these bearings? *(Leave the Γ function as it is in the final result. No need to compute a value.)*
5. Write Julia code to plot pdf of the distributions in the above four questions. **[5 points]**

Bonus Questions

1. Write Julia code and empirically demonstrate the additive property of Chisquared distribution
2. Write Julia code to sample 10,000 data points from an Inverse Chisquared distribution and plot their histogram with 100 bins.
3. Write Julia code to empirically test the additive property of Chi-square distribtuion.
4. Derive the equation for variance of the Beta distribution.
5. Prove that the mean of Gaussian distribution is μ. Start with $E(x)$ for a conitnuous probability distribution and proceed by integration of $xf(x)$.