Lecture 4: In–Class Assignment

1. Prove that $E(aX + b) = aE(X) + b$, where X is a random variable, a and b are constants.

➢ We know that E (X) is called as the expectation of X.

And from the properties of expectations we get to know that,

$E(aX) = aE(X)$ <– Property 1

Also we know that, $E(\sum_i a_i X_i) = \sum_i a_i . E(X_i)$ from Property 2 of expectation

We can write the above equation as follows:

$E(aX + b) = \quad - From\ property\ 2$
$E(aX + b) = aE(X) + b \quad - From\ property\ 1$

2. Write the cumulative distribution function for the probability distribution

p (x = 1) = 0.1
p (x = 2) = 0.3
p (x = 3) = 0.4
p (x = 4) = 0.2

➢ cdf (x) = $\begin{cases} 0.1 & 0 \le x < 1 \\ 0.4 & 1 \le x < 2 \\ 0.8 & 2 \le x < 3 \\ 1 & 3 \le x < 4 \end{cases}$

3. In a study of battery life for laptop computers, researchers found that the probability that the battery life (L) will exceed 5 hours is 0.12. Three such batteries are used in independent laptops and we are interested in finding the probability that some x of the three batteries will last 5 hours or more.

I. Specify the standard probability distribution you will use to model this scenario.
➢ We can use binomial distribution for this problem.

II. What is the state space or the set of possible outcomes for this scenario?
➢ State space for the scenario is {0, 1, 2, 3}

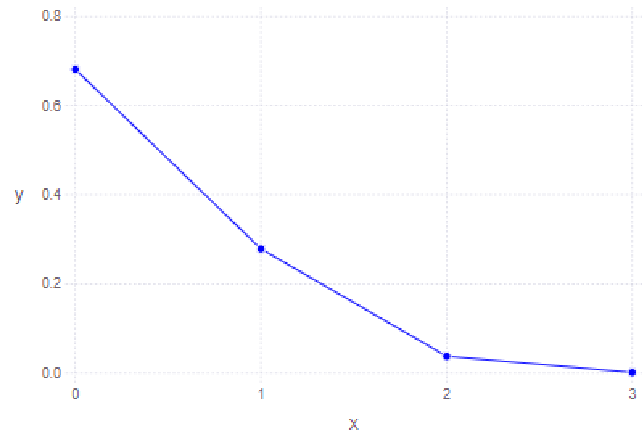
III. Find the probability that only one of the three batteries will last 5 hours or more.
➢ Probability for only one of the 3 batteries will last for 5 hours is 0.278784..

```
In [1]: using Distributions

In [2]: d = Binomial(3, 0.12)
Out[2]: Distributions.Binomial{Float64}(n=3, p=0.12)

In [3]: pdf(d, 1)
Out[3]: 0.278784
```

IV. Write Julia code to plot the true distribution for all possible values of x.

In [9]:
```
df1 = DataFrames.DataFrame(x = 0:3, y = data1)
true_dist = Gadfly.plot(x = 0:3, y = data1, Geom.point, Geom.line, Theme(default_color=colorant"blue"))
```

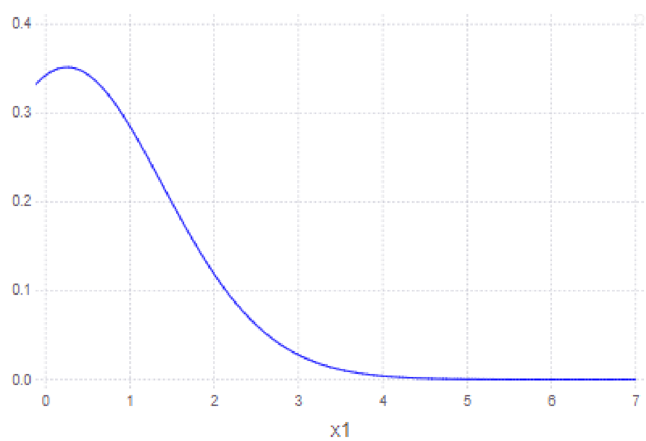
Out[9]:



V. Write Julia code to sample 1000 points from this distribution and plot the empirical distribution

In [31]:
```
true_dist_1 = Gadfly.plot(df, x="x1", Geom.density(bandwidth=1), Theme(default_color=colorant"blue"), Coord.Cartesian(xmin=0))
```

Out[31]:



4. A statistically inclined farmer would like to model the number of grasshoppers per square meter of his rangeland. He is told that typically there are 0.5 grasshoppers per square meter on a rangeland.

I. Specify the standard probability distribution you will use to model this scenario.

➢ We can use Poisson distribution for this particular problem.

II. What is the state space or the set of possible outcomes for this scenario?

➢ State space for following problem is number of grasshoppers per square meter.

III. Find the probability that there are five or more grasshoppers in a randomly selected square meter region.

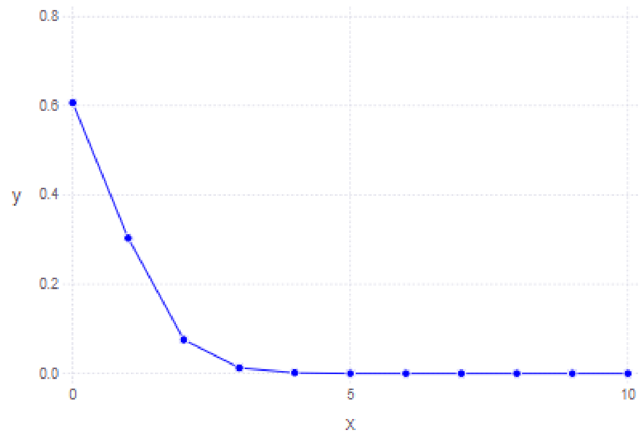
➢ Probability for 5 or more grasshoppers in randomly selected region is 0.00015795069263349796

```
In [14]: d1 = Poisson(0.5)
Out[14]: Distributions.Poisson{Float64}(λ=0.5)
```

```
In [15]: dist = pdf(d1, 5)
Out[15]: 0.00015795069263349796
```

IV. Write Julia code to plot the true distribution for values of x up to 10.

```
In [33]: true_dist = Gadfly.plot(x = 0:10, y = data3, Geom.point, Geom.line, Theme(default_color=colorant"blue"))
Out[33]:
```



V. Write Julia code to sample 1000 points from this distribution and plot the empirical distribution.

```
In [42]: true_dist_2 = Gadfly.plot(df5, x="x1", Geom.density(bandwidth=1), Theme(default_color=colorant"blue"), Coord.Cartesian(xmin=0))
Out[42]:
```