

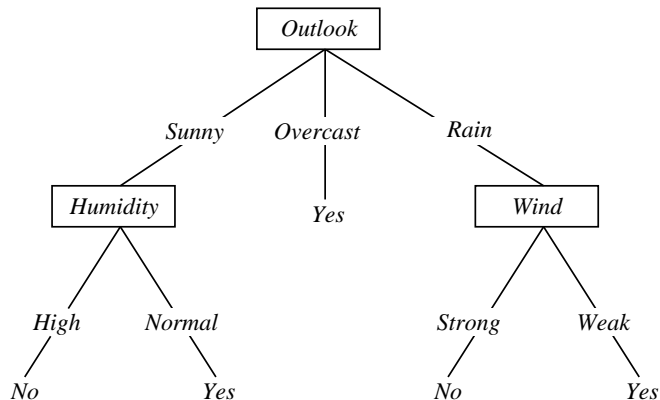
Decision Tree Learning

[read Chapter 3]

[recommended exercises 3.1, 3.4]

- ▶ Decision tree representation
- ▶ ID3 learning algorithm
- ▶ Entropy, Information gain
- ▶ Overfitting

Decision Tree for *PlayTennis*



A Tree to Predict C-Section Risk

Learned from medical records of 1000 women

Negative examples are C-sections

[833+,167-] .83+ .17-

Fetal_Presentation = 1: [822+,116-] .88+ .12-

Previous_Csection = 0: [767+,81-] .90+ .10-

Primiparous = 0: [399+,13-] .97+ .03-

Primiparous = 1: [368+,68-] .84+ .16-

Fetal_Distress = 0: [334+,47-] .88+ .12-

Birth_Weight < 3349: [201+,10.6-] .95+ .05-

Birth_Weight \geq 3349: [133+,36.4-] .78+ .22-

Fetal_Distress = 1: [34+,21-] .62+ .38-

Previous_Csection = 1: [55+,35-] .61+ .39-

Fetal_Presentation = 2: [3+,29-] .11+ .89-

Fetal_Presentation = 3: [8+,22-] .27+ .73-

Decision Trees

Decision tree representation:

- ▶ Each internal node tests an attribute
- ▶ Each branch corresponds to attribute value
- ▶ Each leaf node assigns a classification

How would we represent:

- ▶ \wedge, \vee, XOR
- ▶ $(A \wedge B) \vee (C \wedge \neg D \wedge E)$
- ▶ $M \text{ of } N$

When to Consider Decision Trees

- ▶ Instances describable by attribute–value pairs
- ▶ Target function is discrete valued
- ▶ Disjunctive hypothesis may be required
- ▶ Possibly noisy training data

Examples:

- ▶ Equipment or medical diagnosis
- ▶ Credit risk analysis
- ▶ Modeling calendar scheduling preferences

Top-Down Induction of Decision Trees

Main loop:

A the “best” decision attribute for next node

Assign *A* as decision attribute for *node*

For each value of *A*, create new descendant of *node*

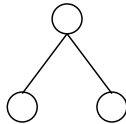
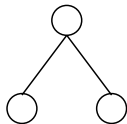
Sort training examples to leaf nodes

If training examples perfectly classified

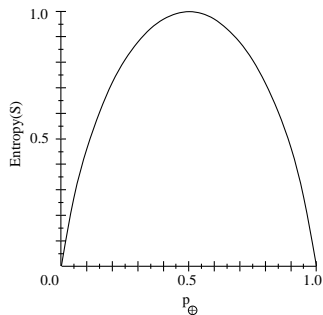
Then STOP

Else iterate over new leaf nodes (1)

Which attribute is best?



Entropy



- ▶ S is a sample of training examples
- ▶ p_{\oplus} is the proportion of positive examples in S
- ▶ p_{\ominus} is the proportion of negative examples in S
- ▶ Entropy measures the impurity of S

$$\text{Entropy}(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

Entropy

$Entropy(S)$ = expected number of bits needed to encode class (\oplus or \ominus) of randomly drawn member of S (under the optimal, shortest-length code)

Why?

Information theory: optimal length code assigns $-\log_2 p$ bits to message having probability p .

So, the expected number of bits to encode \oplus or \ominus of random member of S :

$$p_{\oplus}(-\log_2 p_{\oplus}) + p_{\ominus}(-\log_2 p_{\ominus})$$

$$Entropy(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

Information Gain

$Gain(S, A) =$ expected reduction in entropy due to sorting on A

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

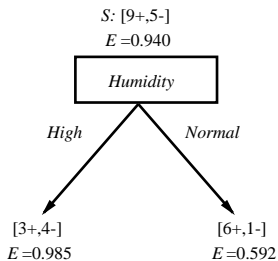


Training Examples

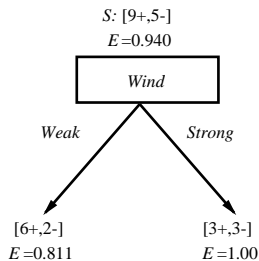
| Day | Outlook | Temperature | Humidity | Wind | PlayTennis |
|-----|----------|-------------|----------|--------|------------|
| D1 | Sunny | Hot | High | Weak | NO |
| D2 | Sunny | Hot | High | Strong | NO |
| D3 | Overcast | Hot | High | Weak | YES |
| D4 | Rain | Mild | High | Weak | YES |
| D5 | Rain | Cool | Normal | Weak | YES |
| D6 | Rain | Cool | Normal | Strong | NO |
| D7 | Overcast | Cool | Normal | Strong | YES |
| D8 | Sunny | Mild | High | Weak | NO |
| D9 | Sunny | Cool | Normal | Weak | YES |
| D10 | Rain | Mild | Normal | Weak | YES |
| D11 | Sunny | Mild | Normal | Strong | YES |
| D12 | Overcast | Mild | High | Strong | YES |
| D13 | Overcast | Hot | Normal | Weak | YES |
| D14 | Rain | Mild | High | Strong | NO |

Selecting the Next Attribute –1

Which attribute is the best classifier?

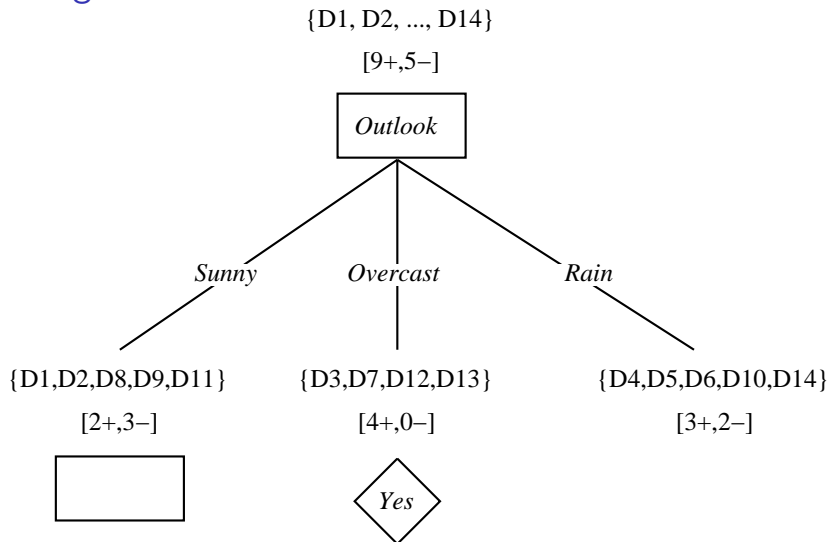


$$\begin{aligned} \text{Gain}(S, \text{Humidity}) &= .940 - (7/14).985 - (7/14).592 \\ &= .151 \end{aligned}$$

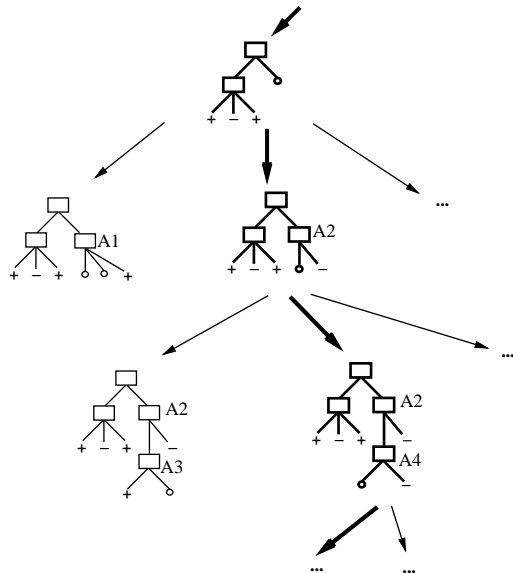


$$\begin{aligned} \text{Gain}(S, \text{Wind}) &= .940 - (8/14).811 - (6/14)1.0 \\ &= .048 \end{aligned}$$

Selecting the Next Attribute – 2



Hypothesis Space Search by ID3



Hypothesis Space Search by ID3

- ▶ **Hypothesis space is complete!**
 - ▶ Target function surely in there...
- ▶ **Outputs a single hypothesis (which one?)**
 - ▶ Can't play 20 questions...
- ▶ **No backtracking**
 - ▶ Local minima...
- ▶ **Statically-based search choices**
 - ▶ Robust to noisy data...
- ▶ **Inductive bias:** approx “prefer shortest tree”

Inductive Bias in ID3

Note H is the power set of instances X

Does that imply Unbiased?

Not really...

- ▶ Preference for short trees, and for those with high information gain attributes near the root
- ▶ Bias is a *preference* for some hypotheses, rather than a *restriction* of hypothesis space H
- ▶ Occam's razor: prefer the shortest hypothesis that fits the data

Occam's Razor

Why prefer short hypotheses?

Argument in favor:

- ▶ Fewer short hyps. than long hyps.
- a short hyp that fits data unlikely to be coincidence
- a long hyp that fits data might be coincidence

Argument opposed:

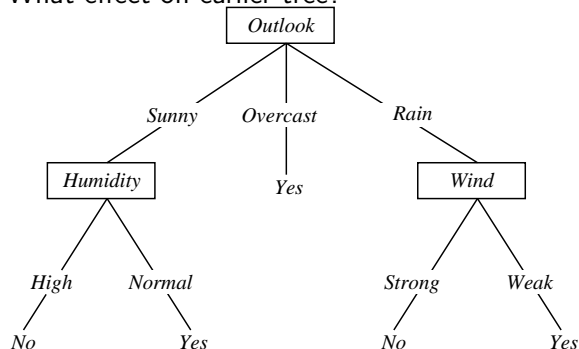
- ▶ There are many ways to define small sets of hyps
- ▶ e.g., all trees with a prime number of nodes that use attributes beginning with "Z"
- ▶ What's so special about small sets based on *size* of hypothesis??

Overfitting in Decision Trees

Consider adding noisy training example #15:

Sunny, Hot, Normal, Strong, PlayTennis = No

What effect on earlier tree?



Overfitting

Consider error of hypothesis h over

- ▶ training data: $error_{train}(h)$
- ▶ entire distribution \mathcal{D} of data: $error_{\mathcal{D}}(h)$

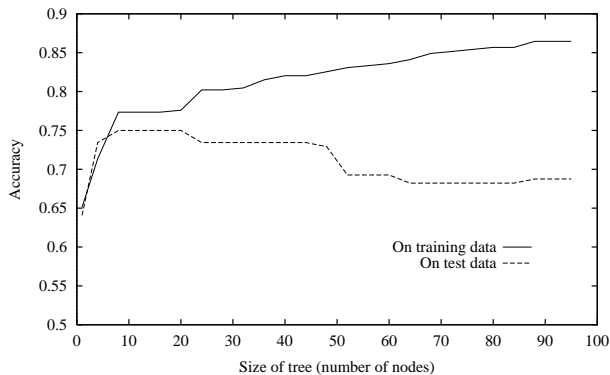
Hypothesis $h \in H$ **overfits** training data if there is an alternative hypothesis $h' \in H$ such that

$error_{train}(h) < error_{train}(h')$ h' : greater error on training data

and

$error_{\mathcal{D}}(h) > error_{\mathcal{D}}(h')$ h' : smaller error on test data

Overfitting in Decision Tree Learning



Avoiding Overfitting

How can we avoid overfitting?

- ▶ Stop growing when data split not statistically significant
- ▶ Grow full tree, then post-prune

How to select “best” tree:

- ▶ Measure performance over training data
- ▶ Measure performance over separate validation data set
- ▶ MDL:

$$\text{MIN}_{\text{tree}} \{ \text{size}(\text{tree}) + \text{size}(\text{misclassifications}(\text{tree})) \}$$

Reduced-Error Pruning

Split data into *training* and *validation* set

Do until further pruning is harmful:

1. Evaluate impact on *validation* set of pruning each possible node (plus those below it).
2. Greedily remove the one that most improves *validation* set accuracy.

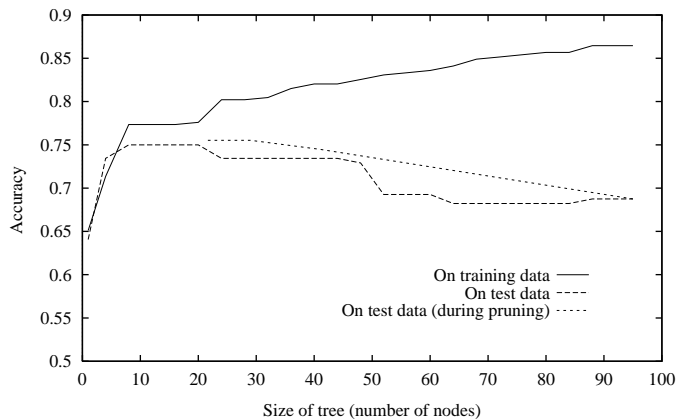
Effect

- ▶ Produces smallest version of most accurate subtree

Q

- ▶ What if data is limited?

Effect of Reduced-Error Pruning

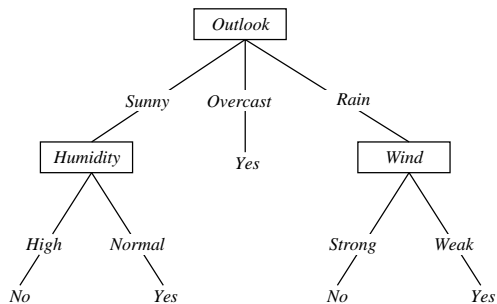


Rule Post-Pruning

1. Convert tree to equivalent set of rules
2. Prune each rule independently of others
3. Sort final rules into desired sequence for use

Perhaps most frequently used method (e.g., C4.5)

Converting A Tree to Rules –1



Converting A Tree to Rules – 2

IF (*Outlook = Sunny*) \wedge (*Humidity = High*)
THEN *PlayTennis = No*

IF (*Outlook = Sunny*) \wedge (*Humidity = Normal*)
THEN *PlayTennis = Yes*

...

Continuous Valued Attributes

Create a discrete attribute to test continuous

- ▶ $Temperature = 82.5$
- ▶ $(Temperature > 72.3) = t, f/1, 0$

| | | | | | | |
|---------------------|----|----|-----|-----|-----|----|
| <i>Temperature:</i> | 40 | 48 | 60 | 72 | 80 | 90 |
| <i>PlayTennis:</i> | No | No | Yes | Yes | Yes | No |

| | | | | | | |
|---------------------|----|----|-----|-----|-----|----|
| <i>Temperature:</i> | 0 | 0 | 0 | 0 | 1 | 1 |
| <i>PlayTennis:</i> | No | No | Yes | Yes | Yes | No |

Attributes with Many Values

Issue:

- ▶ If attribute has many values, *Gain* will select it
- ▶ Imagine using *Date* as attribute

Approach: Use *GainRatio* instead

$$\text{GainRatio}(S, A) \equiv \frac{\text{Gain}(S, A)}{\text{SplitInformation}(S, A)}$$

$$\text{SplitInformation}(S, A) \equiv - \sum_{i=1}^c \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|}$$

where S_i is subset of S for which A has value v_i

Attributes with Costs

Consider

- ▶ medical diagnosis, *BloodTest* has cost \$150
- ▶ robotics, *Width_from_1ft* has cost 23 sec.

How to learn a consistent tree with low expected cost?

One approach: replace gain by

$$\frac{Gain^2(S, A)}{Cost(A)}.$$

$$\frac{2^{Gain(S,A)} - 1}{(Cost(A) + 1)^w}$$

where $w \in [0, 1]$ determines importance of cost

Unknown Attribute Values: some examples missing values of A

1. Use training example anyway, sort through tree
 - ▶ If node n tests A , assign most common value of A among other examples sorted to node n
 - ▶ Assign most common value of A among other examples with same target value
 - ▶ Assign probability p_i to each possible value v_i of A
 - ▶ Assign fraction p_i of example to each descendant in tree
2. Classify new examples in same fashion