ASSIGNMENT 8

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1 Conventions

N = length of the signal array(or time array).
k: In time domain the signal exists from -kπ to kπ.
t = np.linspace(-k*pi,k*pi,N)

2 Finding the Spectrum of $\sin^3(t)$ and $\cos^3(t)$

The transforms for $\sin^3(t)$ and $\cos^3(t)$ contain impulses in frequency domain. But since we are not taking infinite number of samples(In discrete time). The value of impulse will be (finite) equal to number of samples given to the function. So we divide it by N(No of samples) to get its weight(or area).

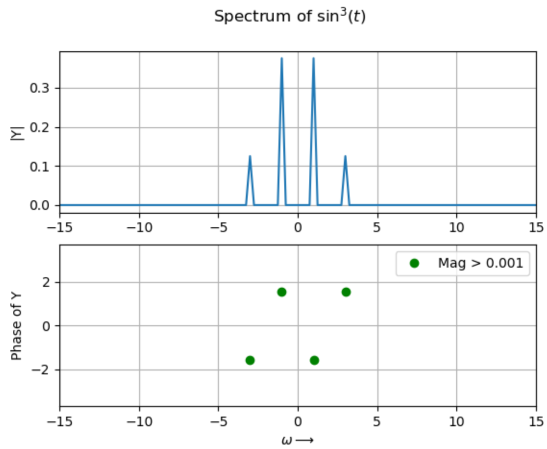
```
Y = np.fft.fftshift(np.fft.fft(y))/N
```

$\sin^3(t)$

$$\sin^3(t) = \frac{3\sin(t) - \sin(3t)}{4}$$

The transform for this will be

$$X(e^{jw}) = \frac{0.25\delta(w+3) - 0.75\delta(w+1) + 0.75\delta(w-1) - 0.25\delta(w-3)}{2j}$$

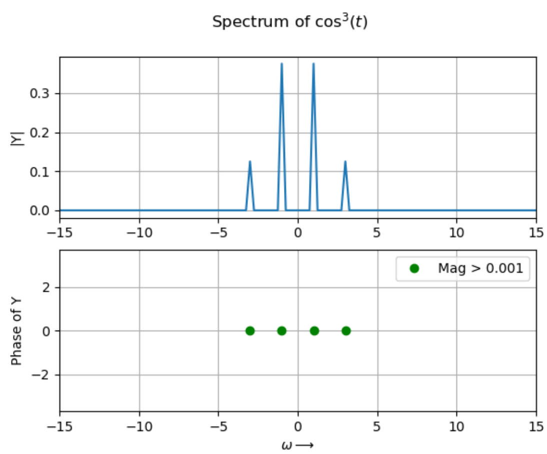


$\cos^3(t)$

$$\cos^3(t) = \frac{3\cos(t) + \cos(3t)}{4}$$

The transform for this will be

$$X(e^{jw}) = \frac{0.25\delta(w+3) + 0.75\delta(w+1) + 0.75\delta(w-1) + 0.25\delta(w-3)}{2}$$



3 Finding the Spectrum of $\cos(20t + 5\cos(t))$

because of the 20t in cos() the principal frequency of the Transform is at 20 and the 5(coefficient of inner cos) inside controls the number of impulses, while frequency of inner cos(t) controls the spacing between the impulses.

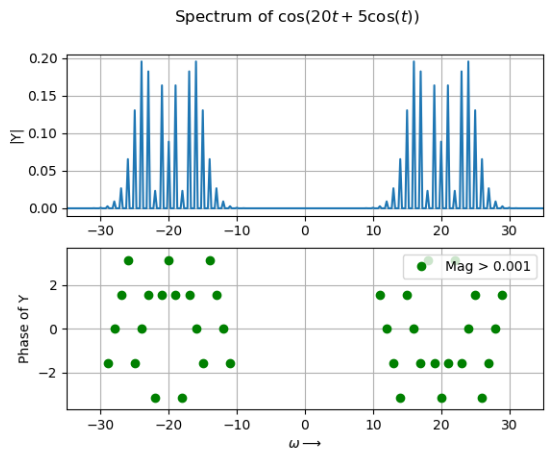


Figure 3: Spectrum of $\cos(20t + 5\cos(t))$

4 Fourier Transform for Gaussian

Gaussian $\exp(-t^2/2)$ is a non periodic band **unLimited** signal.The CTFT of gaussian is also a gaussian.

$$X_c(w) = X(jw) = \sqrt{2\pi}\exp(-w^2/2)$$

If we sample the gaussian with a sampling period T_s, then the Transform of gaussian is $X_s(w)$.
Then within a sampling frequency length in the freq domain

$$X_s(w) = X_c(w)/T_s$$

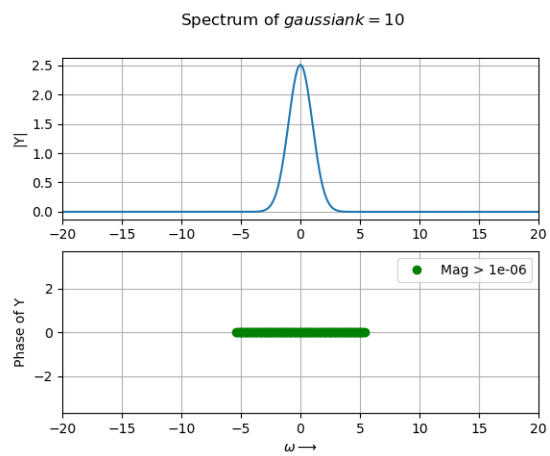
To find the gaussian from fft function(Y = X_s)

```
Y=np.fft.fftshift(np.fft.fft(np.fft.ifftshift(y)))
```

we can use either fftshift or ifftshift to calculate Y. Now to compare the error between The transform obtained from formula and function we multiply y by $T_s = \frac{2k\pi}{N}$.

Now the difference between these two gives us the error in $X_c(w)$.

for K = 10, N=512

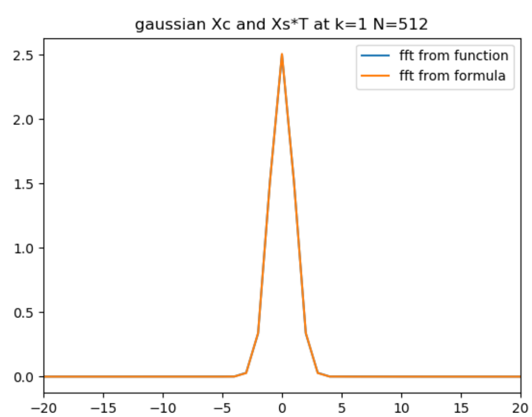


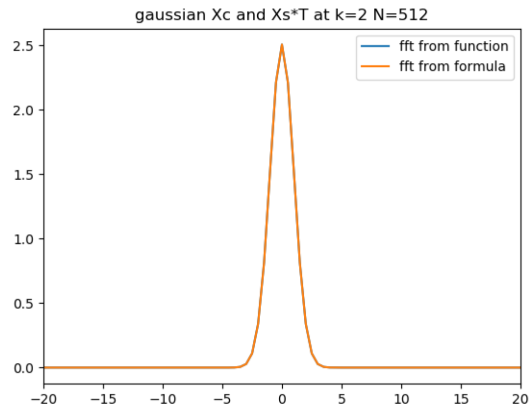
for k = 1,2,3,4 and N=512.

k	error in X_c	error in X_s
1	0.004212495531544125	0.3432650171380575
2	8.331486611723449e-10	3.394553030552918e-08
3	6.66134943357952e-15	1.8093852842949594e-13
4	5.378652970638708e-16	1.0957301855399136e-14
10	2.458215155328783e-15	2.003133916948414e-14

The error is less than 1e-6 for k $\geq$ 2. Even for k=1 the error is in the order of 1e-3 which doesn't show much differnce when you plot them.

Increasing K bring more of the gaussian into consideration and error reduces. But you should increase the N to ensure proper sampling and thus reduce the error.




gaussian Xc and Xs*T at k=2 N=512
fft from function
fft from formula
2.5
2.0
1.5
1.0
0.5
0.0
−20
−15
−10
−5
0
5
10
15
20