EE6143 Matlab assignment 1

Pothuganti Harshavardhan
EE17B061

February 22, 2021

1 Theoretical Error Probability(SER)

Symbol Error Rate(SER) is approximately equal to the probability of error for a constellation for large enough number of symbols.

1.1 Theoretical SER calculation

$$q = Q(d/\sigma_v)$$
$$\sigma_v = \sqrt{N_o/2}$$

2d is the minimum distance between any two symbols in the constellation.

σ_v is the noise variance along one dimension of the constellation.

$$P(c1) = (1-q)^2$$

P(c1) is the probability that there is no symbol error given the symbol is one of the 4 symbols in the corner of M-QAM.

$$P(c2) = (1-q) * (1-2q)$$

P(c2) is the probability that there is no symbol error given the symbol is one of the 4*($\sqrt{M}$-2) symbols on the edges/sides of M-QAM(not including the 4 corners).

$$P(c3) = (1-2q) * (1-2q)$$

P(c3) is the probability that there is no symbol error given the symbol is one of the $(\sqrt{M}-2)^2$ symbols on the inside of M-QAM.

$$P(c) = \frac{4}{M} * P(c1) + \frac{4 * (\sqrt{M}-2)}{M} * P(c2) + \frac{(\sqrt{M}-2)^2}{M} * P(c3)$$

P(c) is the average probability that a received symbol is decoded without error.

$$SER = P(e) = 1 - P(c)$$

to reduce bit error rate **gray coding** is used for assigning bits to each of the symbols so that any two nearest symbols differ only by one bit.

Most of the symbols errors are nearest neighbour errors. i.e, most of the errors make the symbol decoded as one of the nearest symbols. Assuming this is always the case. Number of bit errors = Number of symbol errors. since the nearest symbols differ only by one bit.

number of bit errors ≈ number of symbol errors
total number of bits = $log_2 M$ * number of symbols

$$BER = \frac{\#biterrors}{\#totalbits} \approx \frac{\#symbolerrors}{log_2 M \#totalsymbols} = \frac{SER}{log_2 M}$$

$$BER \approx \frac{SER}{log_2 M}$$

2 Simulation details

Symbols are gray coded to get the least bit error rate for the same symbol error rate. Nearest neighbour decision rule is used for decoding received symbols.

3 Observations

In Figure 1, we can see that theoretical BER and empirical BER are almost same for high $\frac{E_b}{N_o}$.

theoretical BER is lower than empirical BER when $\frac{E_b}{N_o}$ is low and the difference is high for large M. This is because when $\frac{E_b}{N_o}$ is low, noise variance is significant so not all the errors are nearest neighbour symbols which increase bit error rate since non nearest neighbour symbols differ by more than 1 bit. So actual BER is more than our theoretical estimate.

As M increases the minimum distance between symbols decrease for the same $\frac{E_b}{N_o}$ so empirical BER is more different for the same $\frac{E_b}{N_o}$ for large M.

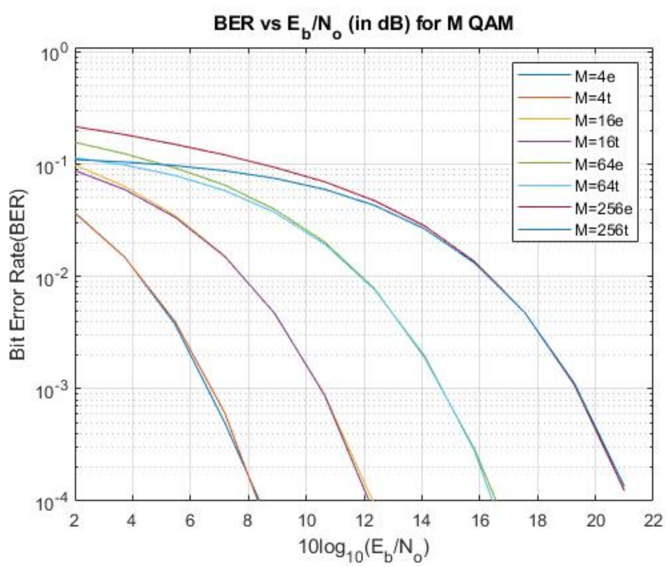

Figure 1: BER Plots

In Figure 1 4e is empirical BER plot for 4-QAM and 4t is theoretical BER plot for 4-QAM.