

A function MRI mind-reading game

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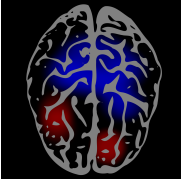

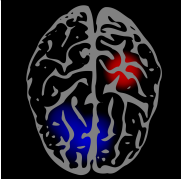
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Section 1

Introduction

Functional MRI

Stimuli	Response
	
	

Functional MRI

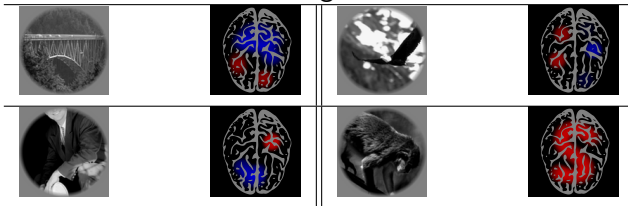
Stimuli x	Response y
$\begin{pmatrix} 1.0 \\ 0 \\ 3.0 \\ 0 \\ -1.2 \end{pmatrix}$	$\begin{pmatrix} 1.2 \\ 0 \\ -1.8 \\ -1.2 \end{pmatrix}$
$\begin{pmatrix} 0 \\ -2.2 \\ -3.1 \\ 4.5 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1.2 \\ -1.9 \\ 0.5 \\ 0.6 \end{pmatrix}$

Encoding vs Decoding

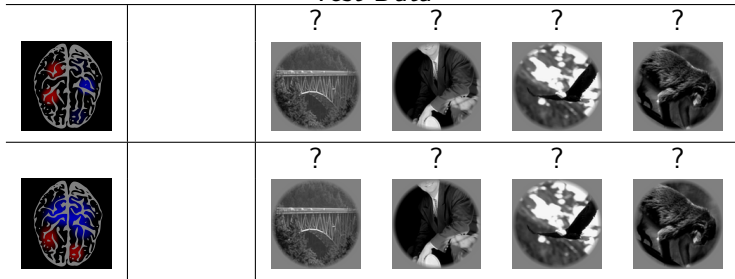
- Encoding: predict y from x .
- Decoding: reconstruct x from y (mind-reading).

A mind-reading game: Classification

Training Data

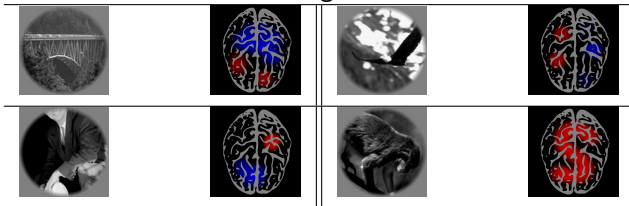


Test Data

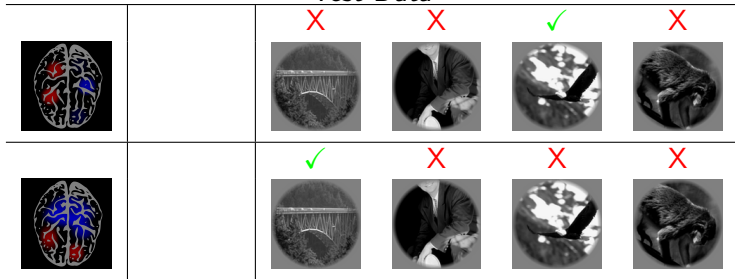


A mind-reading game: Classification

Training Data

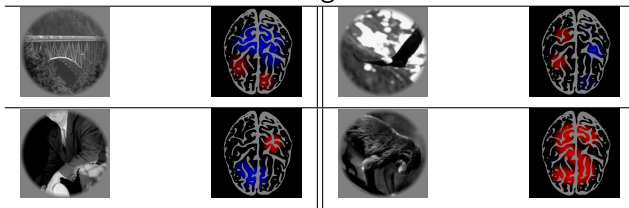


Test Data

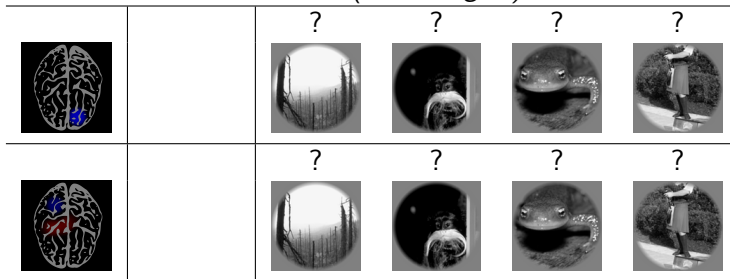


A mind-reading game: Identification

Training Data



Test Data (*new images!*)



Statistical formulation I

Training data.

- Given training classes $S_{\text{train}} = \{\text{train:1}, \dots, \text{train:k}\}$ where each class train:i has features $x_{\text{train:i}}$.
- For $t = 1, \dots, T_{\text{train}}$, choose class label $z_{\text{train:t}} \in S_{\text{train}}$; generate

$$y_{\text{train:t}} = f(x_{z_{\text{train:t}}}) + \epsilon_t$$

where f is an unknown function, and ϵ_t is i.i.d. from a known or unknown distribution.

Test data.

- Given test stimuli $S_{\text{test}} = \{\text{test:1}, \dots, \text{test:l}\}$ with features $\{x_{\text{test:1}}, \dots, x_{\text{test:l}}\}$
- Task: for $t = 1, \dots, T_{\text{test}}$, predict $\hat{z}_{\text{test:t}} \in S_{\text{train}}$ from $y_{\text{test:t}}$; try to minimize misclassification rate

Statistical formulation II

- f is an unknown function
- P is a known or unknown distribution over image features
- *Training data.* Draw $x_{\text{train}:i} \sim P$ for $i = 1 \text{ hdots}, k$.
- *Test data.* Draw $x_{\text{train}:i} \sim P$ for $i = 1 \text{ hdots}, \ell$.
- Theoretical question: Analyze average misclassification rate when classes are generated this way

Toy example I

Model.

- Features x are one-dimensional real numbers, as are responses y . Parameter β is also a real number.
- Suppose we estimated $\hat{\beta}$ from training data.

Test data.

- Generate features $x_{\text{test}:1}, \dots, x_{\text{test}:\ell}$ iid $N(0, \sigma_x^2)$.
- Hidden labels $z_{\text{test}:t}$ are iid uniform from S_{train} . Generate $y_{\text{test}:t} \sim N(\beta x_{z_{\text{test}:t}}, \sigma_\epsilon^2)$
- Estimate $\hat{z}_{\text{test}:t}$ by maximum likelihood assuming $\hat{\beta}$ is correct. Thus:

$$\hat{z}_{\text{test}:t} = \operatorname{argmin}_z (\hat{\beta} x_z - y_{\text{test}:t})^2$$