HW3: INTRODUCTION TO SETS

I know that the great Hilbert said 'We will not be driven out of the paradise Cantor has created for us,' and I reply 'I see no reason for walking in!' —Richard Hamming

Course: CS 5002
Fall 2021
Due: October 2, 2021

PROBLEMS

Problem 1: Set rewrites

Rewrite the following statements using set notation:

(a) (½ point) A is a subset of C. $A \subseteq C$

(b) (½ point) The element 3 is not a member of A. $3 \notin A$

(c) (½ point) F contains all the elements of G. $G \subseteq F$

(d) (½ point) A is not a subset of D. $A \nsubseteq D$

(e) (½ point) The element 42 is a member of B. $42 \in B$

(f) (½ point) S and T contain the same elements. $S = T$

(g) (½ point) Intersection of sets X and Y is an empty set. $X \cap Y = \emptyset$

(h) (½ point) Union of sets A and B is a subset of set C. $(A \cup B) \subseteq C$

(i) (½ point) Number 25 is a member of the interesection of sets M, N and P.
$25 \in (M \cap N \cap P)$

(j) (½ point) Symmetric difference of sets A and B is equal to a union of sets C and D.
$A \oplus B = C \cup D$

Problem 2: Set elements

Please list the elements of the following sets.

(a) (1 point) $A = \{x : x \in \mathbb{N}, 4 < x < 9\}$ 5,6,7,8

(b) (1 point) $B = \{x : x \in \mathbb{N}, x \text{ is odd}, x \leq 11\}$ 1,3,5,7,9,11

(c) (1 point) $C = \{x : x \in \mathbb{N}, 15 + x = 10\}$ $\emptyset$

(d) (1 point) $D = \{x : x \text{ is a vowel}, x \text{ is not "a" or "i"}\}$ "e", "o", "u"

(e) (1 point) $E = \{x : x \in \mathbb{N}, x \text{ is even}, x > 2 \text{ and } x \leq 5\}$ 4

Problem 3: Set equality

(a) (2 points) Which of these sets are equal? $\{d, e, b\}, \{b, d, e\}, \{e, d, b\}, \{b, d, e\}$?
{d, e, b} = {b, d, e} = {e, d, b} = {b, d, e}

(b) (3 points) Consider the following sets:

$$\{5, 15\}$$
$$\{x : x^2 + 2x + 4 = 0\}$$
$$\{x : x \in \mathbb{N}, x \text{ is odd}, 4 < x < 16, x \bmod 5 = 0\}$$

Which of them are equal to $B = \{15, 5\}$?

$\{5, 15\}, \{x : x \in N, x\ is\ odd, 4 < x < 16, x\ mod\ 5 = 0\}$

Problem 4: Set operations

The next question refers to the following sets:

$$A = \{1, 2, 3, 4\}$$
$$B = \{3, 5, 7, 9\}$$
$$C = \{2, 3, 7, 8\}$$
$$\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Please write the members of the resulting sets for the following set operations:

(a) (1 point) $A \cup B$

{1, 2, 3, 4, 5, 7, 9}

(b) (1 point) $A \cup C$

{1, 2, 3, 4, 7, 8}

(c) (1 point) $B \cup C$

{2, 3, 5, 7, 8, 9}

(d) (1 point) $B \cup B$

{3, 5, 7, 9}

(e) (1 point) $(A \cup B) \cup C$

{1, 2, 3, 4, 5, 7, 8, 9}

(f) (1 point) $A \cap C$

{2, 3}

(g) (1 point) A^c

{5, 6, 7, 8, 9}

(h) (1 point) $A \setminus B$

{1, 2, 4}

(i) (1 point) $A \cap (B \cup C)$

{2, 3}

(j) (1 point) $A \cup (B \cap C)$

{1, 2, 3, 4, 7}

Problem 5: Empty sets

The nex questions refer to the following sets:

$$X = \{x : x^2 = 9, 2x = 4\}$$
$$Y = \{x : x \neq x\}$$
$$Z = \{x : x + 8 = 8\}$$

(a) ($1/2$ point) Is X the empty set? .... Yes

(b) ($1/2$ point) Is Z the empty set? .... No

(c) (1 point) Is $Y \cap Z$ the empty set? .... Yes

(d) (1 point) Is $Y \cup Z$ the empty set? .... No

(e) (1 point) Is $X \cap Y \cup Z$ the empty set? .... No

(f) (1 point) Is $Z \cup Z$ the empty set? .... No

Problem 6: Sets and power sets

Consider the set:

$A = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7, 8\}\}$

Determine which of the following are true or false:

(a) (1 point) $\emptyset \in A$ .... True ....

(b) (1 point) $1 \in A$ .... False

(c) (1 point) $\{1, 23\} \subseteq A$ .... False

(d) (1 point) $\{6, 7, 8\} \in A$ .... True

(e) (1 point) $\{\{4, 5\}\} \subseteq A$ .... True

Problem 7: Cartesian product

Let $A = \{a, b, c\}, B = \{x, y\}$ and $C = \{0, 1\}$. Find the following Cartesian product: $A \times B \times C$

$A \times B \times C = \{a, b, c\} \times \{x, y\} \times \{0, 1\}$
$= \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

Problem 8: Cartesian product and air travel

Let A be the set of all airlines (a finite set), and let B and C both be the sets of all cities in the United States (also finite sets). Explain in words what is the Carteasian product $A \times B \times C$, and give one example of how this Cartesian product may be used.

A x B x C represents all the airlines that connect all the cities in pairs. This can be used as a preliminary model of a flight booking company. If there is an airline that connect the two cities, then the value of this choice can be true, otherwise false. If both cities in a pair are the same, then that is an automatic true.

Problem 9: Propositional and set formulas

(a) (5 points) Verify that the propositional formula $(P \wedge \neg Q) \vee (P \wedge Q)$ is equivalent to P.

(a)

$(P \wedge \neg Q) \vee (P \wedge Q)$

$= P \wedge (\neg Q \vee Q)$ *Distributive laws*

$= P \wedge (T)$ *Complement laws*

$= P$ *Identity laws*

Therefore, $(P \wedge \neg Q) \vee (P \wedge Q)$ is equivalent to P

(b) (10 points) Prove that $A = (A - B) \cup (A \cap B)$ for all sets, A, B by showing $x \in A$ iff $x \in (A - B) \cup (A \cap B)$ for all elements x using the equivalence of part (a) in a chain of IFF's.

(b)

Proof:

$A - B = A \cap B^C$

If $x \in (A - B) \cup (A \cap B) = (A \cap B^C) \cup (A \cap B)$.

This is same as saying that $(A \wedge \neg B) \vee (A \wedge B)$

From part (a), we have proved that $(A \wedge \neg B) \vee (A \wedge B) = A$

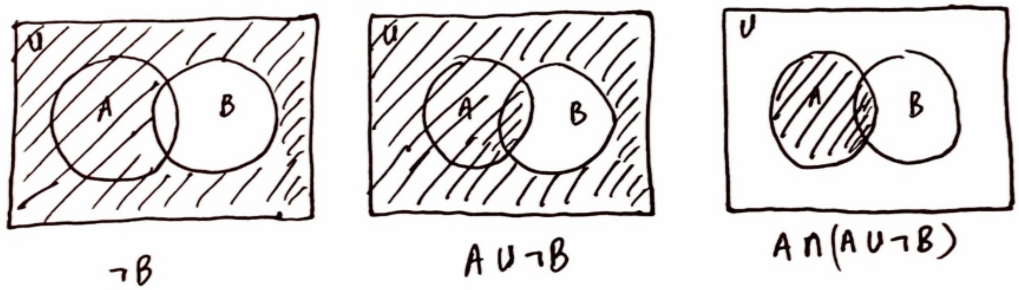
So $(A \cap B^C) \cup (A \cap B)$ is equivalent to A

Therefore, $x \in A$ iff $x \in (A - B) \cup (A \cap B)$. ■

Problem 10: Set formulas and Venn diagrams

(a) (5 points) Prove that $A = A \cap (A \cup \neg B)$ for all sets A and B using Venn diagrams.

Proof:



Therefore, $A = A \cap (A \cup \neg B)$. ■

(b) (5 points) Prove that $A = A \cap (A \cup \neg B)$ for all sets A and B using membership table.

Proof:
(b)

A	B	$\neg B$	$A \cup \neg B$	$A \cap (A \cup \neg B)$
1	1	0	1	1
1	0	1	1	1
0	1	0	0	0
0	0	1	1	0

Therefore, $A = A \cap (A \cup \neg B)$ ■

Problem 11: Sets and word problems

In a group of 100 persons, 72 people love sweets and 43 love grilled food. How many people love sweets only? How many people love grilled food only, and how many love sweets and grilled food?

U = 100
S = 72
G = 43
$S \cap G = (S + G) - U = 15$
$Only\ sweets = S - S \cap G = 72 - 15 = 57$
$Only\ grilled\ food = G - S \cap G = 43 - 15 = 28$

Problem 12: Sets and word problems

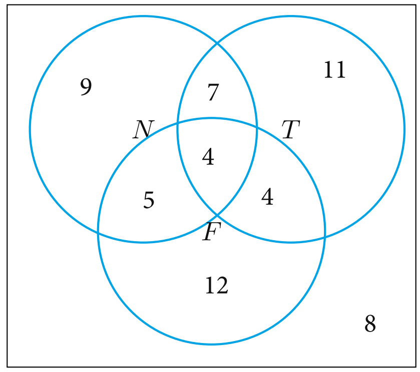
In a survey of 60 people, it was found that 25 read *Newsweek*, 26 read *Time*, and 26 read *Fortune*. Further, 9 read both *Newsweek* and *Fortune*; 11 read both *Newsweek* and *Time*; 8 read both *Time* and *Fortune*. 8 read no magazines at all.

(a) (10 points) Find the number of people who read all three magazines.

Newsweek: N	Time: T	Fortune: F	Set: U
n(N) = 25	n(T) = 26	n(F) = 26	n(U) = 60
$n(N \cap T) = 11$	$n(N \cap F) = 9$	$n(T \cap F) = 8$	$n(N \cup T \cup F)^C = 8$

(a)
$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F)$ (equation (1))
$n(N \cup T \cup F) = n(U) - n(N \cup T \cup F)^C = 60 - 8 = 52$
Plugging $n(N \cup T \cup F) = 52$ into equation (1) to get:
$52 = 25 + 26 + 26 - 11 - 9 - 8 + n(N \cap T \cap F) = 48 + n(N \cap T \cap F)$
Therefore, $n(N \cap T \cap F) = 52 - 49 = 3$

(b) (3 points) Fill in the correct number of people in each of the eight regions of the Venn diagram. N, F, and T denote *Newsweek*, *Fortune* and *Time* respectively.



(c) (5 points) Write the expression that indicates the set of people who read exactly one magazine.

(c)
n(Exactly one magazine)
$=(N \cap T^c \cap F^c) \cup (N^c \cap T \cap F^c) \cup (N^c \cap T^c \cap F)$
$= (n(N) - n(N \cap T) - n(N \cap F) + n(N \cap T \cap F)) + (n(T) - n(T \cap F) - n(N \cap T) + n(N \cap T \cap F))$
$+ (n(F) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F))$
$= n(N) + n(T) + n(F) - 2n(N \cap T) - 2n(N \cap F) - 2n(T \cap F) + 3n(N \cap T \cap F)$

(d) (2 points) Determine the number of people who read exactly one magazine.

(d)
n(Exactly one magazine)
$= 25 + 26 + 26 - 2(11) - 2(9) - 2(8) + 3(3)$
$= 30$

Question	Points	Score
Set rewrites	5	
Set elements	5	
Set equality	5	
Set operations	10	
Empty sets	5	
Sets and power sets	5	
Cartesian product	5	
Cartesian product and air travel	5	
Propositional and set formulas	15	
Set formulas and Venn diagrams	10	
Sets and word problems	10	
Sets and word problems	20	
Total:	100	

SUBMISSION DETAILS

Things to submit:

- Please submit this assignment as a .pdf named "CS5002_[lastname]_HW3.pdf" through Canvas by 11:59pm PST on Saturday, October 2, 2021.
- Please make sure your name is in the document as well (e.g., written on the top of the first page).