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HW1: PROPOSITIONAL LOGIC AND BOOLEAN ALGEBRA

Contrariwise, if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic. —Lewis Carroll

Course: CS 5002
Fall 2021
Due: Sunday, September 19, 2021

PROPOSITIONAL LOGIC PROBLEMS

Problem 1: Which is a proposition?

Which of the following are propositions? If the statement is a proposition, determine its truth value.

(a) What day is it? (1)

(a) no

(b) Number 10 is an even number. (1)

(b) yes, T

(c) It is sunny every day in Hawaii. (1)

(c) yes, F

(d) Have you been to a Kraken game yet? (1)

(d) no

(e) I order you to go on a hike this weekend! (1)

(e) no

Problem 2: Logic statement from sentence

Let p be "The instructor is tired" and let q be "The students is having fun".

Write a logic statement that describes the following sentences.

(a) The instructor is tired and the students are having fun. (1)

(a) $p \wedge q$

(b) The students are not having fun and the instructor is tired. (1)

(b) $\neg q \wedge p$

(c) The instructor is either tired, or the students are having fun, or both. (1)

(c) $p \vee q$

(d) Either the students are having fun or the instructor is tired, but the students are not having fun if the instructor is tired. (1)

(d) $p \oplus q$

Points: ______ out of 9

(e) That the instructor is not tired is necessary and sufficient for the students to have fun. (1)

(e) $\neg p \leftrightarrow q$

Problem 3: Truth values

The *truth value* of a compound statement (that is, a statement composed of multiple statements) is determined by the truth values of the individual statements, together with how the indivdual statements are combined to form the compound statement.

Determine the truth value for each of the following compound statements.

(a) *Paris is in France* and $2 + 2 = 4$. (1)

(a) T

(b) *Paris is in France* and $2 + 2 = 5$. (1)

(b) F

(c) *Paris is in England* or $2 + 2 = 4$. (1)

(c) T

(d) *Paris is in England* implies that $2 + 2 = 5$. (1)

(d) T

Problem 4: Truth tables

For each of the expressions below, provide a truth table.

(a) $p \to \neg p$ (5)

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

(b) $p \oplus (p \vee q)$ (3)

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

(c) $p \leftrightarrow \neg p$ (3)

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

(d) $(p \vee q) \wedge \neg r$ (4)

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

Problem 5: Logically equivalent

Use truth tables to show that $(p \wedge q) \vee \neg(p \wedge \neg q)$ is logically equivalent to $\neg(p \wedge \neg q)$

p	q	$p \wedge q$	$\neg(p \wedge \neg q)$	$(p \wedge q) \vee \neg(p \wedge \neg q)$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

p	q	$(p \wedge \neg q)$	$\neg(p \wedge \neg q)$
T	T	F	T
T	F	T	F
F	T	F	T
F	F	F	T

so $(p \wedge q) \vee \neg(p \wedge \neg q) \equiv \neg(p \wedge \neg q)$

Final truth table:

p	q	$(p \wedge q) \vee \neg(p \wedge \neg q)$	$\neg(p \wedge \neg q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Problem 6: Logically equivalent

Use truth tables to determine if the following statements are logically equivalent.

$$(p \vee q) \wedge r \;??\; p \vee (q \wedge r)$$

p	q	r	(p∨q)	(p∨q)∧r
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

p	q	r	(q∧r)	p∨(q∧r)
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

not equivalent

Final truth table:

p	q	r	$(p \vee q) \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F
T	T	F	F	T
T	F	F	F	T
F	T	F	F	F
F	F	F	F	F

Problem 7: Quantifiers

Determine the truth value for each of the following statements, when the universe of discourse is $\Re$, the set of real numbers.

(a) $\forall x, |x| = x$ (1)

(a) F

(b) $\exists x, x^2 = x.$ (1)

(b) T

(c) $\forall x, x + 1 > x$ (1)

(c) T

(d) $\exists x, x + 2 = x$ (1)

(d) F

(e) $\exists x(x^4 < x^2)$ (1)

(e) T

(f) $\forall x(2x > x)$ (1)

(f) F

(g) $\exists x, 2x > -x$ (1)

(g) T

(h) $\forall x, x + 2 = x$ (1)

(h) F

(i) $\forall x(x^3 > x + 2)$ (1)

(i) F

(j) $\exists x) + 7 = 5$ (1)

$\exists x,\ x + 7 = 5$

(j) T

Problem 8: Quantifiers in English

Let $W(x)$ be the statement "x has visited Utah's Mighty Five". The domain (or, *universe of discourse*) consists of the students enrolled at Northeastern-Seattle. Express these quantifications in English.

(a) $\exists x W(x)$ (1)

(a) Some students have visited Utah's Mighty Five.

(b) $\forall x W(x)$ (1)

(b) Every student has visited Utah's Mighty Five.

(c) $\neg\exists x W(x)$ (1)

(c) No student has visited Utah's Mighty Five.

(d) $\exists x \neg W(x)$ (1)

(d) Some students have not visited Utah's Mighty Five

(e) $\forall x \neg W(x)$ (1)

(e) No student has visited Utah's Mighty Five.

(f) $\forall \neg x W(x)$ (1)

(f) No translation for this statement because only proposition can be negated, not variable.

BOOLEAN ALGEBRA PROBLEMS

Problem 9: Evaluating Boolean expressions

Consider the following Boolean variables, with assigned values:

- $x = 0$
- $y = 1$
- $z = 1$

Please give the value for the Boolean expressions below:

(a) $x + \overline{y} + \overline{z}$ (3)

$\overline{y} = 0$

$\overline{z} = 0$

$x + \overline{y} + \overline{z} = 0 + 0 + 0 = 0$

(b) $x \cdot y + \overline{z}$ (3)

$\bar{z} = 0$

$x \cdot y = 0 \cdot 1 = 0$

$x \cdot y + \bar{z} = 0 + 0 = 0$

(c) $z(x + \overline{y})$ (3)

$\bar{y} = 0$

$x + \bar{y} = 0 + 0 = 0$

$z(x + \bar{y}) = 1 \cdot 0 = 0$

(d) $x \cdot y \cdot \overline{z}$ (3)

$\bar{z} = 0$

$x \cdot y \cdot \bar{z} = 0 \cdot 1 \cdot 0 = 0 \cdot 0 = 0$

(e) $\overline{x}(y + \overline{z})$ (3)

$\bar{x} = 1$ $\quad$ $y + \bar{z} = 1$

$\bar{z} = 0$ $\quad$ $\bar{x}(y + \bar{z}) = 1 \cdot 1 = 1$

Problem 10: Boolean identities

Prove the following Boolean identities:

(a) $\overline{(a \cdot b)} = \overline{a} + \overline{b}$ (5)

a	b	$(a \cdot b)$	$\overline{(a \cdot b)}$	$\bar{a}$	$\bar{b}$	$\bar{a} + \bar{b}$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

so $\overline{(a \cdot b)} = \bar{a} + \bar{b}$

(b) $a + (a \cdot b) = a$ (5)

a	b	$(a \cdot b)$	$a + (a \cdot b)$
1	1	1	1
1	0	0	1
0	1	0	0
0	0	0	0

So $a + (a \cdot b) = a$

Problem 11: Simplifying Boolean expressions

As much as possible, simplify the following Boolean expression:

$$[\overline{a} \cdot \overline{b} \cdot c] + [a \cdot \overline{b} \cdot \overline{c}] + [\overline{a} \cdot b \cdot \overline{c}] + [\overline{a} + \overline{b} + \overline{c}]$$

$= [\bar{a} \cdot \bar{b} \cdot c] + \bar{a} + [a \cdot \bar{b} \cdot \bar{c}] + \bar{b} + [\bar{a} \cdot b \cdot \bar{c}] + \bar{c}$ commutative law

$= \bar{a} \cdot [\bar{b} \cdot c + 1] + \bar{b} \cdot [a \cdot \bar{c} + 1] + \bar{c} \cdot [\bar{a} \cdot b + 1]$ distributive law

$= \bar{a} \cdot 1 + \bar{b} \cdot 1 + \bar{c} \cdot 1$ domination law

$= \bar{a} + \bar{b} + \bar{c}$ identity law

$= \overline{abc}$ De Morgan's law

Question	Points	Score
Which is a proposition?	5	
Logic statement from sentence	5	
Truth values	4	
Truth tables	15	
Logically equivalent	10	
Logically equivalent	10	
Quantifiers	10	
Quantifiers in English	6	
Evaluating Boolean expressions	15	
Boolean identities	10	
Simplifying Boolean expressions	10	
Total:	100	

SUBMISSION DETAILS

Submit your answers to the math problems on Cavas as a pdf. You can do this by printing out this assignment, hand-writing your answers, and then scanning in, or by using a program such as Preview on the Mac to annotate the pdf.
Things to submit:

- Submit the following on Canvas for HW1:
  - The written parts of this assignment as a .pdf named "CS5002_[lastname]_HW1.pdf". For example, my file would be named "CS5002_Bonaci_HW1.pdf". (There should be no brackets around your name).