Homework 6 Solutions

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Problem 1

The *normal equation* for that least square problem is

$$(A^*A + \lambda I)x = A^*b.$$

To solve this, we can

1. compute $A^*A + \lambda I$, which is a bidiagonal matrix,
2. use n Givens rotations to reduce $A^*A + \lambda I$ to a upper triangular matrix R,
3. Apply same Givens rotations to A^*b to get c, and solve upper triangular linear equation $Rx = c$.

Problem 2

By Gauss elimination (may with column pivoting), reduce $[C|d]$ into row echelon form

$$\begin{bmatrix} R & C' & | & d' \end{bmatrix},$$

in which R is a invertible upper triangular matrix.
Partition x into $[x_1|x_2]^T$ corresponding to $[R \quad C']$, then we have

$$x_1 = R^{-1}(d - C'x_2).$$

Partition A into $[A_1|A_2]$ and substitute equation above into $||Ax - b||$, we have a new least square problem with lower size

$$min||(A_2 - A_1R^{-1}C')x_2 - (b - A_1R^{-1}d)||_2.$$

Problem 3

Suppose A has size $m \times n$, C has size $p \times n$ $(p < m)$.Define the objective function:

$$L(x, \lambda) = \frac{1}{2}||Ax - b||_2^2 + \lambda^*(d - Cx), \quad \lambda \in \mathbb{C}^p,$$

and set

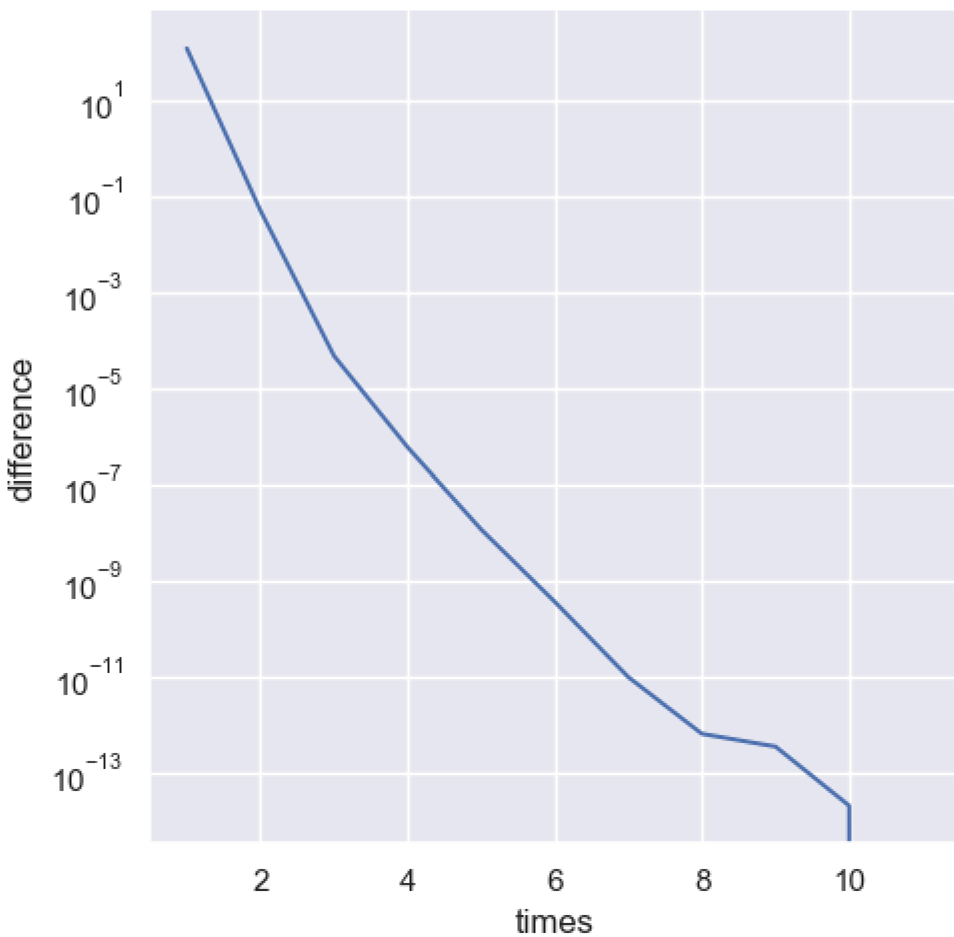
$$L_x(x, \lambda) = A^*(Ax - b) - C^*\lambda = 0.$$

Then we can construct the augmented system

$$\begin{bmatrix} I & A & O \\ A^* & O & C^* \\ O & C & O \end{bmatrix} \begin{bmatrix} r \\ x \\ \lambda \end{bmatrix} = \begin{bmatrix} b \\ O \\ d \end{bmatrix}.$$

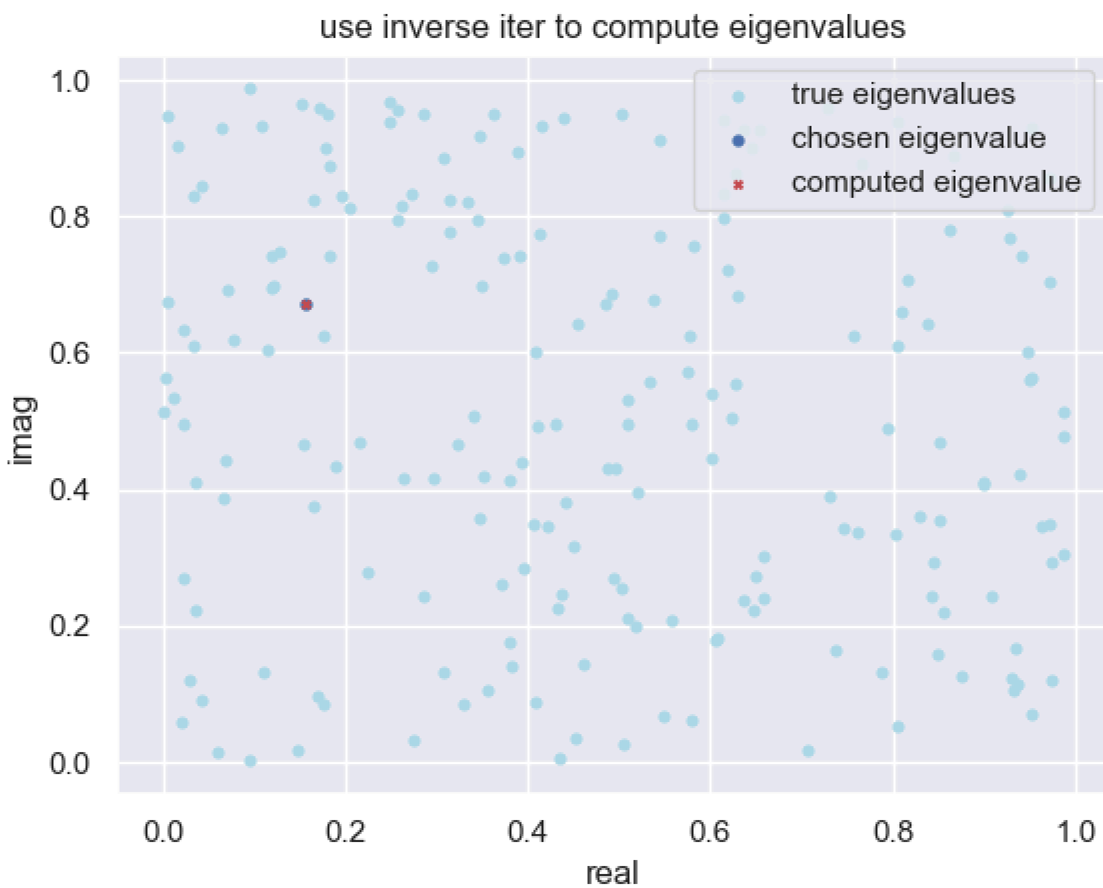
Problem 4

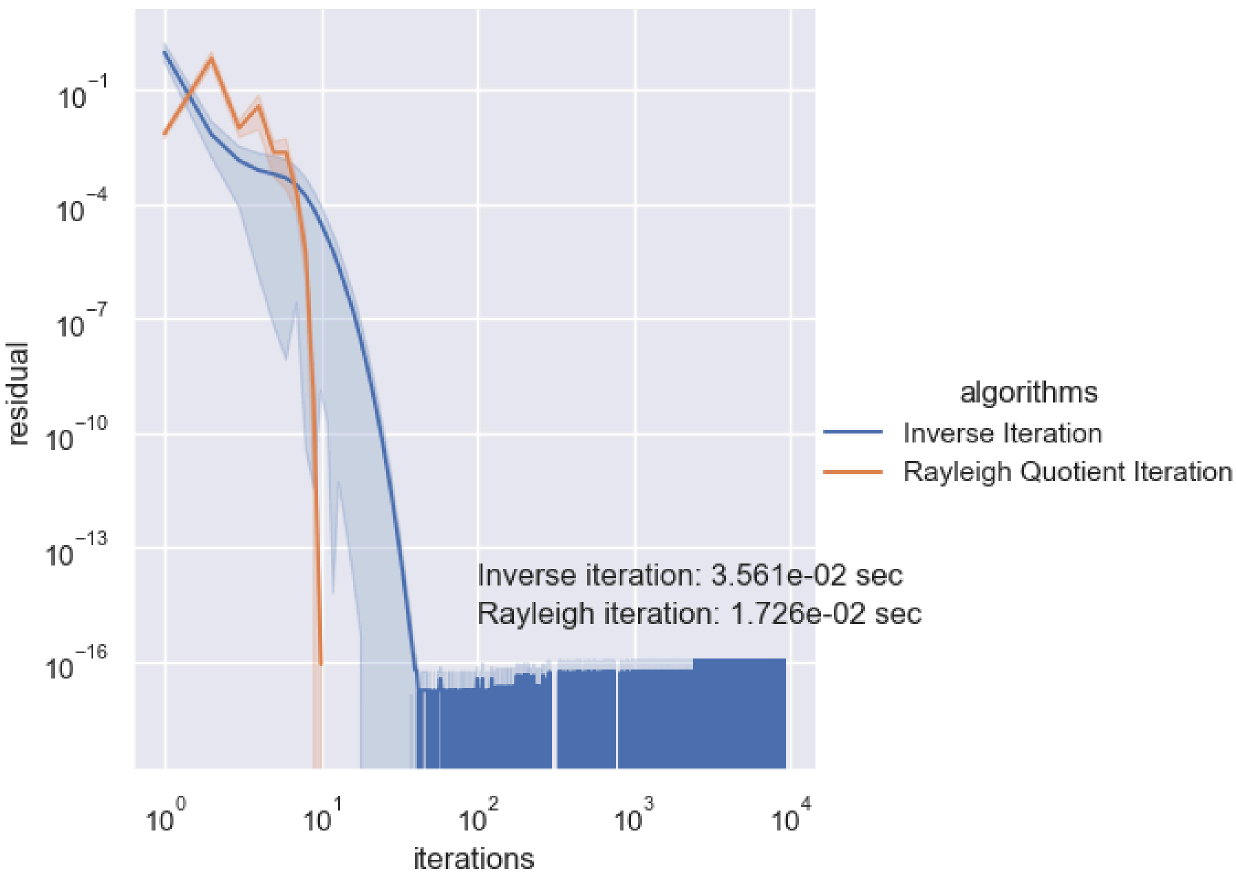
see `powiter.py`.



Problem 5

see `powiter.py`





residual
iterations
algorithms
Inverse Iteration
Rayleigh Quotient Iteration
Inverse iteration: 3.561e-02 sec
Rayleigh iteration: 1.726e-02 sec