Homework 10 Solutions

Weihao Li
Fudan University

Problem 1

This is equivalent to solve a least square problem

$$min_{\lambda}||\lambda^2 Mx + \lambda Cx + Kx||_2.$$

Hence we have the orthogonal condition

$$(2\lambda Mx + Cx)^*(\lambda^2 Mx + \lambda Cx + Kx) = 0,$$

which is a scalar cubic equation of λ. Then we can find three approximate $\hat{\lambda}$, and choose one that has the smallest $||\lambda^2 Mx + \lambda Cx + Kx||_2$.

Problem 2

Let $B = LL^T$ be the cholesky decomposition of B.
$AB = ALL^T$ is similar to $L^T AL$, which is a real symmetric matrix, hence diagonalizable.
To find all the eigenvalue and eigenvector of AB, just compute $B = LL^T$ first and find the all the eigenvalue and eigenvector of real symmetric matrix $L^T AL$ (using symmetric QR, Jacobi, etc.).

Problem 3

see `prob03.py`.
(The "optimum choices" of some norm of A is from the table 1 in [1])

Problem 4

Since Hermitian matrices that commute are also simultaneously diagonalizable, there is some unitary U that

$$U^*AU = D_A, U^*EU = D_E,$$

in which we denote $D_A = diag(a_1, a_2, \dots, a_n), D_E = diag(e_1, e_2, \dots, e_n)$. Then

$$\begin{aligned} ||exp(A+E) - exp(A)||_2 &= ||\exp(D_A + D_E) - exp(D_A)||_2 \\ &= max_{1\le i\le n}|\exp(a_i)(exp(e_i) - 1)| \\ &\le max_{1\le i\le n}|\exp(a_i)|max_{1\le i\le n}|(exp(e_i) - 1)| \\ &= \exp(||A||_2)\exp(||E||_2 - 1). \end{aligned}$$

Problem 5

see `prob05.py` and `prob05output.xlsx`.

[1]C. B. Moler and C. F. Van Loan, *Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later*, SIAM Rev., 45 (2003), pp. 3–49.