Oct 31, 2023 (Due: 08:00 Nov 7, 2023)

1. Let (λ, x) be a normalized eigenpair of a Hermitian matrix A. Suppose that $\hat{x}$ is an approximate eigenvector satisfying $\|\hat{x} - x\|_2 = O(\epsilon)$. Show that

$$\frac{\hat{x}^* A \hat{x}}{\hat{x}^* \hat{x}} - \lambda = O(\epsilon^2).$$

2. Given x, $y \in \mathbb{R}^n$. Describe in detail how to construct a rotation matrix Q such that the columns of $[x, y]Q$ are orthogonal to each other.

3. Implement the Jacobi diagonalization algorithm for real symmetric matrices. Visualize the performance for a few matrices with different sizes. Visualize the convergence history for one example. You are encouraged to make observations on *entry-wise* convergence.

You are also encouraged to try the "wrong" choice of Jacobi rotations for the cyclic Jacobi algorithm.

4. Have a quick glance at the paper "From Random Polygon to Ellipse: An Eigenanalysis" by A. N. Elmachtoub and C. F. Van Loan (available on eLearning). Reproduce the experiments in this paper.

5. (optional) When bidiagonalizing an $m \times n$ matrix with $m > n$, there are two common options: bidiagonalization after QR factorization vs. direct bidiagonalization. Suppose that both left and right orthogonal transformations need to be accumulated. Calculate the cost in terms of number of floating-point operations for these options, and determine the crossover point.

Will the crossover point change if orthogonal transformations are not accumulated?