Nov 14, 2023 (Due: 08:00 Nov 21, 2023)

1. Suppose you are given an approximate eigenvector $\hat{x}$ of the quadratic eigenvalue problem

$$(\lambda^2 M + \lambda C + K)x = 0.$$

How to extract the corresponding approximate eigenvalues?

2. Let A and B be $n \times n$ real symmetric matrices. Suppose that B is positive definite. Show that AB is diagonalizable.

Design an algorithm to compute all eigenvalues and eigenvectors of AB.

3. Implement the scaling-and-squaring algorithm (combined with truncated Taylor series) for computing the matrix exponential. Test the accuracy of your algorithm by a few diagonalizable matrices with known spectral decomposition.

(optional) Implement the Schur–Parlett algorithm and compare the accuracy.

4. Let A and E be Hermitian matrices with $AE = EA$. Try to give an upper bound on

$$\|\exp(A+E) - \exp(A)\|_2.$$

Make sure your upper bound tends to zero when $\|E\|_2 \to 0$.

5. Implement a random projection algorithm to compute the full rank decomposition of a low rank matrix with known rank. Test your implementation with a few low rank matrices.

6. (optional) In the lecture we have discussed how to solve Sylvester matrix equation $AX - XB = C$ using the Bartels–Stewart algorithm (through Schur decompositions). What happens the matrices are real and if only real Schur decompositions are permitted?

7. (optional) Use truncated SVD to compress some grayscale images. If you only have colored images, you can convert them to grayscale using

$$\text{gray} = \alpha \cdot \text{red} + \beta \cdot \text{green} + \gamma \cdot \text{blue}.$$

Common choices of the constants are $(\alpha, \beta, \gamma) = (0.299, 0.587, 0.114)$ and $(\alpha, \beta, \gamma) = (0.2126, 0.7152, 0.0722)$.

You are also encouraged to think about how to compress colored images.