Sep 5, 2023 (Due: 08:00 Sep 12, 2023)

1. Let $\hat{x}$ be an approximation to x. In practice it is often much easier to estimate $\tilde{E}_{\text{rel}}(\hat{x}) = |x - \hat{x}|/|\hat{x}|$ compared to $E_{\text{rel}}(\hat{x}) = |x - \hat{x}|/|x|$. What is the relationship between E_{rel} and $\tilde{E}_{\text{rel}}$?

2. How to evaluate $f(x) = \tan x - \sin x$ for $x \approx 0$ such that numerical cancellation is avoided?

3. You are given $A \in \mathbb{R}^{m\times n}$ and $x \in \mathbb{R}^n$, both already stored in floating-point format. Show that there exists a "small" matrix $E \in \mathbb{R}^{m\times n}$ such that $\text{fl}(Ax) = (A + E)x$. Try to bound the entries of E as tight as you can. You may assume that there is no overflow or (gradual) underflow in the calculation.

4. Let $L \in \mathbb{R}^{n\times n}$ be lower triangular and nonsingular. Provide two different implementations for solving the linear system $Lx = b$, where $b \in \mathbb{R}^n$ is a given vector.

5. Implement Gaussian elimination for solving nonsingular linear systems. You may assume that no divide-by-zero error is encountered. Measure the execution time of your program in terms of matrix dimensions and visualize the result by a log–log scale plot. (You may generate your test matrices with with normally distributed random elements.)

6. (optional) Analyze the rounding error for complex arithmetic. You may assume that there is no overflow or (gradual) underflow in the calculation.