Homework 5 Solutions

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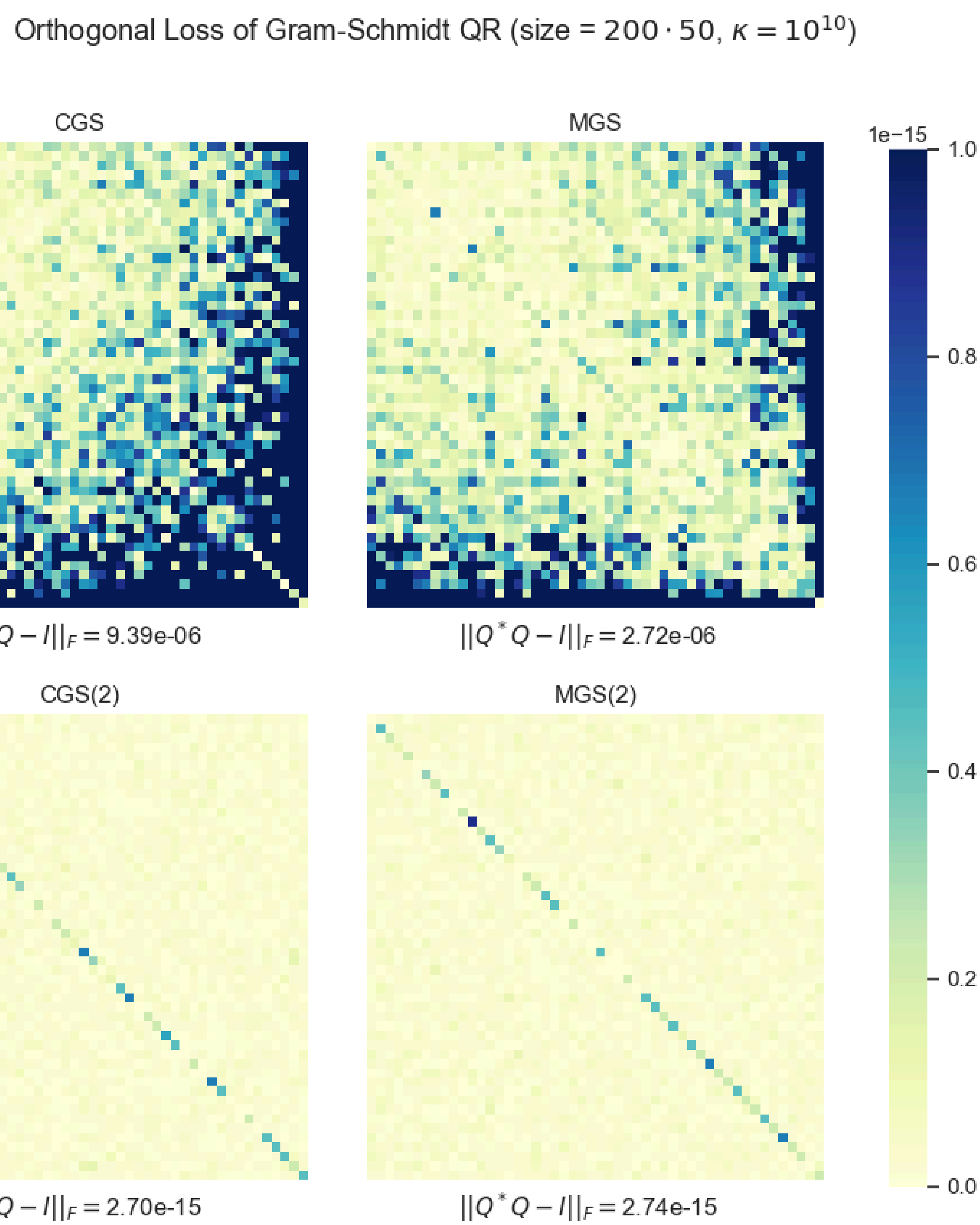
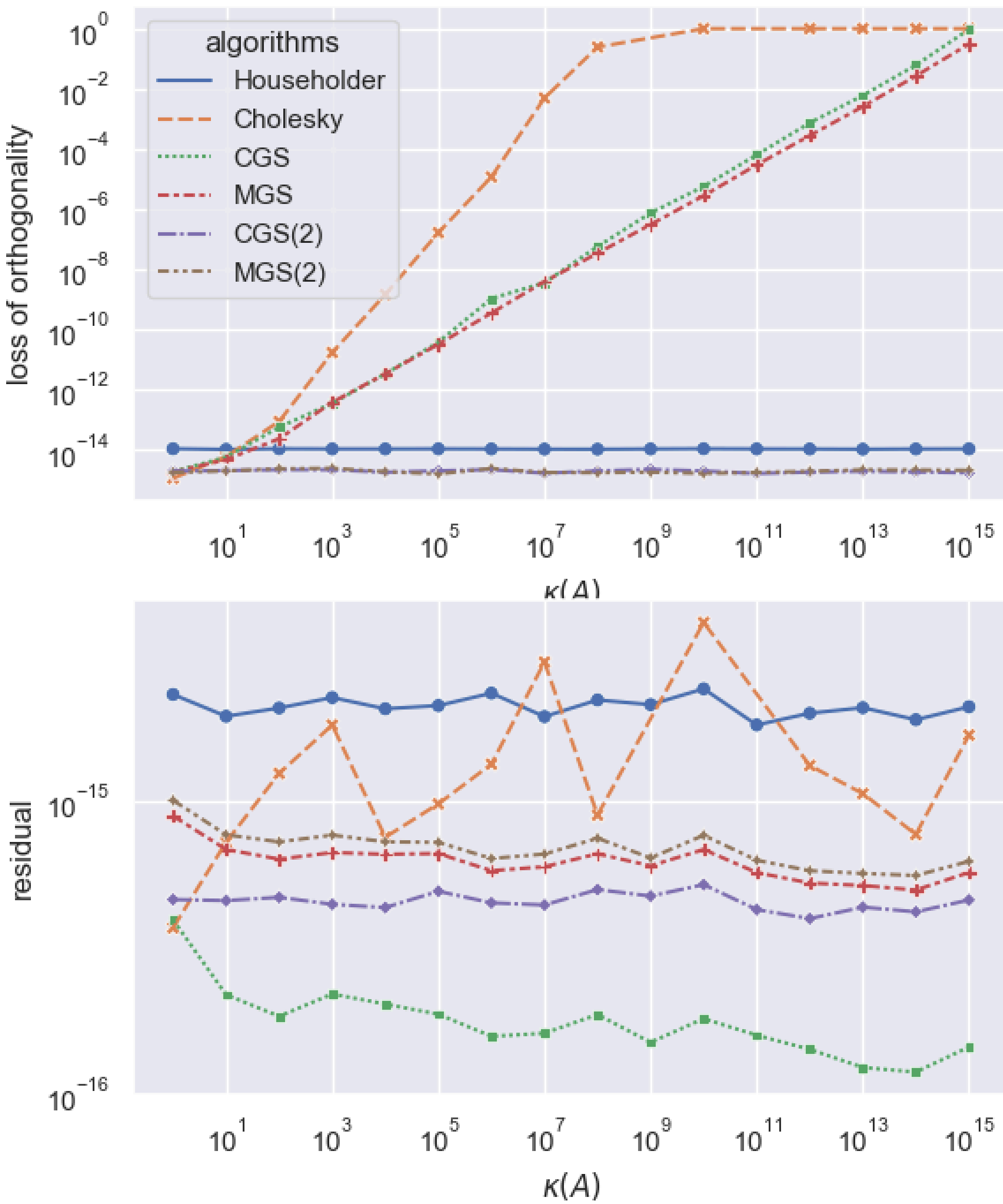
Problem 1

See `qrdecomp.py`.



Problem 2

See `qrdecomp.py`.



Problem 3

For all minimizer x of the least square problem $min||Ax - b||_2$, x must satisfy

$$A^*Ax = A^*b.$$

Hence for any $b \in \mathbb{C}^m$,

$$A^*AXb = A^*b.$$

Let b run through standard basis in $\mathbb{C}^m$, we see that every column of A^*AX and A^* is the same, hence

$$A^*AX = A^*, \tag{3.1}$$

then we also have

$$\begin{aligned} (AX)^*A &= A, \\ X^*A^*AX &= X^*A^*, \end{aligned}$$

combine results above, we have

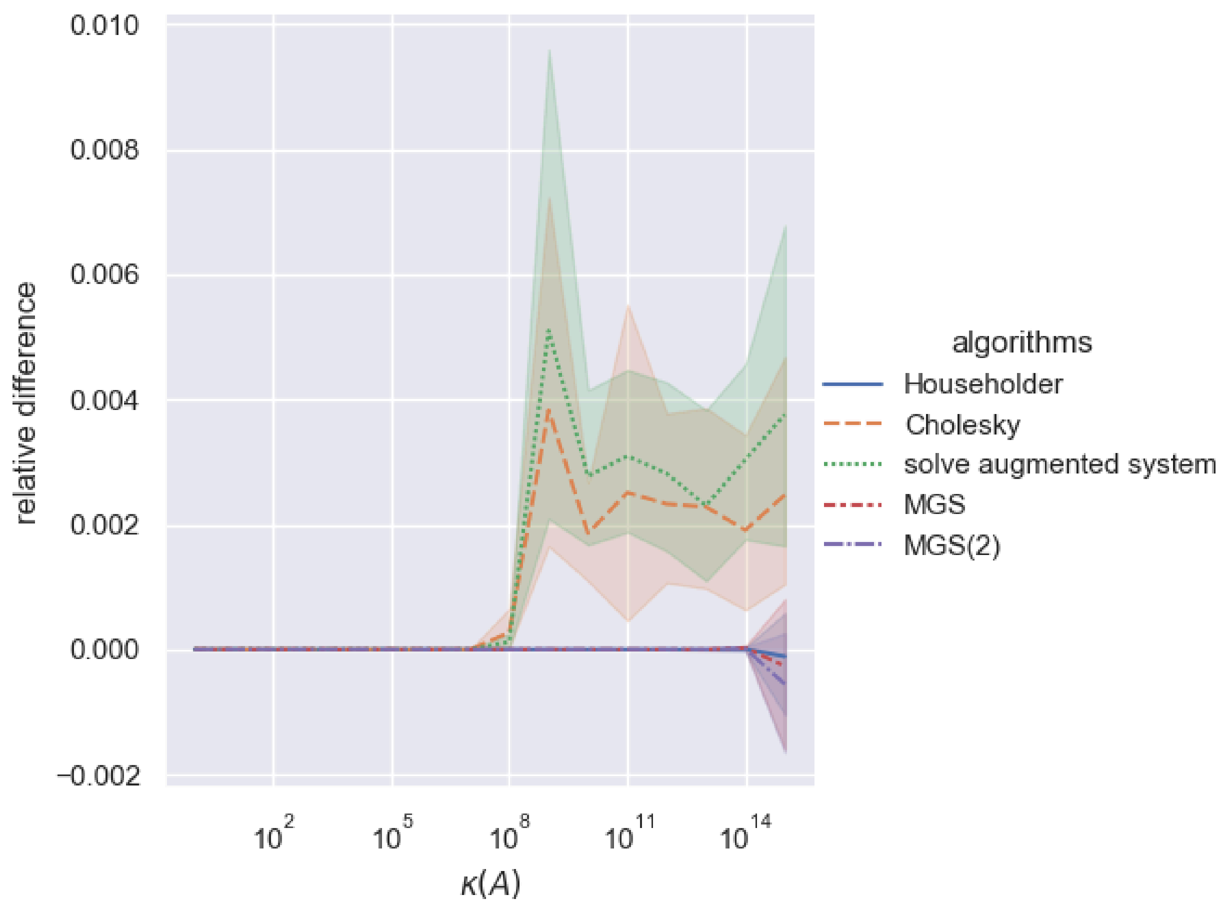
$$AX = (AX)^*AX = X^*A^*AX = X^*A^* = (AX)^*,$$

and

$$AXA = (AX)^*A = A. \qquad \square$$

Problem 4

See `lstsq.py`.
For a residual r from given solution, the "relative difference" means $\frac{r-r_0}{r_0}$, in which r_0 is the residual from the solution given by `np.linalg.lstsq`.



Problem 5

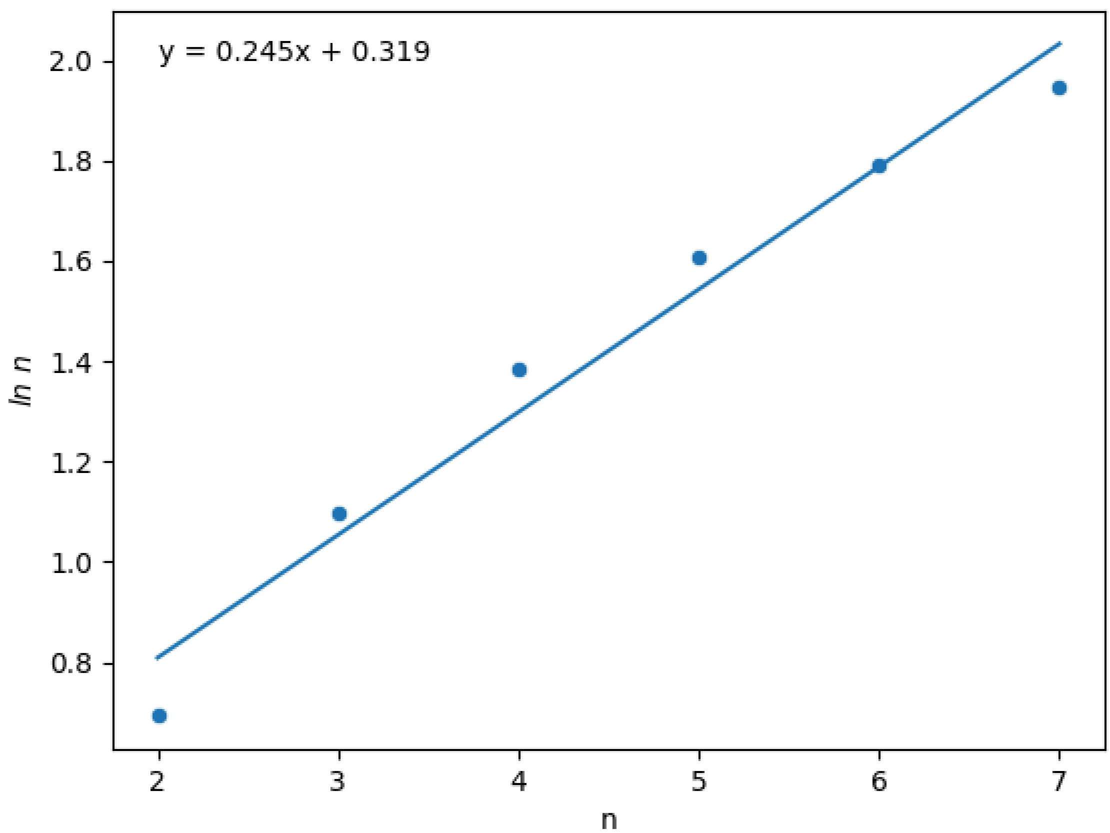
Let the approximate straight line be $\hat{y} = a_1 x + a_2$, and we use

$$min \sum_{i=1}^{6} (y - \hat{y})^2$$

to determine whether the solution is best. This can be easily transformed in to a least square problem $min||Ax - b||_2$, in which

$$A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \\ x_5 & 1 \\ x_6 & 1 \end{bmatrix}, b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}.$$

For the solution and visualization, see `lstsq.py`.


y = 0.245x + 0.319
2.0
1.8
1.6
1.4
1.2
1.0
0.8
ln n
2
3
4
5
6
7
n