Determining the coefficient of convective heat transfer

Instructor: Mah Noor Jamil[†]

[†]*Integrated Sciences and Mathematics, Dhanani School of Science and Engineering, Habib University (mahnoor.jamil@sse.habib.edu.pk)*

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1 Objective

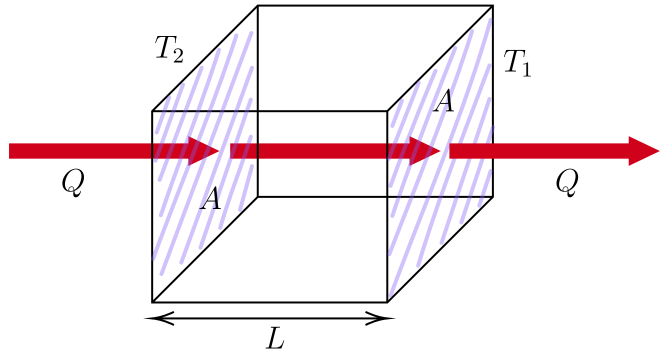
In this experiment we will measure the change of temperature of a body as heat flows from it through the surrounding through convective losses and thus determine the coefficient of convective heat transfer from the given data. The data is collected by first heating two metallic cylinders up to a certain temperature and then noting the time taken by them to return to the atmospheric temperature. The temperatures are measured using thermocouples and CASSY.

2 Theoretical Background

2.1 Thermal Conduction

Conduction is the process whereby heat energy is transferred through a medium from a region of high temperature to low temperature with the help of collisions between neighboring particles. Just like how there are good and bad conductors of electricity, we also have good and bad conductors of heat. They can be differentiated based on how fast or slow do they conduct heat. In fact, we can quantify this through an equation for the rate of conductive heat transfer through a medium.

For that, consider the following slab of copper metal. One side of it is heated to a temperature T_2, while the other side is maintained at a lower temperature T_1. In accordance with the second law of thermodynamics, heat energy must flow from the hotter side towards the colder side. Suppose this heat energy Q flows perpendicularly through both sides as shown below:



Here, A is the area of the surfaces with temperatures T_1 and T_2, and L is the thickness of the slab itself.

The equation for the rate of conductive heat transfer P is:

$$P \equiv \frac{dQ}{dt} = -kA\frac{T_2 - T_1}{L},$$

where k is the thermal conductivity of the material, which is a measure of the ability of a material to conduct heat.

2.2 Thermal Convection

Convection is the process in which heat is transferred by the movement of particles from one region to another within a fluid. This movement occurs because of a difference in densities within a fluid, which can arise when there are stark temperature differences within sections of the fluid. For instance, heating up the base of a beaker of water will cause water molecules nearest to the base to heat up, expand and rise due to their lower density. In doing so, they push the surrounding denser - and colder - water molecules towards the heat source, which get heated up next. This cycle of sorts continues, causing there to be bulk movement of a stream of warmer water with lower density around the entire liquid, until the entire beaker of water has the same temperature.

2.3 Specific Heat Capacity

Specific heat capacity is the amount of heat energy required to raise the temperature of a mass of 1 kg by $1°C$ or 1 K. It is a constant for materials, and is given as:

$$c = \frac{Q}{m\Delta T},$$

where Q is the heat energy transferred to or from the substance, m is its mass, and ΔT is the temperature difference.

Rearranging this equation gets us the the following expression:

$$Q = mc\Delta T.$$

If heat is being lost by the object with time then one can calculate the rate of energy lost as a function of time by taking the time derivative of the above formula to get the following:

$$\frac{dQ}{dt} = -mc\frac{dT}{dt}.$$

Here, notice that m and c do not appear within the time derivative because they can be treated as constants. Furthermore, a negative sign implies that heat is being lost by the object; i.e. its temperature is dropping.

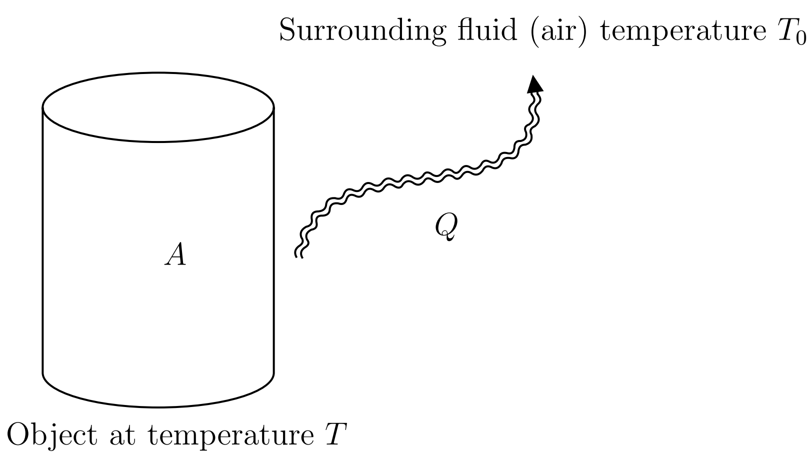
2.4 Newton's Law of Cooling

Newton's law of cooling states that the rate at which an object cools is directly proportional to the difference in temperature between the object and the object's surroundings. It predominantly focuses on convection as the main mode of heat transfer. This makes sense because heat energy must be carried away from the surface to the surroundings with the help of convection through air. Mathematically it is given as:

$$\frac{dQ_{\text{conv}}}{dt} = hA(T - T_0),$$

where h is the coefficient of convective heat transfer (i.e. the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference), A is the area of the surface through which heat energy is being lost, and T and T_0 are the temperatures of the object and the surrounding environment's respectively.

In your Lab Notebook: Indicate the SI units of each term in the above equation.



2.5 Mathematical Modelling

Suppose amount Q heat is provided to cylindrical rod of a certain material to raise its temperature by an amount $\Delta T = T - T_0$. When the rod cools down through convection in air currents the same amount of energy is lost with time to convection. Hence, the rate of heat lost must be the same.

In your Lab Notebook: Use the principle of conversation of energy to arrive at the following differential equation:

$$\frac{1}{(T - T_0)} dT = -\frac{hA}{mc} dt.$$

This is a first-order separable differential equation and can be solved to arrive at an equation of the temperature difference as a function of time $X(t)$:

$$X(t) = X_0 \exp\left(\frac{hAt}{mc}\right),$$

where $X(t) = T(t) - T_0$ and X_0 is the initial temperature difference at $t = 0$.

In your Lab Notebook for BONUS MARKS: Try deriving the expression for $X(t)$ yourself by solving this differential equation.

3 Procedure and Apparatus

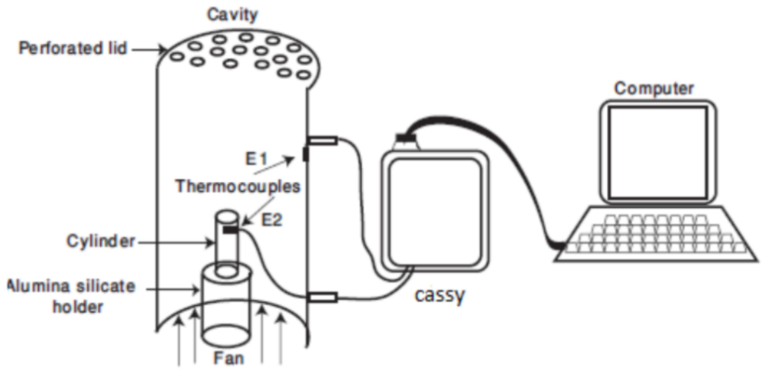
3.1 Equipment

Equipment	Use
2 metal cylinders	The body under observation ($c = 510$ J/kg °C)
Cavity	To provide a closed environment to the cylinder
Hot plate	To heat the cylinders
2 Thermocouples and CASSY	To observe temperature change inside the cavity

3.2 Experimental Set-up

Note: Please read all the instructions before starting the experiment. Please ask the instructor if the following explanation is unclear before performing the experiment.

A schematic of the experimental set-up is shown below:



3.3 Safety Precautions

- DO NOT touch the surface of the cylinder and hot plate with your bare hands. These surfaces are extremely hot!
- Handle the cylinder with tongs and thermal gloves.

3.4 Procedure

1. Measure the mass m, diameter d and length L of the cylinder. Note these values down in your lab notebook.
2. Place both cylinders in the steel container, and cover completely with graphite powder. Avoid spillage, as this needs to be reused.
3. Carefully place the container on the hot plate, and set its temperature to 350°C. Heat it for 45 min.
4. In the meantime,
    - Set up your CASSY sensor. For that, you have to connect the following:
        - **Thermocouple 1 to "U" in Input B**. This will measure the temperature of the air surrounding the cylinder. Insert the thermocouple's exposed wires inside the cavity as shown in the diagram above.
        - **Thermocouple 2 to "U" in Input A**. This will measure the temperature of the cylinder by using the clasp.

      Thermocouples give out voltage readings U, which can then be converted to temperature T through an equation that's decided when it is calibrated. Fortunately, the thermocouples provided are already calibrated and the relationship between T and U is the following:

      $$T(U) = 25097U + 18.844.$$

    - Locate a file on your desktop called Newton_Law_Cooling. Double-clicking it will open up CASSY Lab 2. Ensure that you can display the following windows: U_A, U_B, T, T_0 and X.

  - In “Settings”, click on “Standard”, and then $T(t)$. Change $T(t)$ to X so that X can be displayed on the y axis. Keep time t on the x axis.
  - If there is an old graph in display, click on “New” to remove it. This should not get rid of the windows you’ve opened.

5. Once 45 min have passed, switch the hot plate off. Wear thermal gloves before opening the lid of the container. Use tongs to grab one cylinder from its end. **Keep the other cylinder covered with graphite so that it stays warm.** Clamp Thermocouple 2 on its other end and place it inside the cavity.

6. Make sure the cavity’s fan is OFF before placing the lid on top.

7. Start measurement on CASSY for up to 600 s. Your graph should have a roughly exponentially decaying shape. **Ask RA for help if needed!**

8. Once completed, save the data of X and t as a Text File (.txt).

9. Repeat steps 5 to 7 for the second cylinder, except this time switch the cavity’s fan ON.

4 Evaluation and Analysis of Results

By the end of this experiment, you should have two text files: one for FAN OFF and the other for FAN ON. Open each file on MATLAB, and use its Curve Fitting Tool to fit the following equation:

$$X(t) = X_0 \exp\left(-\kappa t\right).$$

In your Lab Notebook: Conduct the following steps:

- **Print out both graphs and paste them in your lab notebook. Both graphs should have properly labeled axes and correct titles.**
- **Use the Curve Fitting Tool to find the values of X_0 and κ for each graph.**
- **Use the values of diameter d and length L to calculate the curved surface area A of each cylinder.**
- **With the help of A, m and c, calculate values of h for both experiments (with fan OFF and fan ON).**

Finally, complete the following analyses in your Lab Notebook:

- **Calculate the type B uncertainties of all the equipments used.**
- **Compare and comment on the values of h in both conditions.**

- **What are the limitations of the equation for Newton's Law of Cooling that we have used? You might require some research for this question. Justify your reasoning.**
- **Identify sources of error and suggest relevant improvements.**