3.2 (a) The proof of Thm 3.2(ii) uses axiom A3 that a + 0 = a. However, in order for it to be useful in the proof, axiom A2 must first be applied so that a+0 = 0+a = a. Only then can we apply the result of Thm 3.2(i) that a(0)+a(0) = 0+a(0) implies a(0) = 0.

(b) In the proof of Thm 3.2(iii), the text reaches the equation that ab+(-a)b = ab+(-(ab)). In order to apply Thm 3.2(i), axiom A2 must be applied so that ab+(-a)b = (-a)b+ab and ab+(-(ab)) = -(ab)+ab and hence (-a)b = -(ab) as desired.

3.4 v *Proof.* We will use proof by contradiction. Suppose that $0 \geq 1$. Then, by Thm 3.2(iv) that $0 \leq a^2$ for all a, it follows that $1 \leq 0 \leq a^2$ for all a and hence $1 \leq a^2$ for all a. However, if $a = \frac{1}{2}, a^2 = \frac{1}{4}$, which is less than 1. Contradiction. Therefore, $0 < 1$. ☐

vii *Proof.* If $0 < a < b$, then we have that $0 < a$ and $0 < b$. From Thm 3.2(vi), we can conclude that $0 < a^{-1}$ and $0 < b^{-1}$. Thus, we just need to show that $b^{-1} < a{-1}$.
We will use proof by contradiction. Suppose $a < b$ and $a^{-1} < b^{-1}$. Since a and b are both positive, $ab > 0$. Thus, $a^{-1} < b^{-1}$ implies that $a^{-1}(ab) < b^{-1}(ab)$ so $b < a$. Contradiction. Therefore, $0 < b^{-1} < a{-1}$ ☐

3.6 (a) *Proof.* Let a,b,c be some real numbers. By axiom, $|a + b + c| = |a + (b + c)|$. By the triangle inequality, we then have that $|a+(b+c)| \leq |a|+|b+c|$. We can then apply the triangle inequality again to $|b+c|$ to get that $|b+c| \leq |b|+|c|$. As a result, we have that $|a|+|b+c| \leq |a|+|b|+|c|$ and so $|a+b+c| \leq |a|+|b|+|c|$. Since a, b, and c were arbitrary, $|a+b+c| \leq |a|+|b|+|c|$ for all $a, b, c \in \mathbb{R}$. ☐

(b) *Proof.* We will prove by induction. Note that $|a_1| = |a_1|$ so the claim holds for n = 1. Now suppose that claim holds for some $n \in \mathbb{N}$. Then, $|a_1 + a_2 + \cdots + a_n + a_{n+1}| = |(a_1 + a_2 + \cdots + a_n) + a_{n+1}| \leq |a_1 + a_2 + \cdots + a_n| + |a_{n+1}|$ by the triangle inequality. By the inductive hypothesis, $|a_1 + a_2 + \cdots + a_n| + |a_{n+1}| \leq |a_1| + |a_2| + \cdots + |a_n| + |a_{n+1}|$ so the claim holds for n+1. Since n was arbitrary, the claim holds for n+1 whenever it holds for n. Therefore by induction, the claim holds for all $n \in \mathbb{N}$. ☐

3.8 *Proof.* We will use proof by contradiction. Suppose that $a \leq b_1$ for every $b_1 > b$ and $a > b$. Then, $a - b > 0$ and $b_1 - b > 0$ so $a - b_1 > 0$. Thus, we have that $a > b_1$. Contradiction. Therefore, if $a \leq b_1$ for every $b_1 > b$, then $a \leq b$. ☐

4.8 (a) If $s \leq t$ for all $s \in S$ and $t \in T$, then clearly there is some $s_0 \in T$ such that $s \leq s_0$ for all $s \in S$ and so S is bounded above. Likewise, there is some $t_0 \in S$ such that $t \geq t_0$ for all $t \in T$ and so T is bounded below.

(b) We will use proof by contradiction. Let s_0 be sup$S$ and t_0 be inf$T$. Suppose $s_0 > t_0$. By the Denseness of $\mathbb{R}$, there is some $s_1 \in S$ such that $t_0 < s_1 < s_0$. Likewise, there is some $t_1 \in T$ such that $t_0 < t_1 < s_1 < s_0$ and so $t_1 < s_1$. Contradiction. Therefore, $s_0 \leq t_0$.

(c) If S = (0,1] and T = [1,2), then $S \cap T = \{1\}$.

(d) If S = (0,1) and T = (1,2), then sup$S$ = inf$T$ and $S \cap T$ is the empty set.

4.10 *Proof.* $a > 0$ and $1 > 0$, so by Archimedean Principle, there is some natural number n such that $na > 1$ Thus, $a > \frac{1}{n}$. Likewise, since $1 > 0$ and $a > 0$, so by Archimedean Principle there is some natural number n such that $1(n) > a$. Thus, $n > a$. Therefore, if $a > 0$, then there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < a < n$. ☐

4.16 *Proof.* Let $S = \{r \in \mathbb{Q} \mid r < a\}$. By definition, a is an upper bound for S since S is defined as the set of all rational numbers which are less than a. By the Completeness Axiom, we know that sup$S$ exists and is a real number. Thus, we want to show that a is a least upper bound for S. We will use proof by contradiction.
Let s_0 be an upper bound for S such that $s_0 < a$. By the denseness of of $\mathbb{Q}$, there is some $s_1 \in S$ such that $s_0 < s_1` < a$ so s_0 is not an upper bound. Contradiction. Since s_0 was an arbitrary element less than a, we can conclude that a is the least upper bound.
Since a was arbitrary, a = sup$S$ for each a $\in \mathbb{R}$. ☐

5.6 *Proof.* $S \subseteq T$ so every element in S is also an element in T. S and T are non-empty, so $infS \leq supS$ and $infT \leq supT$. Let $t_0 = infT$. As $t_0 \leq t$ for all $t \in T$, then clearly $t_0 \leq s$ for all $s \in S$ as well. Thus, t_0 is a lower bound for S but $t_0 \leq infS$ since inf$S$ is the greatest lower bound for S.
Likewise, let $t_1 = supT$. As $t_1 \geq t$ for all $t \in T$, then clearly $t_1 \geq s$ for all $s \in S$ as well. Thus, t_1 is an upper bound for S but $t_1 \geq supS$ since sup$S$ is the least upper bound for S.
Therefore, $infT \leq infS \leq supS \leq supT$. □

7.2 (a) The terms of the sequence converge to 0 as n increases.
(b) The terms of the sequence converge to $\frac{3}{4}$ as n increases.
(c) The terms of the sequence converge to 0 as n increases.
(d) The terms of the sequence do not converge as n increases.

7.4 (a) $< x_n > = < \frac{\pi}{n} >$ converges to 0 as n increases
(b) $< x_n > = < (1 + \frac{1}{n})^n >$ converges to e as n increases.