

1. Let $U_{(1)}, \dots, U_{(n)}$ be the values of n independent uniform $(0,1)$ random variables arranged in increasing order. Let $0 \leq x < y < 1$.

- a) Find a simple formula for $P(U_{(1)} > x \text{ and } U_{(n)} < y)$
b) Find a formula for $P(U_{(1)} \leq x \text{ and } U_{(n)} < y)$
Pitman 5.1.8

2. Calculate the following double integrals:

- a) $\iint_R y + \frac{x}{y^2} dA$ where $R = \{(x, y) \mid 0 \leq x \leq 2, 1 \leq y \leq 2\}$
b) $\iint_R \frac{xy^2}{x^2+1} dA$ where $R = \{(x, y) \mid 0 \leq x \leq 1, -3 \leq y \leq 3\}$
Stuart 15.1.28, 15.1.29

3. Let random variables X, Y have joint density function $f_{X,Y}(x, y)$ over the region $\{(x, y) : -1 < x < 1, 0 < y < 3\}$. For each part below, (i) set up (but do not evaluate) an integral for the given expression, and (ii) compute the exact answer in the case where the density is uniform over the region.
- a) $P(X < Y)$
 - b) $P(X + 2Y > 1)$
 - c) $P(X^2 < Y)$
 - d) $P(\sqrt{X^2 + Y^2} > 1)$

Stat 134 Fall 2017 Joint Distributions Final Review Worksheet Problem 1