Data 8, Lab 8

The Central Limit Theorem, Sample Means, and Correlation

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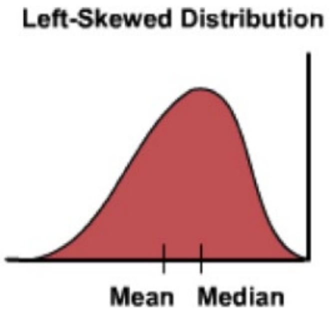
Agenda

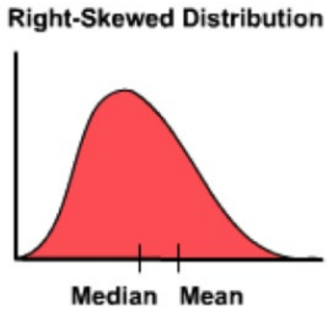
1. Skewness
2. Variability
3. Chebyshev's Bounds
4. Standard Units
5. Normal Distribution
6. Central Limit Theorem
7. Distribution of Sample Means
8. Correlation

Skewness

- Left skew
  - Long left tail
  - Mean < Median



- Right skew
  - Long right tail
  - Mean > Median



Variability

- Variance: How spread out is the data?
- Standard Deviation: Square root of the variance
  - Same unit as the data
  - The larger the SD, the more spread out the data is

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Chebyshev's Bounds

- **Regardless of the distribution**, the proportion of values in the range "average ± z SDs" is at least 1 - 1/z^2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888…%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)
average ± 5 SDs	at least 1 - 1/25 (96%)

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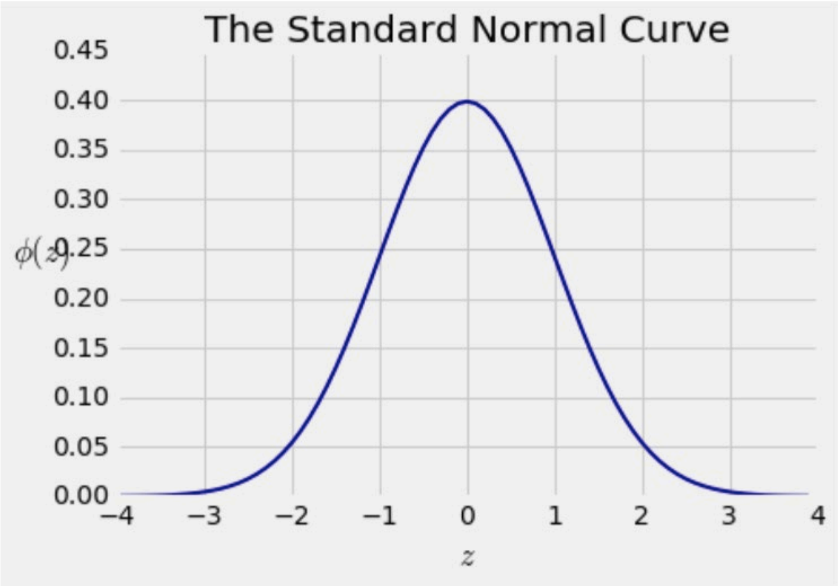
Standard Units

- Standard Unit: Number of SD's above or below average
- Allows us to easily compare different distributions and units
- Z = (value-average)/SD
- Average of standard units is always 0
- SD of standard units is always 1

Normal Distribution

- An extremely common distribution in statistics, shaped like a bell curve
- Most of the data is within a few SD's of the mean



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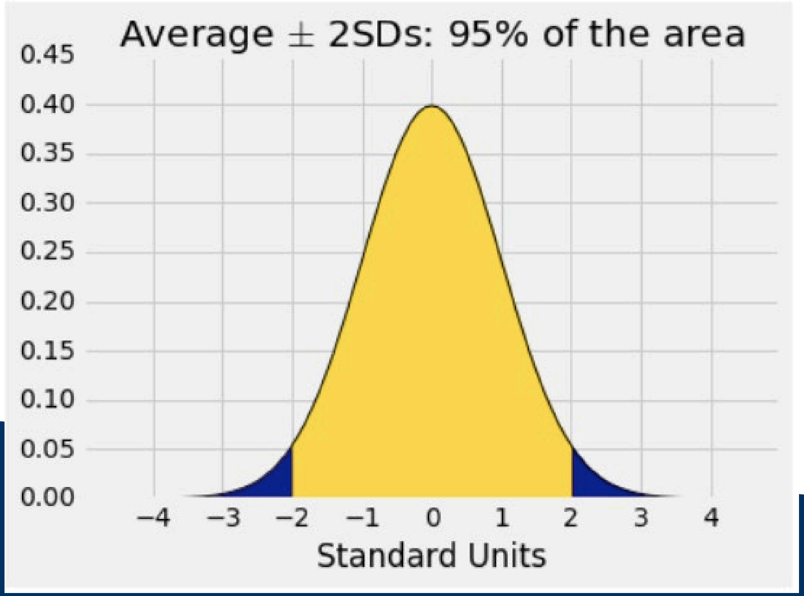
Normal Distribution (cont'd)

Range	All Distributions	Normal
average ± 1 SDs	at least 0%	68%
average ± 2 SDs	at least 75%	95%
average ± 3 SDs	at least 88.9%	99.7%



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Central Limit Theorem

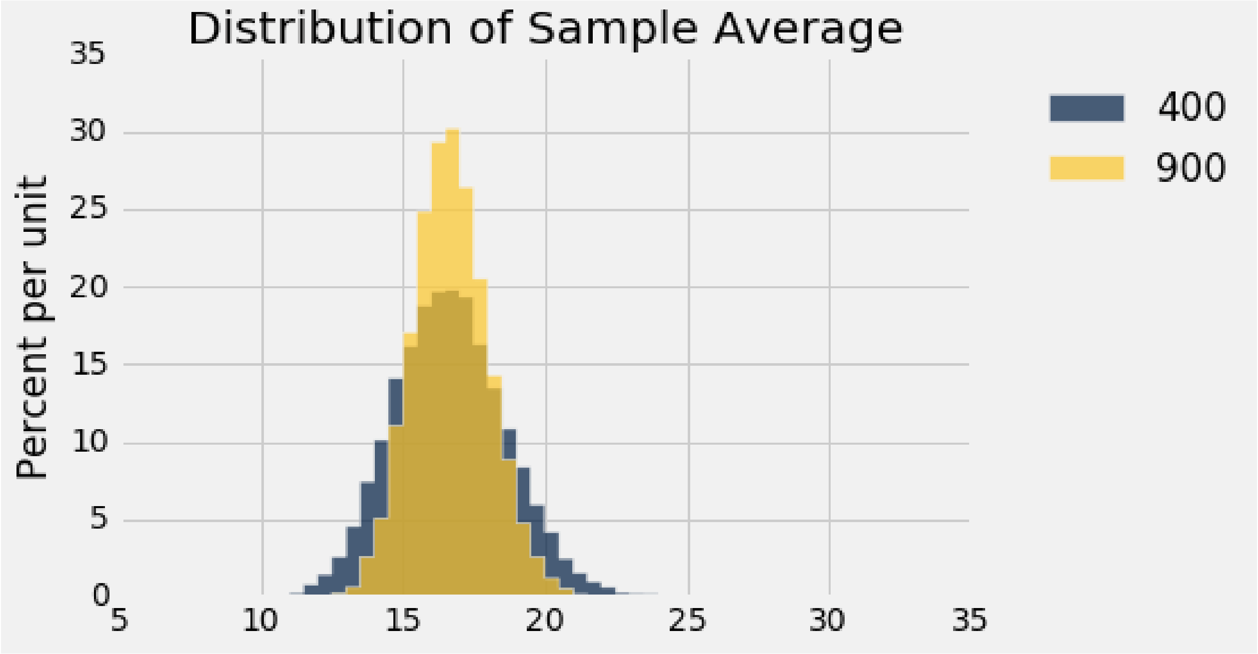
- If the sample is large and drawn at random with replacement
- Regardless of the distribution of the population, the distribution of the sample sum or average is **roughly normal**
- Distribution of the sample sum/average:
  - Many possible random samples of the same size
  - Distribution is based on the sum/average of different samples

Distribution of Sample Mean

- As the sample size increases, the sample mean is more likely to be closer to the population mean
- As a result, the distribution of sample means will have lower SD – a "narrower bell shape" when the sample size increases

$$SD\ of\ Sample\ Means = \frac{Population\ SD}{\sqrt{Sample\ Size}}$$

Distribution of Sample Mean



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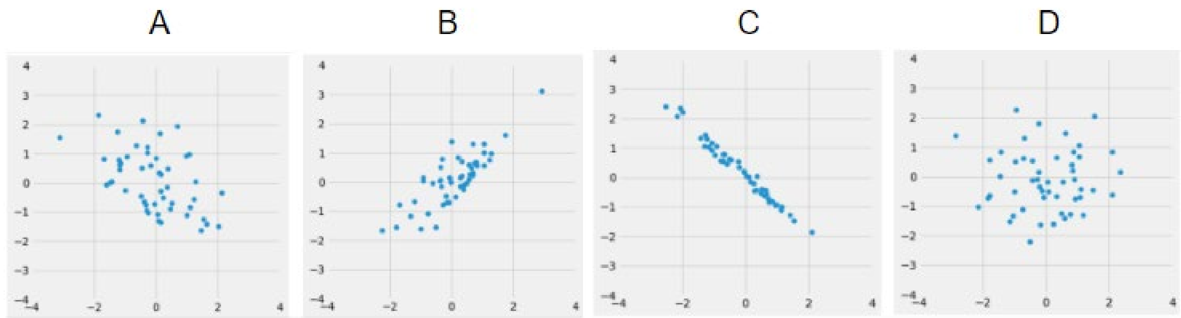
Correlation

- Measure the strength of the linear relationship between two variables
- Correlation must be between -1 and 1 (inclusive)
- When r is positive, there is a **positive linear association** between the two variables
- When r is negative, there is a **negative linear association** between the two variables
- Correlation of x and y is the same as correlation of y and x

Calculating Correlation

- Calculated as r = average of element-wise product of two variables in standard units
- Algorithm:
    1. Given two numeric arrays `x` and `y` of the same length
    2. Convert both `x` and `y` into standard units `x_su` and `y_su`
    3. Calculate array of the product of the arrays in standard units
       `xy_product = x_su*y_su`
        1. The ith element in this product array is the product of the ith element in the `x_su` array and the ith element in the `y_su` array
    4. Correlation is the mean of this array `np.mean(xy_product)`

Correlation Example: Worksheet Q6



A. Small negative correlation: Weak negative linear association since the points are not clustered tightly around a line
B. Positive correlation: Positive linear association since the points are clustered somewhat tightly around a line
C. Strong negative correlation: Strong negative linear association since the points are clustered tightly around a line
D. No correlation: No visible trend since the points are just a blob

Announcements

- Project 2 Checkpoint 2 is due today 4/10
  - Entire project is due next Friday 4/17, bonus point for early submission by 4/16
  - If you're working with a partner make sure you add their email onto Okpy so you both receive credit
- HW10 due on 4/16. Bonus point for early submission by 4/15
- Lab 8 extension by one day to Saturday 4/11 at 11:59PM for this week only
- This semester's lecture on privacy has been cancelled, but you can watch last year's version here. The material covered in that lecture will be on the final.