Data 8, Lab 9

The Central Limit Theorem and Sample Means

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Agenda

1. Skewness
2. Variability
3. Chebyshev's Bounds
4. Standard Units
5. Normal Distribution
6. Central Limit Theorem
7. Distribution of Sample Means

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Skewness

- Left skew
  - Long left tail
  - Mean < Median
- Right skew
  - Long right tail
  - Mean > Median

Variability

- Variance: How spread out is the data?
- Standard Deviation: Square root of the variance
  - Same unit as the data
  - The larger the SD, the more spread out the data is

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Chebyshev's Bounds

- Regardless of the distribution, the proportion of values in the range "average ± z SDs" is at least 1 - 1/z^2

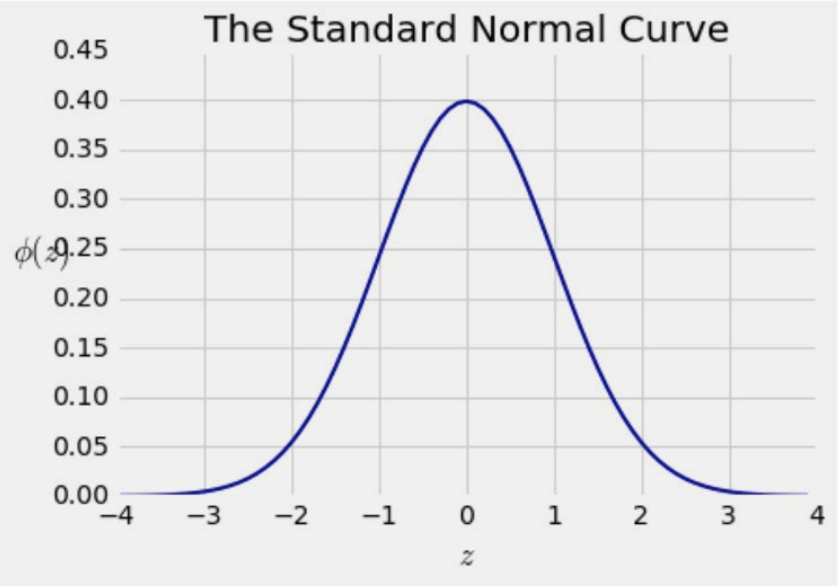
Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)
average ± 5 SDs	at least 1 - 1/25 (96%)

Standard Units

- Standard Unit: Number of SD's above or below average
- Allows us to easily compare different distributions and units
- Z = (value-average)/SD
- Average of standard units is always 0
- SD of standard units is always 1

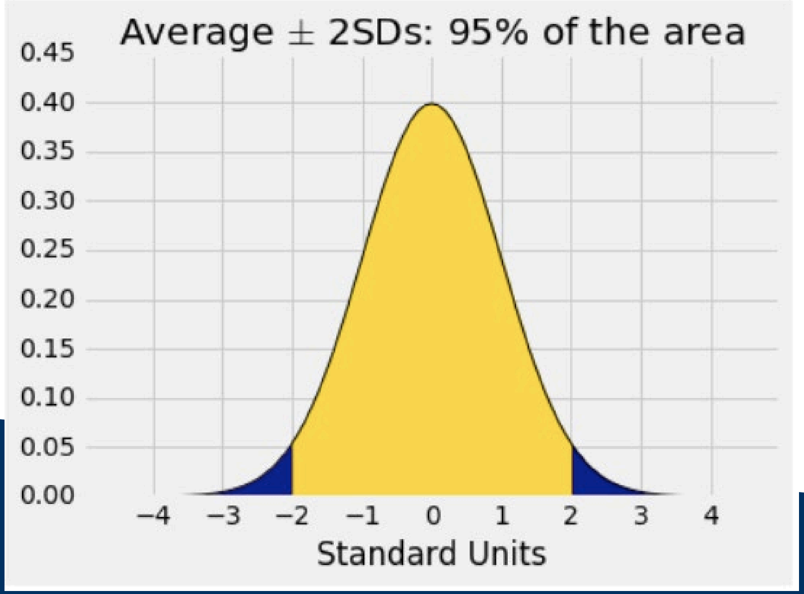
Normal Distribution

- An extremely common distribution in statistics, shaped like a bell curve
- Most of the data is within a few SD’s of the mean



Normal Distribution (cont'd)

Range	All Distributions	Normal
average ± 1 SDs	at least 0%	68%
average ± 2 SDs	at least 75%	95%
average ± 3 SDs	at least 88.9%	99.7%



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Central Limit Theorem

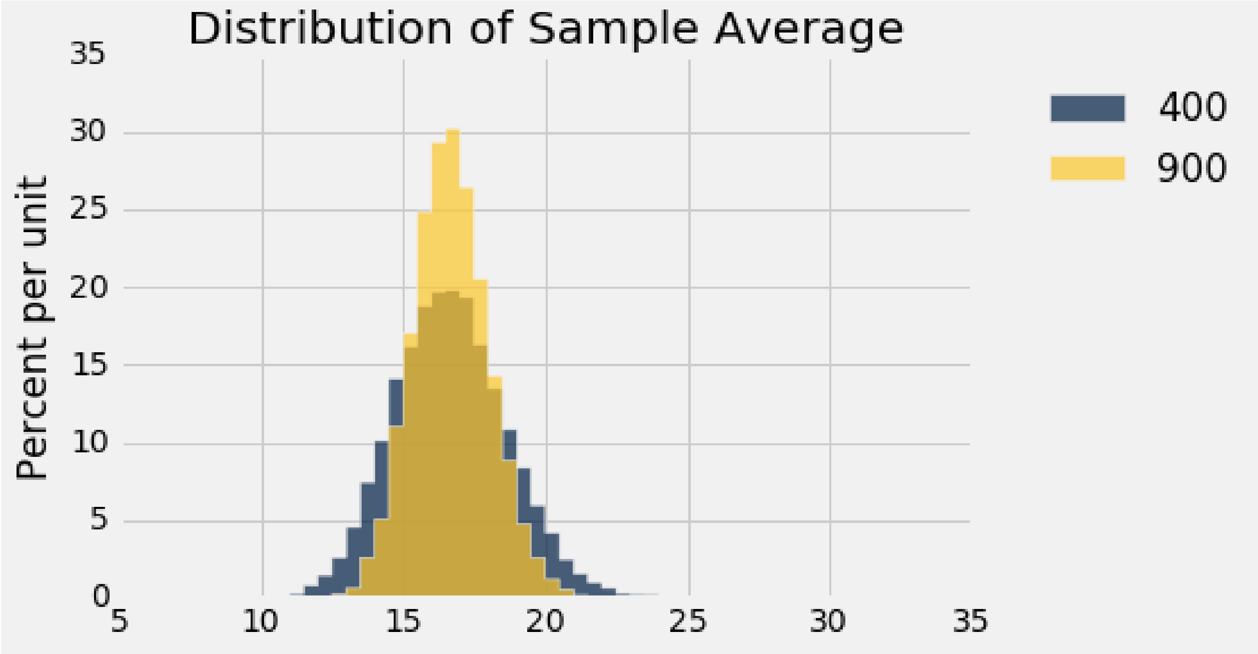
- If the sample is large and drawn at random with replacement
- Regardless of the distribution of the population, the distribution of the sample sum or average is **roughly normal**
- Distribution of the sample sum/average:
  - Many possible random samples of the same size
  - Distribution is based on the sum/average of different samples

Distribution of Sample Mean

- As the sample size increases, the sample mean is more likely to be closer to the population mean
- As a result, the distribution of sample means will have lower SD – a "narrower bell shape" when the sample size increases

$$SD\ of\ Sample\ Means = \frac{Population\ SD}{\sqrt{Sample\ Size}}$$

Distribution of Sample Mean



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Announcements

- Checkpoint 2 of Project 2 due today (11/8)
- Homework 10 due next Thursday (11/14)
- No class next Monday (11/11)