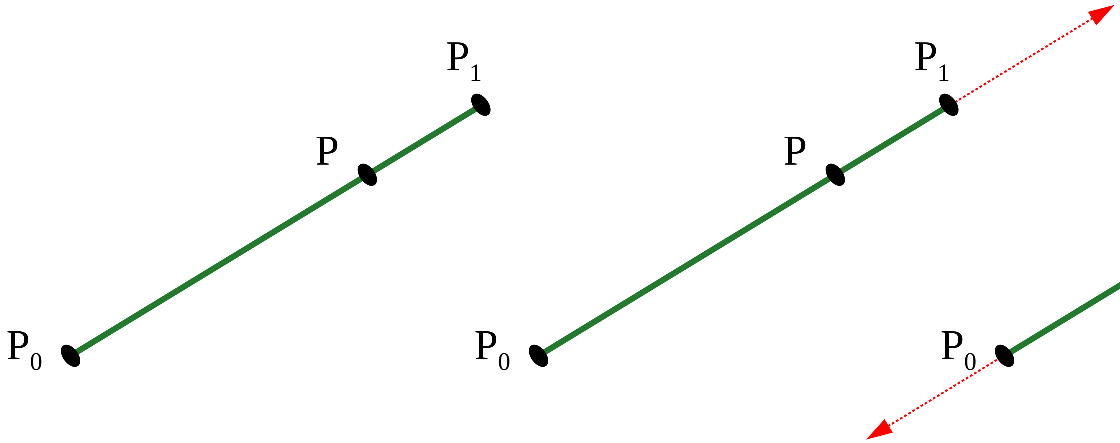
Unidad 6

Intersecciones y Ordenamiento Espacial

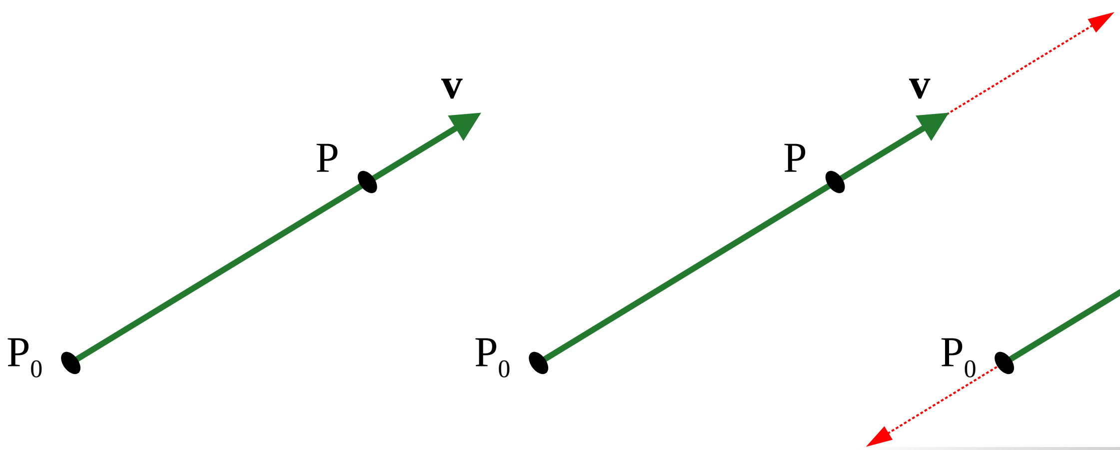
UNL FICH
Universidad Nacional del Litoral
Facultad de Ingeniería y Ciencias Hídricas

Recta vs Rayo vs Segmento



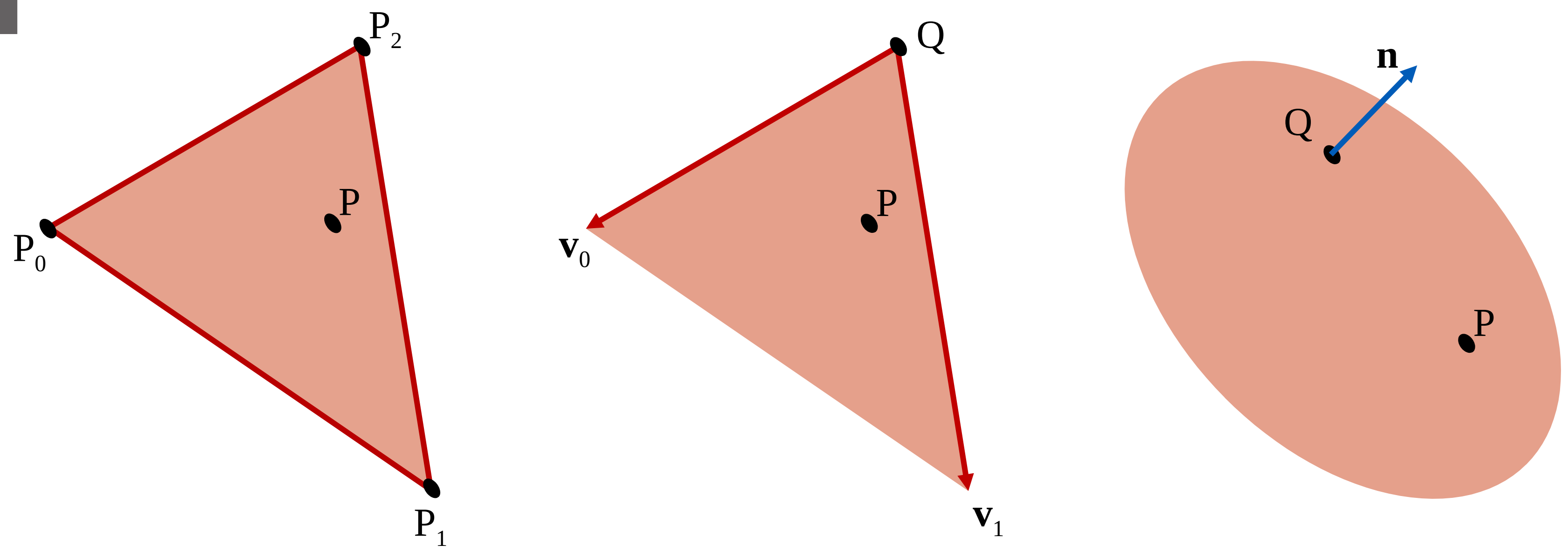
$P = P_0 + \alpha \mathbf{v} = P_0 + \alpha(P_1 - P_0) = (1-\alpha)P_0 + \alpha P_1 = \alpha^0 P_0 + \alpha^1 P_1$ (Siempre: $\alpha^0 + \alpha^1 = 1$)

Segmento: $\alpha \in [0,1]$ Rayo: $\alpha >= 0$ Recta $\forall \alpha \in R$



$\alpha = \alpha^1 = (P - P_0) \cdot \mathbf{v} / v^2$

Plano - Triángulo



Plano $\{P_0, P_1, P_2\}$ $\{Q, \mathbf{v}_0, \mathbf{v}_1\}$ $\{Q, \mathbf{n}\}$ $\{a, b, c, d\}$

$P = P_2+\alpha^0(P_0-P_2)+\alpha^1(P_1-P_2) = \alpha^0P_0+\alpha^1P_1+(1\text{-}\alpha^0\text{-}\alpha^1)P_2 = \alpha^0P_0+\alpha^1P_1+\alpha^2P_2 = Q+\alpha^0\mathbf{v}_0+\alpha^1\mathbf{v}_1$

Triángulo: $\alpha^i \in [0,1]$ plano $\alpha^i \in R$ siempre: $\alpha^0+\alpha^1+\alpha^2 = 1$

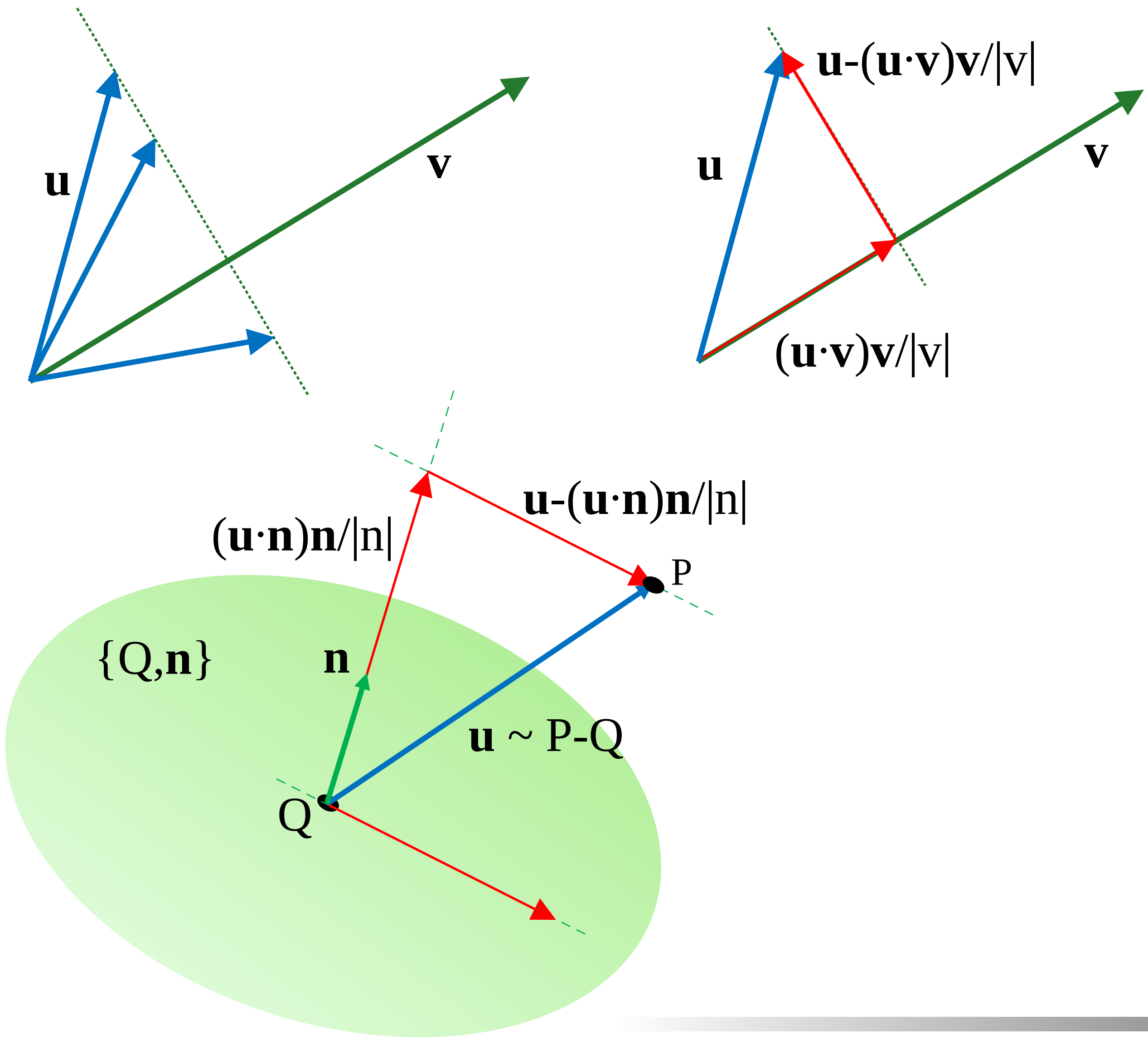
$\{Q,\mathbf{n}\} \setminus (P\text{-}Q)\cdot\mathbf{n} = 0$

$Q = P_2$; $\mathbf{n} = \mathbf{v}_0 \times \mathbf{v}_1 = (P_0\text{-}P_2)\times(P_1\text{-}P_2)$

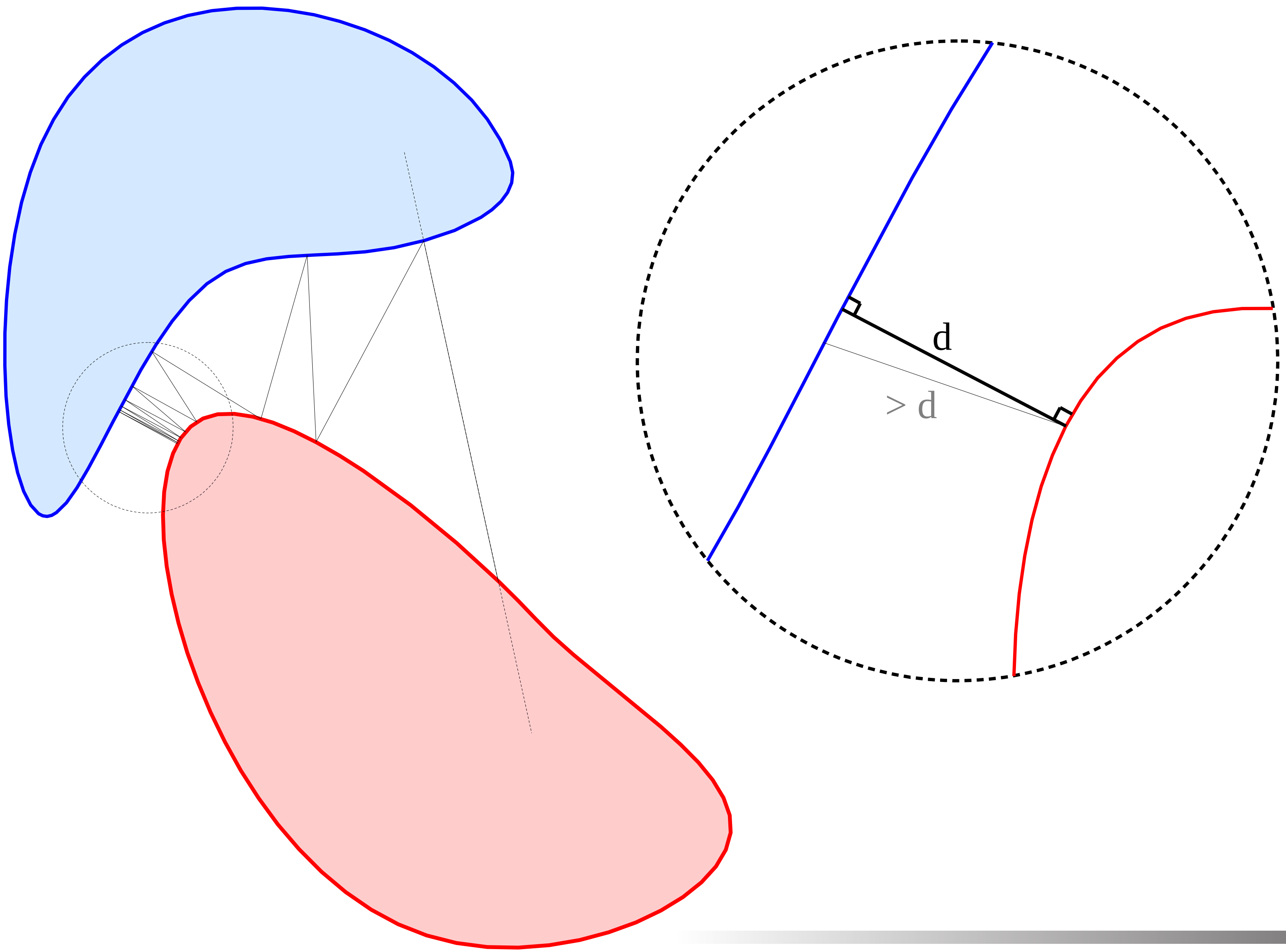
$\{a,b,c,d\} \setminus aP^x+bP^y+cP^z+d = 0$

$\mathbf{n} = \{a,b,c\}$; $Q = \text{-}d\mathbf{n}/n^2$

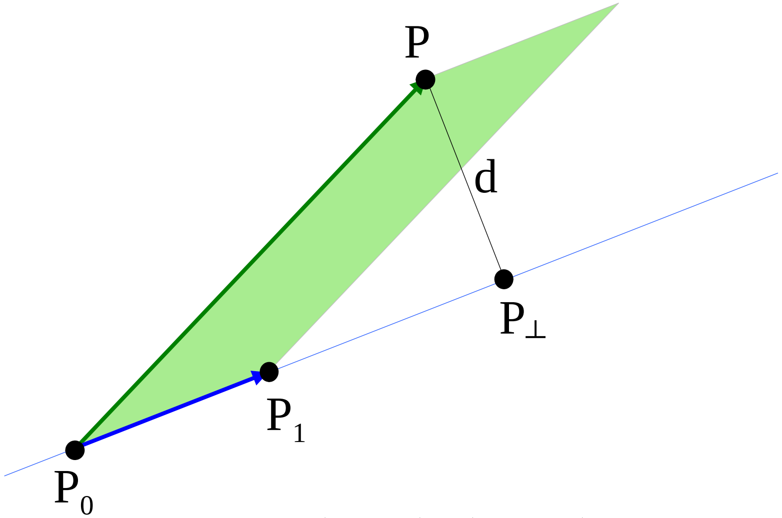
Proyecciones Ortogonales

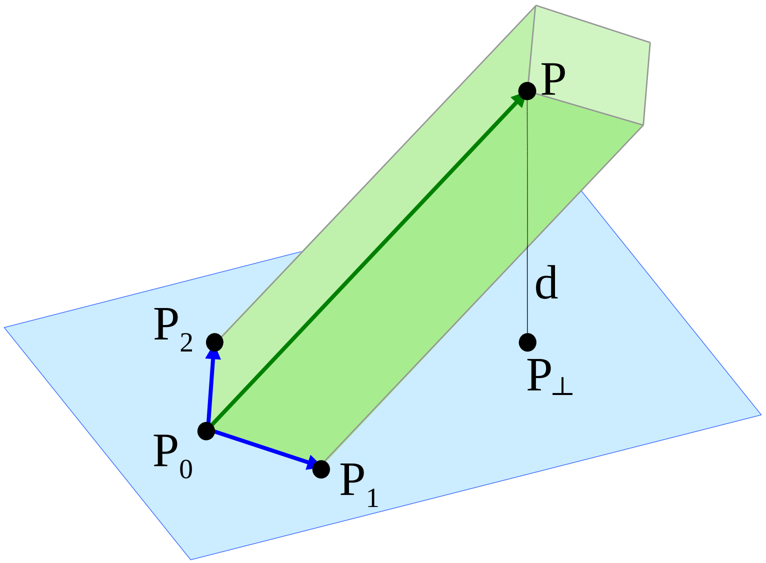


Distancia (Mínima) Entre Objetos



Distancia de Punto a Recta/Plano





$$\mathbf{d}(P,\{P_0,P_1\}) = \frac{(P-P_0)\times(P_1-P_0)}{\|P_1-P_0\|} =$$

$$= (P-P_0)\times\frac{(P_1-P_0)}{\|P_1-P_0\|} =$$

$$= (P-P_0)\times\mathbf{t}$$

$$\mathbf{t} = \frac{(P_1-P_0)}{\|P_1-P_0\|}$$

$$(P_\perp-P_0) = [(P-P_0)\cdot\mathbf{t}]\ \mathbf{t}$$

$$d^2 = (P-P_0)^2 - [(P-P_0)\cdot\mathbf{t}]^2$$

$$d(P,\{P_0,P_1,P_2\}) = \frac{(P-P_0)\cdot(P_1-P_0)\times(P_2-P_0)}{\|(P_1-P_0)\times(P_2-P_0)\|} =$$

$$= (P-P_0)\cdot\frac{(P_1-P_0)\times(P_2-P_0)}{\|(P_1-P_0)\times(P_2-P_0)\|} =$$

$$= (P-P_0)\cdot\mathbf{n}$$

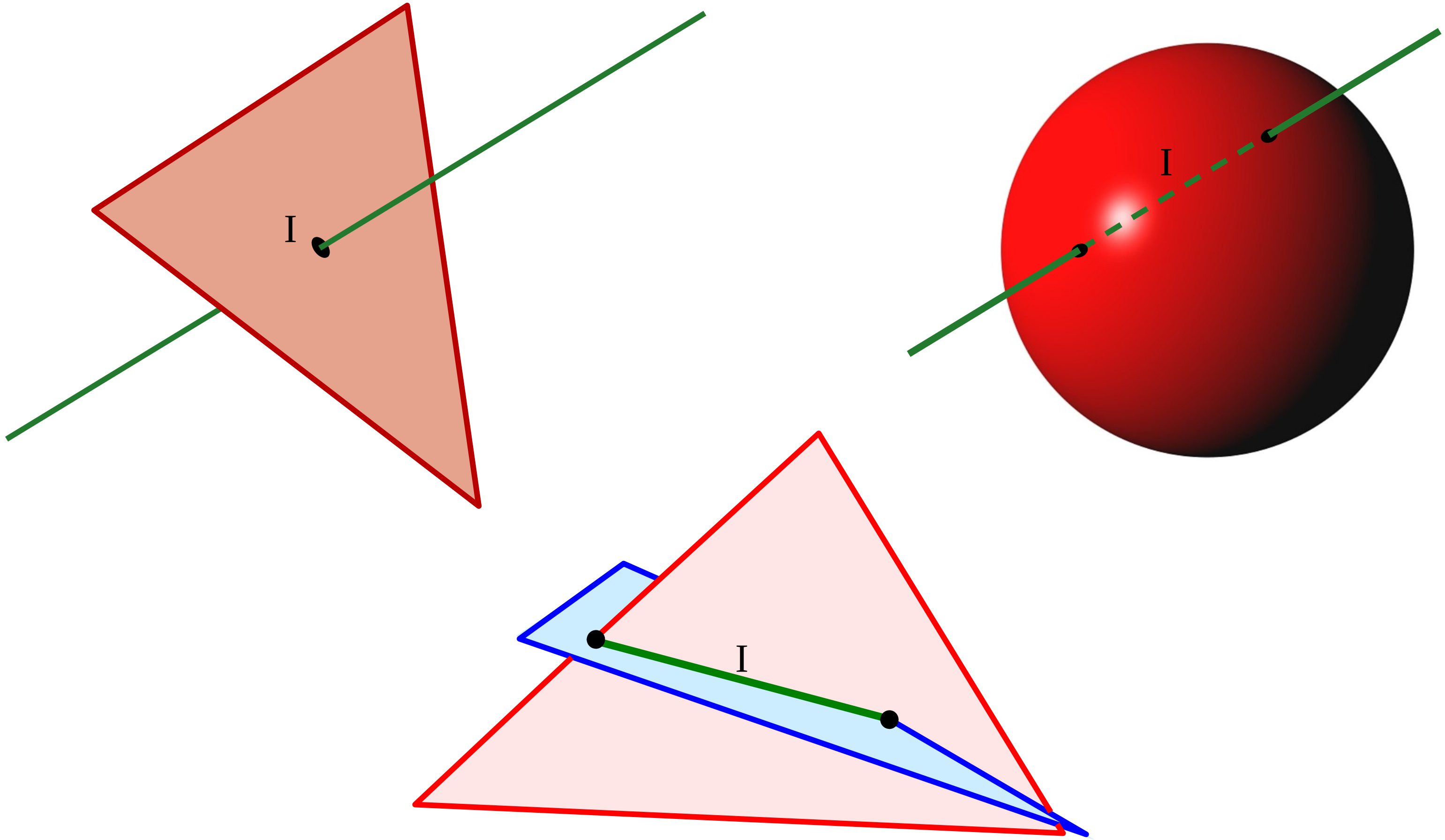
$$\mathbf{n} = \frac{(P_1-P_0)\times(P_2-P_0)}{\|(P_1-P_0)\times(P_2-P_0)\|}$$

$$(P_\perp-P_0) = (P-P_0) - [(P-P_0)\cdot\mathbf{n}]\ \mathbf{n}$$

$$d^2 = [(P-P_0)\cdot\mathbf{n}]^2$$

Parte 1: Intersecciones

Nos interesan sólo: Rayos contra objetos – Clipping – Detección de Colisiones



Intersección de Segmentos

$$\left.\begin{aligned} I &= (1-\alpha)P_0+\alpha P_1 = (1-\beta)Q_0+\beta Q_1 \\ I &= P_0+\alpha(P_1-P_0) = Q_0+\beta(Q_1-Q_0) \\ I &= P_0+\alpha\mathbf{\Delta}P = Q_0+\beta\mathbf{\Delta}Q \\ I &= P_0+\alpha\mathbf{t}_P = Q_0+\beta\mathbf{t}_Q \end{aligned}\right\}\ \text{2e/2i en el plano} \rightarrow \alpha,\beta \in [0,1]\ ?$$

$\mathbf{n} = \mathbf{\Delta}P\times\mathbf{\Delta}Q$, $n^2 = \mathbf{n}\cdot\mathbf{n}$.

Si $n^2 = 0 \Rightarrow$ paralelas o coincidentes

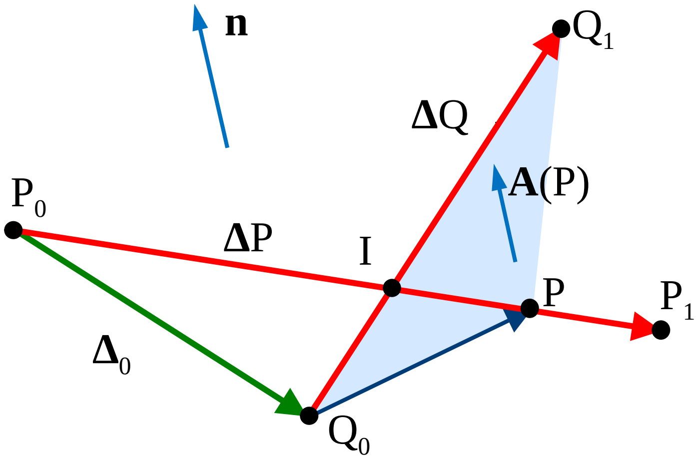
$\mathbf{A}(P) = (P-Q_0)\times\mathbf{\Delta}Q/2$ \ $\mathbf{A}(I) = 0 \Rightarrow A(P) = 2\mathbf{A}(P)\cdot\mathbf{n}$

$A(I) = 0 = (P-Q_0)\times\mathbf{\Delta}Q\cdot\mathbf{n} = (P_0+\alpha\mathbf{\Delta}P-Q_0)\times\mathbf{\Delta}Q\cdot\mathbf{n}$

$= \alpha\, n^2 - \mathbf{\Delta}_0\times\mathbf{\Delta}Q\cdot\mathbf{n}$

$\Rightarrow \quad \alpha = \mathbf{n}\times\mathbf{\Delta}_0\cdot\mathbf{\Delta}Q/n^2 \quad \alpha \in [0,1]\ ?$

Del mismo modo: $\beta = \mathbf{n}\times\mathbf{\Delta}_0\cdot\mathbf{\Delta}P/n^2 \quad \beta \in [0,1]\ ?$



Algoritmo:

a) $\mathbf{\Delta}P = P_1-P_0$, $\mathbf{\Delta}Q = Q_1-Q_0$, $\mathbf{n} = \mathbf{\Delta}P\times\mathbf{\Delta}Q$, $n^2 = \mathbf{n}\cdot\mathbf{n}$ Si $n^2 \neq 0 \Rightarrow$ b)

b) $\mathbf{\Delta}_0 = Q_0-P_0$

c) $\mathbf{b} = \mathbf{n}\times\mathbf{\Delta}_0$, $\alpha' = \mathbf{b}\cdot\mathbf{\Delta}Q$, $\beta' = \mathbf{b}\cdot\mathbf{\Delta}P$ Si $\{\alpha',\beta'\}\in[0,n^2]^2 \Rightarrow$ d)

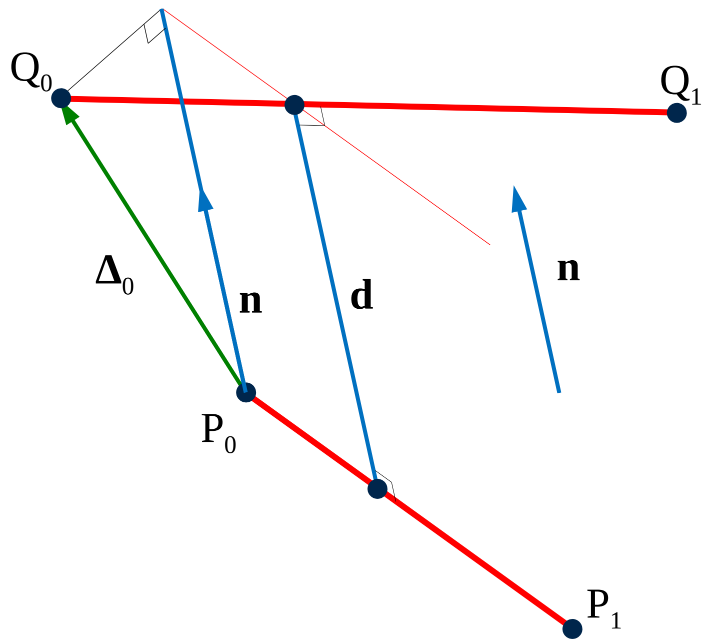
d) $I = P_0 + \alpha'/n^2\ \mathbf{\Delta}P$ ($n^2\neq 0$, comprobado antes)

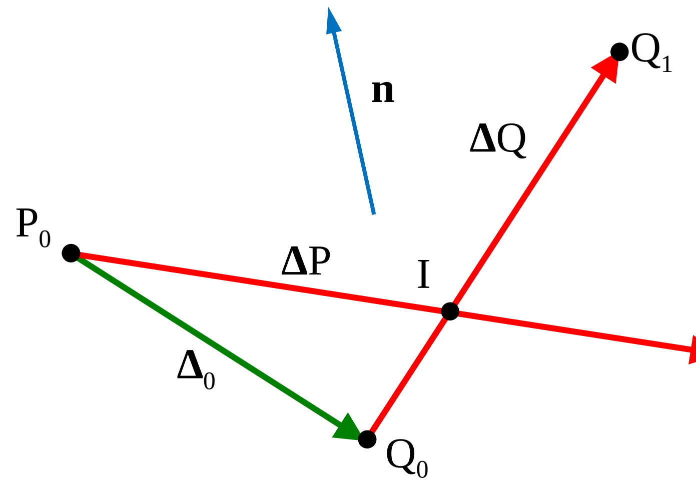
Intersección de Segmentos

$$I = (1-\alpha)P_0 + \alpha P_1 = (1-\beta)Q_0 + \beta Q_1 \rightarrow \alpha\ ,\ \beta$$

2e/2i en el plano

3e/2i en el espacio





Algoritmo:

a) $\mathbf{\Delta}P = P_1 - P_0$, $\mathbf{\Delta}Q = Q_1 - Q_0$, $\mathbf{n} = \mathbf{\Delta}P \times \mathbf{\Delta}Q$, $n^2 = \mathbf{n} \cdot \mathbf{n}$ Si $n^2 \neq 0 \Rightarrow$ b)

b) $\mathbf{\Delta}_0 = Q_0 - P_0$

c) $\mathbf{b} = \mathbf{n} \times \mathbf{\Delta}_0$, $\alpha' = \mathbf{b} \cdot \mathbf{\Delta}Q$, $\beta' = \mathbf{b} \cdot \mathbf{\Delta}P$ Si $\{\alpha', \beta'\} \in [0, n^2]^2 \Rightarrow$ d)

d) $I = P_0 + \alpha'/n^2\ \mathbf{\Delta}P$ ($n^2 \neq 0$, comprobado antes)

Intersección entre Segmento y Triángulo

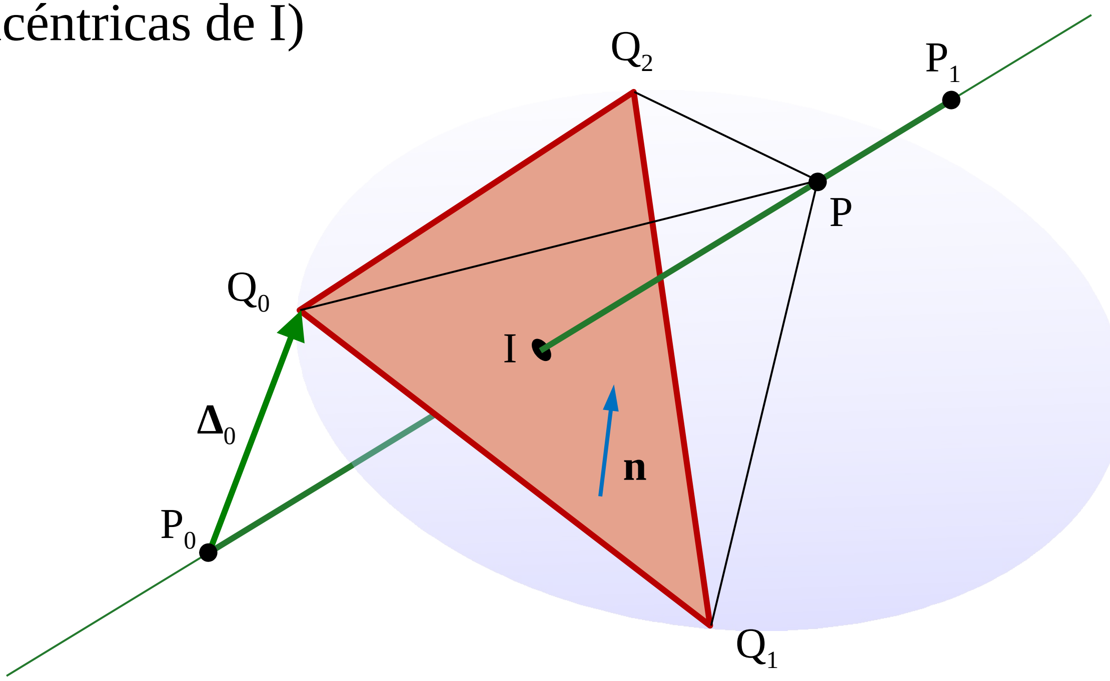
$I = (1-\alpha)P_0+\alpha P_1 = (1-\beta-\gamma)Q_0+\beta Q_1+\gamma Q_2$ 3e/3i $\rightarrow$ α,β,γ ($\in [0,1]$?)

$I \setminus vol\_tetraedro(P,Q_0,Q_1,Q_2) = 0$ equiv. : $I \setminus dist(P, plano(Q_0,Q_1,Q_2)) = 0$

$\mathbf{n} = (Q_1-Q_0) \times (Q_2-Q_0)$

$(P_0+\alpha\mathbf{\Delta}P-Q_0)\cdot\mathbf{n} = 0 \quad\Rightarrow\quad \alpha = \mathbf{\Delta}_0\cdot\mathbf{n}/\mathbf{\Delta}P\cdot\mathbf{n} \quad\Rightarrow\quad I = P_0+\alpha\mathbf{\Delta}P$

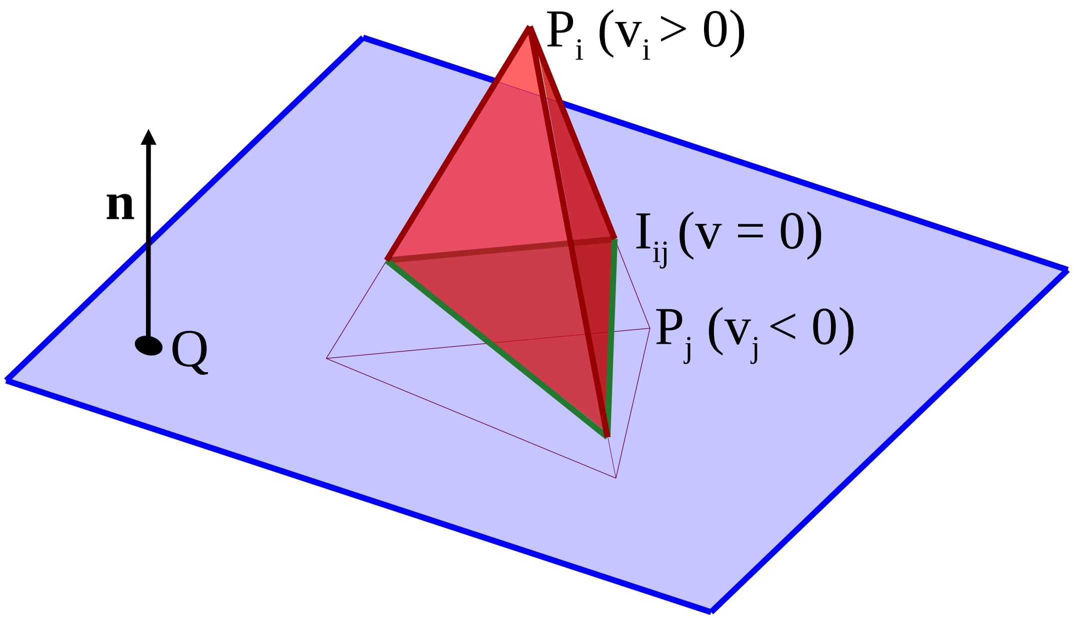
(β,γ por coordenadas baricéntricas de I)



¿Potenciales problemas numéricos? ¿Consecuencias?

Intersección entre un Plano y Muchos Segmentos

Un plano vs. muchos segmentos/triángulos/tetraedros

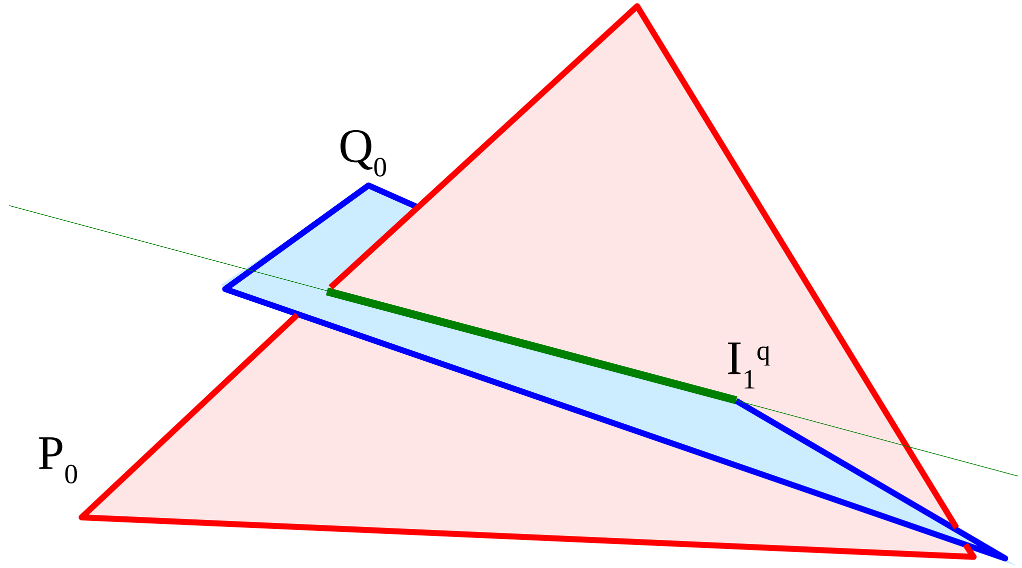


Plano {Q,**n**} o {Q_0,Q_1,Q_2} con $\mathbf{n} = (Q_1 - Q_0) \times (Q_2 - Q_0)$ (no hace falta normalizar)

Para cada vértice P_i: $v_i = |\mathbf{n}| d_i = (P_i - Q) \cdot \mathbf{n} = P_i \cdot \mathbf{n} - Q \cdot \mathbf{n}$ ($Q \cdot \mathbf{n}$ = cte.)

En cada arista (P_i,P_j) cuyos extremos tengan valores de distinto signo ($v_i\, v_j < 0$), el punto en que v=0 es: $I_{ij} = P_j + \underbrace{\boxed{v_j/(v_j - v_i)}}_{\alpha} (P_i - P_j)$

Intersección de Triángulos



b) Hasta dos intersecciones I_i^p de aristas P dentro de Q al menos una $\Rightarrow$ c)

c) Hasta dos intersecciones I_i^q de aristas Q dentro de P al menos una $\Rightarrow$ d)

d) Los I están alineados, dos "centrales" forman la intersección:

$(I_1^p - I_0^p)^2 > (I_1^q - I_0^q)^2$? (elegimos el más largo)

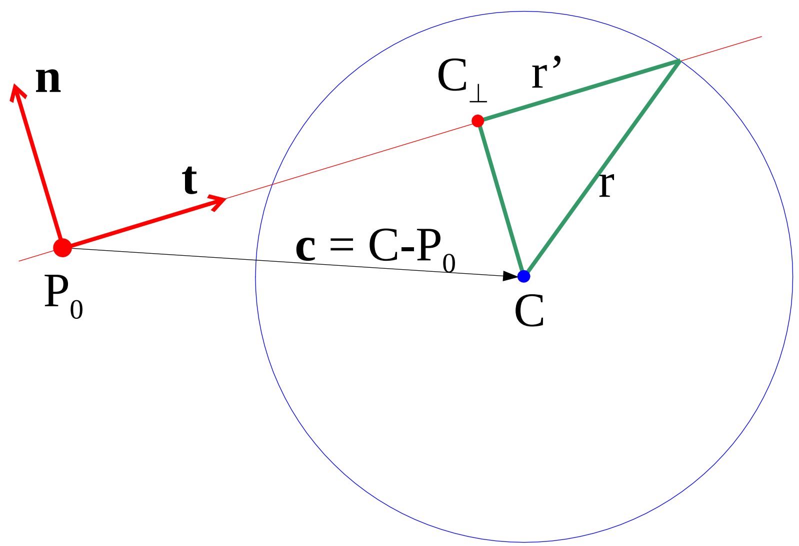
Si $\Rightarrow \mathbf{t} = I_1^p - I_0^p$; $\alpha_p^0 = 0$; $\alpha_p^1 = 1$; $\alpha_q^0 = (I_0^q - I_0^p) \cdot \mathbf{t}$; $\alpha_q^1 = (I_1^q - I_0^p) \cdot \mathbf{t}$

No $\Rightarrow \mathbf{t} = I_1^q - I_0^q$; $\alpha_q^0 = 0$; $\alpha_q^1 = 1$; $\alpha_p^0 = (I_0^p - I_0^q) \cdot \mathbf{t}$; $\alpha_p^1 = (I_1^p - I_0^q) \cdot \mathbf{t}$

e) $[\alpha^0_I, \alpha^1_I] = [\alpha_p^0, \alpha_p^1] \cap [\alpha_q^0, \alpha_q^1]$ $\neq \varnothing \Rightarrow I = I_?^0(\alpha{=}0) + \alpha^i_I\, \mathbf{t}$

Intersección de Rayo o Plano con Esfera

a) Esfera {C, r} y plano (P_0,**n**) en el espacio, vemos el plano de canto.
b) Esfera {C, r} y recta (P_0,**t**) en el espacio, vemos el plano que forman la recta y el centro.
d) Circunferencia {C, r} y recta (P_0,**t**) en el plano.



1) Plano: $C_\perp = P_0 + \mathbf{c} - (\mathbf{c}\cdot\mathbf{n})\mathbf{n}/n^2$ Recta: $C_\perp = P_0 + (\mathbf{c}\cdot\mathbf{t})\mathbf{t}/t^2$

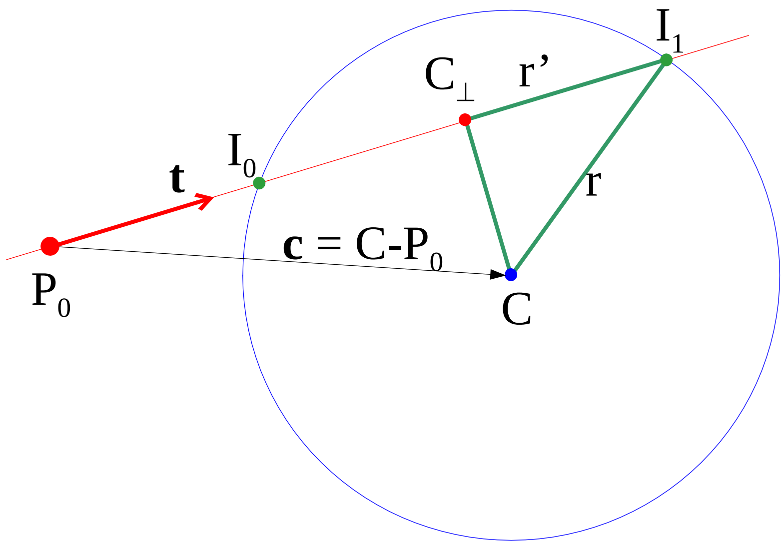
2) $r'^2 = r^2 - (C_\perp - C)^2 > 0$

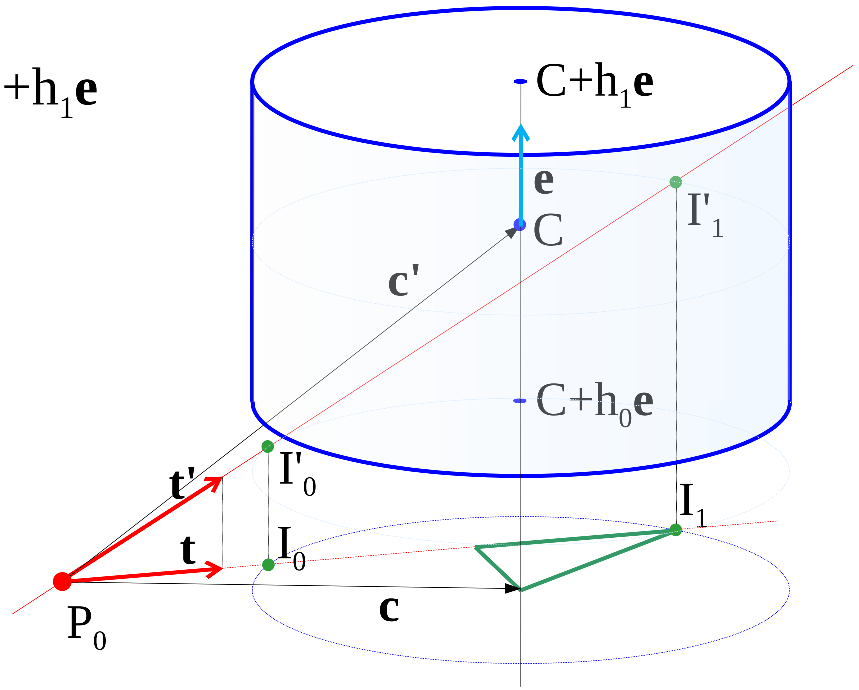
3) Plano: I = Disco {$C_\perp$,r',**n**} en el plano Recta: $I_i = C_\perp \pm r'\mathbf{t}/|\mathbf{t}|$

Intersección de Rayo con Cilindro

Recta $\{P_0, \mathbf{t}'\}$ contra Cilindro $\{C, \mathbf{e}, r\}$

entre $C+h_0\mathbf{e}$ y $C+h_1\mathbf{e}$





1) Proyección: $\mathbf{c} = \mathbf{c}'\text{-}(\mathbf{c}'\cdot\mathbf{e})\mathbf{e}/e^2 \neq \mathbf{0}$ $\quad \mathbf{t} = \mathbf{t}'\text{-}(\mathbf{t}'\cdot\mathbf{e})\mathbf{e}/e^2 \neq \mathbf{0}$ $\quad C_\perp = P_0+(\mathbf{c}\cdot\mathbf{t})\mathbf{t}/t^2$

2) $r'^2 = r^2\text{-}(C_\perp\text{-}C)^2 > 0$

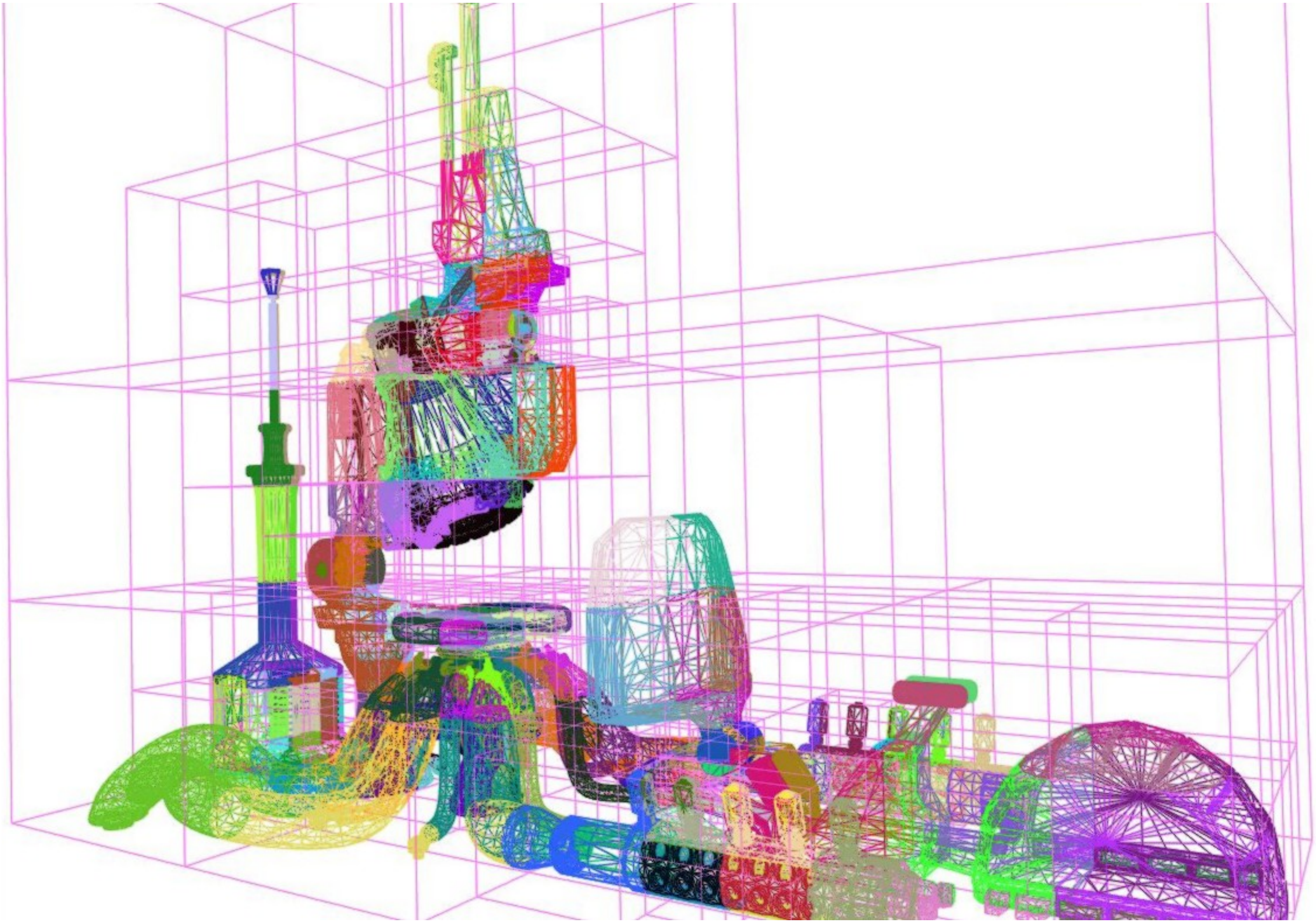
3) Intersecciones Proyectadas: $I_i = C_\perp \pm r'\mathbf{t}/|\mathbf{t}|$

4) $\alpha^i = (I_i\text{-}P_0)\cdot\mathbf{t}/t^2$ Intersecciones: $I'_i = P_0+\alpha^i\,\mathbf{t}'$

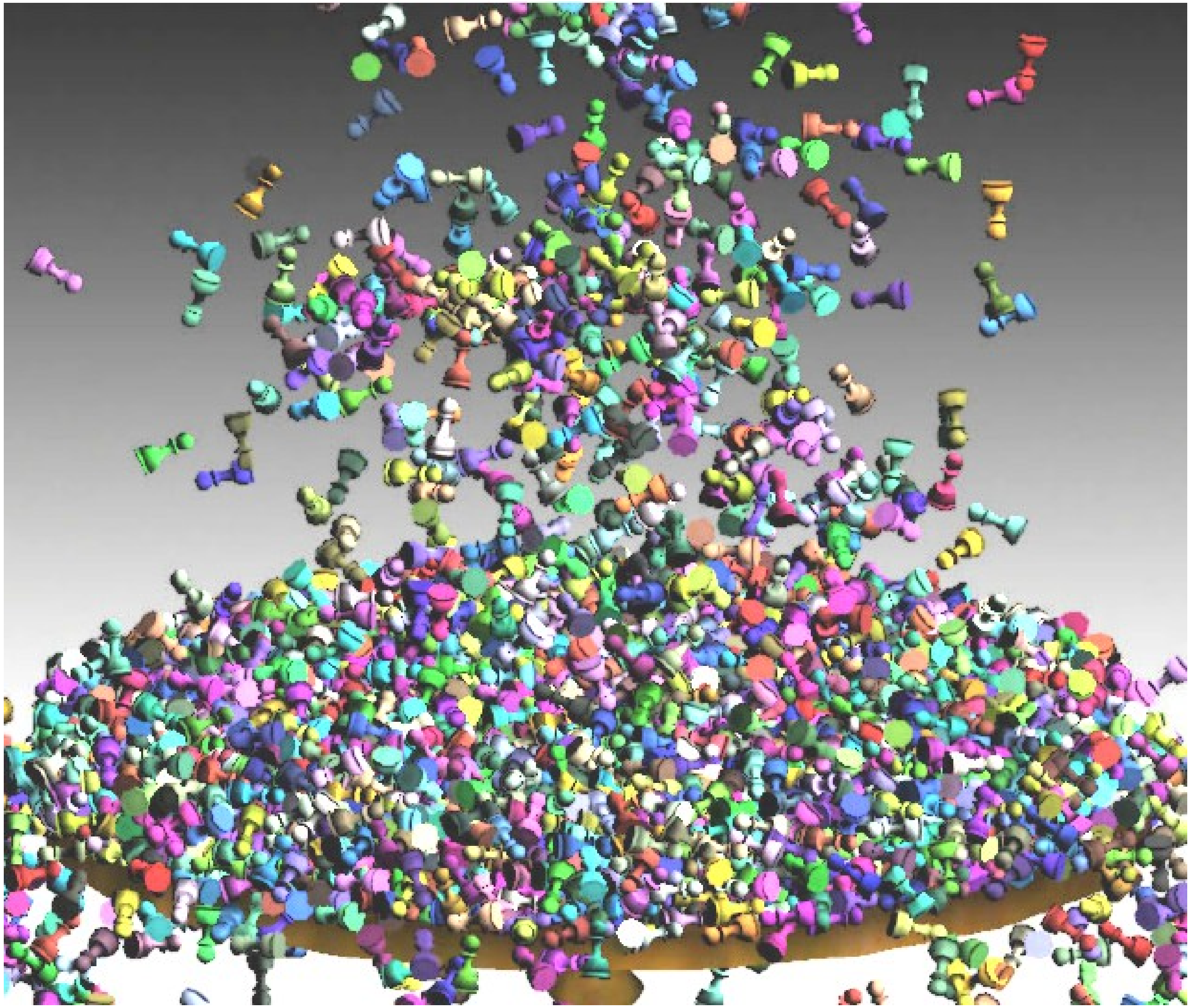
5) Verificar: $\min(h_0,h_1) < (I'_i-C)\cdot\mathbf{e}/e^2 < \max(h_0,h_1)$

(Normalmente, $\mathbf{e}=\mathbf{e}_z \Rightarrow \mathbf{c}=\{c'^x,c'^y\}$; $\mathbf{v}=\{v'^x,v'^y\}$; $h_{min} < I'^z_i < h_{max}$)

Parte 2: Optimización

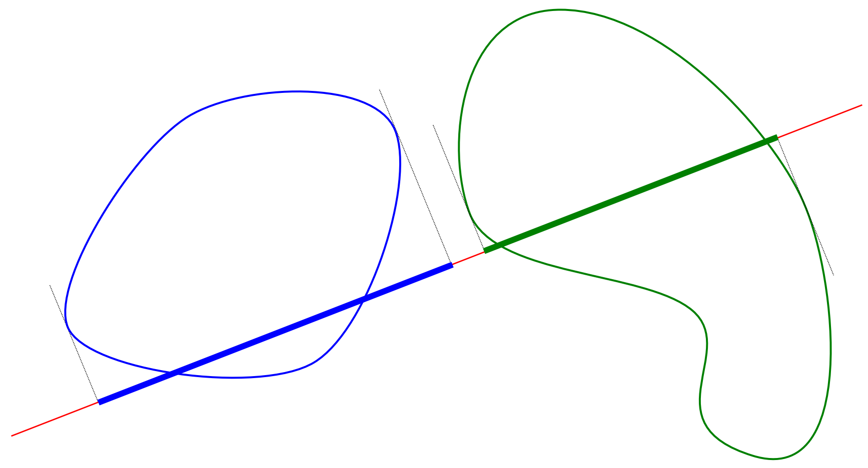


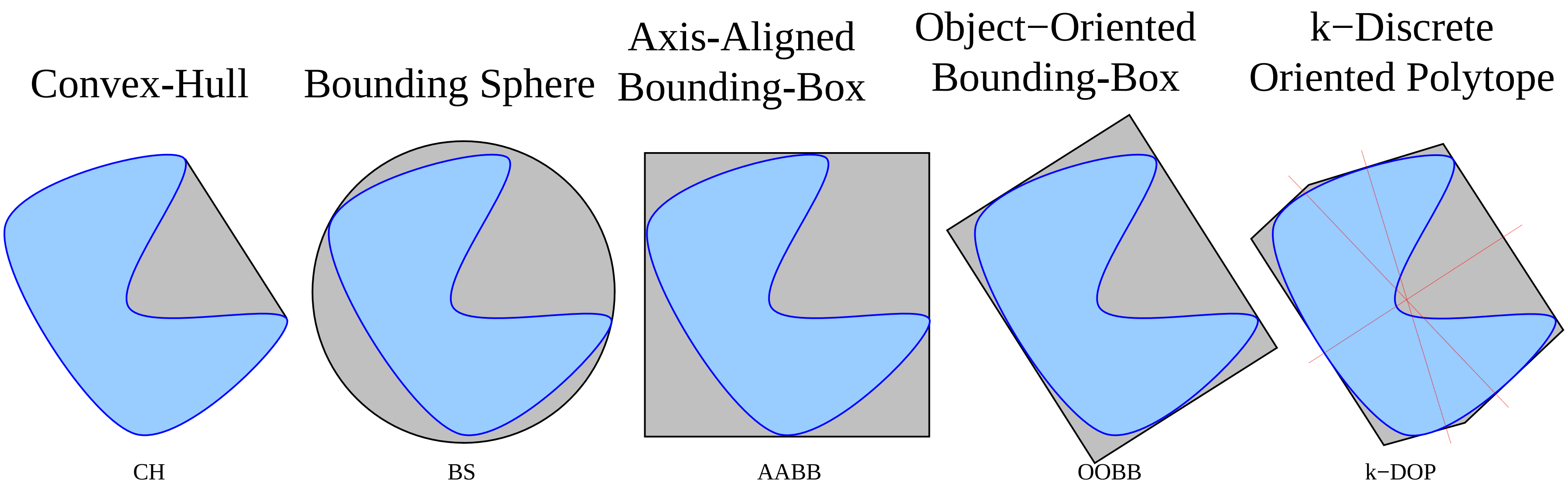
Ejemplo: Detección de Colisiones



https://developer.nvidia.com/gpugems/GPUGems3/gpugems3_ch29.html

Linea Separadora y Envoltorio Convexo

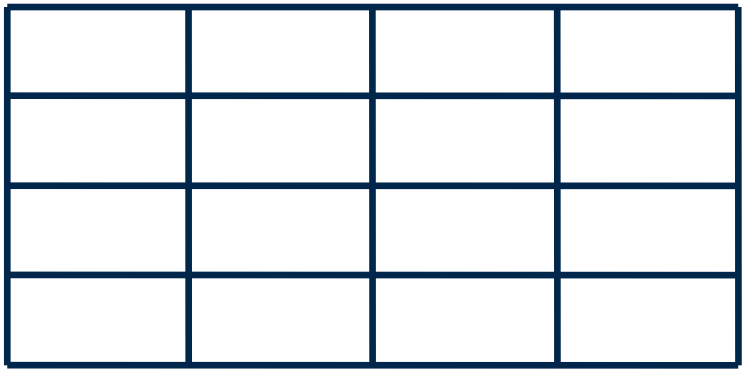




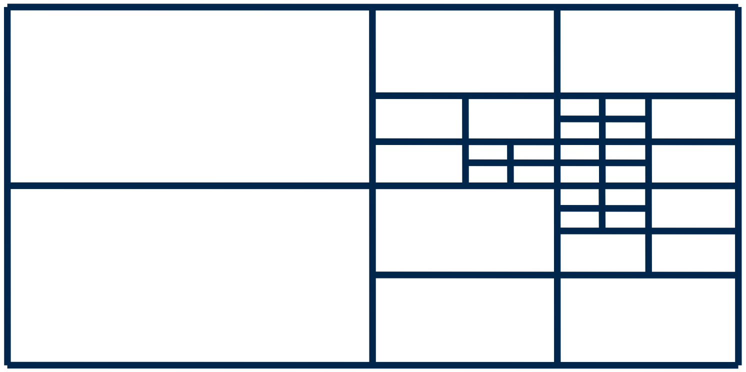
Ordenamiento Espacial

Divide and Conquer: $\sum$pocos*pocos << todos*todos

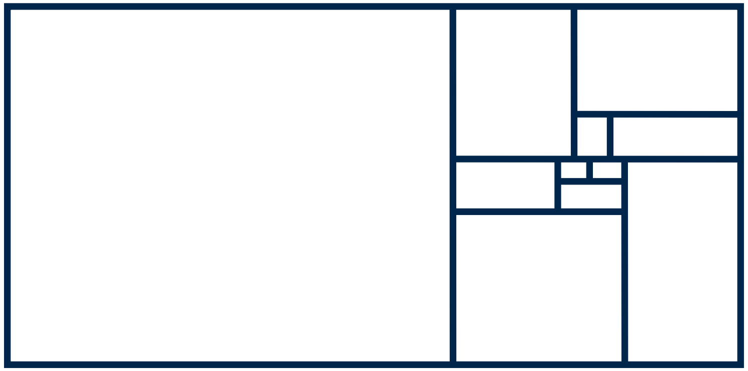
Partición



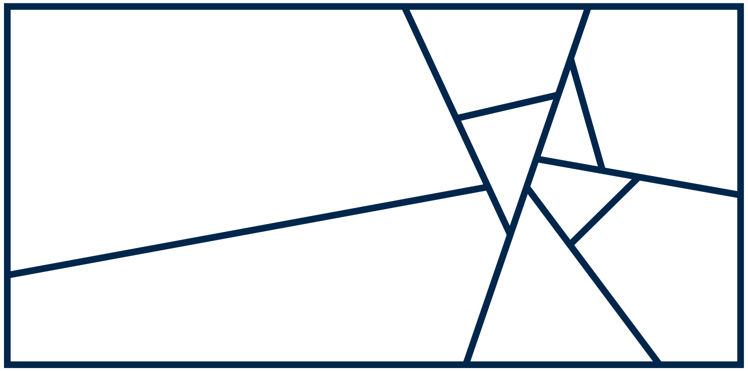
Quadtree



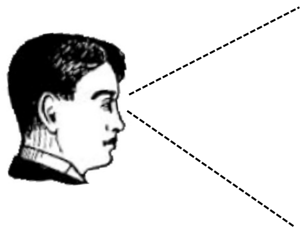
k-d Tree

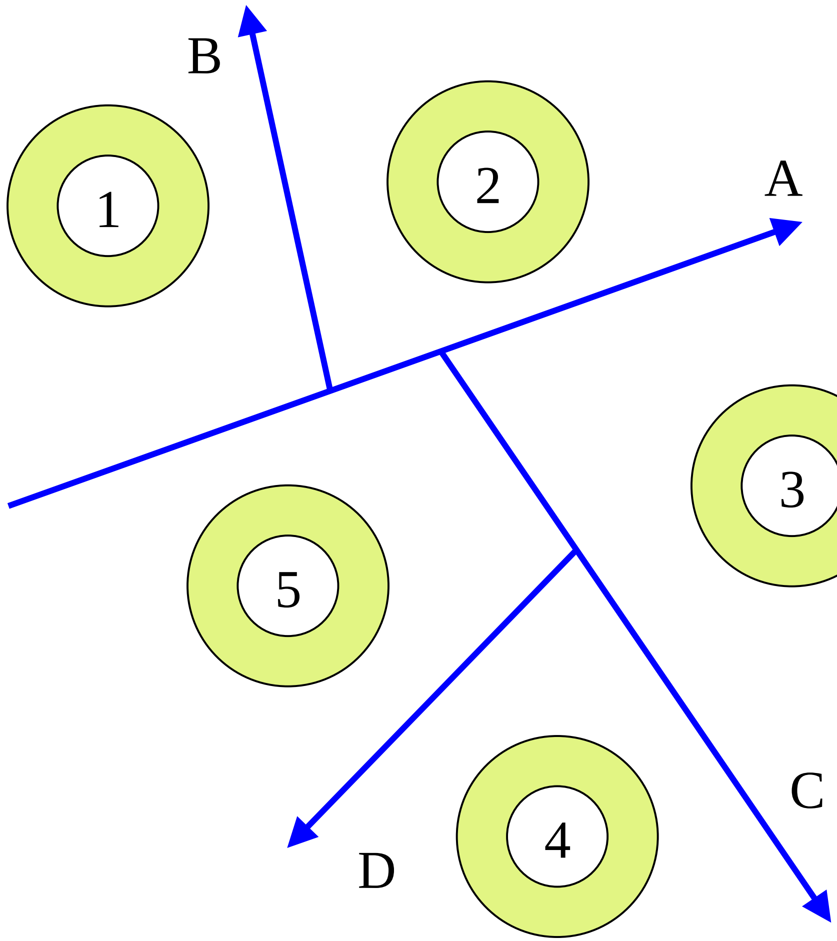


BSP Tree

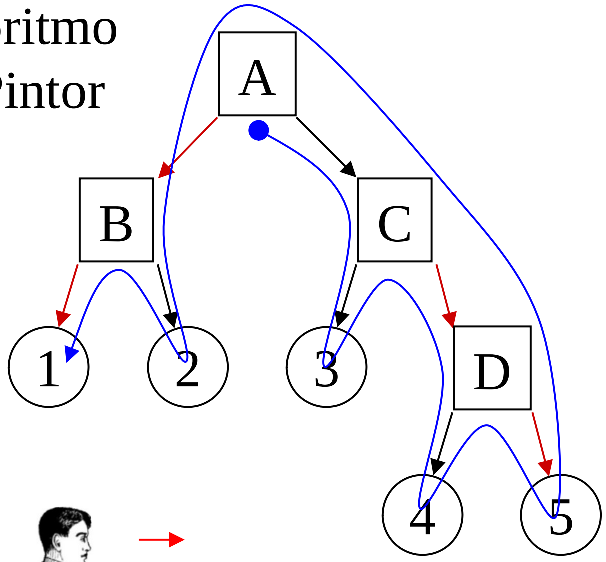


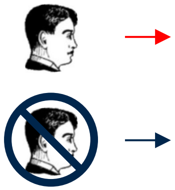
Ejemplo: Rendering Ordenado con BSP





Algoritmo
del Pintor





Nota: 5 no está mas atrás que 2, pero nunca puede ocluirlo.

Nearest Neighbour K-D Tree

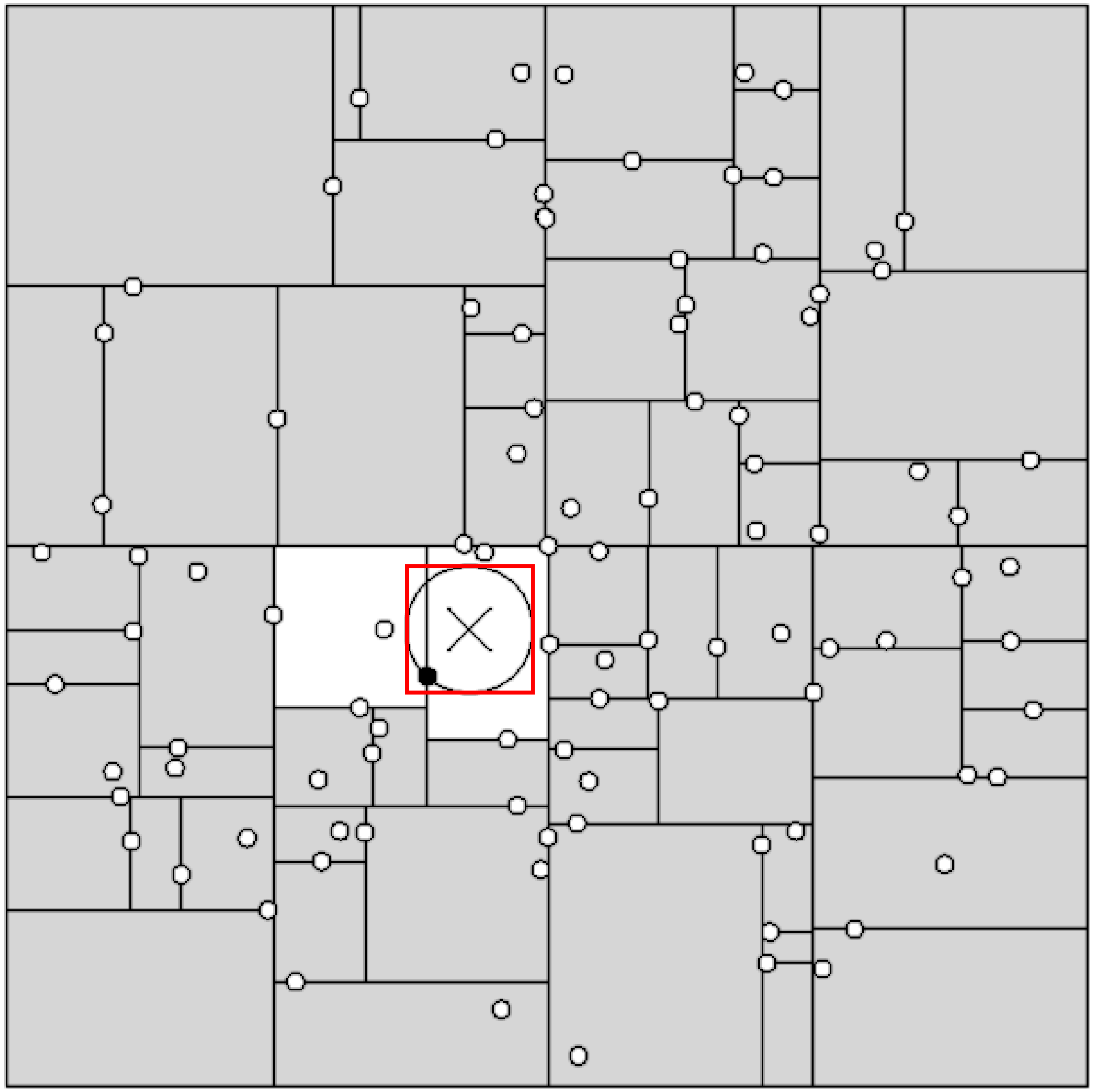
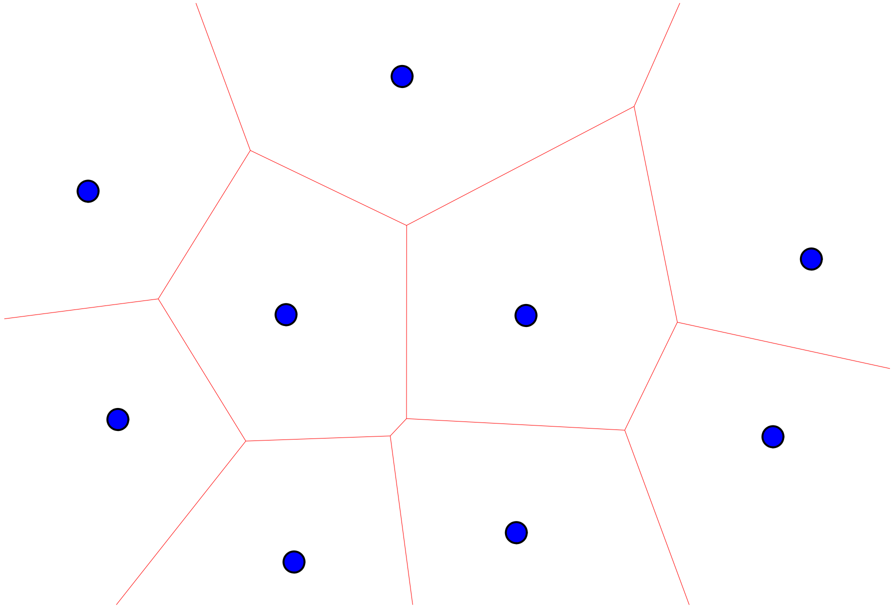
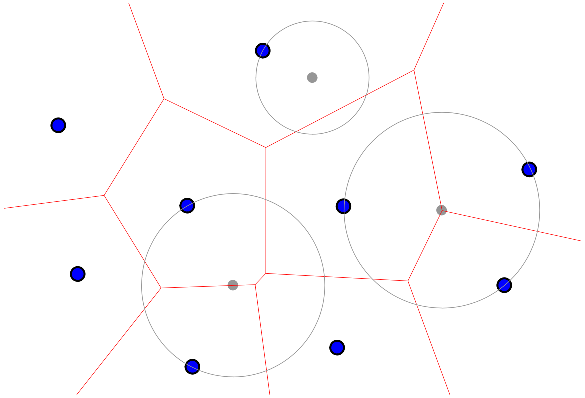
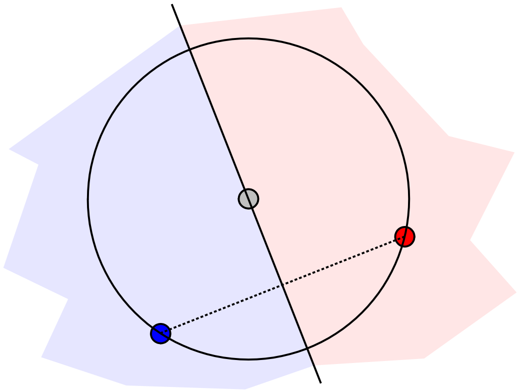


Diagrama de Voronoï







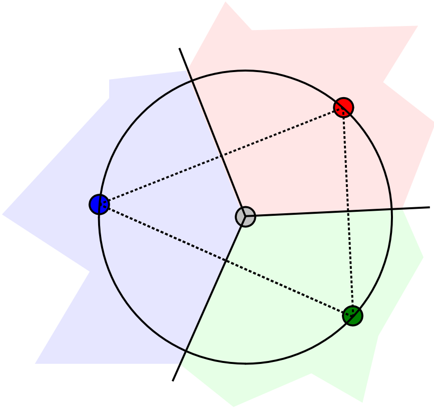
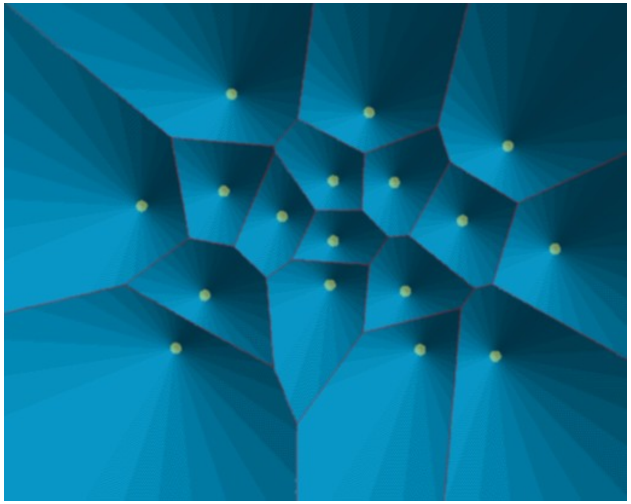
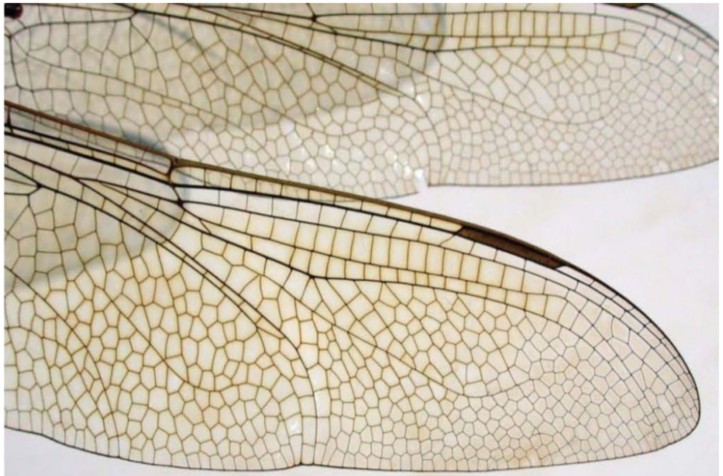
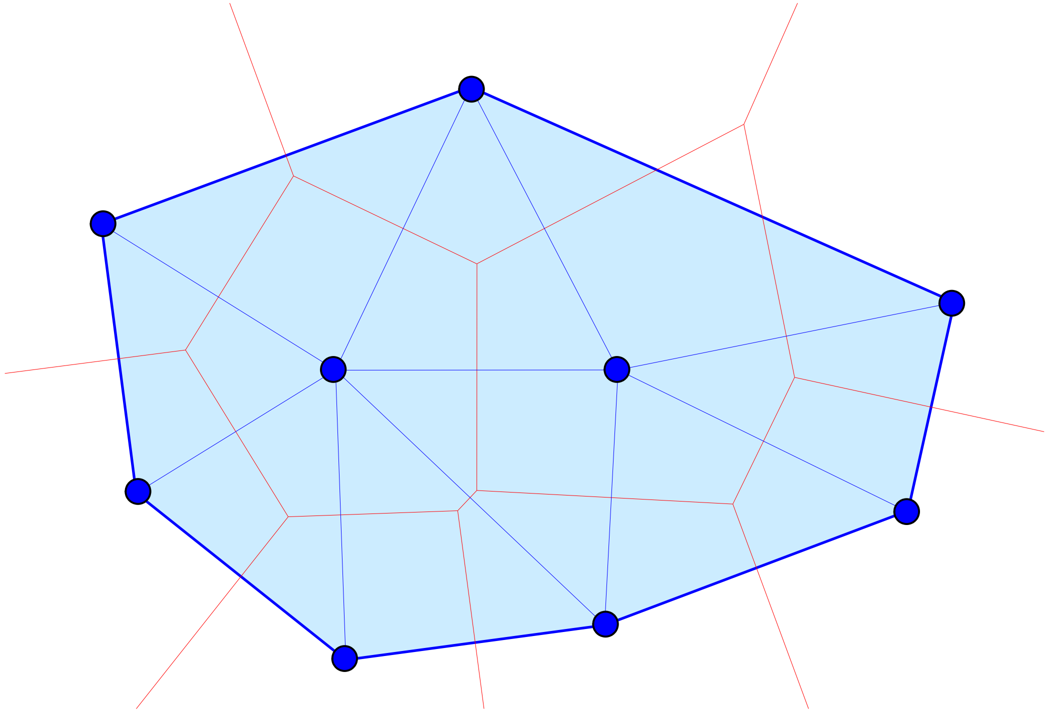


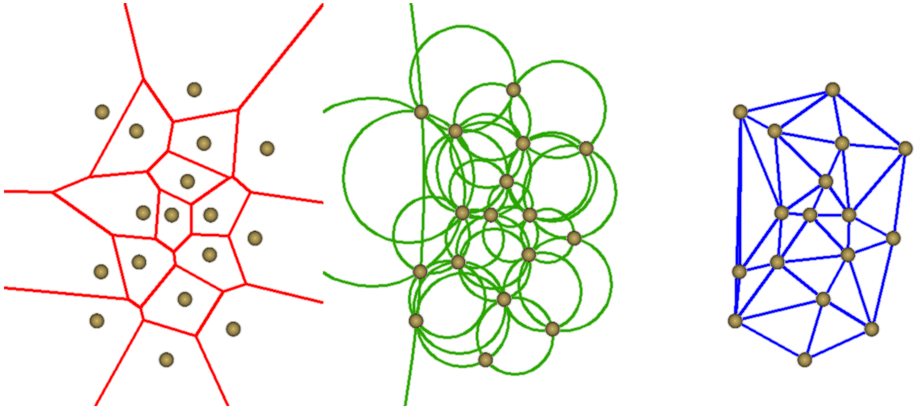
Diagrama de Voronoï

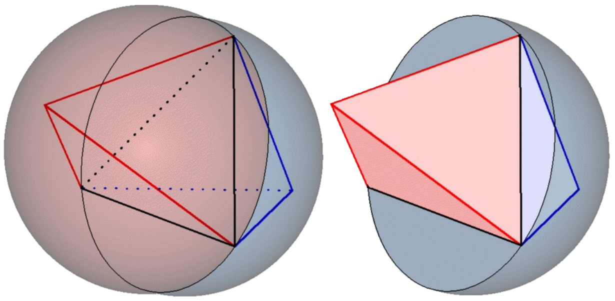




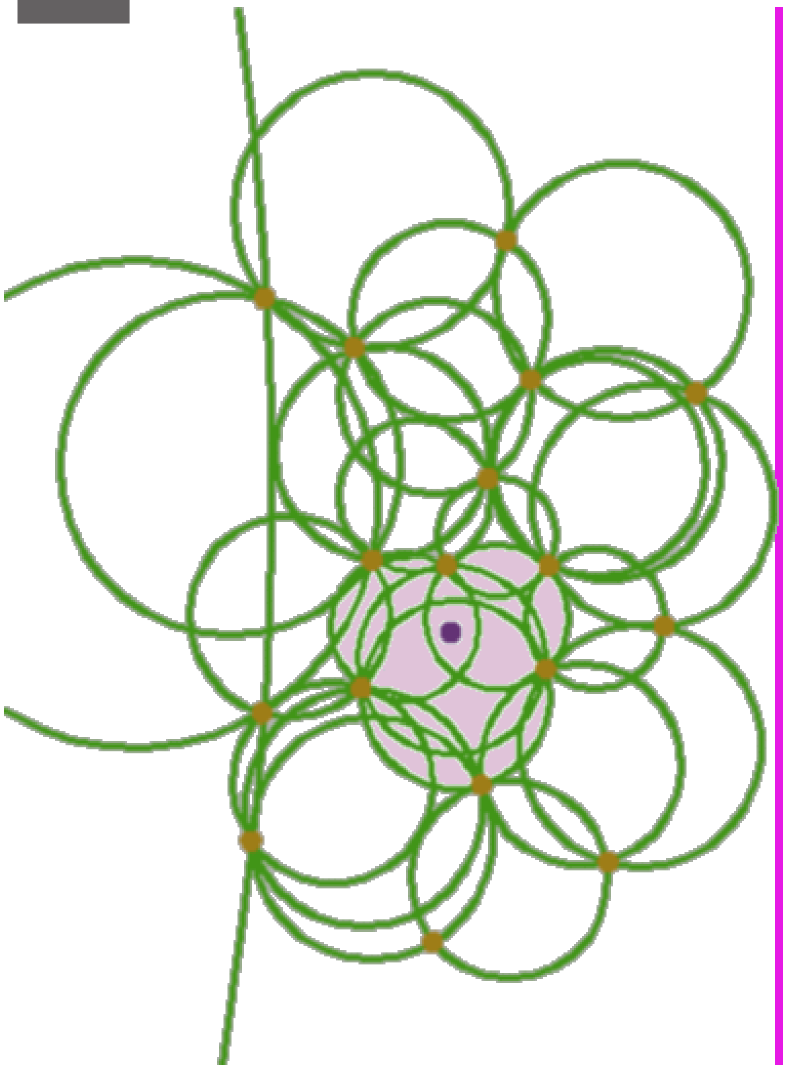

Triangulación de Delaunay



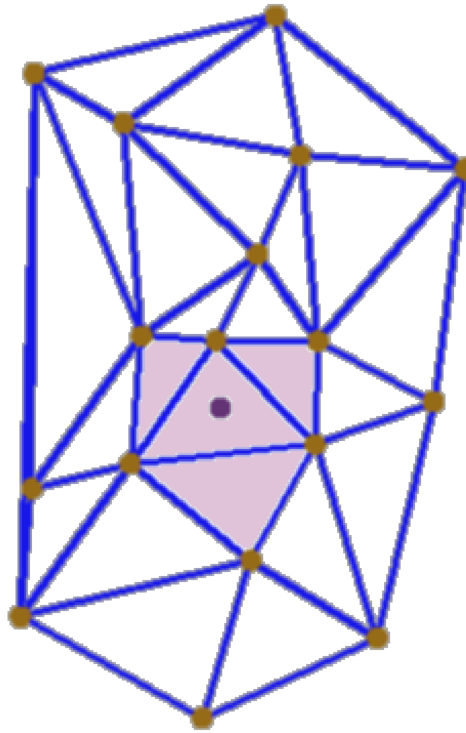




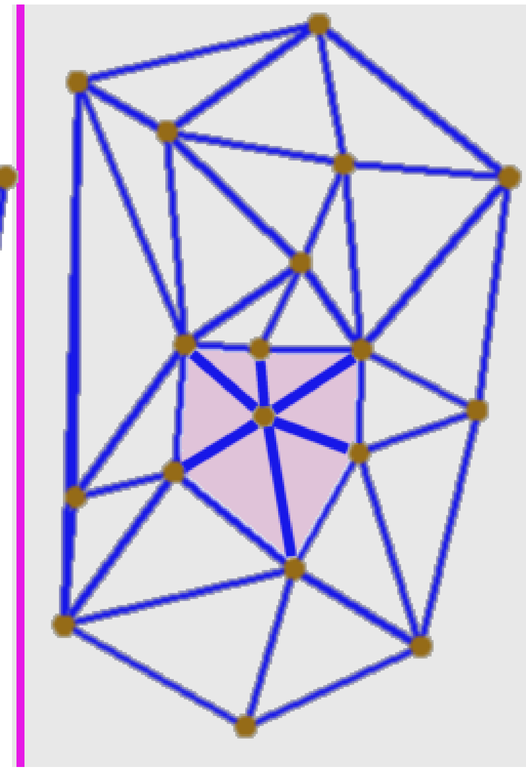
Construcción de la Triangulación Delaunay



Entorno Natural del nodo a agregar

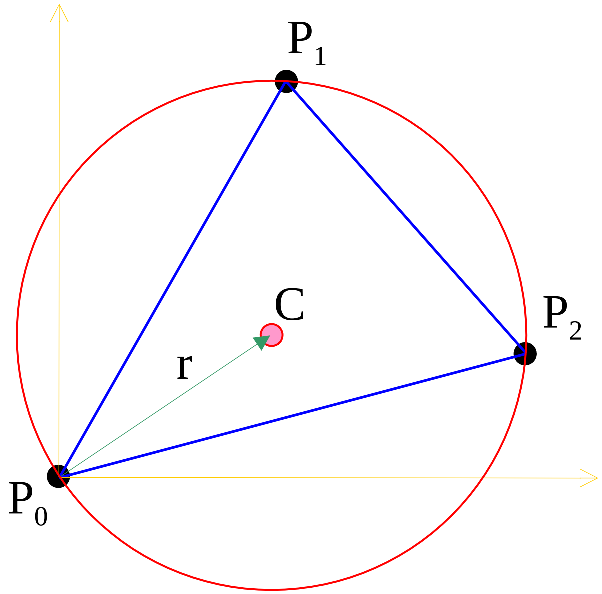


Cavidad



Retriangulación

Centro de Esfera o Circunsferencia



$r^2 = (C-P_i)^2 = C^2 - 2\, C \cdot P_i + P_i^{\,2}$

Con origen en P_2; $\mathbf{c} = C - P_2$; $\mathbf{p}_i = P_i - P_2$:

$r^2 = (C-P_2)^2 = c^2$

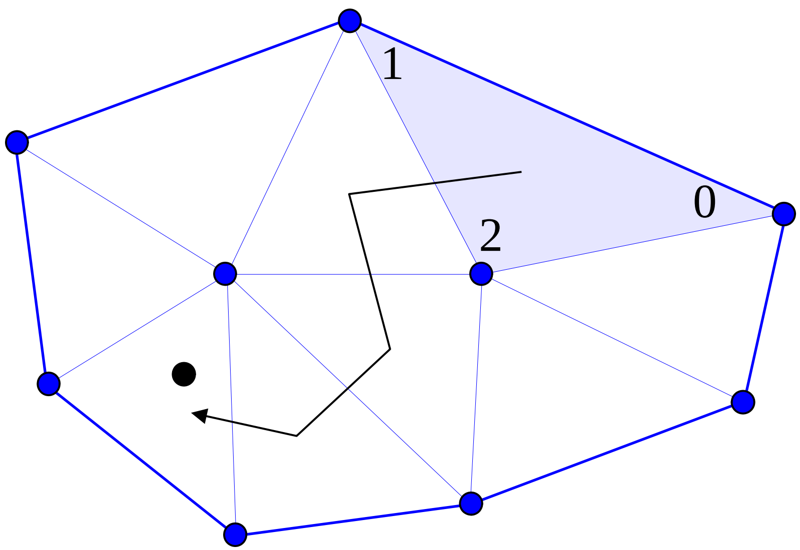
$c^2 = c^2 - 2\, \mathbf{c} \cdot \mathbf{p}_i + p_i^{\,2} \quad \Rightarrow \quad 2\, \mathbf{c} \cdot \mathbf{p}_i = p_i^{\,2} \quad (i \in \{0,1\} \text{ o } i \in \{0,1,2\} \text{ en 3D})$

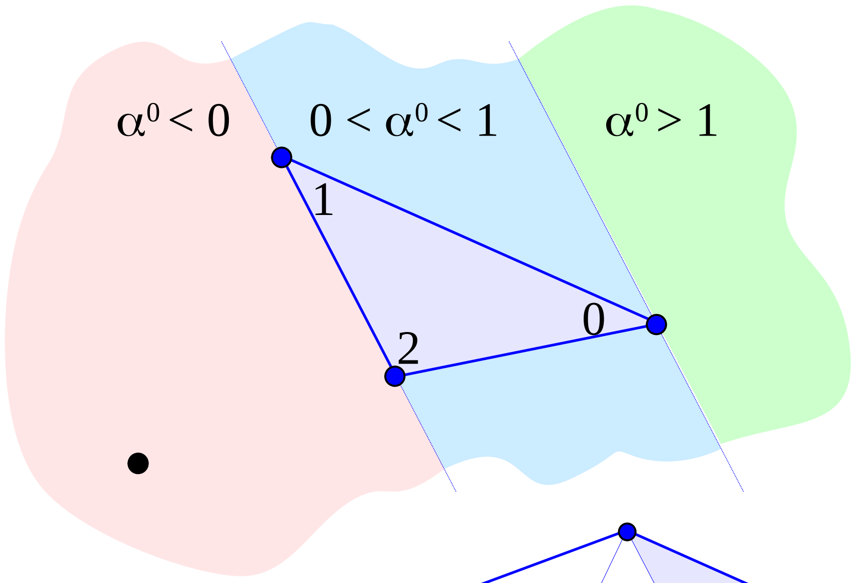
Ecuaciones cerradas (sin sistema de ecuaciones):

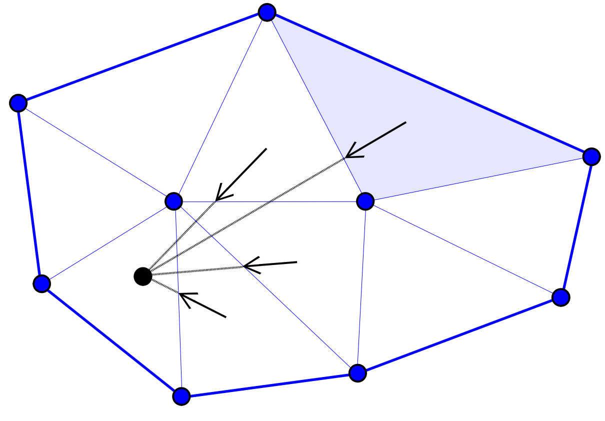
$2a^2\, \mathbf{c} = \mathbf{a} \mathrm{x} (p_1^{\,2}\, \mathbf{p}_0 - p_0^{\,2}\, \mathbf{p}_1)$ (Circunferencia, se restó P_2; $\mathbf{a} = \mathbf{p}_0 \mathrm{x} \mathbf{p}_1$)

$2v\, \mathbf{c} = \Sigma\, p_i^{\,2}\, \mathbf{p}_i^{\,*}$ (Esfera: se restó P_3; $\mathbf{p}_i^{\,*} = \mathbf{p}_{(i+1)\%3} \mathrm{\ x\ } \mathbf{p}_{(i+2)\%3}$; $v = \mathbf{p}_0 \mathrm{x} \mathbf{p}_1 \cdot \mathbf{p}_2$)

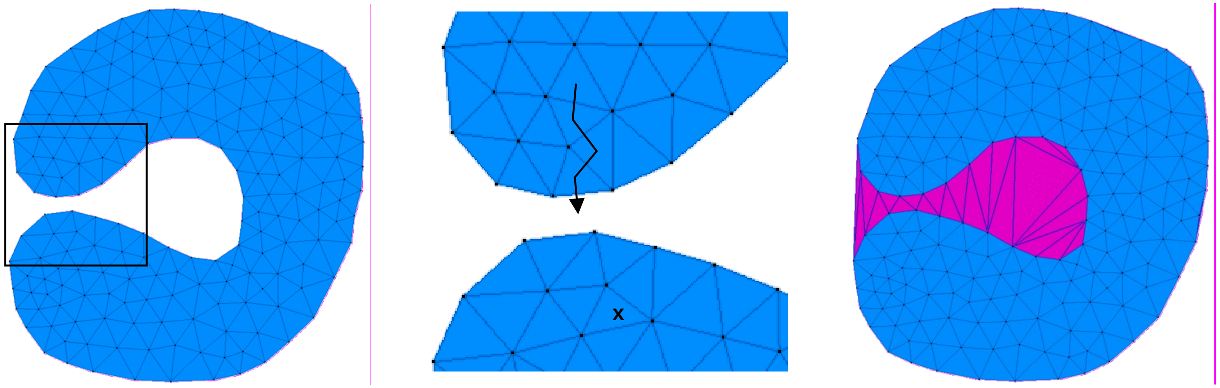
Lineal Walk / Búsqueda Lineal



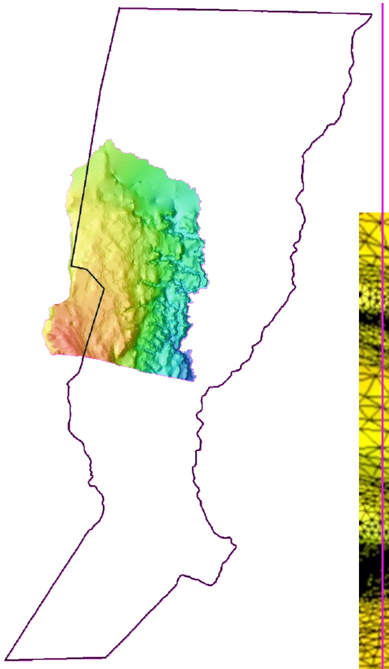




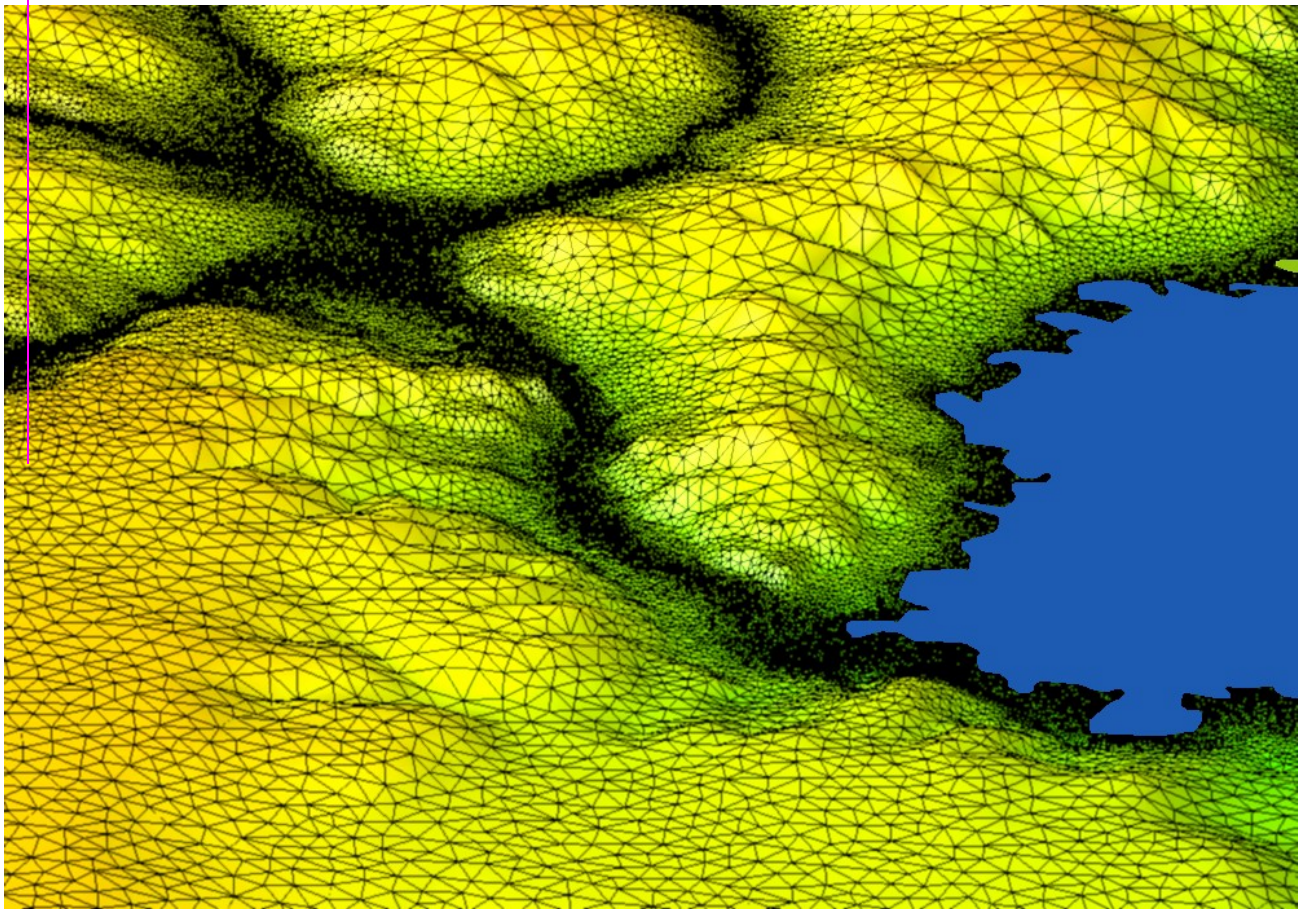
La triangulación debe ser convexa



Ejemplo: Triangulación en Interpolación



Escurrimiento de aguas de lluvia



Ejemplo: Free-Form Deformation