Unidad 9

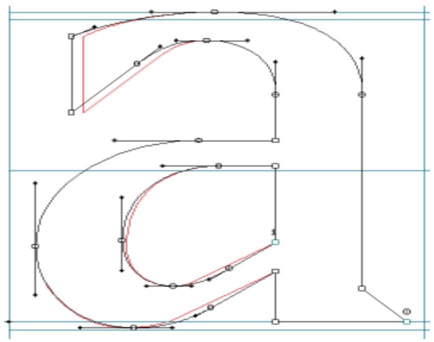
Curvas y Superficies

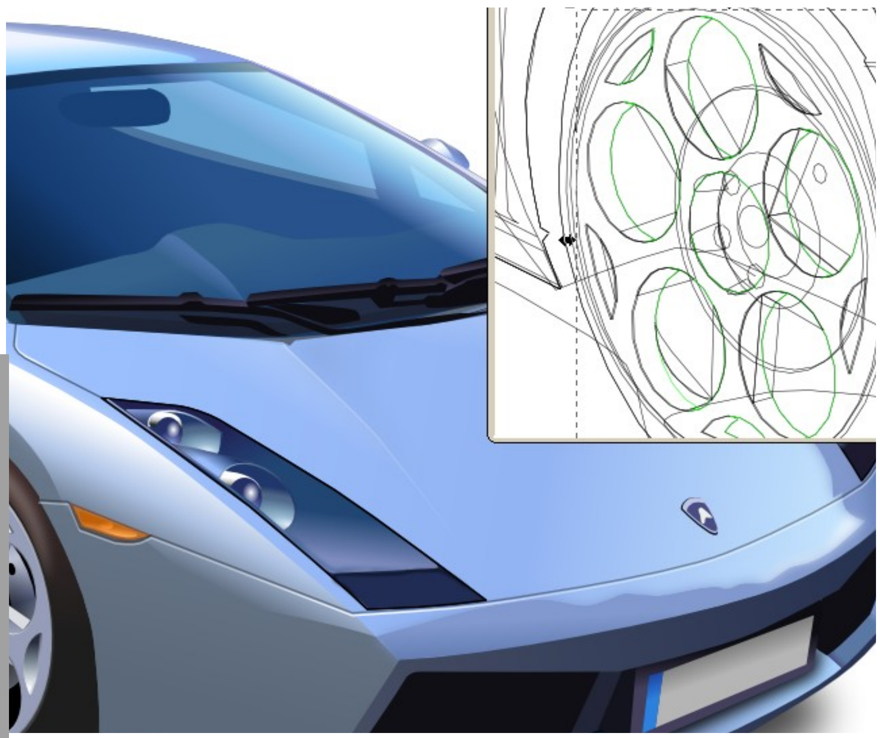
Universidad Nacional del Litoral
Facultad de Ingeniería y Ciencias Hídricas

Curvas y Superficies en CG

Uso Curvas En Computación Gráfica

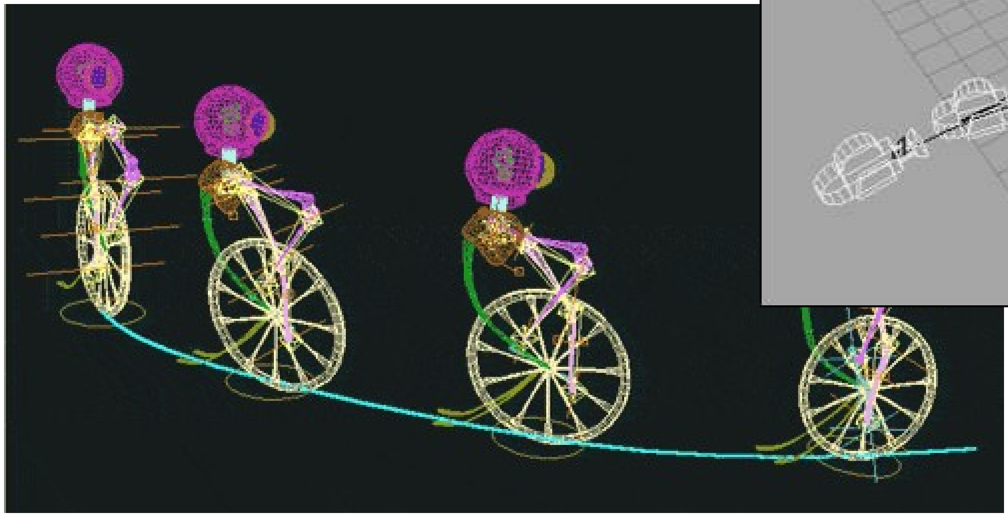
Modelar objetos “suaves” u orgánicos

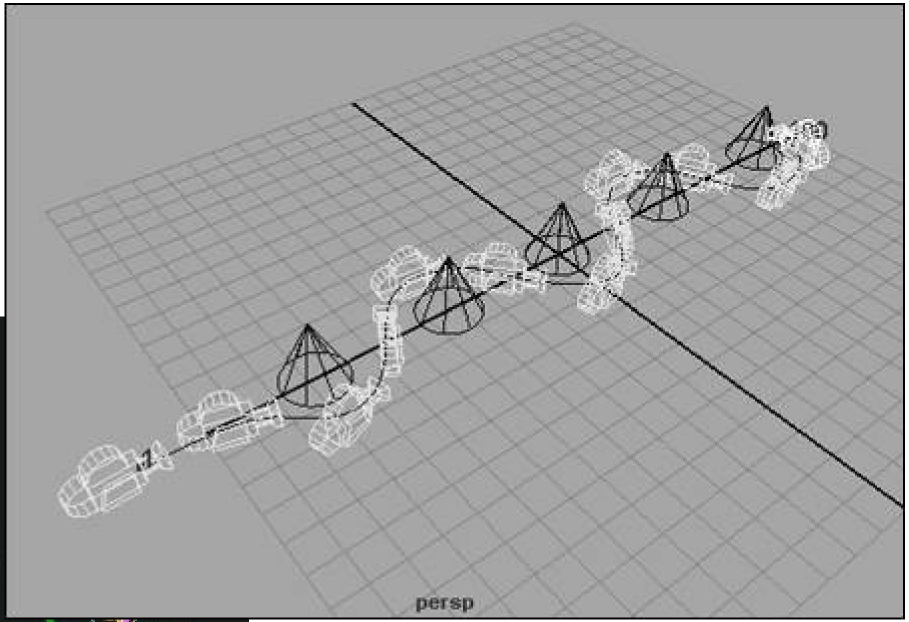




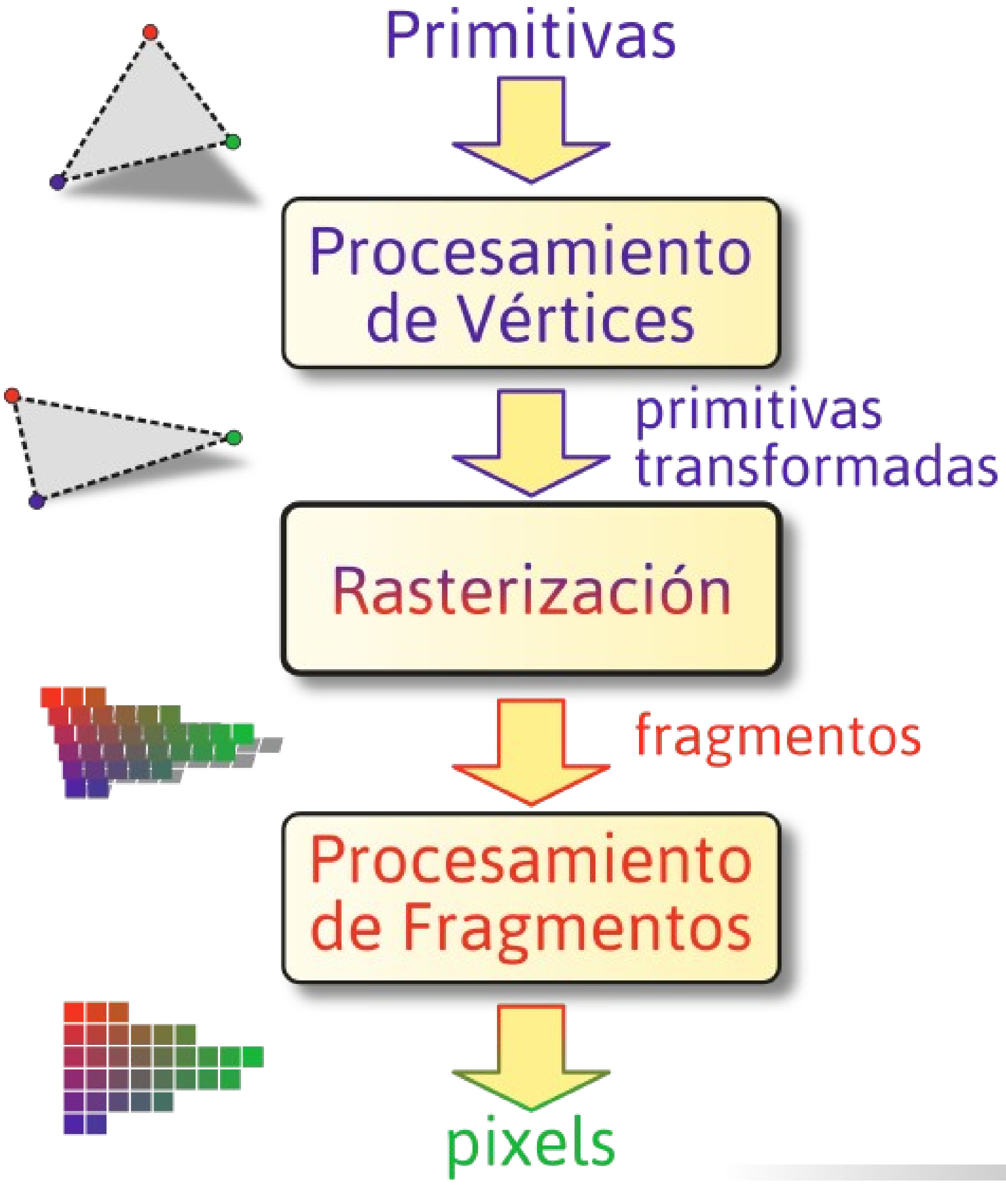
Uso Curvas En Computación Gráfica

Definir trayectorias para movimientos, cámaras, etc.





Usted NO está aquí



Definición y Representación

Algunos aspectos a tener en cuenta

Cómo define el “usuario” la forma de la curva

Qué información se guarda y qué información se calcula

Cómo se rasteriza o cómo se encuentran los puntos de la curva

Ecuaciones Explícitas

Mapeo directo entre una coordenada y la(s) otra(s):

$$y = f(x), \qquad x_0 \geq x \geq x_1$$

Ejemplo: Recta: $y = mx + b$

+ Fácil de graficar
– No todas pueden representarse

Se utilizan en la visualización de resultados de procesos experimentales

Ecuaciones Implícitas

Los puntos de la curva son los puntos que satisfacen una ecuación:

$$f(x, y) = 0$$

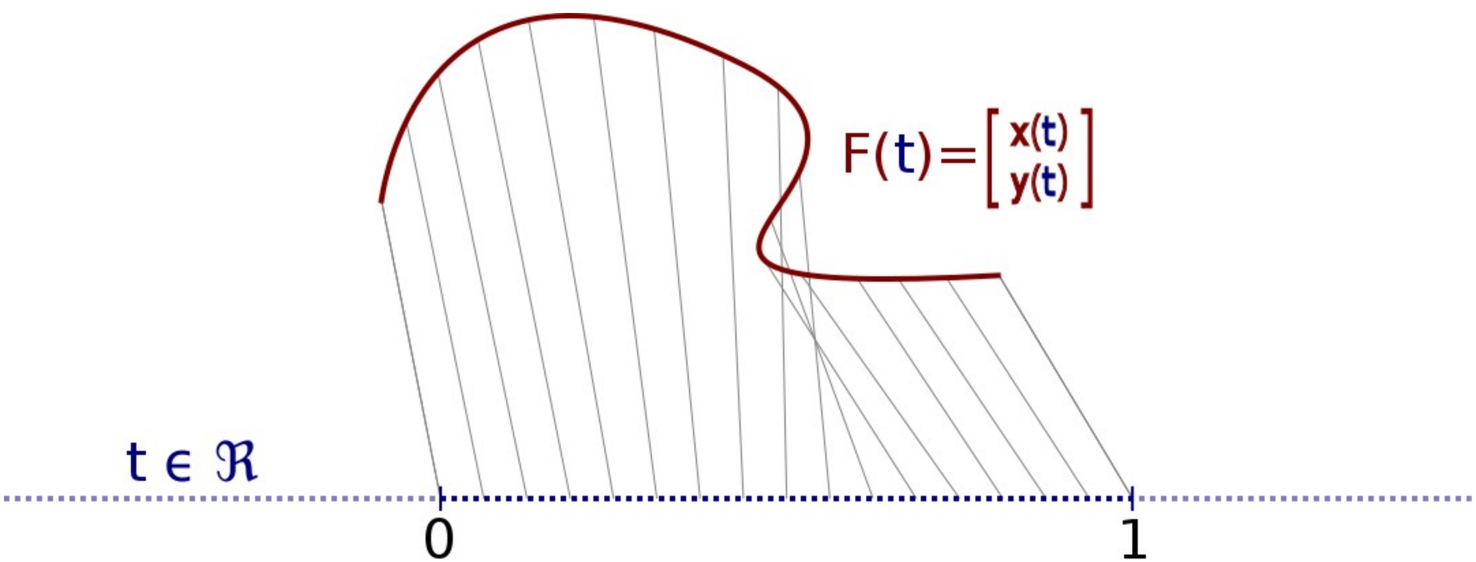
Ejemplo: Círculo: $x^2 + y^2 - r^2 = 0$

+ Es fácil asociar los miembros y coeficientes de la ecuación con propiedades de la curva
+ Es fácil saber si un punto pertenece o no
+ Son "orientables"
– Difícil de graficar en CG

Ecuaciones Paramétricas

Mapeo entre un parámetro arbitrario t y los puntos de la curva:

$$\begin{array}{l} x = f(t) \\ y = g(t) \end{array}, \qquad t_0 \leq t \leq t_1$$



Ecuaciones Paramétricas

Mapeo entre un parámetro arbitrario t y los puntos de la curva:

$$\begin{matrix} x = f(t) \\ y = g(t) \end{matrix}, \qquad t_0 \leq t \leq t_1$$

Ejemplo: Recta: $x = t, \quad y = mt + b, \quad x_0 \leq t \leq x_1$

Ejemplo: Círculo: $x = r\cos(t), \quad y = r\,sen(t),$

$$0 \leq t < 2\pi$$

+ Fácil de graficar

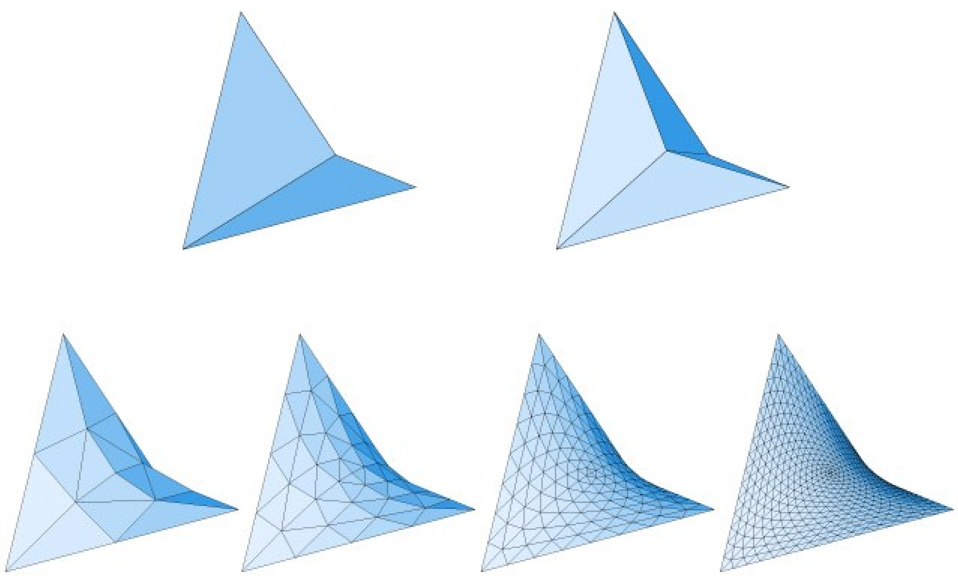
+ Pueden representar todo tipo de curvas

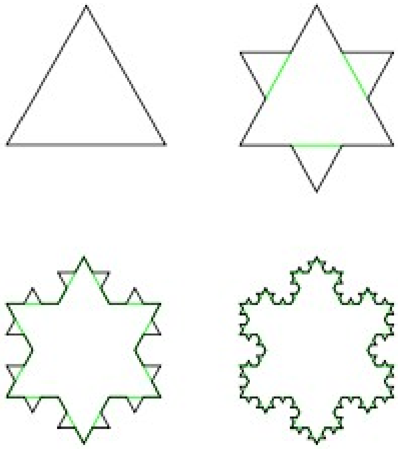
Definición y Representación

Generativas o Procedurales

Proveen un procedimiento para generar los puntos de la curva

Ejemplos: Subdivisiones, Fractales





$$\begin{aligned} x &= f(t) \\ y &= g(t), \\ z &= h(t) \end{aligned} \quad \Rightarrow \quad \boldsymbol{x} = \boldsymbol{F}(t) = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix}, \qquad t_0 \leq t \leq t_1$$

t puede verse como el tiempo:

F(*t*) responde a ¿Donde está el lápiz en el tiempo t?

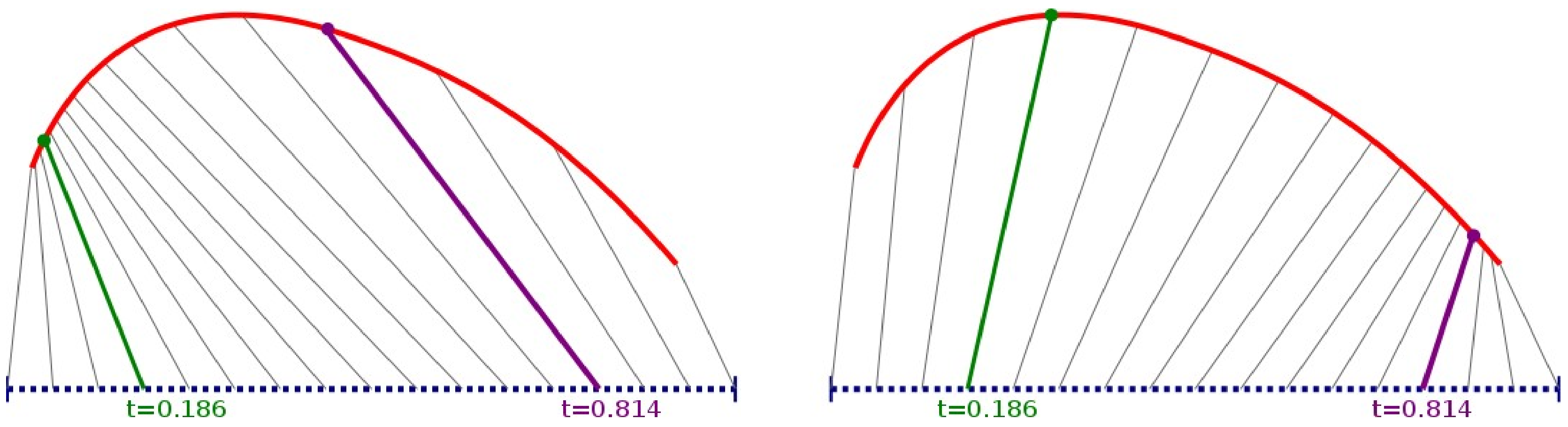
Introducen un elemento “invisible”: velocidad

¿Donde está el punto para *t*=0.5?

Curvas Paramétricas

$$\begin{aligned} x &= f(t) \\ y &= g(t), \\ z &= h(t) \end{aligned} \quad \Rightarrow \quad \boldsymbol{x} = \boldsymbol{F}(t) = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix}, \qquad t_0 \leq t \leq t_1$$

t puede verse como el tiempo:



Reparametrización

Normalización del dominio:

$$t \in [t_0, t_1] \;\Rightarrow\; t = (1-u)t_0 + ut_1 = t_0 + u(t_1 - t_0)$$

Velocidad constante = por longitud de arco

Implica resolver para t:

$$u = \int_0^t \sqrt{[\boldsymbol{f}'(v)]^2}\,dv$$

Curvas Paramétricas

Forma de la funciones f,g,h:

- Necesidad de definir y editar fácilmente
  - Pocos parámetros
  - Significado de los parámetros
  - Eficiencia

- Más usadas:
  - **Polinomios/Series de Potencias**
  - Trigonométricas

Series de Potencia y Funciones Base

Polinomios de grado N:

$$\boldsymbol{F}(t)=\sum_{i=0}^{N} \boldsymbol{a}_{\boldsymbol{i}}\, t^{i}$$

N+1 parámetros (orden)

Ejemplo: Segmento de Recta

$$\boldsymbol{F}(\alpha)=(1-\alpha)\,\boldsymbol{p}_{\boldsymbol{0}}+\alpha\,\boldsymbol{p}_{\boldsymbol{1}} \;\Rightarrow\; \boldsymbol{F}(t)=\underbrace{\boldsymbol{a}_{\boldsymbol{0}}}_{\boldsymbol{p}_0} t^{0}+\underbrace{\boldsymbol{a}_{\boldsymbol{1}}}_{\boldsymbol{p}_1-\boldsymbol{p}_0} t^{1}$$

Series de Potencia y Funciones Base

Es más fácil definirlas mediante

- puntos de control (o polígono de control)
- funciones mezcladoras (blending functions)

$$\boldsymbol{F}(t)=\sum_{k=0}^{N} \boldsymbol{p_k} B_k(t)$$

Orden = Cantidad de puntos de control = Grado+1

Blending functions:
(polinomios de Bernstein)

$$B_i^n(u) = C_i^n u^i (1-u)^{n-i}$$

$$C_i^n = \frac{n!}{i!(n-i)!}$$

Ejemplo orden=3, grado=n=2:

$$B_0^2(u) = \frac{2}{0!(2-0)!} u^0 (1-u)^{2-0} = \frac{2}{1(2)} u^0 (1-u)^2 = \mathbf{(1-u)^2}$$

$$B_1^2(u) = \frac{2!}{1!(2-1)!} u^1 (1-u)^{2-1} = \frac{2}{1(1)} u^1 (1-u)^1 = \mathbf{2u(1-u)}$$

$$B_2^2(u) = \frac{2!}{2!(2-2)!} u^2 (1-u)^{2-2} = \frac{2}{2(1)} u^2 (1-u)^0 = \mathbf{u^2}$$

Blending functions:
(polinomios de Bernstein)

$$B_i^n(u) = C_i^n u^i (1-u)^{n-i}$$

$$C_i^n = \frac{n!}{i!(n-i)!}$$

Ejemplo orden=4, grado=n=3:

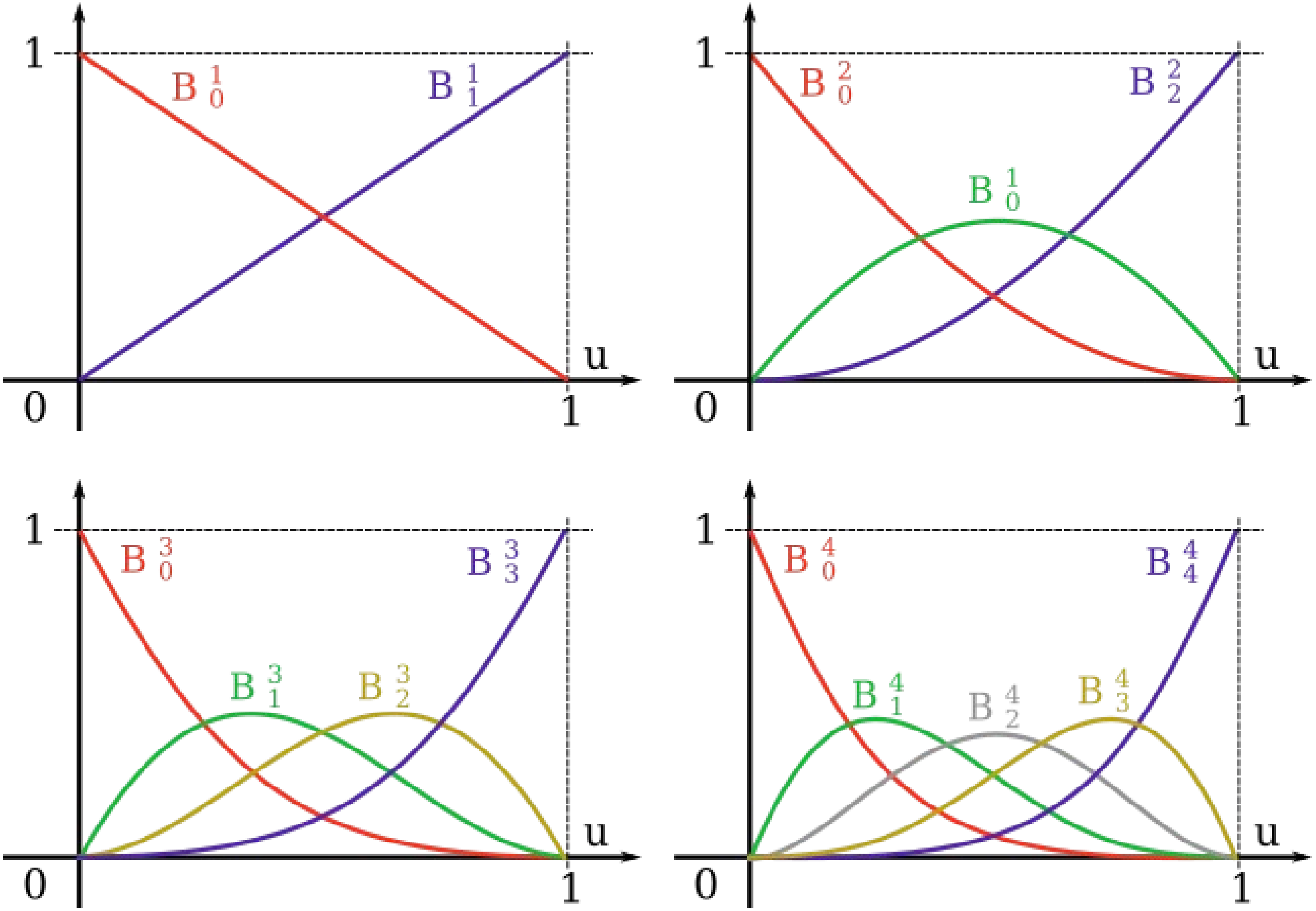
$$B_0^3(u) = \frac{3!}{0!(3-0)!} u^0 (1-u)^{3-0} = \frac{6}{1(6)} u^0 (1-u)^3 = \mathbf{(1-u)^3}$$

$$B_1^3(u) = \frac{3!}{1!(3-1)!} u^1 (1-u)^{3-1} = \frac{6}{1(2)} u^1 (1-u)^2 = \mathbf{3u(1-u)^2}$$

$$B_2^3(u) = \frac{3!}{2!(3-2)!} u^2 (1-u)^{3-2} = \frac{6}{2(1)} u^2 (1-u)^1 = \mathbf{3u^2(1-u)}$$

$$B_3^3(u) = \frac{3!}{3!(3-3)!} u^3 (1-u)^{3-3} = \frac{6}{6(1)} u^3 (1-u)^0 = \mathbf{u^3}$$

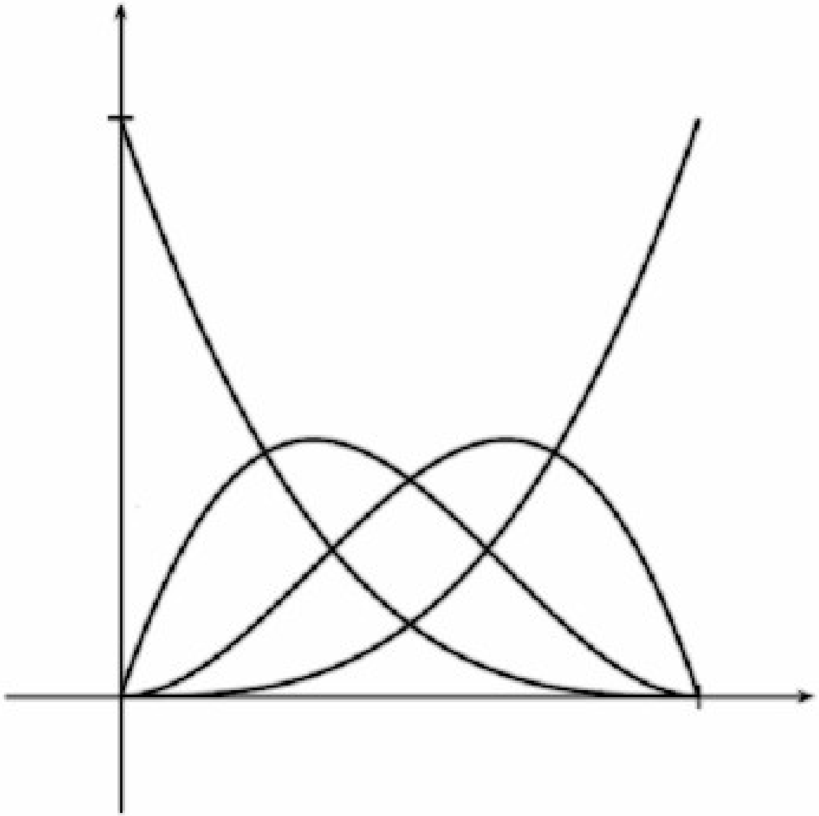
Curvas de Bezier: Blending Functions



Blending functions:
(polinomios de Bernstein)

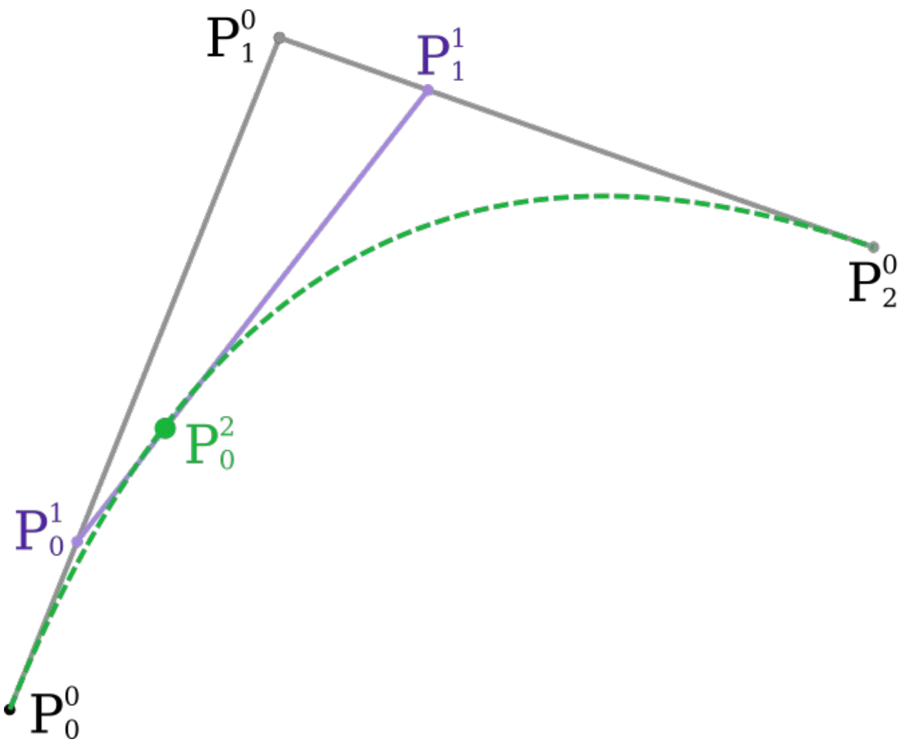
$$B_i^n(u)=C_i^n u^i(1-u)^{n-i}$$

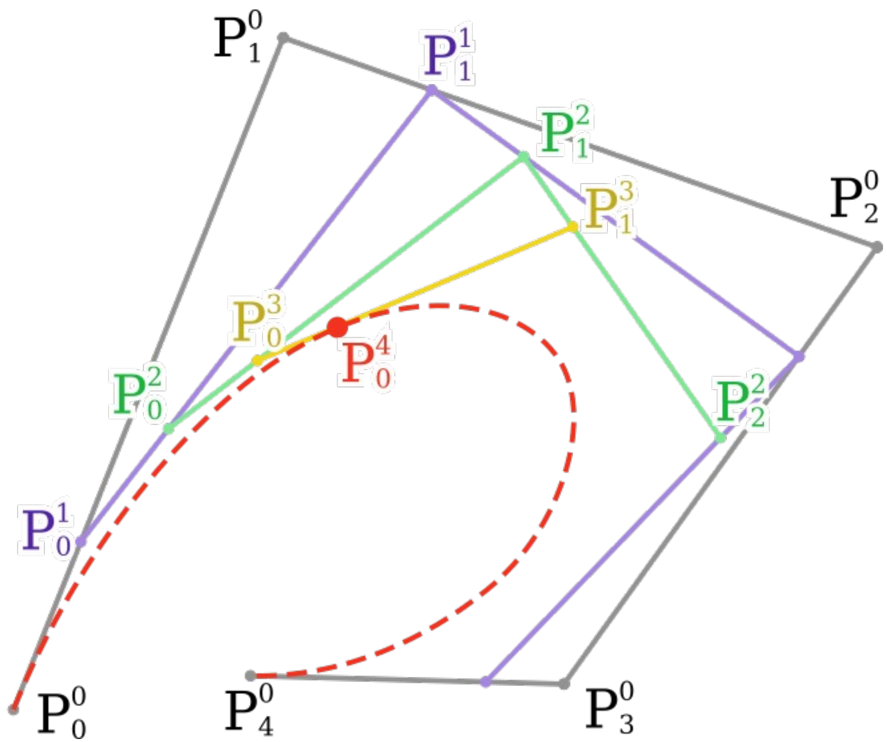
$$C_i^n=\frac{n!}{i!(n-i)!}$$



- Simetría respeto a u=0.5
- Interpola puntos extremos
- Aproxima puntos interiores
- Control global
- Los puntos de la curva son combinaciones afines convexas de los puntos de control
- Unicidad
- Suavidad

Algoritmo De Casteljau





¿Qué tiene que ver con los polinomios de Bernstein?

Algoritmo De Casteljau

Ejemplo n=3:

$$P_{01}=(1-u)P_0+uP_1 \qquad P_{12}=(1-u)P_1+uP_2$$

$$P(u)=(1-u)\underbrace{((1-u)P_0+uP_1)}_{P_{01}}+u\underbrace{((1-u)P_1+uP_2)}_{P_{12}}$$

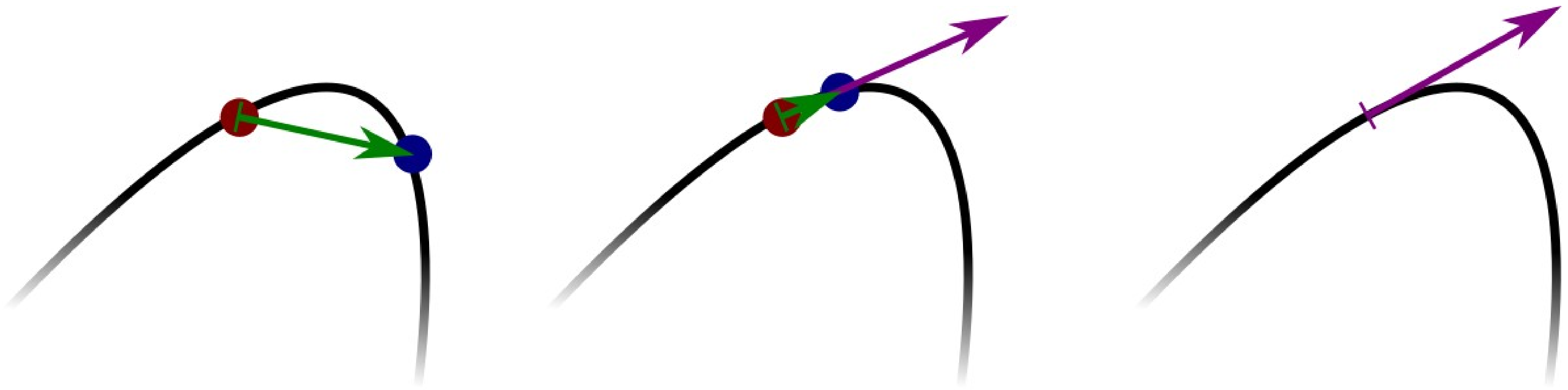
$$P(u)=(1-u)^2P_0+(1-u)uP_1+u(1-u)P_1+u^2P_2$$

$$P(u)=\underbrace{\mathbf{(1-u)^2}}_{B_0^2}P_0+\underbrace{\mathbf{2u(1-u)}}_{B_1^2}P_1+\underbrace{\mathbf{u^2}}_{B_2^2}P_2$$

Derivada de una Curva

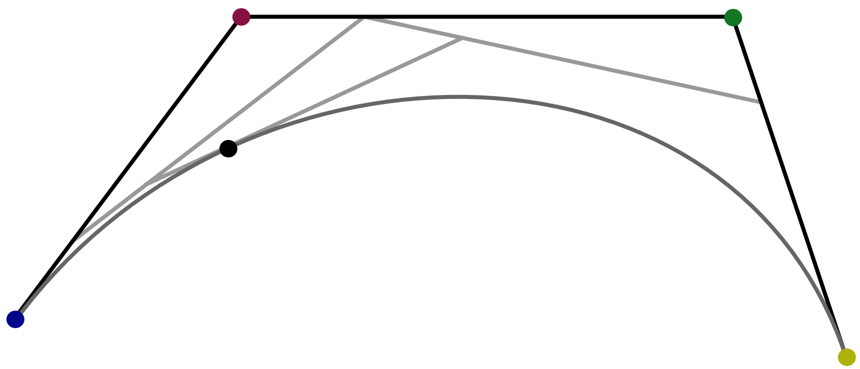
¿**Qué es** la derivada de un curva?

$$\frac{dP(u)}{du} = \lim_{\Delta u \to 0} \frac{P(u+\Delta u) - P(u)}{\Delta u}$$



Derivada de una Curva de Bezier

$$\frac{dP}{du} = d\,\frac{\left(\overbrace{B_0^3}^{(1-u)^3} P_0 + \overbrace{B_1^3}^{3(1-u)^2 u} P_1 + \overbrace{B_2^3}^{3(1-u)u^2} P_2 + \overbrace{B_3^3}^{u^3} P_3\right)}{du}$$



Derivada de una Curva de Bezier

$$\frac{dP}{du} = d\,\frac{\left(\overbrace{B_0^3}^{(1-u)^3} P_0 + \overbrace{B_1^3}^{3(1-u)^2 u} P_1 + \overbrace{B_2^3}^{3(1-u)u^2} P_2 + \overbrace{B_3^3}^{u^3} P_3\right)}{du}$$

$$\frac{dP}{du} = d\,\frac{\left((1-u)^3 P_0 + 3(1-u)^2 u P_1 + 3(1-u)u^2 P_2 + u^3 P_3\right)}{du}$$

$$\frac{dP}{du} = \frac{d(1-u)^3}{du} P_0 + 3\frac{d(1-u)^2 u}{du} P_1 + 3\frac{d(1-u)u^2}{du} P_2 + \frac{du^3}{du} P_3$$

$$\frac{dP}{du} = \left(-3(1-u)^2\right)P_0 + 3\left(-2(1-u)u + 1(1-u)^2\right)P_1 +$$

$$+\ 3\left(2u(1-u) - u^2\right)P_2 + \left(3u^2\right)P_3$$

Derivada de una Curva de Bezier

$$\frac{dP}{du} = \left(-3\underbrace{(1-u)^2}_{B_0^2}\right)P_0 + 3\left(-\underbrace{2(1-u)u}_{B_1^2}+1\underbrace{(1-u)^2}_{B_0^2}\right)P_1 +$$

$$+ 3\left(\underbrace{2u(1-u)}_{B_1^2}-\underbrace{u^2}_{B_2^2}\right)P_2 + \left(3\underbrace{u^2}_{B_2^2}\right)P_3$$

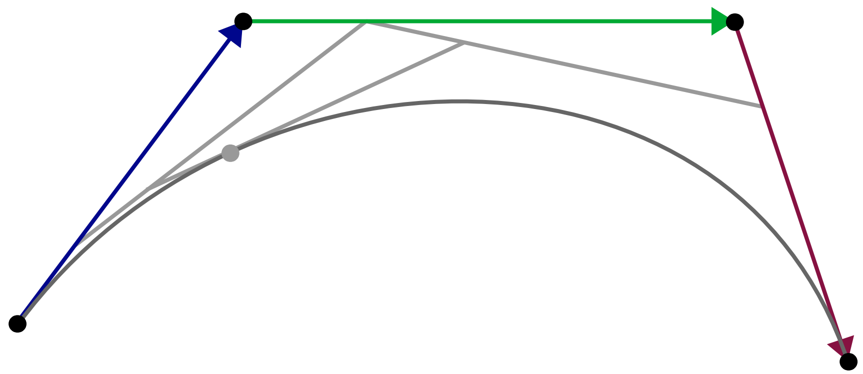
$$\frac{dP}{du} = -3B_0^2(P_0) + -3B_1^2(P_1) + 3B_0^2(P_1) +$$

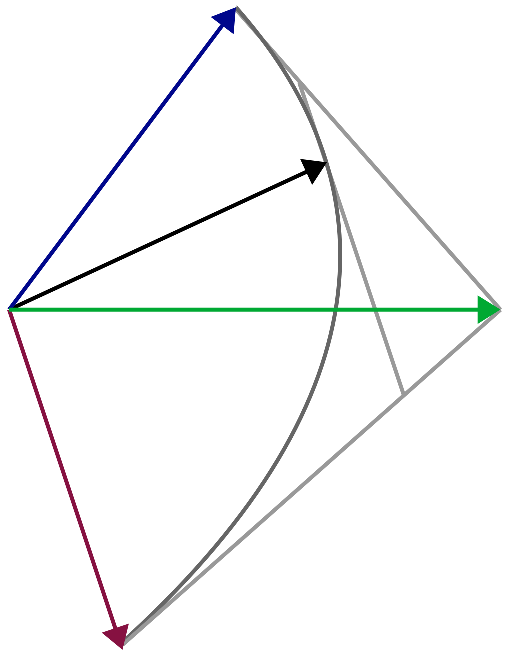
$$+ 3B_1^2(P_2) + -3B_2^2(P_2) + 3B_2^2(P_3)$$

$$\frac{dP}{du} = 3\left(B_0^2(P_1-P_0) + B_1^2(P_2-P_1) + B_2^2(P_3-P_2)\right)$$

Derivada de una Curva de Bezier

$$\frac{dP}{du} = 3\left(B_0^2\left(P_1 - P_0\right) + B_1^2\left(P_2 - P_1\right) + B_2^2\left(P_3 - P_2\right)\right)$$





Derivada de una Curva de Bezier

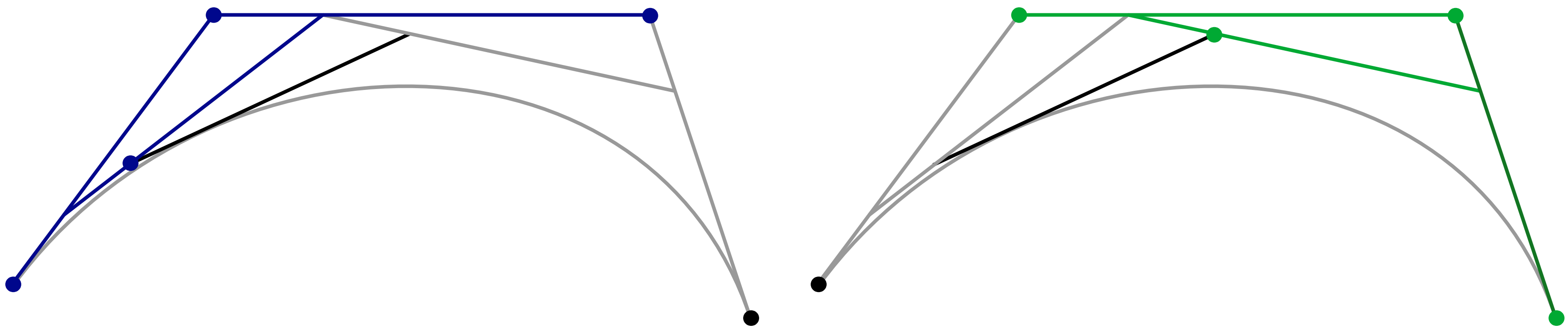
$$\frac{dP}{du} = 3\left(B_0^2\left(P_1 - P_0\right) + B_1^2\left(P_2 - P_1\right) + B_2^2\left(P_3 - P_2\right)\right)$$

$$\frac{dP}{du} = 3\left(B_0^2 P_1 - B_0^2 P_0 + B_1^2 P_2 - B_1^2 P_1 + B_2^2 P_3 - B_2^2 P_2\right)$$

$$\frac{dP}{du} = \underbrace{3}_{n}\left(\underbrace{\left(B_0^2 P_1 + B_1^2 P_2 + B_2^2 P_3\right)}_{P_{123}} - \underbrace{\left(B_0^2 P_0 + B_1^2 P_1 + B_2^2 P_2\right)}_{P_{012}}\right)$$

Derivada de una Curva de Bezier

$$\frac{dP}{du} = \underbrace{3}_{n}\left(\underbrace{\left(B_0^2 P_1 + B_1^2 P_2 + B_2^2 P_3\right)}_{P_{123}} - \underbrace{\left(B_0^2 P_0 + B_1^2 P_1 + B_2^2 P_2\right)}_{P_{012}}\right)$$



Derivada de una Curva de Bezier

Globalmente:

$$\frac{d}{du}P = n\sum_{i=0}^{n-1} B_i^{n-1}\Delta_i$$

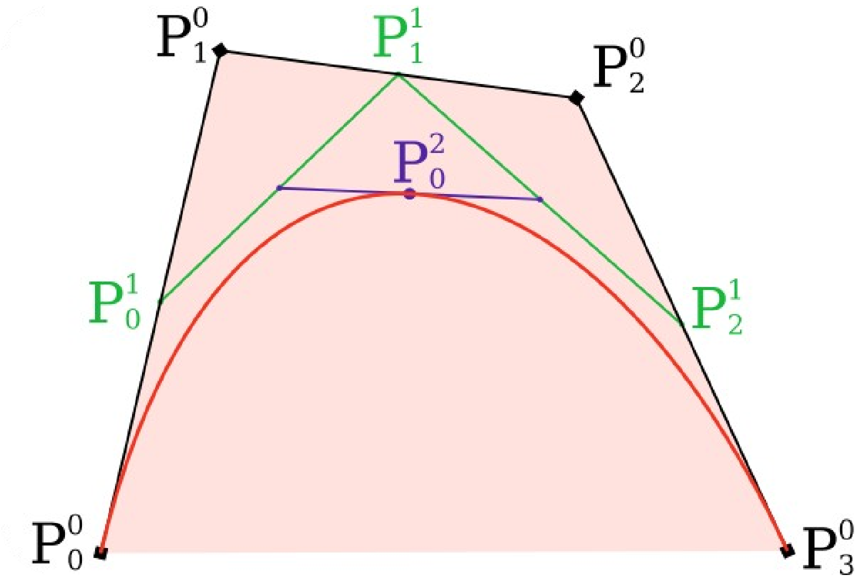
Propiedad: Variation Diminishing

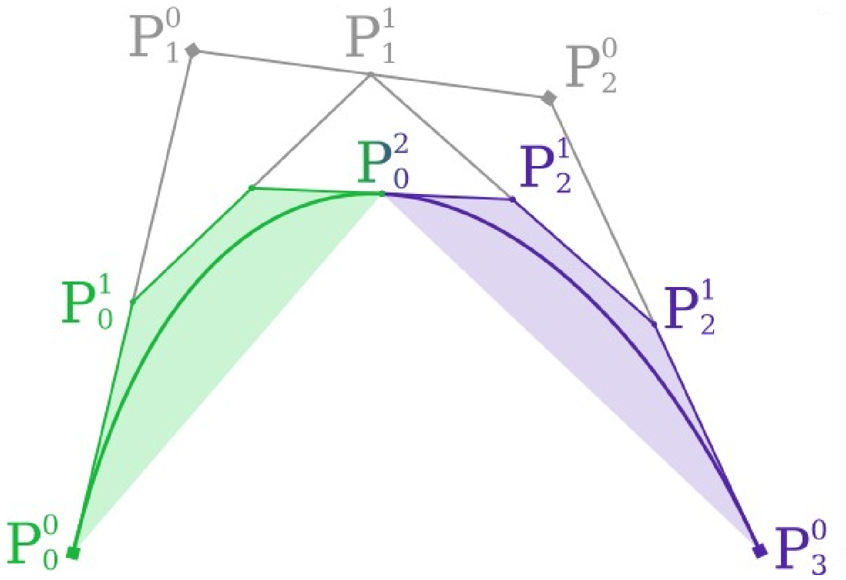
Localmente:

$$\frac{d}{du}P_0^n = n\Delta_0^{n-1} = n\left(P_i^{n-1} - P_0^{n-1}\right)$$

Último segmento de DC tangente a la curva

Subvidisión de Curvas de Bezier

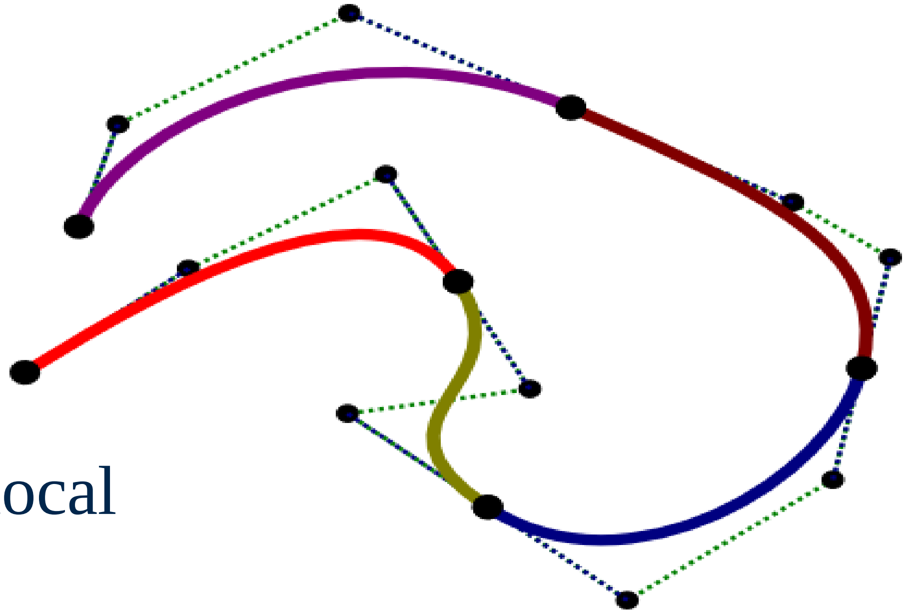




De Casteljau genera los nuevos puntos de control

El tamaño del Convex-Hull se reduce cuadráticamente

Piecewise Bézier Curves (Bézier por tramos)



+ Se tiene control local

¿Cuantos tramos usar?
¿Cuan bien aproximo la forma que busco?
¿Cuán complicado es cada tramo?

Continuidad Paramétrica vs Geométrica

¿Cómo se pegan?

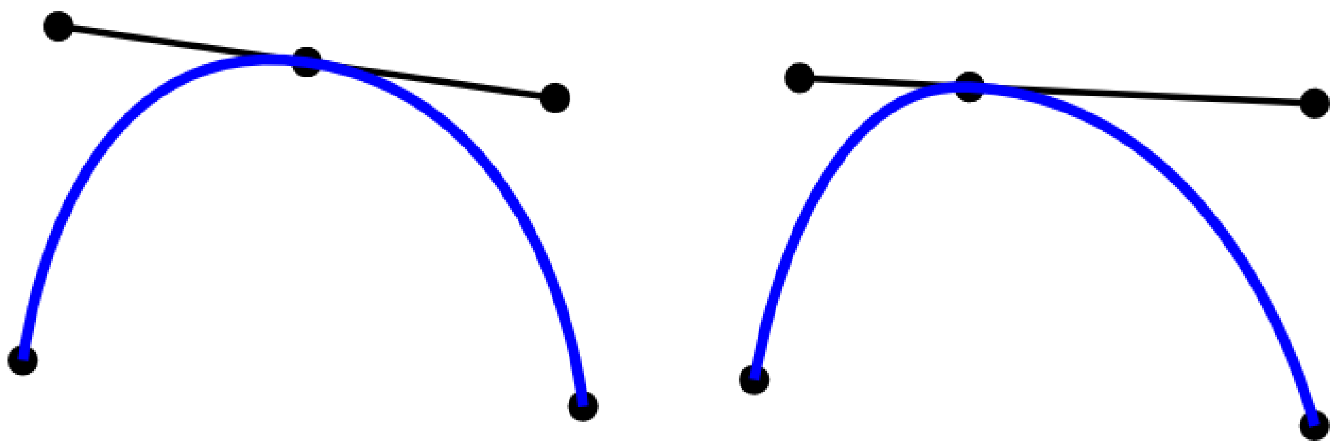
Continuidad **Paramétrica**

Derivadas iguales

Continuidad **Geométrica**

Derivadas Proporcionales

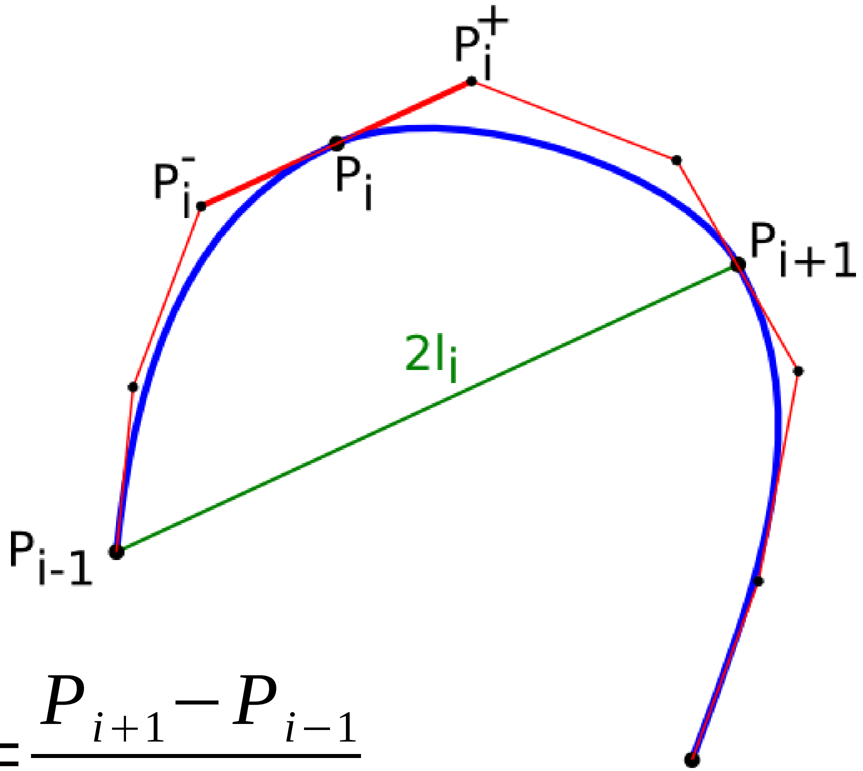
Independiente de la parametrización



Splines de Catmull-Rom

$$P_i^- = P_i - \frac{l_i}{3}$$

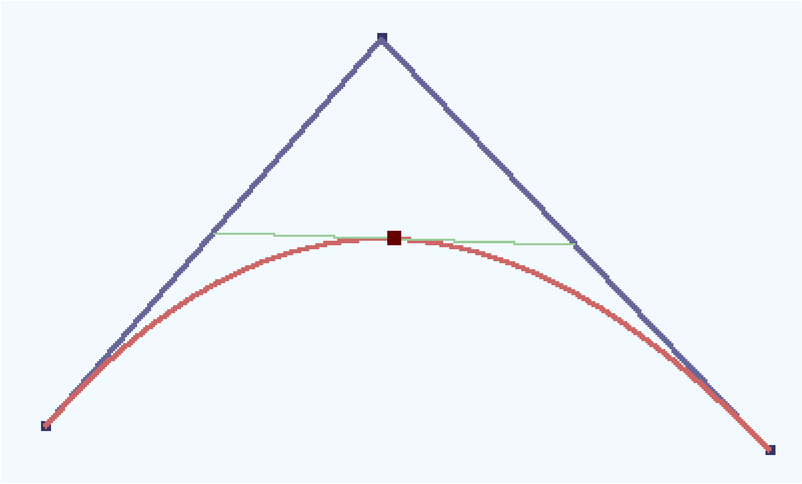
$$P_i^+ = P_i + \frac{l_i}{3}$$

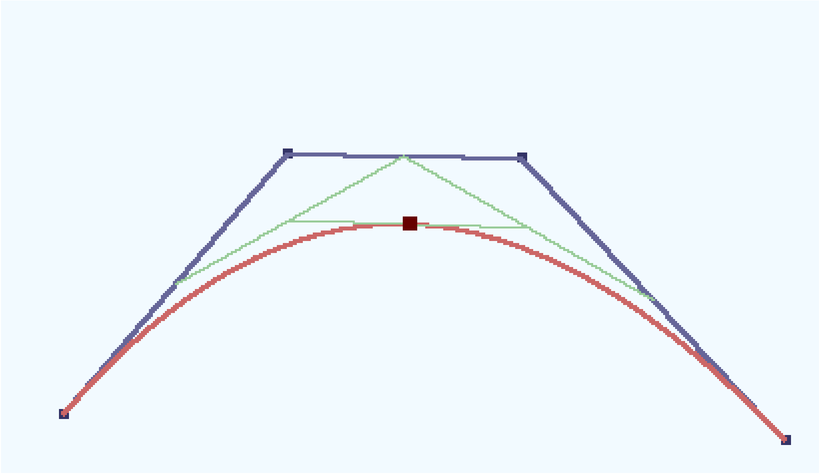


$$l_i = \frac{dP(i)}{du} = \frac{P_{i+1} - P_{i-1}}{(i+1)-(i-1)} = \frac{P_{i+1} - P_{i-1}}{2}$$

Elevación de Grado

$$P = \overbrace{(1-u)^2}^{B_0^2} P_0 + \overbrace{2(1-u)u}^{B_1^2} P_1 + \overbrace{u^2}^{B_2^2} P_2$$





$$P = \underbrace{(1-u)^3}_{B_0^3} \hat{P}_0 + \underbrace{3(1-u)^2 u}_{B_1^3} \hat{P}_1 + \underbrace{3(1-u)u^2}_{B_2^3} \hat{P}_2 + \underbrace{u^3}_{B_3^3} \hat{P}_3$$

Elevación de Grado

$$P = \overbrace{(1-u)^2}^{B_0^2} P_0 + \overbrace{2(1-u)u}^{B_1^2} P_1 + \overbrace{u^2}^{B_2^2} P_2 \qquad P = u\,P + (1-u)\,P$$

$$P = \underbrace{u(1-u)^2 P_0 + 2(1-u)u^2 P_1 + u^3 P_2}_{u\,P} +$$

$$+ \underbrace{(1-u)^3 P_0 + 2(1-u)^2 u\,P_1 + (1-u)u^2 P_2}_{(1-u)\,P}$$

$$P = \overbrace{(1-u)^3}^{B_0^3} \underbrace{P_0}_{\hat{P}_0} + \overbrace{u(1-u)^2}^{B_1^3} \underbrace{(P_0 + 2P_1)}_{3\hat{P}_1} + \overbrace{(1-u)u^2}^{B_2^3} \underbrace{(2P_1 + P_2)}_{3\hat{P}_2} + \overbrace{u^3}^{B_3^3} \underbrace{P_2}_{\hat{P}_3}$$

$$P = \underbrace{(1-u)^3}_{B_0^3} \hat{P}_0 + \underbrace{3(1-u)^2 u}_{B_1^3} \hat{P}_1 + \underbrace{3(1-u)u^2}_{B_2^3} \hat{P}_2 + \underbrace{u^3}_{B_3^3} \hat{P}_3$$

B-Splines

Base polinómica (Cox/de Boor):

$$B_{k,1}(t) = \begin{cases} 1 & \text{si } u_k \leq t \leq u_{k+1} \\ 0 & \text{en otro caso} \end{cases}$$

$$B_{k,d}(t) = \frac{t - u_k}{u_{k+d-1} - u_k} B_{k,d-1}(t) + \frac{u_{k+d} - t}{u_{k+d} - u_{k+1}} B_{k+1,d-1}(t)$$

u = vector de **knots**

knot= valor de t donde cambian las funciones activas

Continuidad C^{n-1} (ahorrando puntos de control)

Suma 1

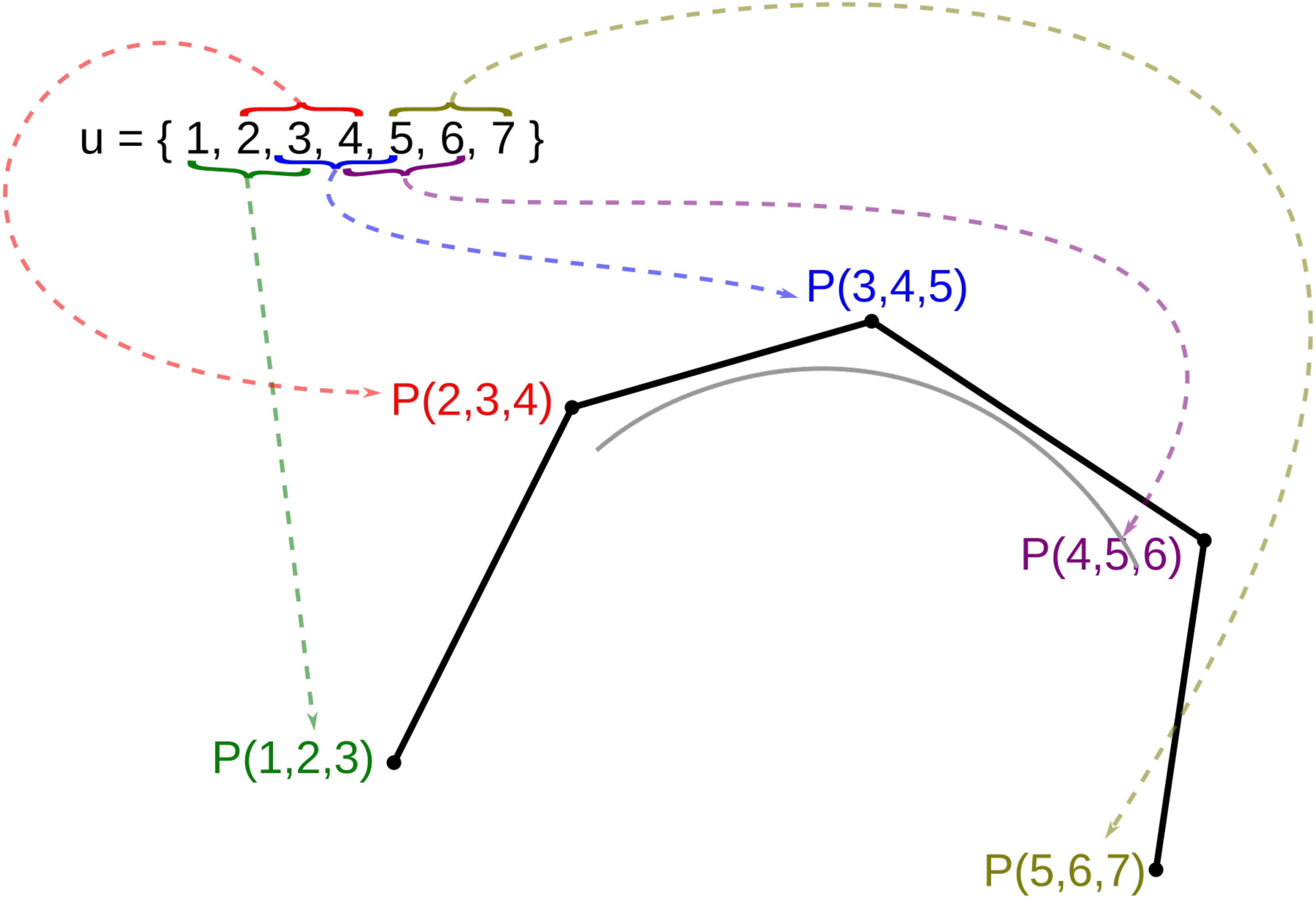
No interpolantes*

Control local

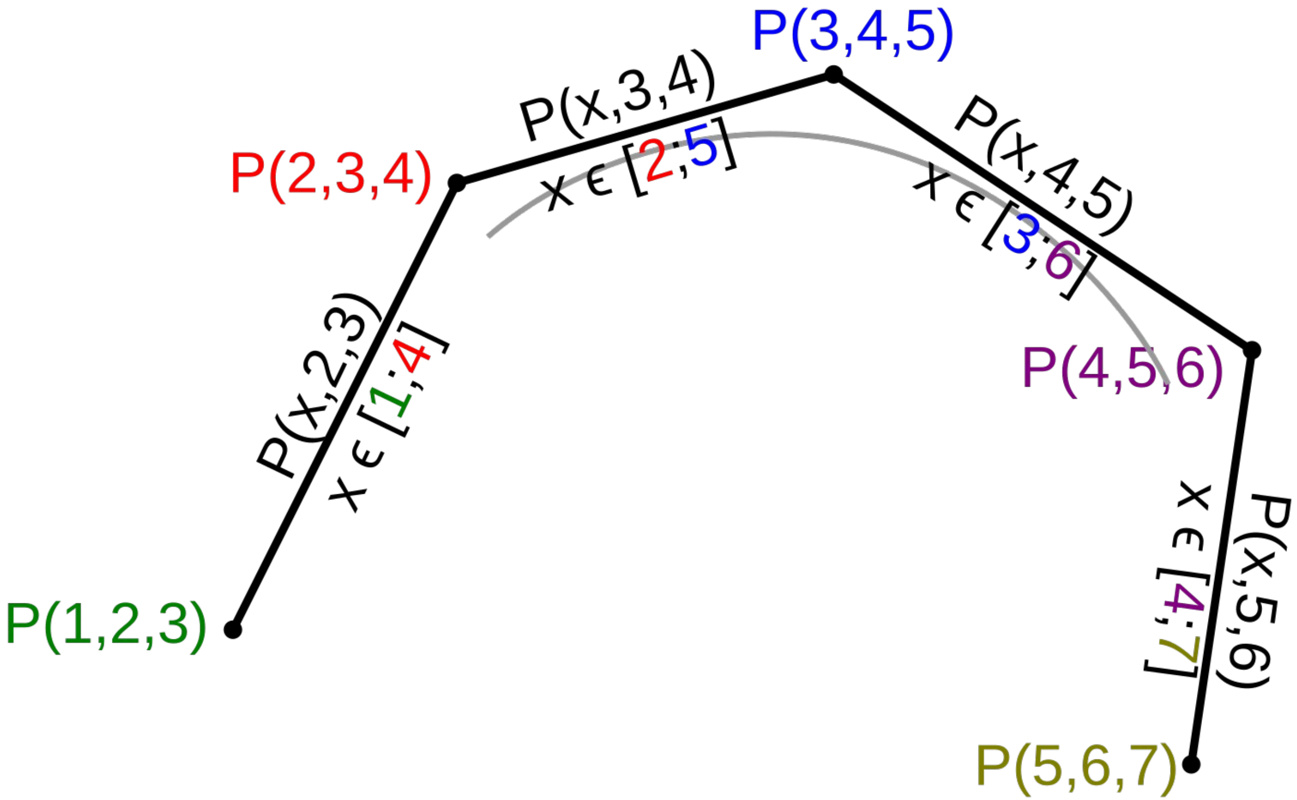
B-Splines: Bloosoming

knots = { 1, 2, 3, 4, 5, 6, 7 }

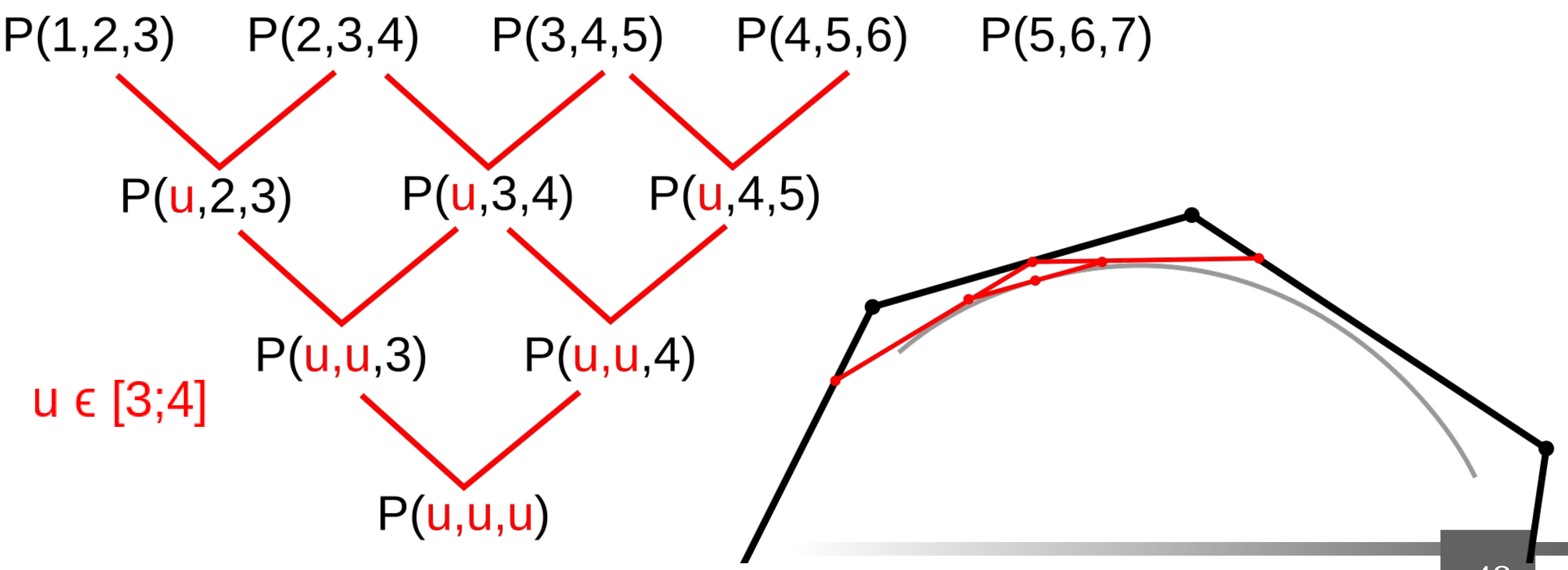
B-Splines: Bloosoming



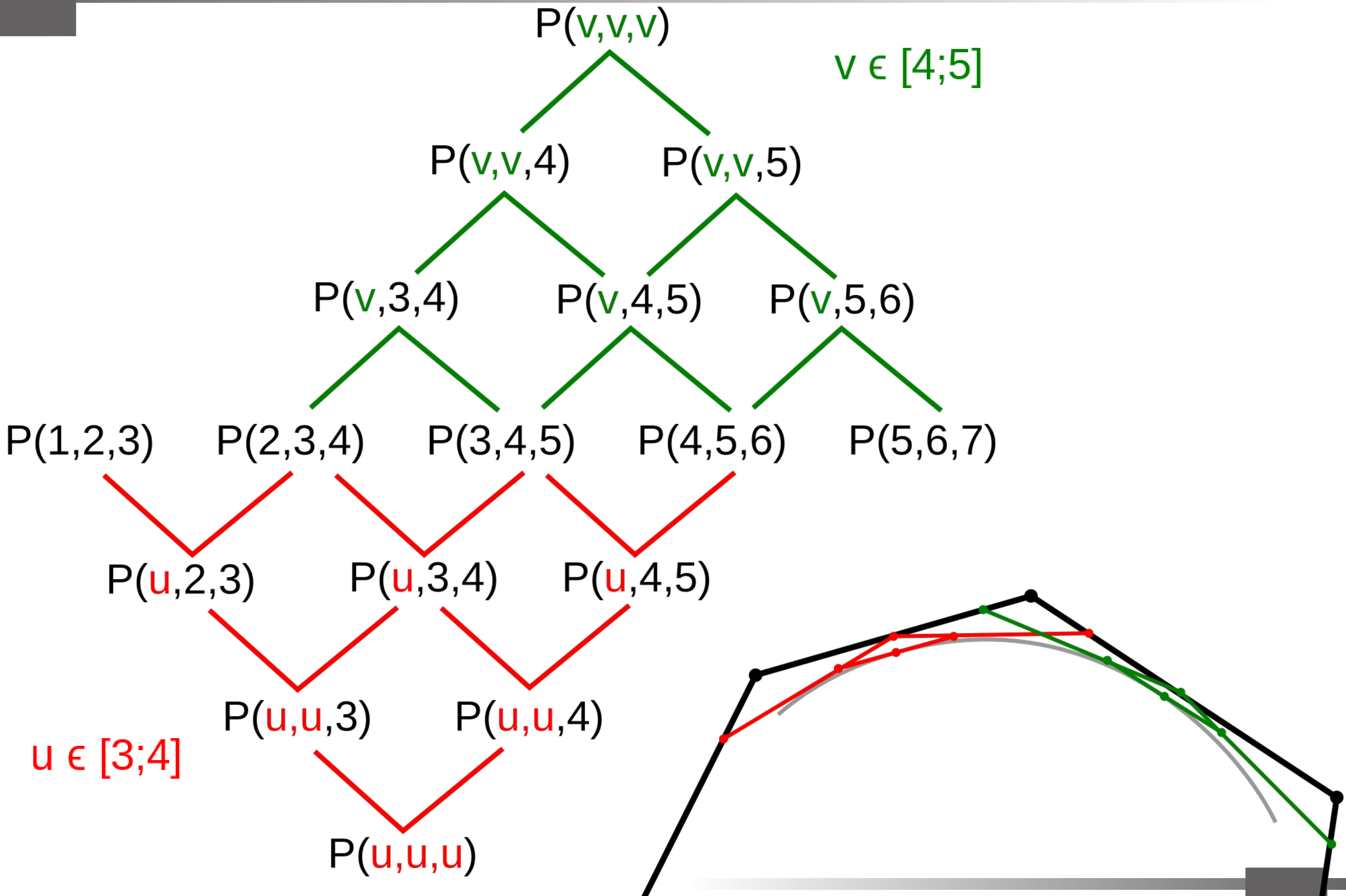
B-Splines: Bloosoming



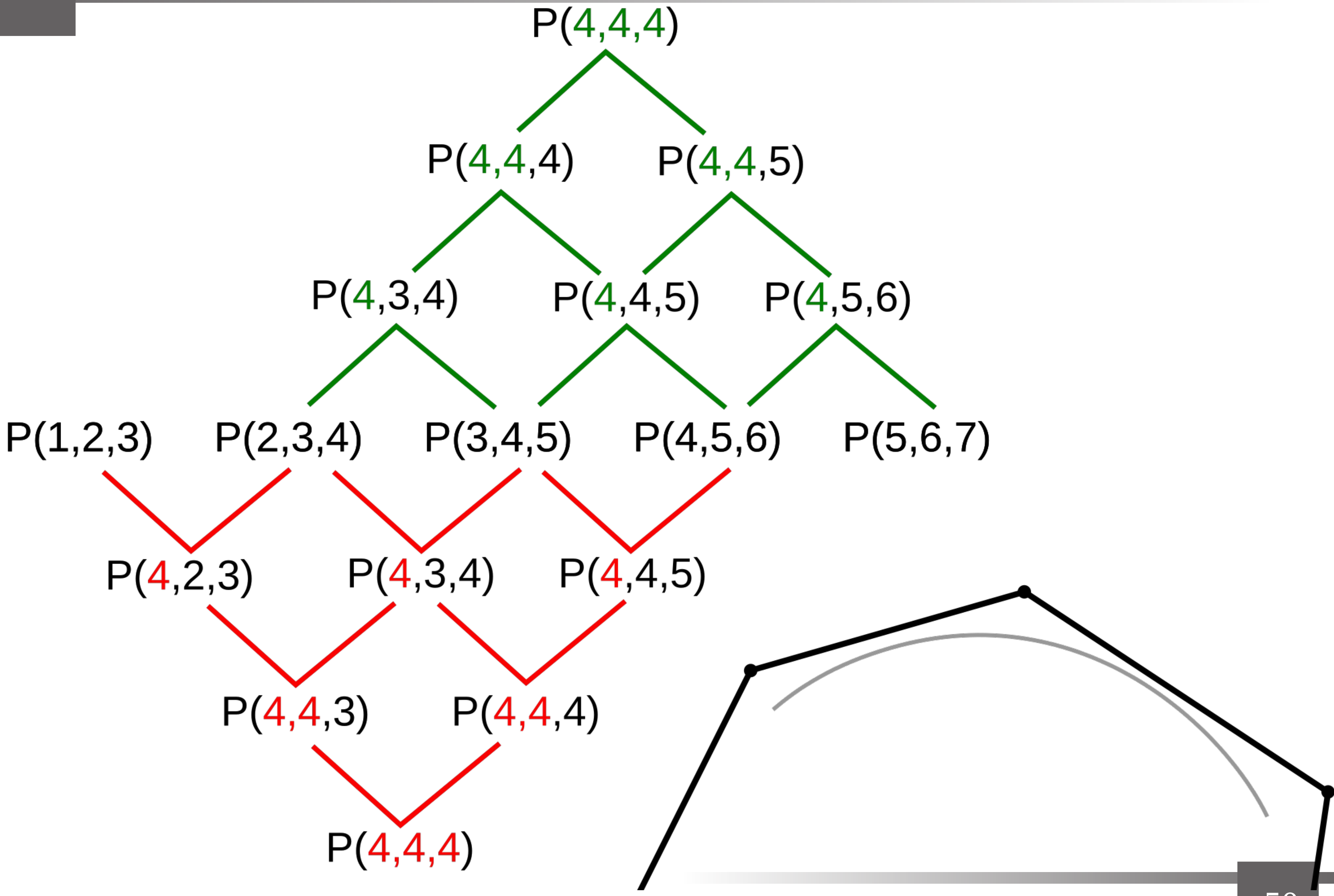
B-Splines: Bloosoming



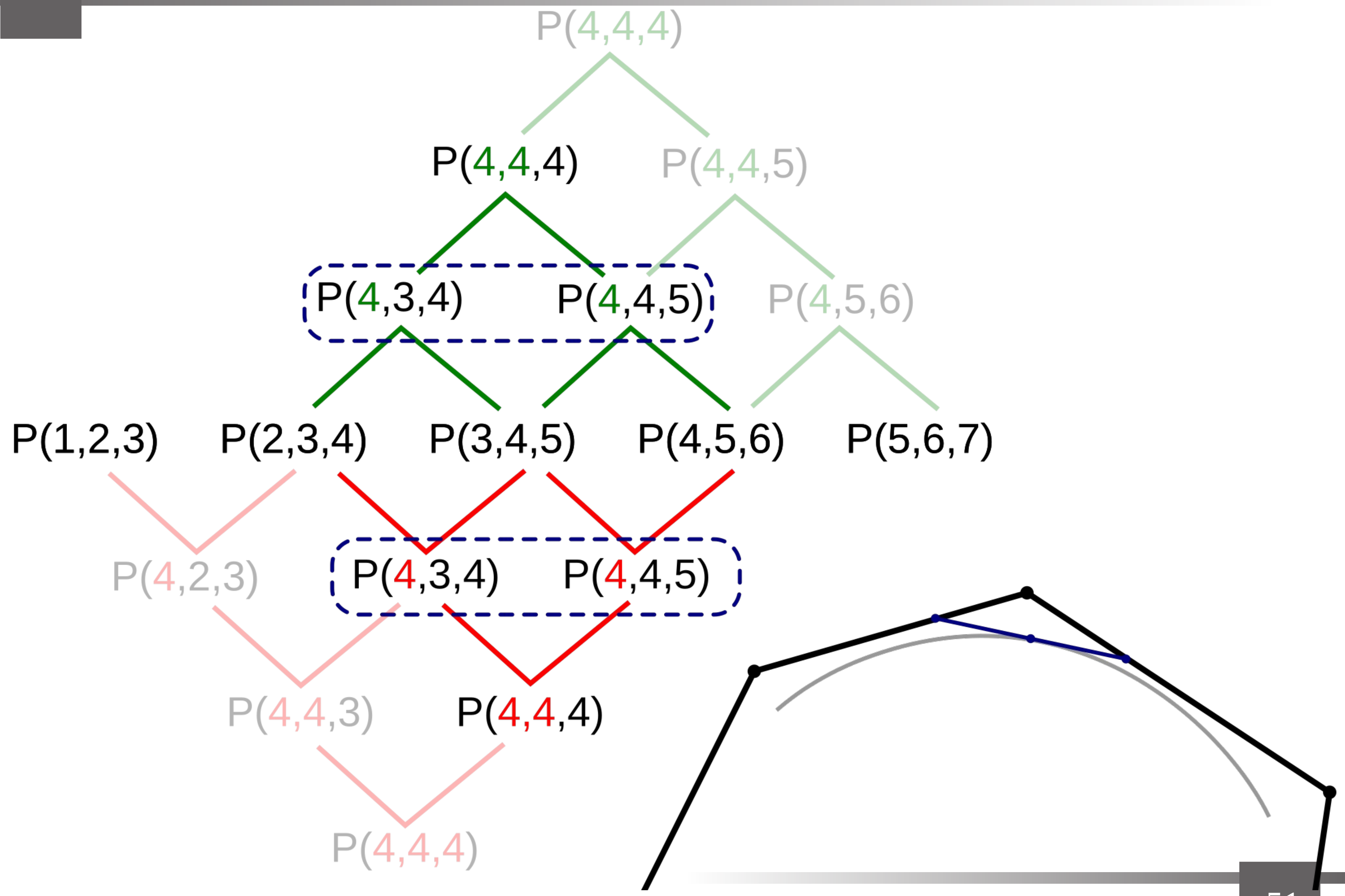
B-Splines: Bloosoming



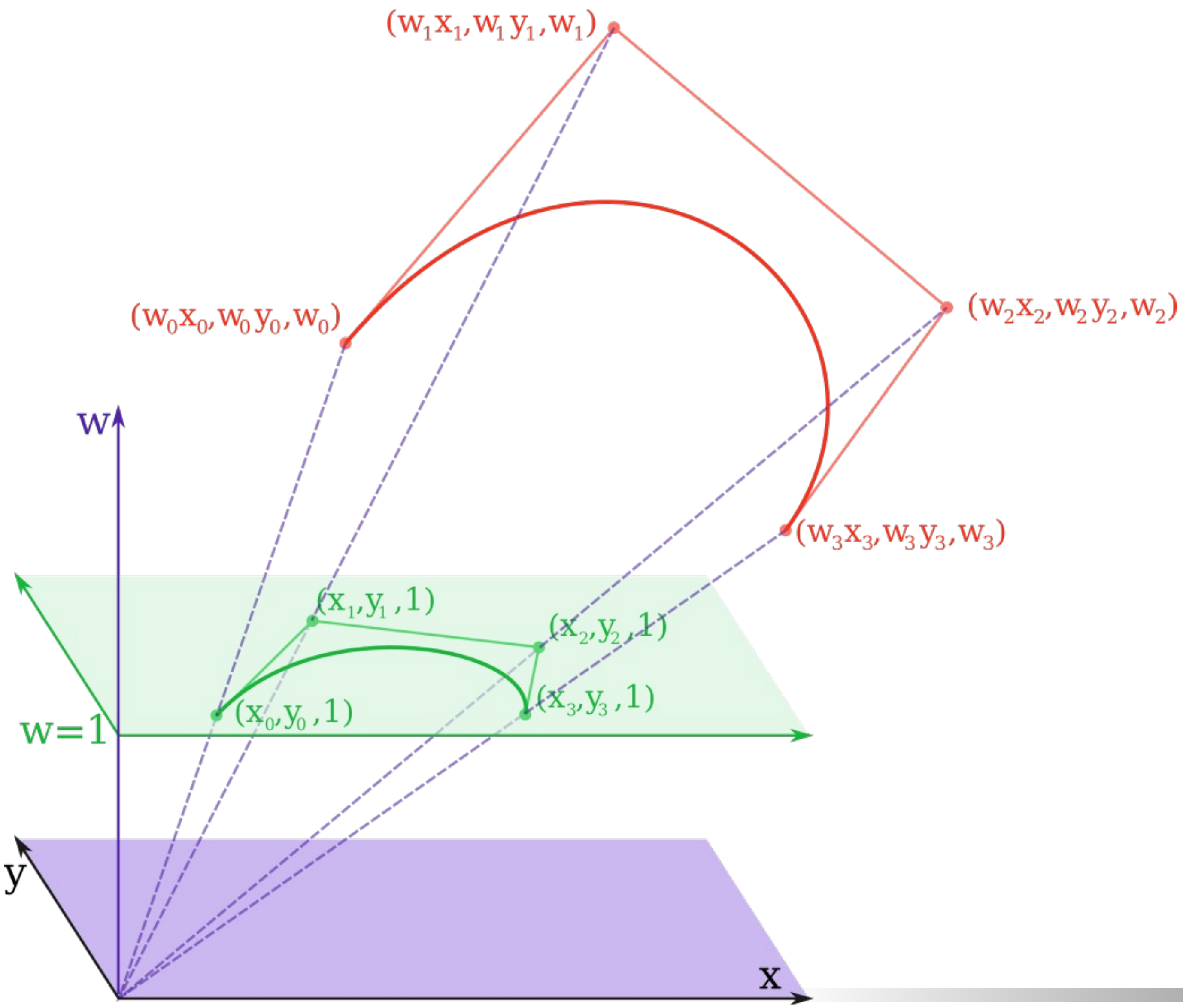
B-Splines: Bloosoming



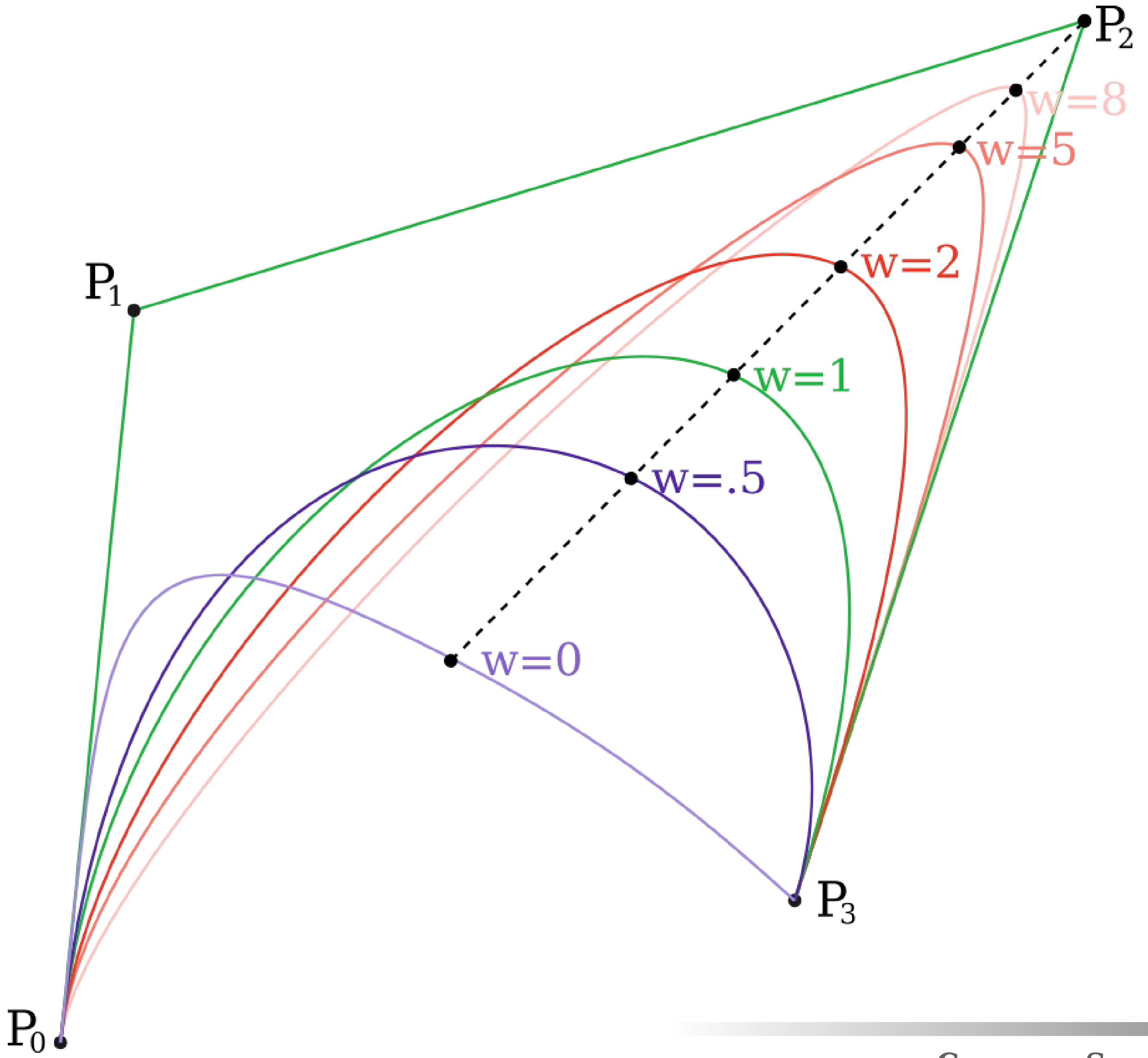
B-Splines: Bloosoming



Curvas Racionales



Curvas Racionales



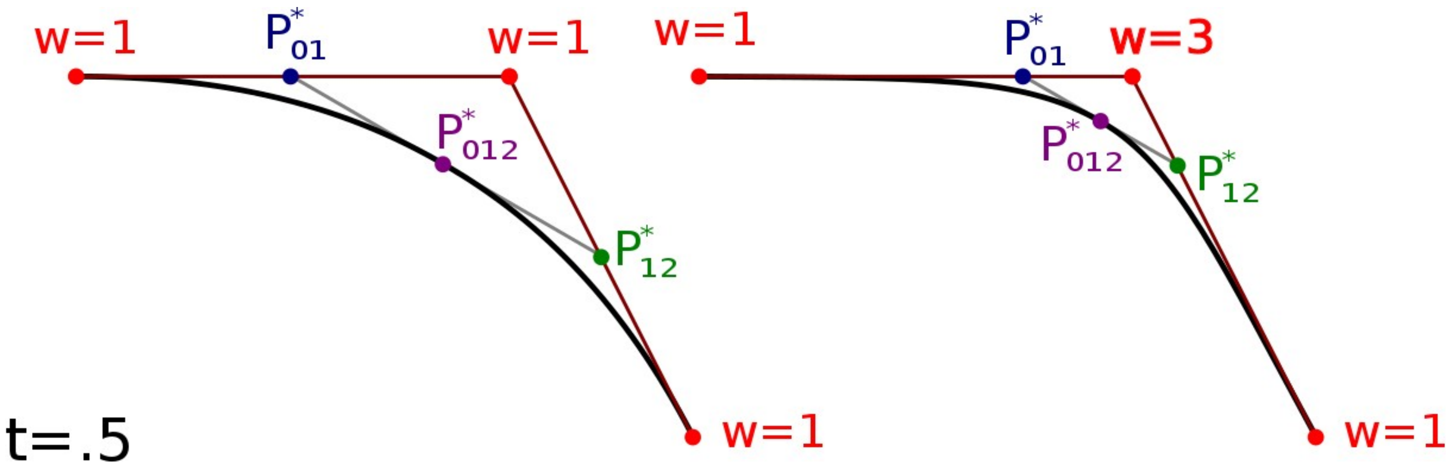
$$P=\{\underbrace{wx,wy,wz}_{wX},w\}=\{wX,w\} \quad \Rightarrow \quad P^*=\frac{wX}{w}$$

$$\underbrace{\{wX_{01},w_{01}\}}_{P_{01}} = (1-u)\underbrace{\{w_0X_0,w_0\}}_{P_0} + u\underbrace{\{w_1X_1,w_1\}}_{P_1}=$$

$$= \{ \underbrace{(1-u)w_0X_0+uw_1X_1}_{w_{01}X_{01}} , \underbrace{(1-u)w_0+uw_1}_{w_{01}} \}$$

$$P_{01}^* = \frac{(1-u)w_0X_0+uw_1X_1}{w_{01}} = \frac{(1-u)w_0}{w_{01}}X_0+\frac{uw_1}{w_{01}}X_1$$

Curvas Racionales



$$P_{01} = \{ \underbrace{(1-u)w_0 X_0 + u w_1 X_1}_{w_{01}X_{01}} , \underbrace{(1-u)w_0 + u w_1}_{w_{01}} \}$$

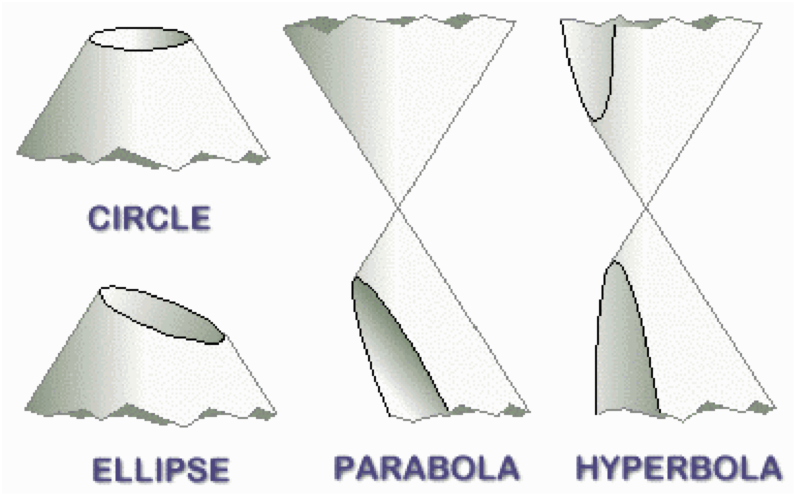
$$P_{12} = \{ \underbrace{(1-u)w_1 X_1 + u w_2 X_2}_{w_{12}X_{12}} , \underbrace{(1-u)w_1 + u w_2}_{w_{12}} \}$$

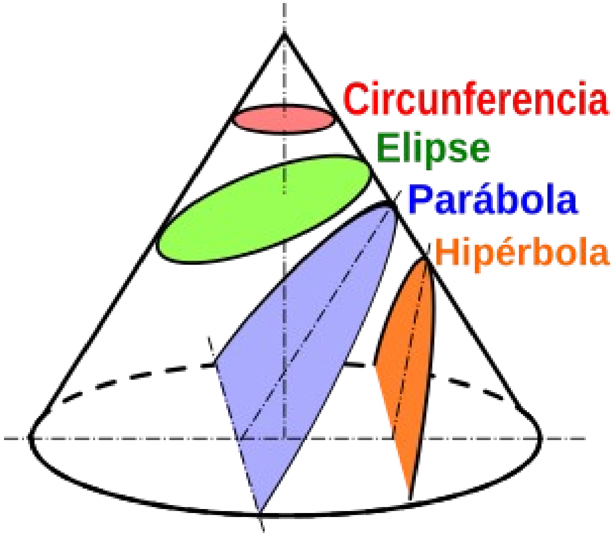
$$P^*_{012} = \frac{(1-u)w_{01}}{w_{012}} X_{01} + \frac{u\, w_{12}}{w_{012}} X_{12} \qquad w_{012} = (1-u)w_{01} + u\, w_{12}$$

Coordenadas Homogéneas

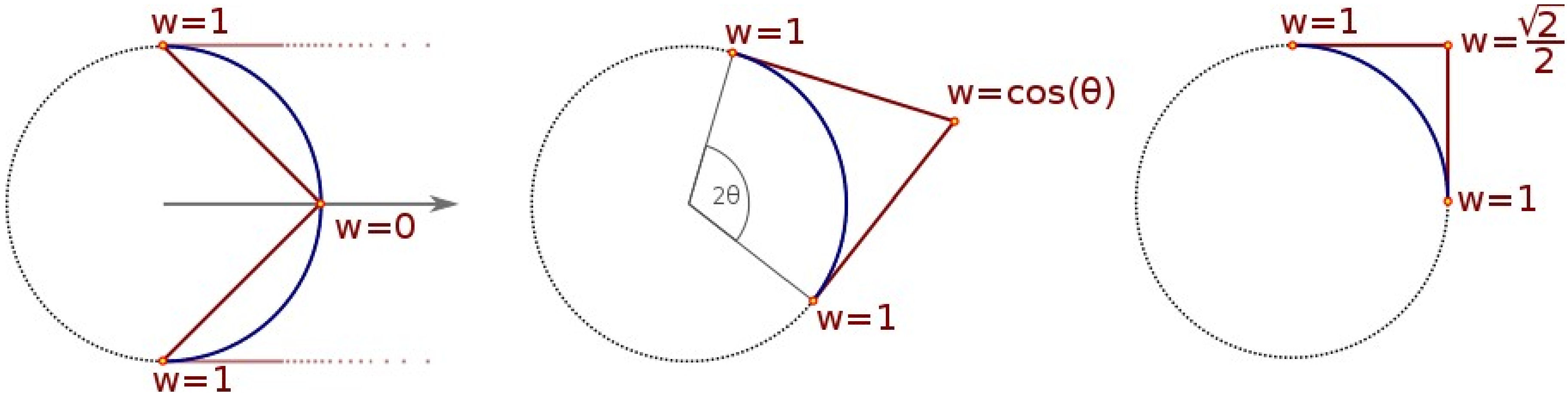
$$\boldsymbol{P}_{\boldsymbol{i}}^{*}=\{w_i x_i, w_i y_i, w_i z_i, w_i\}=\{w_i \boldsymbol{P}_{\boldsymbol{i}}, w_i\}$$

$$\boldsymbol{P}=\frac{\sum B_i^n w_i \boldsymbol{P}_{\boldsymbol{i}}}{\sum B_j^n w_j}=\sum \frac{w_i B_i^n}{\sum B_j^n w_j}\boldsymbol{P}_{\boldsymbol{i}}$$





Representación de Circunferencias



Curvas NURBS

Knot vector arbitrario

Coordenadas Homogéneas

$$\boldsymbol{P}_{\boldsymbol{i}}^{*}=\{w_i x_i, w_i y_i, w_i z_i, w_i\}=\{w_i \boldsymbol{P}_{\boldsymbol{i}}, w_i\}$$

$$\boldsymbol{P}=\frac{\sum B_i^n w_i \boldsymbol{P}_{\boldsymbol{i}}}{\sum B_j^n w_j}=\sum \frac{w_i B_i^n}{\sum B_j^n w_j} \boldsymbol{P}_{\boldsymbol{i}}$$

NURBS:

NU: Non-Uniform

R: Rational

BS: B-Splines

Base polinómica (Cox/de Boor):

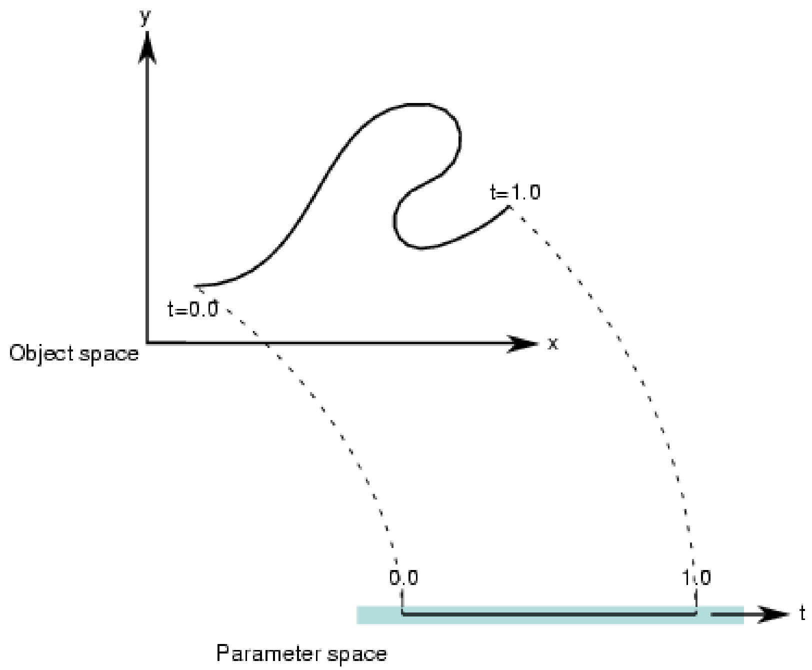
$$B_{k,1}(t)=\begin{cases} 1 & \text{si } u_k \le t \le u_{k+1} \\ 0 & \text{en otro caso} \end{cases}$$

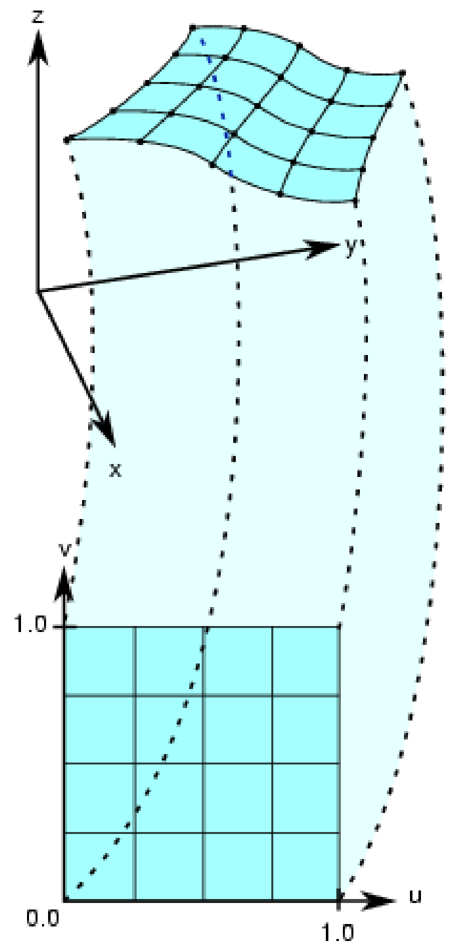
$$B_{k,d}(t)=\frac{t-u_k}{u_{k+d-1}-u_k} B_{k,d-1}(t)+\frac{u_{k+d}-t}{u_{k+d}-u_{k+1}} B_{k+1,d-1}(t)$$

Superficies Paramétricas

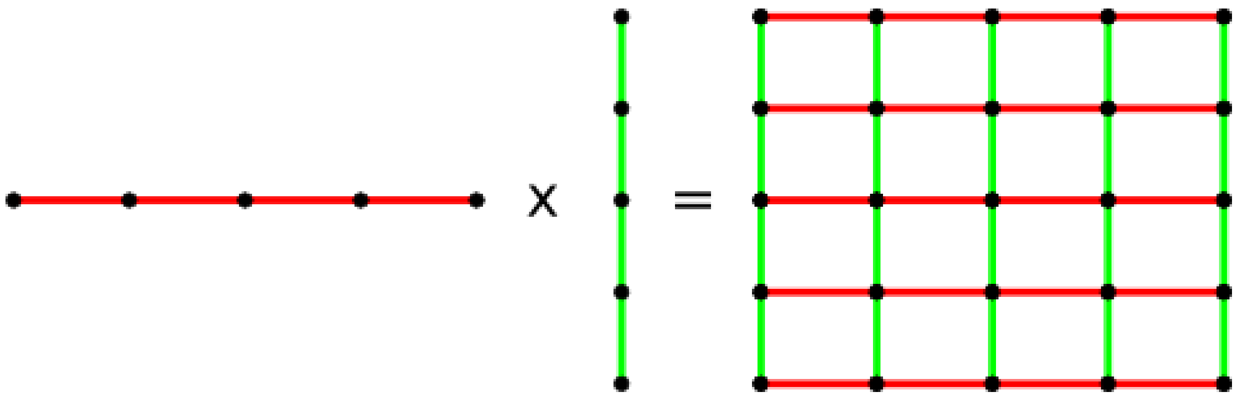
Curva: $\boldsymbol{x} = \boldsymbol{F}(u), \quad u_0 \le u \le u_1$

Superficie: $\boldsymbol{x} = \boldsymbol{F}(u, v), \quad \begin{array}{l} u_0 \le u \le u_1 \\ v_0 \le v \le v_1 \end{array}$





En base al producto cartesiano o tensorial:



$$P(u,v)=\sum_{i}^{n}\sum_{j}^{m}B^{n}(u)B^{m}(v)P_{i,j}=$$

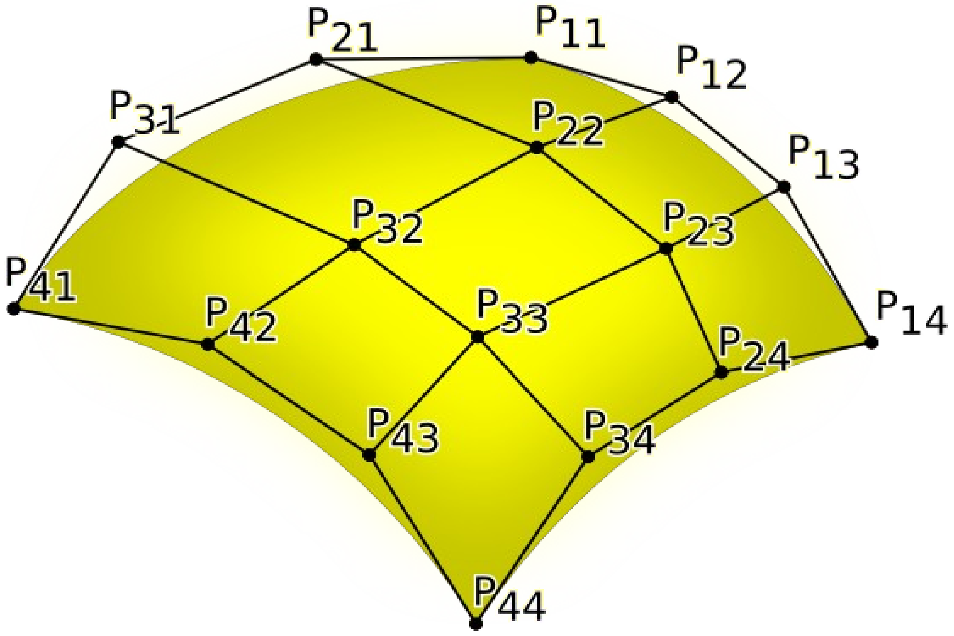
$$=\sum_{i}^{n}B^{n}(u)\left(\sum_{j}^{m}B^{m}(v)P_{i,j}\right)=\sum_{j}^{m}B^{m}(v)\left(\sum_{i}^{n}B^{n}(u)P_{i,j}\right)$$

Pts de Control: grilla regular de (n+1)x(m+1)

$$P(u,v)=\sum_{i}^{n}\sum_{j}^{m} B^{n}(u)\,B^{m}(v)\,P_{i,j}=$$

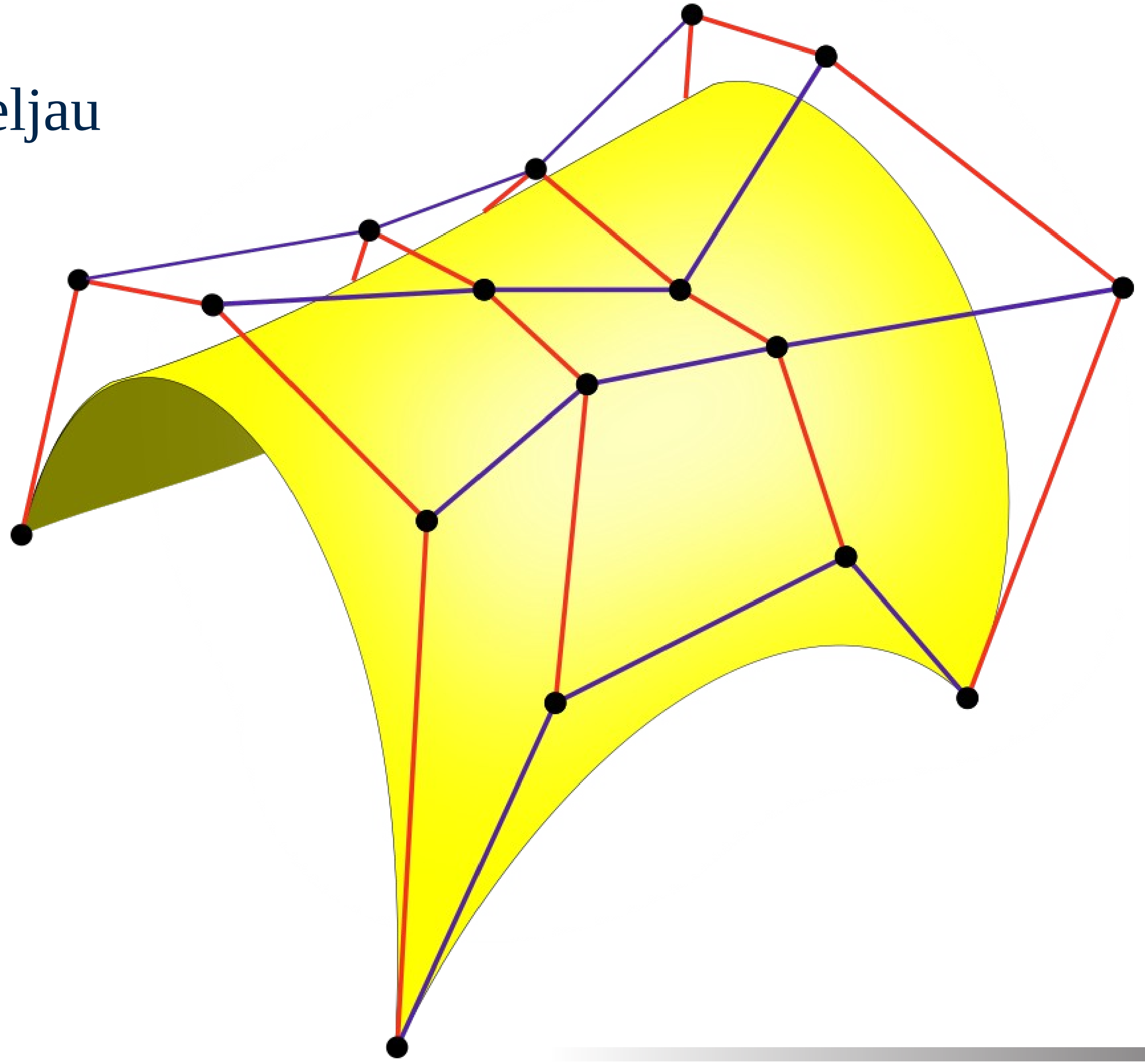
$$=\sum_{i}^{n} B^{n}(u)\left(\sum_{j}^{m} B^{m}(v)\,P_{i,j}\right)$$

$$=\sum_{j}^{m} B^{m}(v)\left(\sum_{i}^{n} B^{n}(u)\,P_{i,j}\right)$$

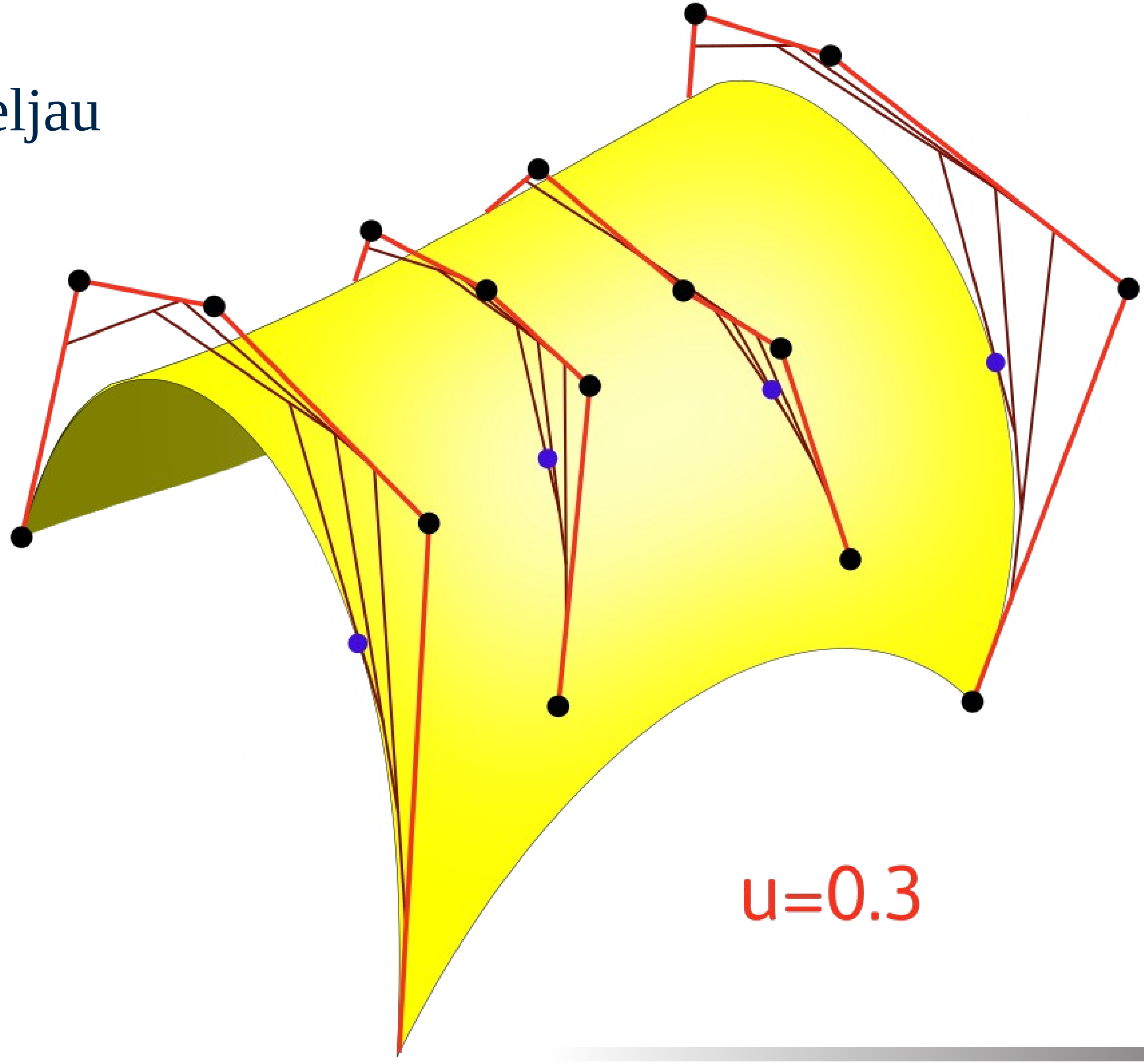


Superficies Paramétricas

De Casteljau

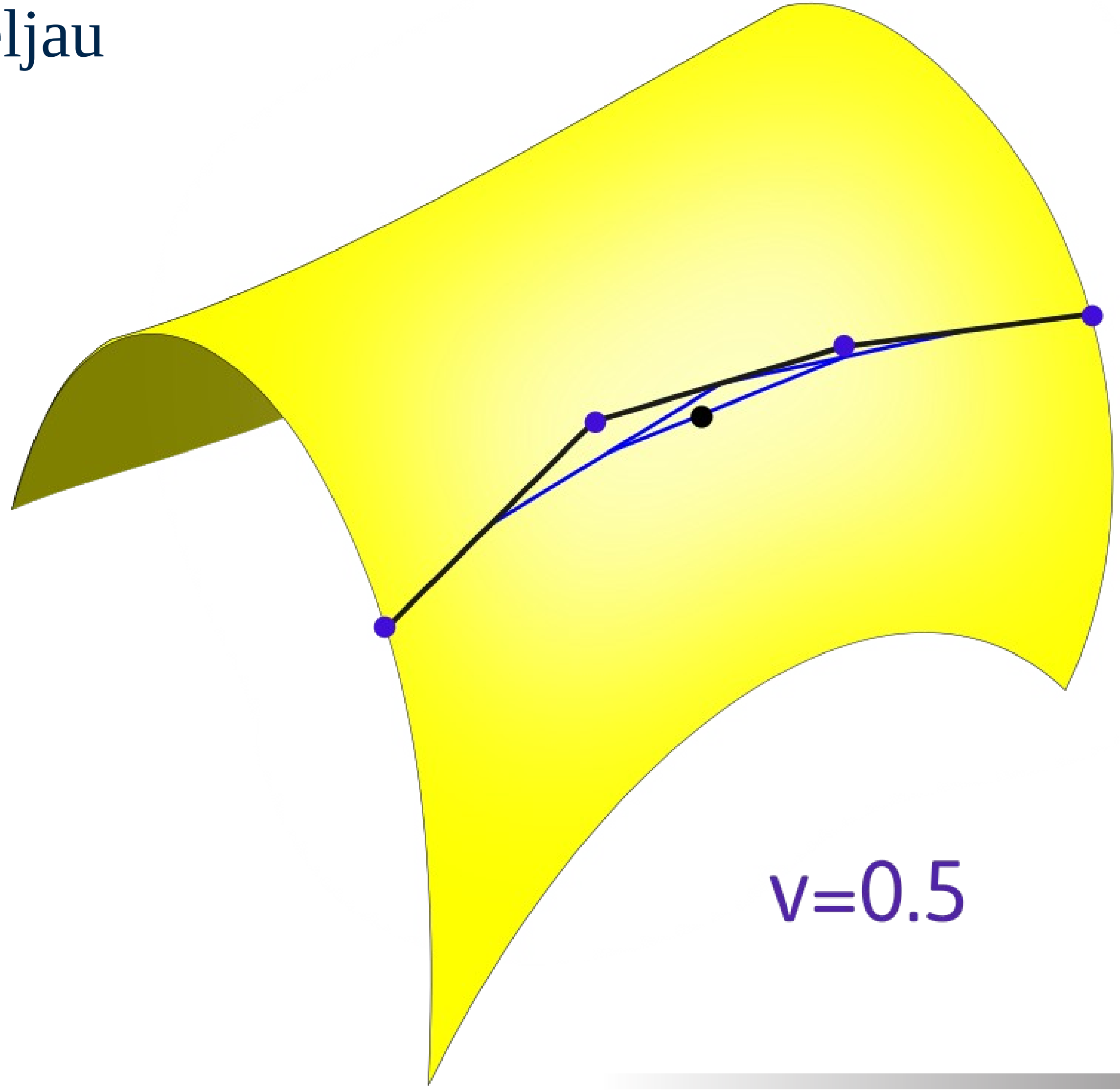


De Casteljau

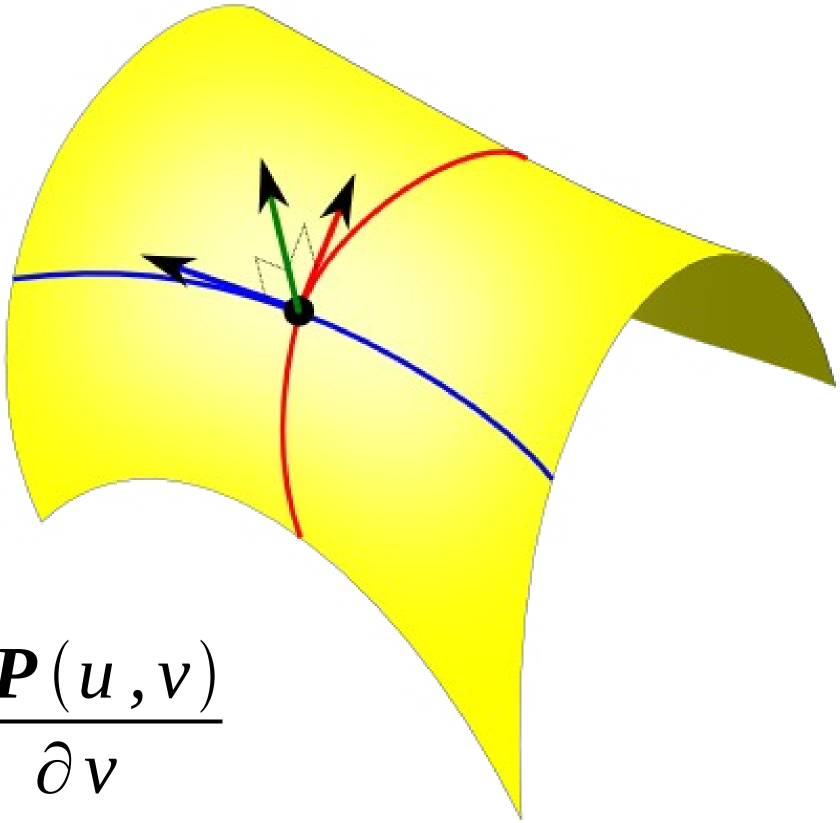


Superficies Paramétricas

De Casteljau



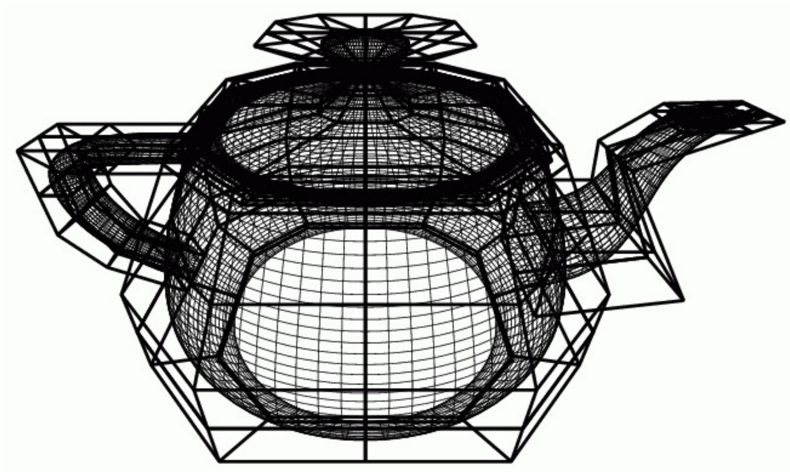
Tangentes y Normales:

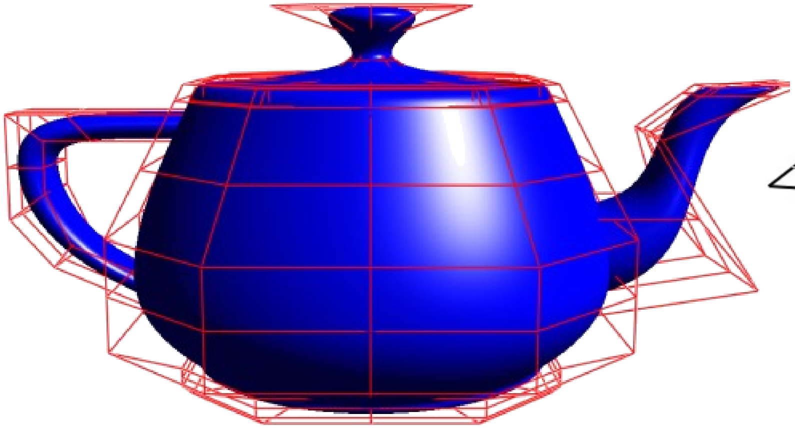


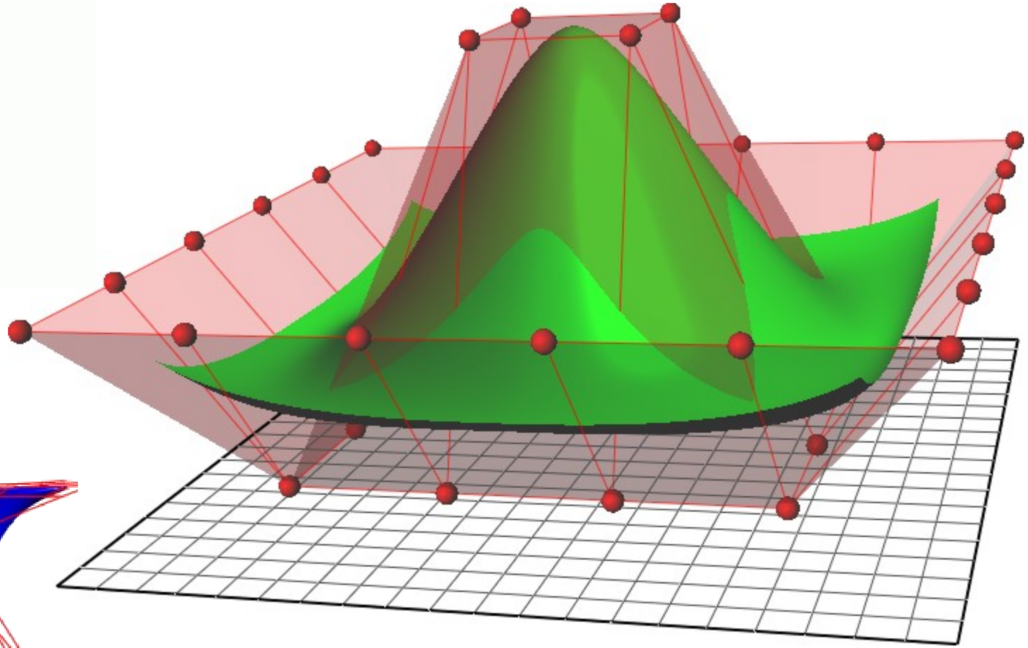
$$\hat{\mathbf{N}}(u,v)=\frac{\partial \mathbf{P}(u,v)}{\partial u}\times\frac{\partial \mathbf{P}(u,v)}{\partial v}$$

$$\mathbf{N}(u,v)=\frac{\hat{\mathbf{N}}(u,v)}{\left|\hat{\mathbf{N}}(u,v)\right|}$$

Superficies Paramétricas

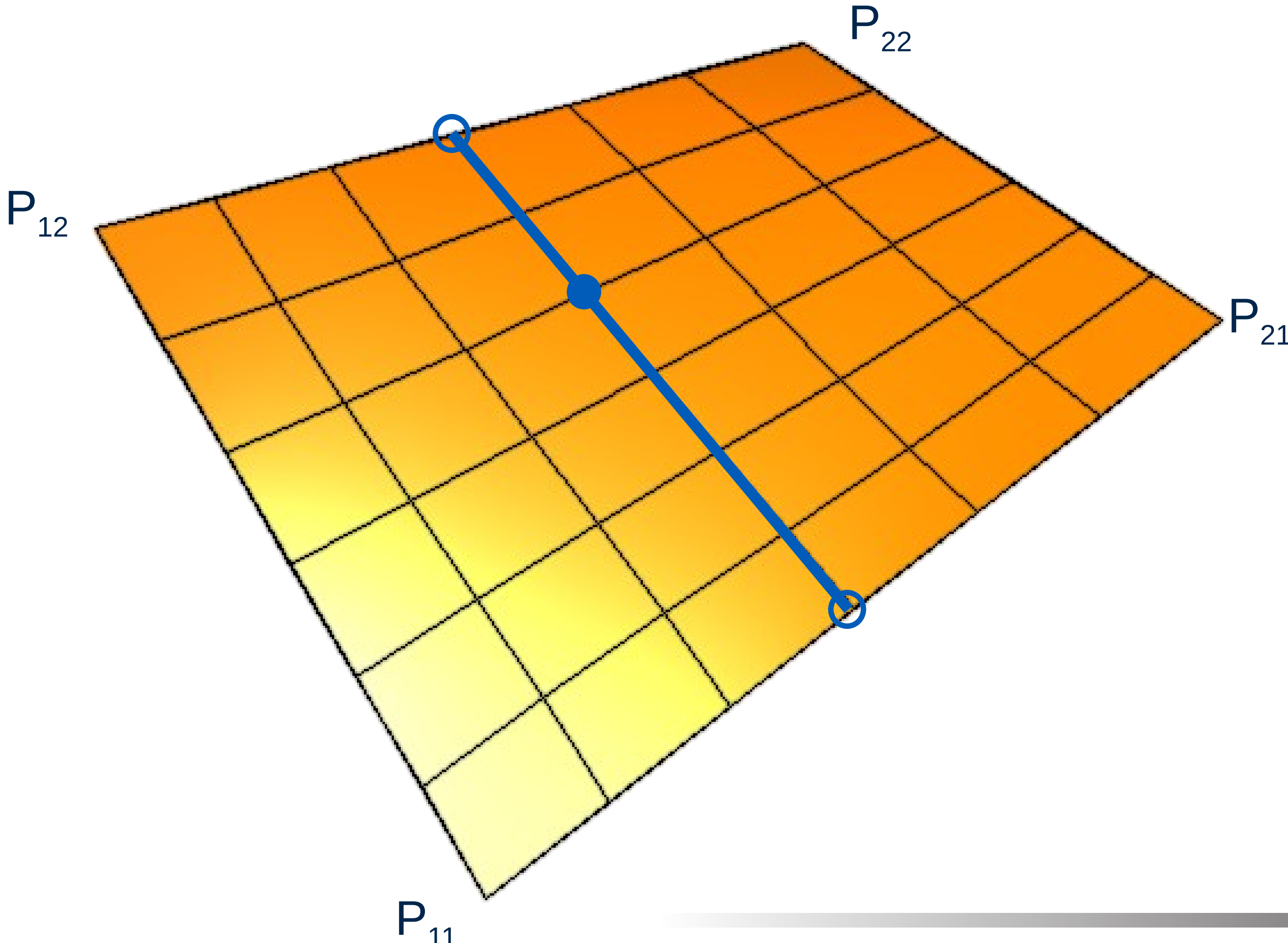






Superficie Bilineal

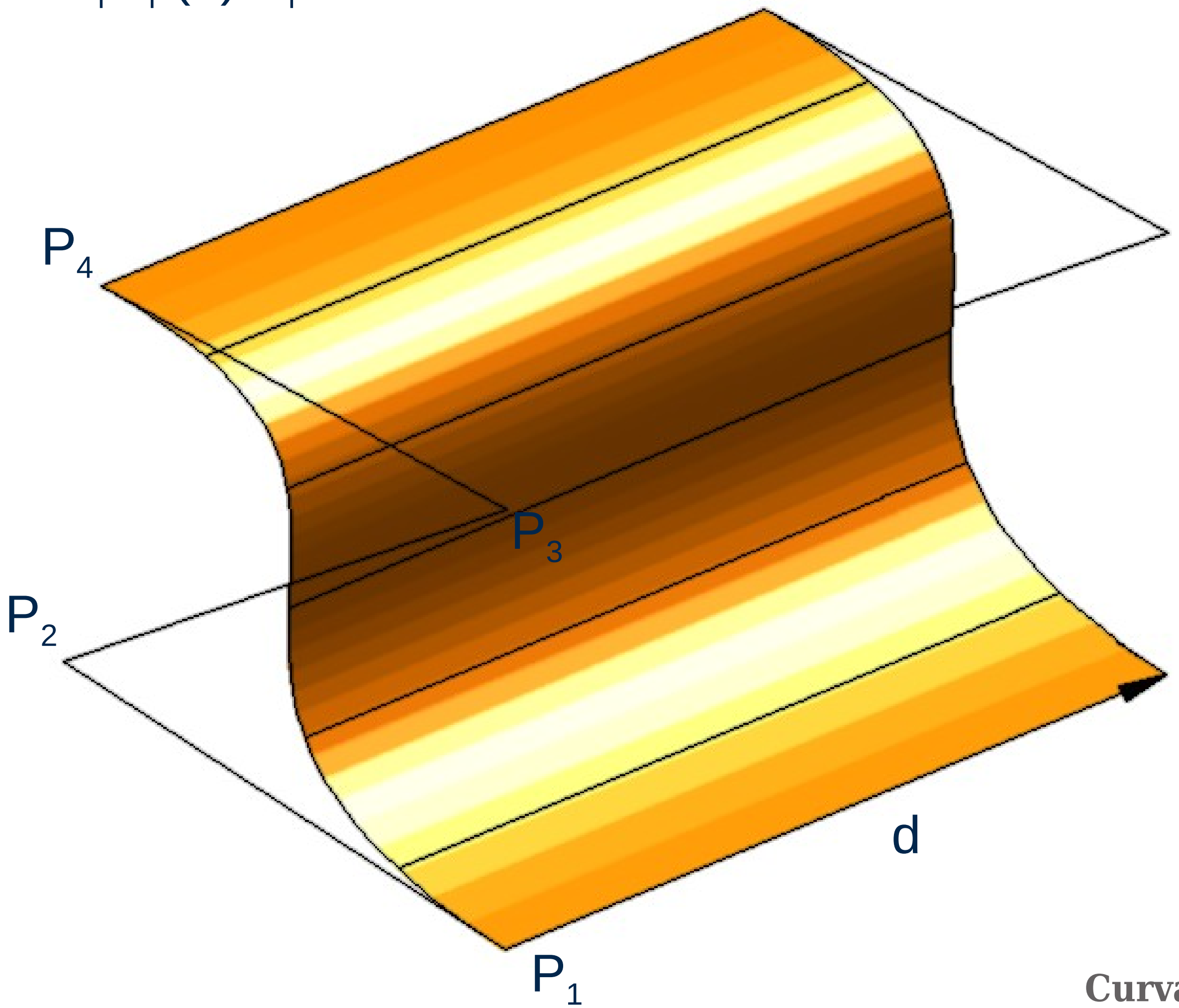
$$P(u,v) = (1-v)\,[(1-u)\,P_{11} + u\,P_{12}] + v\,[(1-u)\,P_{21} + u\,P_{22}]$$



Superficie Cilídrica

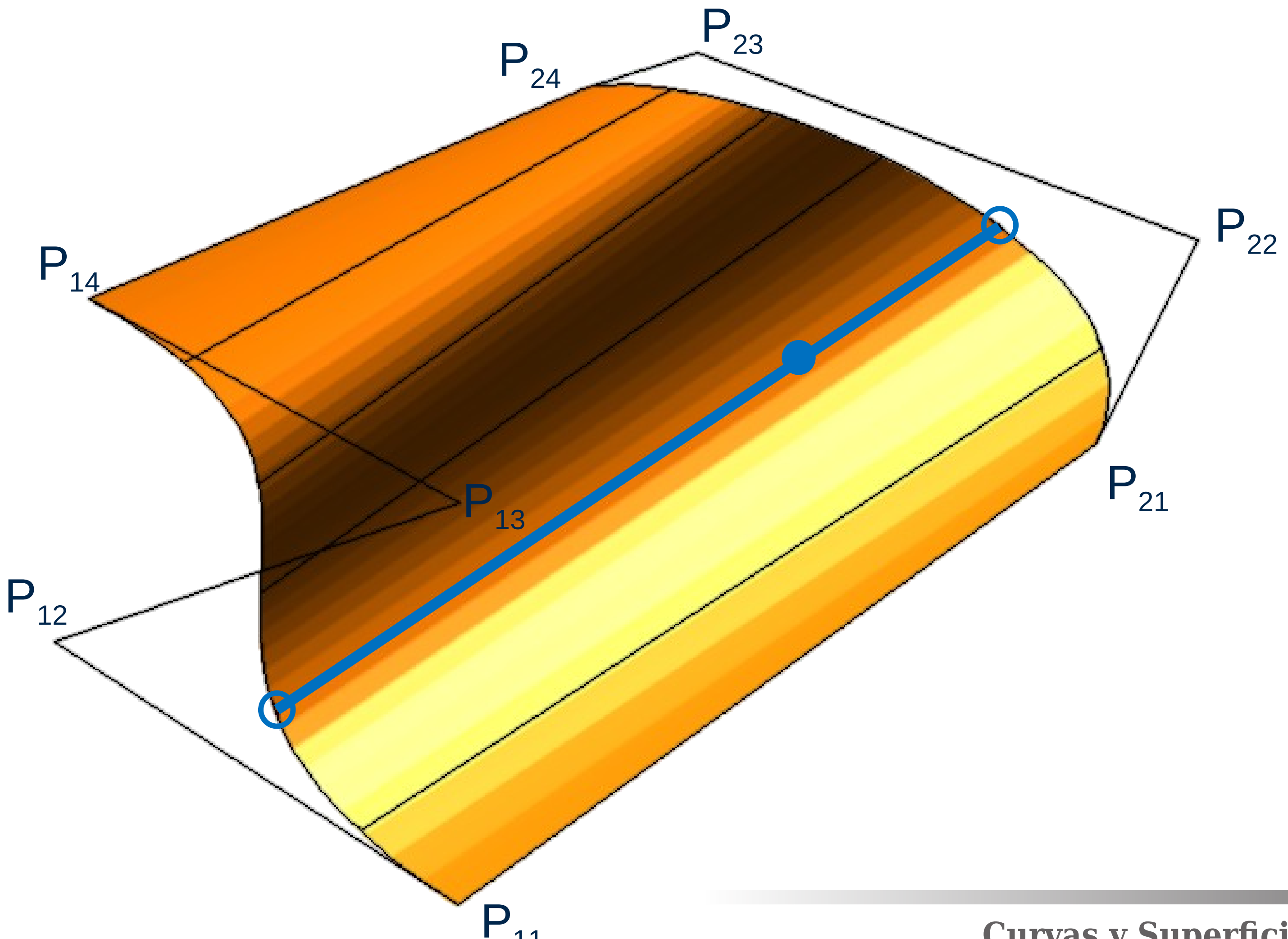
$$P(u,v) = (1-v)\sum_i B_i^n(u)\, P_i + v \sum_i B_i^n(u)\,(P_i+d)$$

$$= \sum_i B_i^n(u)\, P_i + v\, d$$



Superficie Reglada

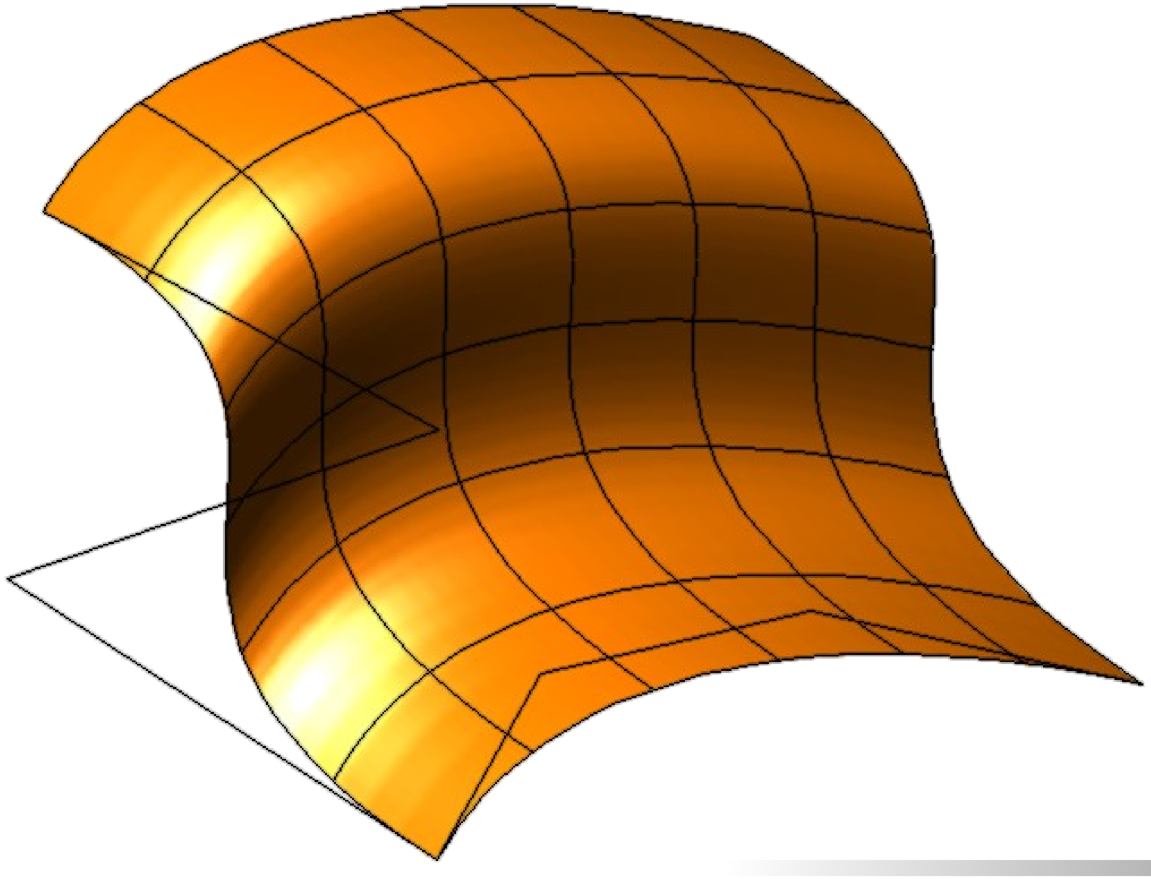
$$P(u,v) = (1-v)\, \Sigma_i\, B_i^n(u)\; P_{1,i} + v\, \Sigma_i\, B_i^n(u)\; P_{2,i}$$



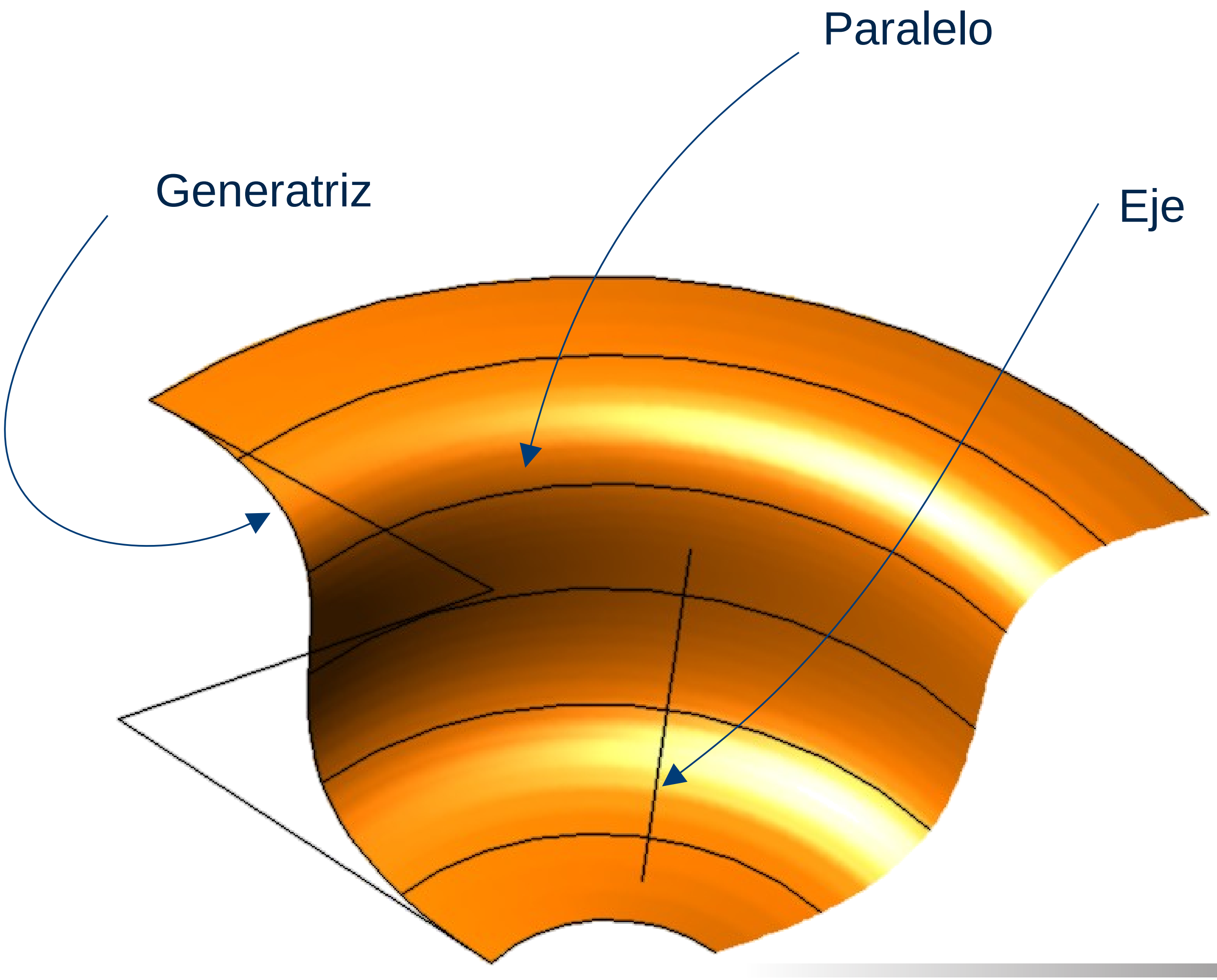
Superficie Traslacional

$P_{ij} - P_{kj} = cte \;\; \forall\, j$ $\qquad$ $P_{ij} - P_{ik} = cte \;\; \forall\, i$

Copia por traslación paralela de cada curva sobre la otra

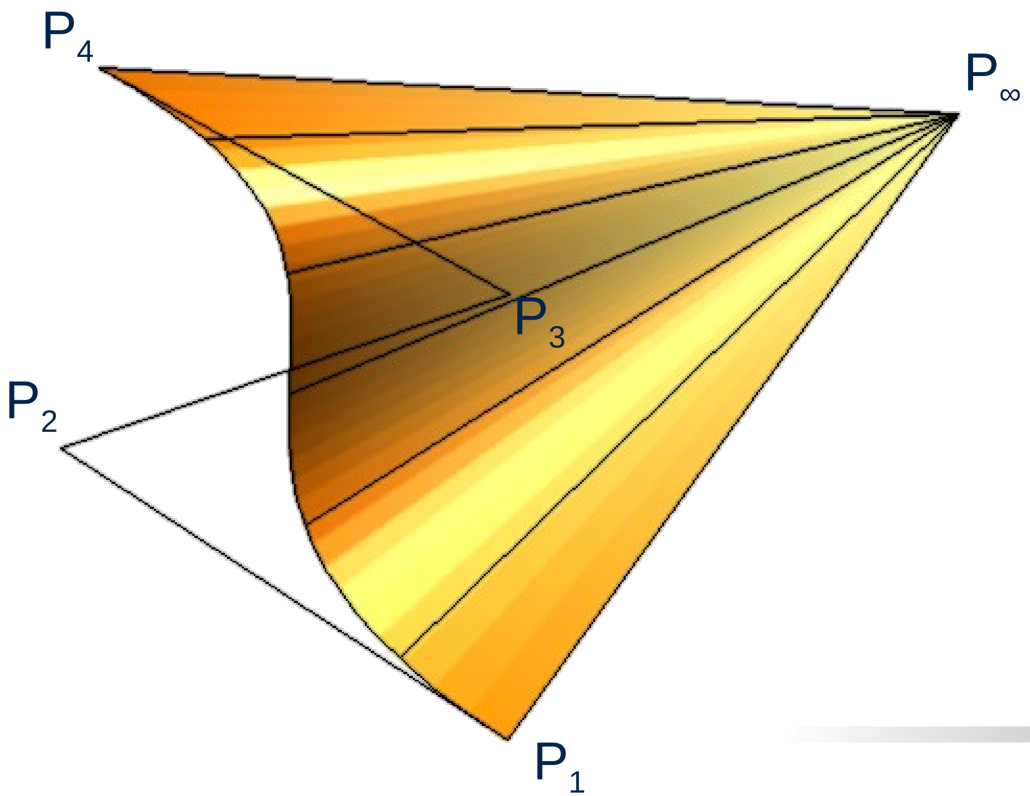


Superficie de Revolución

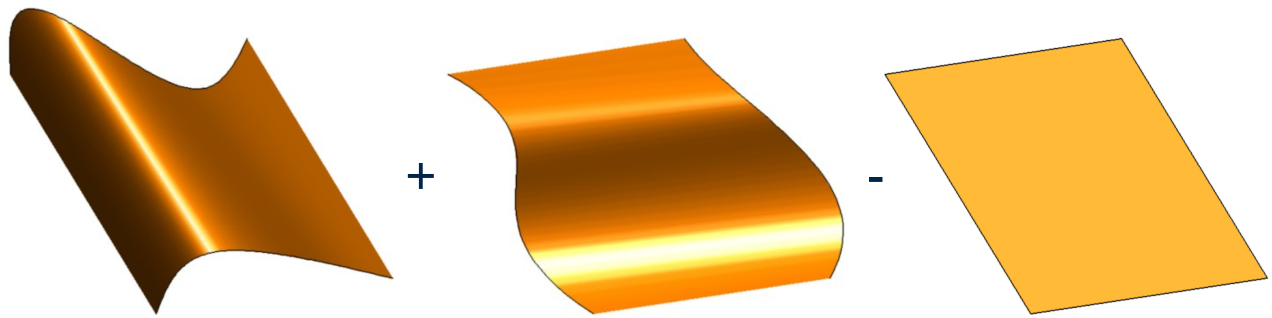


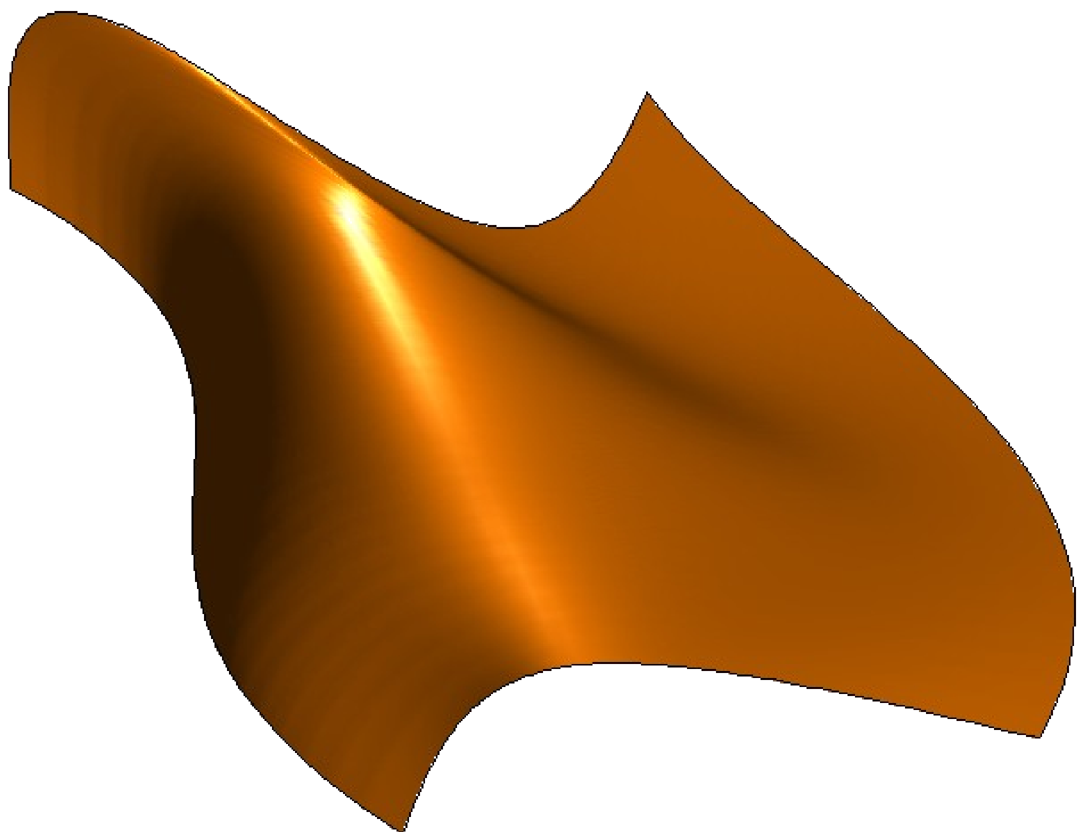
Superficie Cónica

$$P(u,v) = (1-v) \sum_i B_i^n(u)\, P_i + v\, P_\infty$$

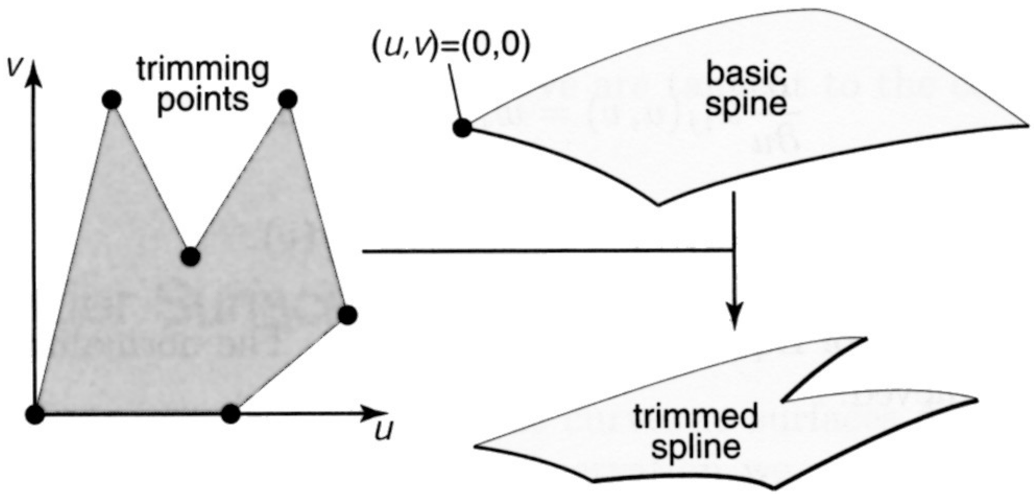


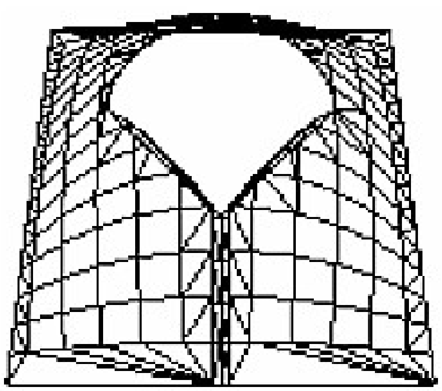
Patch de Coons

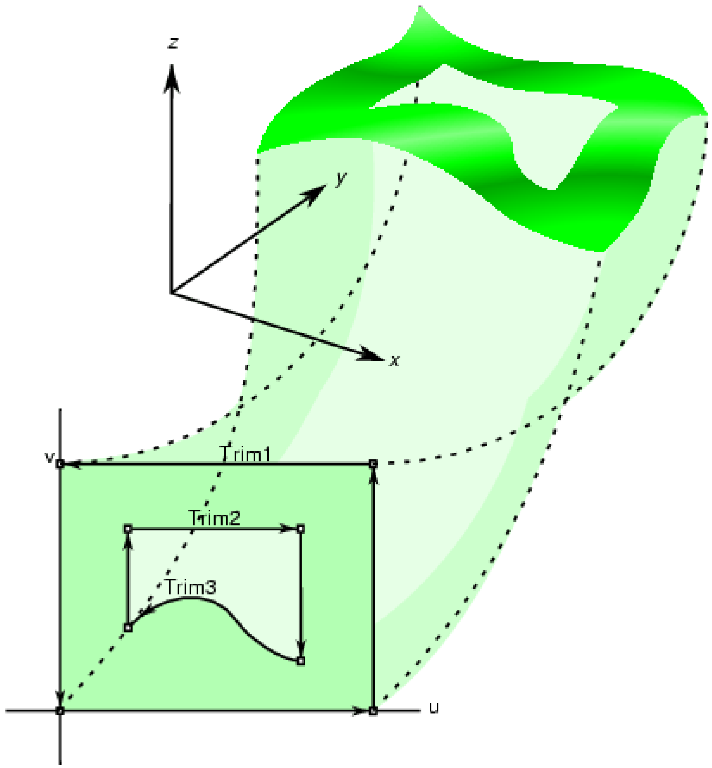




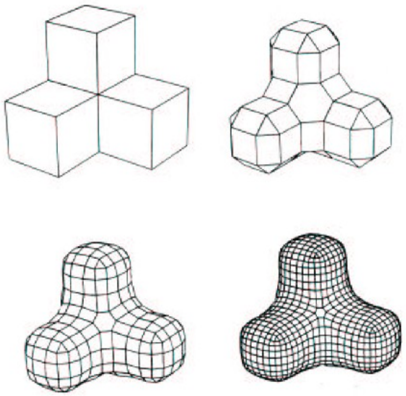
Superficies Recortadas ("Trimmed")

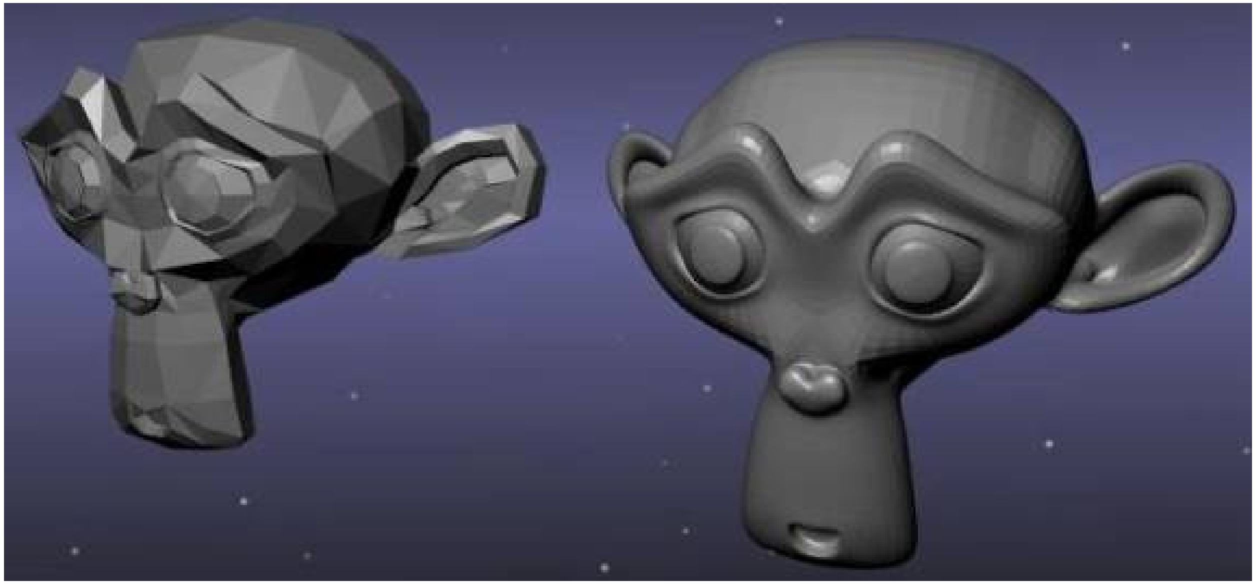


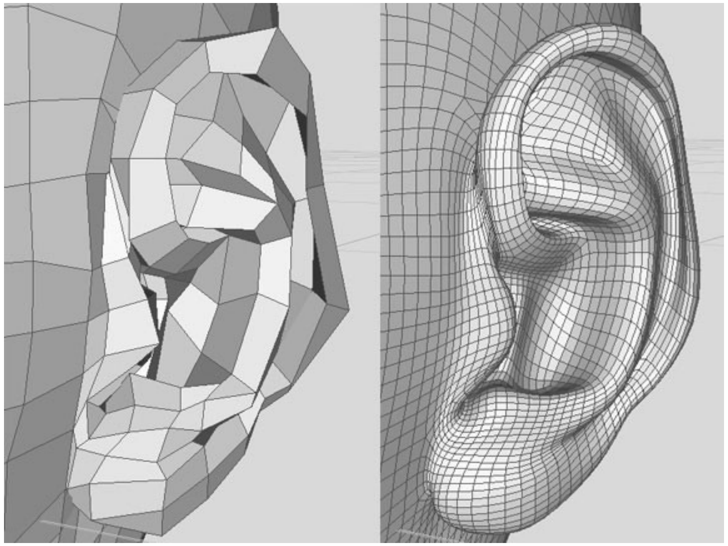




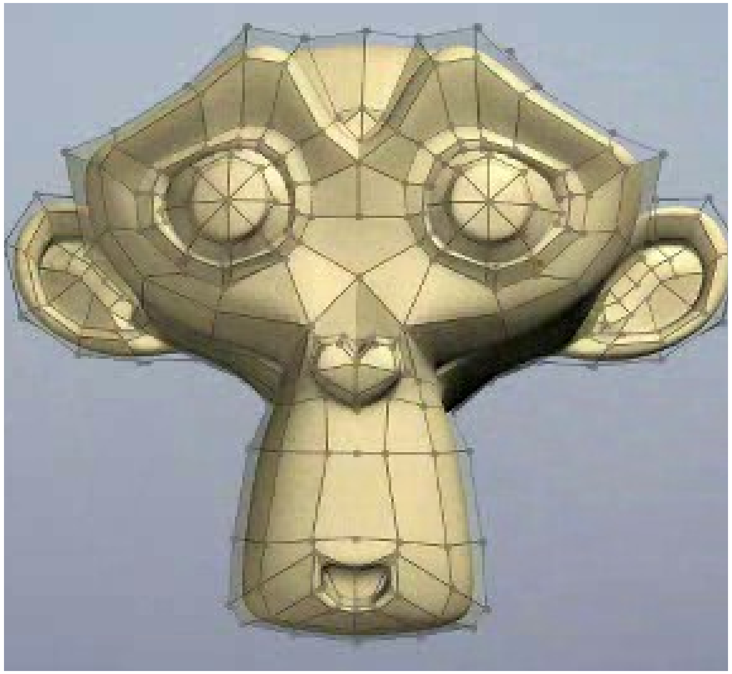
Subdivision Surfaces

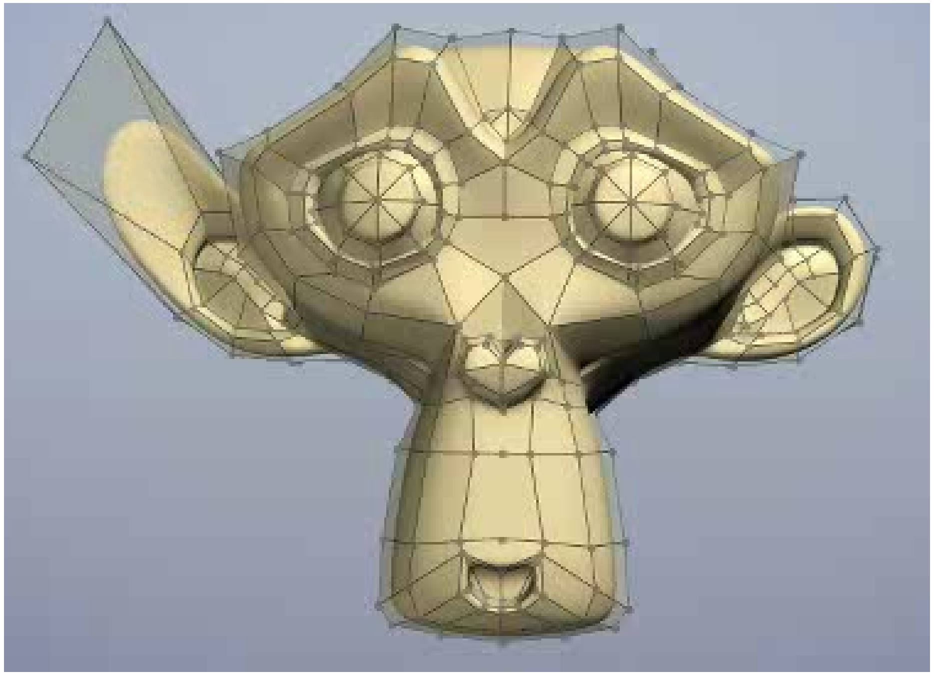


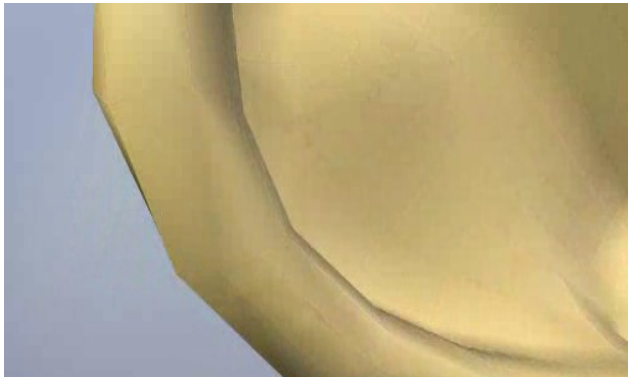


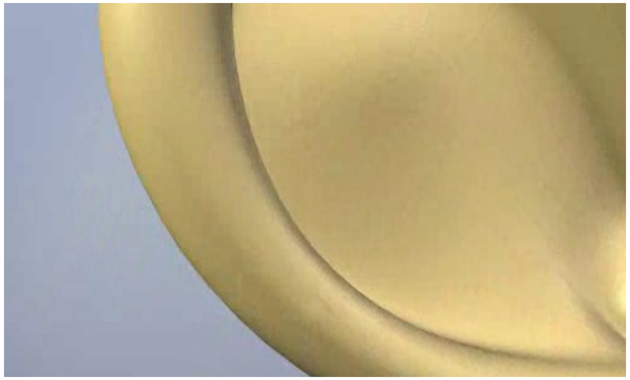


Subvidision Surfaces



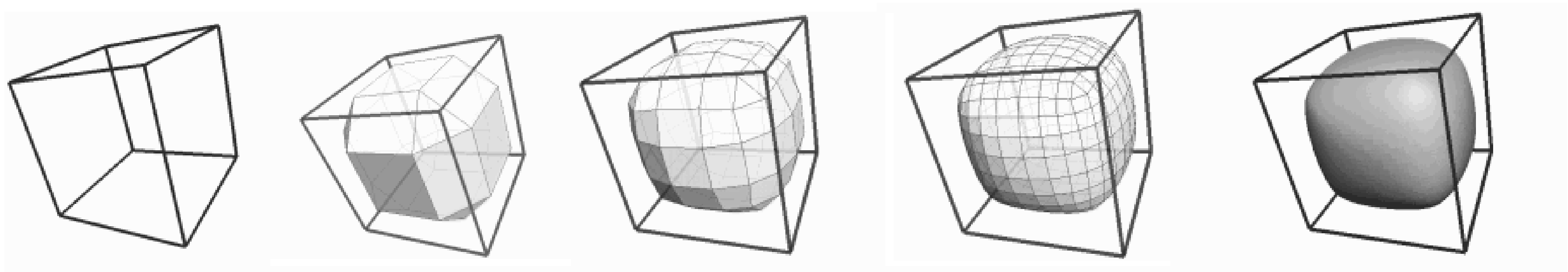




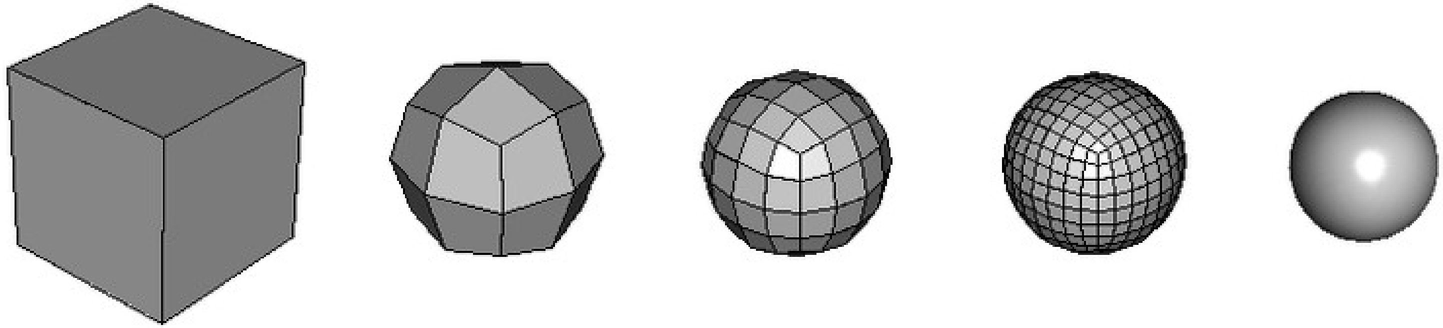


Subdivision Surfaces

Doo-Sabin

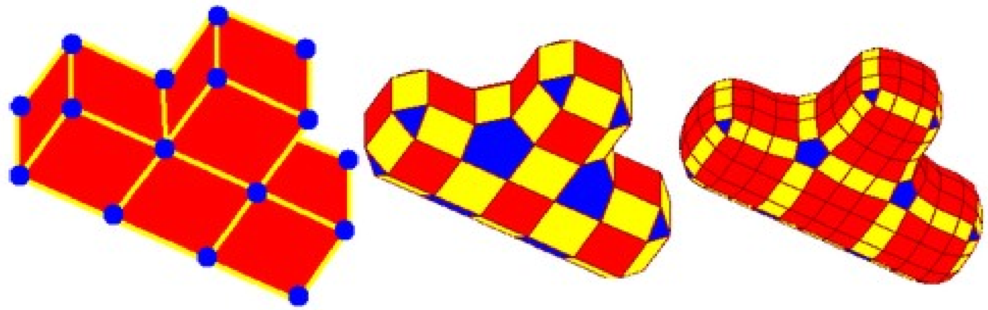


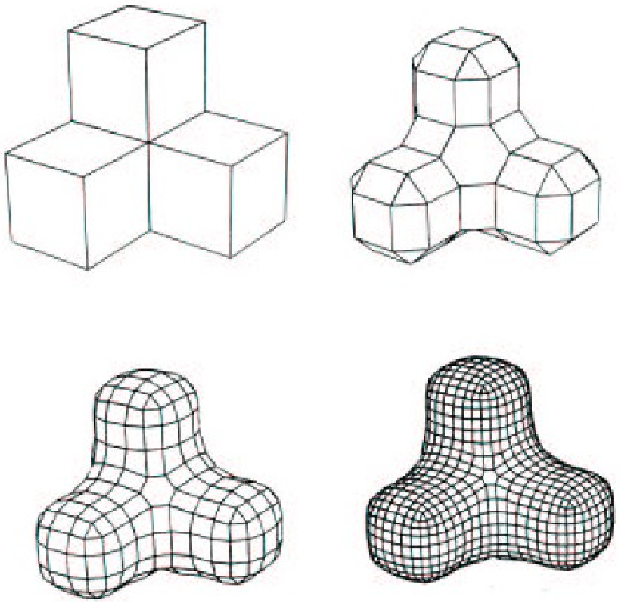
Catmull-Clark

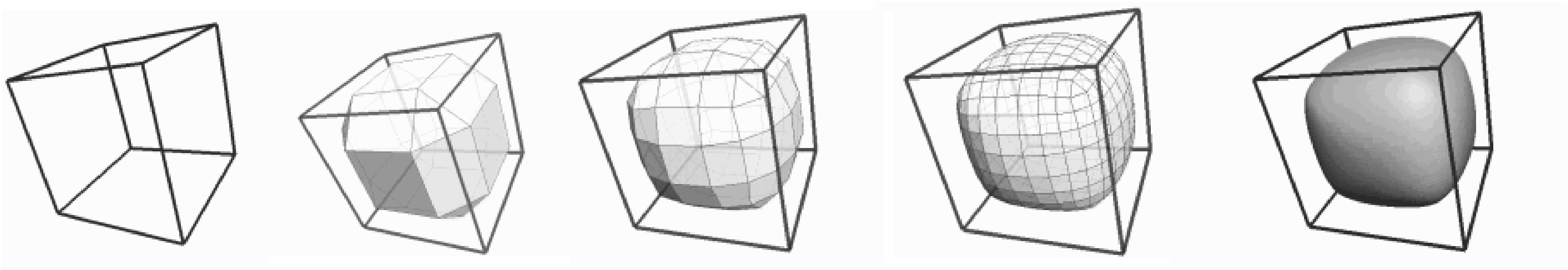


Butterfly, Loop, Midedeg, Kobbelt, etc...

Doo-Sabin



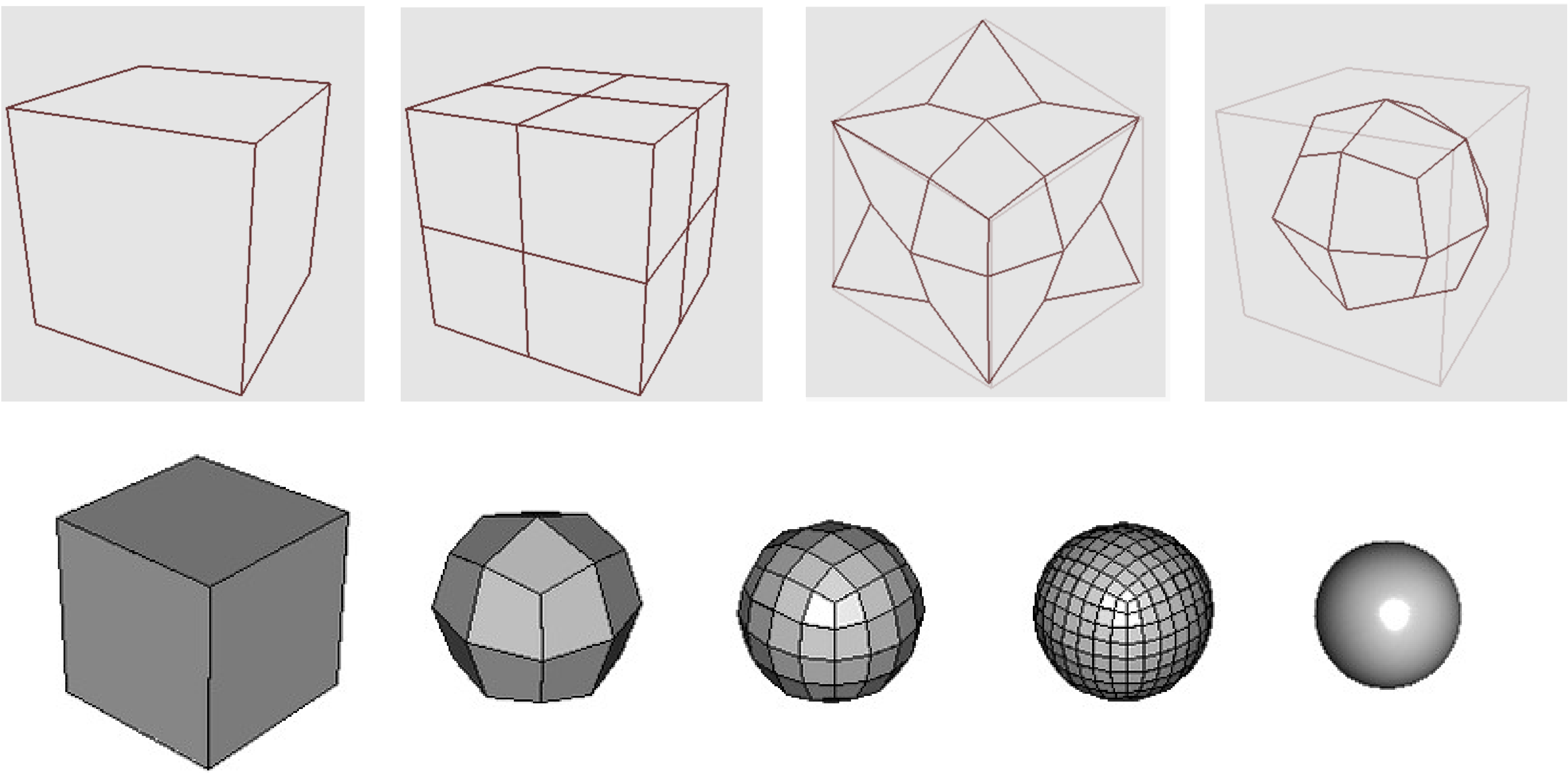




Subdivision Surfaces

Catmull-Clark

$$P'=\frac{F+2R+(n-3)P}{n}=\frac{\frac{4R'}{n}-\frac{F}{n}+P*(n-3)}{n}$$



Subdivision Surfaces