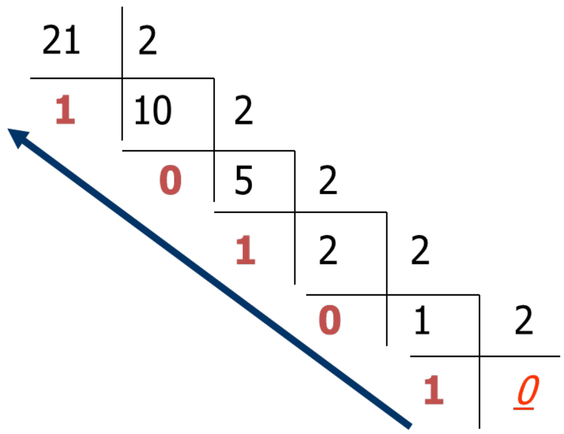
Numeral Systems

1. Convert Base 10 {0; 1; 2; 3; 4; 5; 6; 7; 8; 9} → Base K

1.1. Base 10 → Base 2 {0; 1}

21.125 $_{(10)}$ → X $_{(2)}$

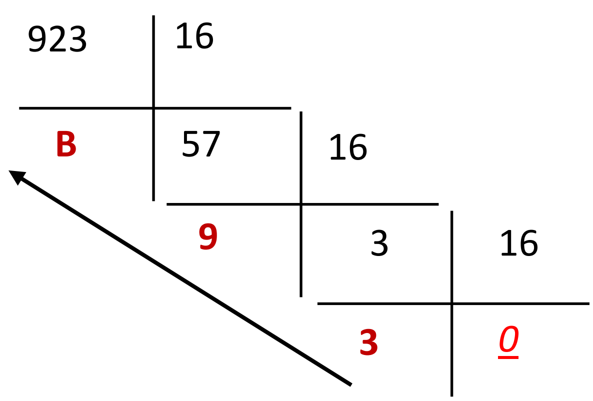


0.125 x 2 = **0**.25
0.25 x 2 = **0**.5
0.5 x 2 = **1**.0

Result: 21.125 $_{(10)}$ = 10101.001 $_{(2)}$

1.2. Base 10 → Base 16 {0; 1; 2; 3; 4; 5; 6; 7; 8; 9; A; B; C; D; E; F}

923 $_{(10)}$ → Y $_{(16)}$

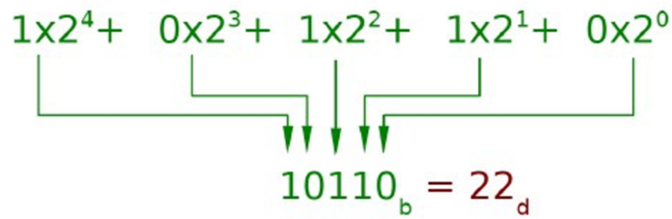


Result: 923 $_{(10)}$ = 39B $_{(16)}$

1.3. Base 10 → Base 8 {0; 1; 2; 3; 4; 5; 6; 7}

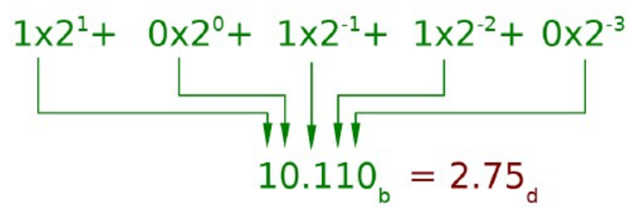
2. Convert Base 2 → Base 10

10110 $_{(b)}$ → X $_{(d)}$



Result: 10110 $_{(b)}$ = 22 $_{(d)}$

10.110 $_{(b)}$ → Y $_{(d)}$

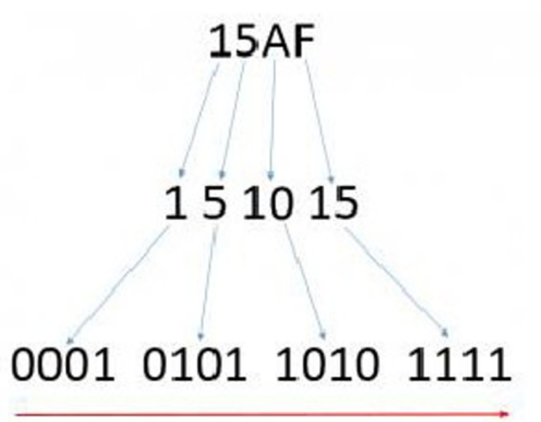


Result: 10.110 $_{(b)}$ = 2.75 $_{(d)}$

3. Convert Base 2 ←→ Base 16

3.1. Base 16 → Base 2

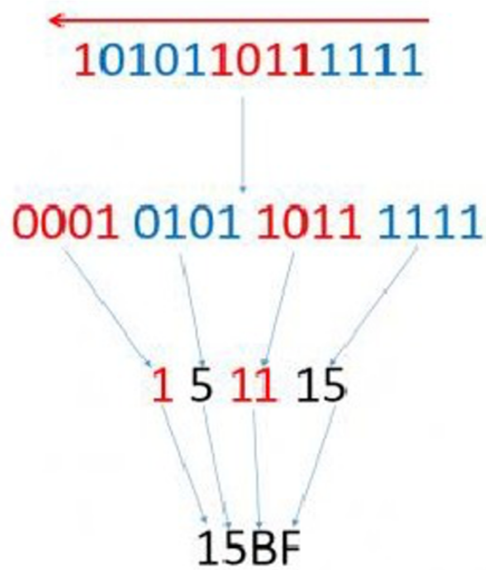
15AF $_{(16)}$ → X $_{(2)}$



Result: 15AF $_{(16)}$ = 1010110101111 $_{(2)}$

3.2. Base 2 → Base 16

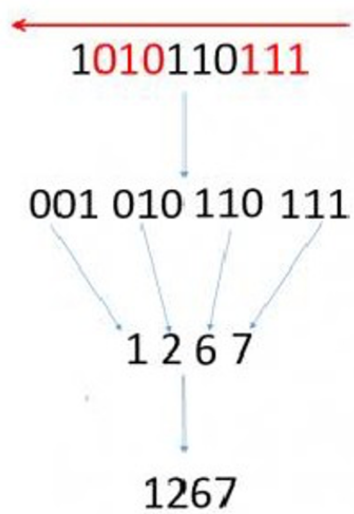
$1010110111111_{(2)} \rightarrow Y_{(16)}$



Result: $1010110111111_{(2)} = 15BF_{(16)}$

4. Convert Base 2 ←→ Base 8

$1010110111_{(2)} \rightarrow X_{(8)}$



Result: $1010110111_{(2)} = 1267_{(8)}$

5. Addition

$1110_{(2)} + 1000_{(2)} = ?$

$$\begin{array}{rrrrrr} & & 1 & 1 & 1 & 0 \\ + & & 1 & 0 & 0 & 0 \\ \hline & 1 & 0 & 1 & 1 & 0 \end{array}$$

Result: $1110_{(2)} + 1000_{(2)} = 10110_{(2)}$

$11.011_{(2)} + 10.110_{(2)} = ?$

$$\begin{array}{rrrrrrr} & & 1 & 1 & .0 & 1 & 1 \\ + & & 1 & 0 & .1 & 1 & 0 \\ \hline & 1 & 1 & 0 & .0 & 0 & 1 \end{array}$$

Result: $11.011_{(2)} + 10.110_{(2)} = 110.001_{(2)}$

$82BA_{(16)} + B781_{(16)} = ?$

$$\begin{array}{rrrrrr} & & 8 & 2 & B & A \\ + & & B & 7 & 8 & 1 \\ \hline & 1 & 3 & A & 3 & B \end{array}$$

Result: $82BA_{(16)} + B781_{(16)} = 13A3B_{(16)}$

6. Multiple

$1011_{(2)} \times 10_{(2)} = ?$

$$
\begin{array}{cccccc}
 & & 1 & 0 & 1 & 1 \\
\times & & & & 1 & 0 \\
\hline
 & & 0 & 0 & 0 & 0 \\
 & 1 & 0 & 1 & 1 & \\
\hline
 & 1 & 0 & 1 & 1 & 0
\end{array}
$$

Result: $1011_{(2)} \times 10_{(2)} = 10110_{(2)}$

$234A_{(16)} \times AB_{(16)} = ?$

$$
\begin{array}{ccccccc}
 & & & 2 & 3 & 4 & A \\
 & \times & & & & A & B \\
\hline
 & & 1 & 8 & 4 & 2 & E \\
 & 1 & 6 & 0 & E & 4 & \\
\hline
 & 1 & 7 & 9 & 2 & 6 & E
\end{array}
$$

Result: $234A_{(16)} \times AB_{(16)} = 17926E_{(16)}$

7. Subtract

$1010_{(2)} - 111_{(2)} = ?$

$$
\begin{array}{ccccc}
 & 1 & 0 & 1 & 0 \\
- & 0 & 1 & 1 & 1 \\
\hline
 & 0 & 0 & 1 & 1
\end{array}
$$

Result: $1010_{(2)} - 111_{(2)} = 11_{(2)}$

F121 $_{(16)}$ – B781 $_{(16)}$ = ?

$$\begin{array}{ccccc} & F & 1 & 2 & 1 \\ - & B & 7 & 8 & 1 \\ \hline & 3 & 9 & A & 0 \end{array}$$

Result: F121 $_{(16)}$ – B781 $_{(16)}$ = 39A0 $_{(16)}$

8. Divide

11101 $_{(2)}$: 101 $_{(2)}$ = ?

$$\begin{array}{cccccc|ccc} - & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ \cline{7-9} & 1 & 0 & 1 & & & 1 & 0 & 1 \\ \cline{2-4} & 0 & 1 & 0 & 0 & & & & \\ & - & 0 & 0 & 0 & & & & \\ \cline{3-5} & & 1 & 0 & 0 & 1 & & & \\ & & - & 1 & 0 & 1 & & & \\ \cline{4-6} & & & 1 & 0 & 0 & \leftarrow \text{Số dư} & & \end{array}$$

Result: 11101 $_{(2)}$: 101 $_{(2)}$ = 101 (remainder = 100).

9. Two's complement notation systems

Leftmost bit: sign bit

One's complement: Changing all the 0s to 1s, all the 1s to 0s.

Two's complement: Add 1 to the one's complement.

-5 $_{(10)}$ → Binary?

5 (1 byte)	0	0	0	0	0	1	0	1
One's complement of 5	1	1	1	1	1	0	1	0
+								1
Two's complement of 5	1	1	1	1	1	0	1	1

Result: $-1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = -5$

$5 - 5 = 5 + (-5) = 0$

5 (1 byte)		0	0	0	0	0	1	0	1
One's complement of 5		1	1	1	1	1	0	1	0
+									1
Two's complement of 5		1	1	1	1	1	0	1	1
+ 5		0	0	0	0	0	1	0	1
Result	1	0	0	0	0	0	0	0	0

6 bit - sized interger.

001100 (12) → 110100 (-12)

001001 (9) → 110111 (-9)

$$
\begin{array}{r cccccc l}
 & 1 & 1 & 0 & 1 & 0 & 0 & (-12) \\
+ & 1 & 1 & 0 & 1 & 1 & 1 & (-9) \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 1 & (-21)
\end{array}
$$

$$
\begin{array}{r cccccc l}
 & 0 & 0 & 1 & 0 & 0 & 1 & (9) \\
- & 0 & 0 & 1 & 1 & 0 & 0 & (12) \\
\hline
 & 1 & 1 & 1 & 1 & 0 & 1 & (-3)
\end{array}
\qquad
\begin{array}{r cccccc l}
 & 0 & 0 & 1 & 0 & 0 & 1 & (9) \\
+ & 1 & 1 & 0 & 1 & 0 & 0 & (-12) \\
\hline
 & 1 & 1 & 1 & 1 & 0 & 1 & (-3)
\end{array}
$$