

실습 보고서

[실습번호: 05]
[실습제목: 5 장 행렬식/determinant 파트 2]



과 목 명	선형대수
교 수 명	이 선우
학 번	20237107
작 성 자	하태영
제 출 일	2025.10.15

한림대학교

가. 5 장 연습문제 풀기

19. (a) $M = \begin{pmatrix} 3 & 2 & 5 & 6 \\ 6 & 5 & 6 & 7 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 2 & 5 \end{pmatrix}$ $A = \begin{pmatrix} 3 & 2 \\ 6 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 8 \\ 2 & 5 \end{pmatrix}$ $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $D = \begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$

$$M = \begin{pmatrix} A & D \\ O & B \end{pmatrix}, \det(M) = \det(A) \cdot \det(B) = \{(3 \cdot 5) - (2 \cdot 6)\} \cdot \{(4 \cdot 5) - (8 \cdot 2)\} = (15 - 12) \cdot (20 - 16) = 3 \cdot 4 = 12 \quad \therefore \det(M) = 12$$

(b) $M = \begin{pmatrix} 0 & 5 & 2 & 3 & -1 \\ -5 & 0 & 6 & 7 & 2 \\ -2 & -6 & 0 & 4 & 8 \\ -3 & -7 & -4 & 0 & 1 \\ 1 & -2 & -8 & -1 & 0 \end{pmatrix}$ 정의 5-19. 반대칭행렬의 행렬식
홀수 차, 즉 $(2n+1)$ 차 정방행렬이 반대칭행렬이면 행렬식은 0이다.
 M 은 5차 반대칭행렬이므로. $\therefore \det(M) = 0$

(c) $M = \begin{pmatrix} 5 & 6 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 1 & 2 & 2 & 1 \\ 3 & 1 & 3 & 7 \end{pmatrix}$ $A = \begin{pmatrix} 5 & 6 \\ 3 & 7 \end{pmatrix}$ $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 2 & 1 \\ 3 & 7 \end{pmatrix}$
 $M = \begin{pmatrix} A & O \\ C & D \end{pmatrix}$ $\det(M) = \{(5 \cdot 7) - (6 \cdot 3)\} \cdot \{(2 \cdot 7) - (1 \cdot 3)\} = (35 - 18) \cdot (14 - 3) = 187$

$\det(M) = 187$

(d) $M = \begin{pmatrix} 4 & 5 & 2 & 0 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & 2 & 6 \\ 0 & 1 & 1 & 5 \end{pmatrix}$ $A = \begin{pmatrix} 4 & 5 \\ 3 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 2 & 6 \\ 1 & 5 \end{pmatrix}$

$$\det(A) = 4 \cdot 1 - 5 \cdot 3 = -11 \quad \det(M) = \det(A) \det(D - CA^{-1}B)$$

$$= -11 \cdot \frac{1232}{121} = -\frac{1232}{11} = -112$$

$$A^{-1} = -\frac{1}{11} \begin{pmatrix} 1 & -5 \\ -3 & 4 \end{pmatrix} \quad A^{-1}B = -\frac{1}{11} \begin{pmatrix} 1 & -5 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = -\frac{1}{11} \begin{pmatrix} 2 & -10 \\ -6 & 8 \end{pmatrix}$$

$$CA^{-1}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{2}{11} & \frac{10}{11} \\ \frac{6}{11} & -\frac{8}{11} \end{pmatrix} = \begin{pmatrix} -\frac{2}{11} & \frac{10}{11} \\ \frac{6}{11} & -\frac{8}{11} \end{pmatrix} \quad D - CA^{-1}B = \begin{pmatrix} \frac{22}{11} & \frac{66}{11} \\ \frac{11}{11} & \frac{35}{11} \end{pmatrix} - \begin{pmatrix} -\frac{2}{11} & \frac{10}{11} \\ \frac{6}{11} & -\frac{8}{11} \end{pmatrix} = \begin{pmatrix} \frac{24}{11} & \frac{56}{11} \\ \frac{5}{11} & \frac{63}{11} \end{pmatrix}$$

$$\det(D - CA^{-1}B) = \left\{ \left(\frac{24}{11} \times \frac{63}{11} \right) - \left(\frac{5}{11} \times \frac{56}{11} \right) \right\} = \frac{1232}{121}$$

$$\frac{24}{11} \cdot \frac{63}{11} - \frac{280}{121} = \frac{1512}{121}$$

$\therefore \det(M) = -112$

19 (c)

$$M = \begin{pmatrix} 4 & 2 & 0 & 4 \\ 1 & 5 & 2 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A = \begin{pmatrix} 4 & 2 \\ 1 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 4 \\ 2 & 3 \end{pmatrix} \quad O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\det(M) = \det(A) \cdot \det(D) = \{4 \cdot 5\} - (2 \cdot 1) \cdot \{2 \cdot 1\} - (0 \cdot 0) = 18 \cdot 2 = 36$$

$$M = \begin{pmatrix} A & B \\ O & D \end{pmatrix} \quad \therefore \det(M) = 36$$

21. $(0,0), (4,2), (5,6), (9,8)$

정리 5-26. 광역식변환의 성질과 계산법

$$S = \left| \det \begin{pmatrix} 4 & 5 \\ 2 & 6 \end{pmatrix} \right| = \left| \{4 \cdot 6 - (5 \cdot 2)\} \right| = |24 - 10| = 14$$

$$\therefore S = 14$$

 $(0,0), (a,c), (b,d), (a+b, c+d)$

$$\left| \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right| = |ad - bc|$$

23.

$$\det \begin{vmatrix} x & y & z & 1 \\ 1 & 2 & -1 & 1 \\ -2 & 2 & 1 & 1 \\ 3 & 1 & 2 & 1 \end{vmatrix} = x \begin{vmatrix} 2 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} + y \begin{vmatrix} 1 & -1 & 1 \\ -2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} + z \begin{vmatrix} 1 & 2 & 1 \\ -2 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & -1 \\ -2 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$$(-1)^{1+1} \cdot \begin{vmatrix} 2 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + (-1) \cdot (-1)^{1+2} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 2 \cdot [-(-1) + 1 \cdot 1 + 1 \cdot 3] = -2 + 1 + 3 = 2 \cdot (-1)^{1+1} = 2$$

$$(-1)^{1+2} \cdot \begin{vmatrix} 1 & -1 & 1 \\ -2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 1 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + (-1) \cdot (-1)^{1+2} \cdot \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+3} \cdot \begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \cdot (-1) + 1 \cdot (-5) + 1 \cdot (-7) = -1 - 5 - 7 = -13 \cdot (-1)^{1+2} = 13$$

$$(-1)^{1+3} \cdot \begin{vmatrix} 1 & 2 & 1 \\ -2 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 1 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 2 \cdot (-1)^{1+2} \cdot \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+3} \cdot \begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 1 \cdot 1 + (-2) \cdot (-5) + 1 \cdot (-8) = 1 + 10 - 8 = 3 \cdot (-1)^{1+3} = 3$$

$$(-1)^{1+4} \cdot \begin{vmatrix} 1 & 2 & -1 \\ -2 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 2 \cdot (-1)^{1+2} \cdot \begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} + (-1) \cdot (-1)^{1+3} \cdot \begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 1 \cdot 3 + (-2) \cdot (-7) + (-1) \cdot (-8) = 3 + 14 + 8 = 25 \cdot (-1)^{1+4} = -25$$

$$2x + 3y + 3z = 25$$

$$25. (a) \begin{pmatrix} 2 & 1 & 3 \\ -1 & -2 & 0 \\ 2 & -2 & 1 \end{pmatrix} \quad \text{adj}A = \begin{pmatrix} -2 & 1 & 6 \\ -7 & -4 & 6 \\ 6 & 3 & -3 \end{pmatrix}^T = \begin{pmatrix} -2 & -7 & 6 \\ 1 & -4 & 3 \\ 6 & 6 & -3 \end{pmatrix} \quad \boxed{\therefore \text{adj}A = \begin{pmatrix} -2 & -7 & 6 \\ 1 & -4 & 3 \\ 6 & 6 & -3 \end{pmatrix}}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} = -2 \quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1 \cdot (-1) = 1 \quad A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = 6$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = (-1) \cdot 7 = -7 \quad A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = -4 \quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -1 \cdot (-6) = 6$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 3 \\ -2 & 0 \end{vmatrix} = 6 \quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} = -1 \cdot (-3) = 3 \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix} = -3$$

$$(b) \begin{pmatrix} 2 & -3 & 1 \\ 4 & 2 & 2 \\ 1 & 0 & -2 \end{pmatrix} \quad \text{adj}A = \begin{pmatrix} -4 & 10 & -2 \\ -6 & -5 & -3 \\ -8 & 0 & 16 \end{pmatrix}^T = \begin{pmatrix} -4 & -6 & -8 \\ 10 & -5 & 0 \\ -2 & -3 & 16 \end{pmatrix} \quad \boxed{\text{adj}A = \begin{pmatrix} -4 & -6 & -8 \\ 10 & -5 & 0 \\ -2 & -3 & 16 \end{pmatrix}}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 2 & 2 \\ 0 & -2 \end{vmatrix} = -4 \quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} = -1 \cdot (-10) = 10 \quad A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 4 & 2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 1 \\ 0 & -2 \end{vmatrix} = -1 \cdot 6 = -6 \quad A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -5 \quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix} = -1 \cdot 3 = -3$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} -3 & 1 \\ 2 & 2 \end{vmatrix} = -8 \quad A_{32} = 0 \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 2 & -3 \\ 4 & 2 \end{vmatrix} = 1 \cdot 16 = 16$$

$$27. (a) \begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix} \quad \text{det}(A) = (-1) \cdot (-1)^{2+1} \cdot \begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} + (-1) \cdot (-1)^{2+2} \cdot \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = 1 \cdot (-5) + (-1) \cdot (-4) = -1 \quad \text{adj}A = \begin{pmatrix} -3 & 3 & -2 \\ 5 & -4 & 2 \\ 5 & -5 & 3 \end{pmatrix}^T = \begin{pmatrix} -3 & 5 & 5 \\ 3 & -4 & -5 \\ -2 & 2 & 3 \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} = -3 \quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} = 3 \quad A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix} = -2$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} = 5 \quad A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = -4 \quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & 5 \\ 2 & 4 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix} = 5 \quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix} = -5 \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} = 3$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = -1 \cdot \begin{pmatrix} -3 & 3 & -2 \\ 5 & -4 & 2 \\ 5 & -5 & 3 \end{pmatrix} \quad \boxed{\therefore A^{-1} = \begin{pmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{pmatrix}}$$

$$27. (b) \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix} \det(A) = 1 \cdot (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = 1 + (-2) = -1 \quad \text{adj}A = \begin{pmatrix} 1 & 2 & -2 \\ 0 & -1 & 0 \\ -1 & -1 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & -1 \\ -2 & 0 & 1 \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 2 \quad A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0 \quad A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \quad A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1 \quad A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = (-1) \cdot \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & -1 \\ -2 & 0 & 1 \end{pmatrix} \quad \therefore A^{-1} = \boxed{\begin{pmatrix} -1 & 0 & 1 \\ -2 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}}$$

$$(c) \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} \det(A) = 2 \cdot (-1)^{1+1} \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} + 1 \cdot (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} + 2 \cdot (-1)^{1+3} \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \quad \text{adj}A = \begin{pmatrix} 2 & -7 & 4 \\ 1 & 4 & -3 \\ -2 & 2 & 1 \end{pmatrix}^T$$

$$= 2 \cdot 2 + (-1) \cdot 7 + 2 \cdot 4 = 4 - 7 + 8 = 5 \quad = \begin{pmatrix} 2 & 1 & -2 \\ -7 & 4 & 2 \\ 4 & -3 & 1 \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = 2 \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = -7 \quad A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1 \quad A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 4 \quad A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -3$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = -2 \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} = 2 \quad A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{5} \begin{pmatrix} 2 & 1 & -2 \\ -7 & 4 & 2 \\ 4 & -3 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{7}{5} & \frac{4}{5} & \frac{2}{5} \\ \frac{4}{5} & -\frac{3}{5} & \frac{1}{5} \end{pmatrix}$$

$$28 \quad \det(A) = 2, \quad \det(\text{adj}A) = 32 \quad |\text{adj}A| = |A|^{n-1} \Rightarrow 32 = 2^5 \quad n-1=5 \quad \therefore n=6$$

행렬 연습 5장

$$29. A = \begin{pmatrix} 1 & 0 & 7 \\ 1 & 1 & 1 \\ 7 & 1 & 1 \end{pmatrix} \quad \text{adj}A = \begin{pmatrix} -6 & 48 & -6 \\ 7 & -48 & -1 \\ -7 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} -6 & 7 & -7 \\ 48 & -48 & 0 \\ -6 & -1 & 1 \end{pmatrix} \quad \therefore \text{adj}A \cdot A = \begin{pmatrix} -48 & 0 & 0 \\ 0 & -48 & 0 \\ 0 & 0 & -48 \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 7 \\ 1 & 1 \end{vmatrix} = 1 \cdot (1-7) = -6 \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 7 \\ 7 & 1 \end{vmatrix} = -1 \cdot (1-49) = 48$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} = 1 \cdot (1-7) = -6 \quad A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 7 \\ 1 & 1 \end{vmatrix} = (-1) \cdot (-7) = 7$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 7 \\ 7 & 1 \end{vmatrix} = 1 \cdot (1-49) = -48 \quad A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 7 & 1 \end{vmatrix} = -1 \cdot 1 = -1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 7 \\ 1 & 7 \end{vmatrix} = 1 \cdot (-7) = -7 \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 7 \\ 1 & 7 \end{vmatrix} = -1 \cdot (7-7) = 0$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$\text{adj}A \cdot A = \begin{pmatrix} -6 & 7 & -7 \\ 48 & -48 & 0 \\ -6 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 7 \\ 1 & 1 & 1 \\ 7 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -6+7-49 & 0+7-7 & -42+49-7 \\ 48-48+0 & 0-48+0 & 336-336+0 \\ -6-1+7 & 0-1+1 & -42-7+1 \end{pmatrix} = \begin{pmatrix} -48 & 0 & 0 \\ 0 & -48 & 0 \\ 0 & 0 & -48 \end{pmatrix}$$

$$\frac{5}{286} \\ \frac{7}{336}$$

나. 구글 코랩 이용하여 matrix determinant 계산하기

5장 연습문제

19. 다음 행렬의 행렬식을 구하라. (np.linalg.det() 사용)

```
▶ 1 import numpy as np
2
3 # 19. (a)
4 matrix_19a = np.array([
5     [3, 2, 5, 6],
6     [6, 5, 6, 7],
7     [0, 0, 4, 8],
8     [0, 0, 2, 5]
9 ])
10 det_19a = np.linalg.det(matrix_19a)
11 print(f"19. (a) 행렬: \n{matrix_19a}")
12 print(f"19. (a) 행렬식: {det_19a}\n")
13
14 # 19. (b)
15 matrix_19b = np.array([
16     [0, 5, 2, 3, -1],
17     [-5, 0, 6, 7, 2],
18     [-2, -6, 0, 4, 8],
19     [-3, -7, -4, 0, 1],
20     [1, -2, -8, -1, 0]
21 ])
22 det_19b = np.linalg.det(matrix_19b)
23 print(f"19. (b) 행렬: \n{matrix_19b}")
24 print(f"19. (b) 행렬식: {det_19b}\n")
25
26 # 19. (c)
27 matrix_19c = np.array([
28     [5, 6, 0, 0],
29     [3, 7, 0, 0],
30     [1, 2, 2, 1],
31     [3, 1, 3, 7]
32 ])
33 det_19c = np.linalg.det(matrix_19c)
34 print(f"19. (c) 행렬: \n{matrix_19c}")
35 print(f"19. (c) 행렬식: {det_19c}\n")
```

```
37 # 19. (d)
38 matrix_19d = np.array([
39     [4, 5, 2, 0],
40     [3, 1, 0, 2],
41     [1, 0, 2, 6],
42     [0, 1, 1, 5]
43 ])
44 det_19d = np.linalg.det(matrix_19d)
45 print(f"19. (d) 행렬: \n{matrix_19d}")
46 print(f"19. (d) 행렬식: {det_19d}\n")
47
48
49 # 19. (e)
50 matrix_19e = np.array([
51     [4, 2, 0, 4],
52     [1, 5, 2, 3],
53     [0, 0, 2, 0],
54     [0, 0, 0, 1]
55 ])
56 det_19e = np.linalg.det(matrix_19e)
57 print(f"19. (e) 행렬: \n{matrix_19e}")
58 print(f"19. (e) 행렬식: {det_19e}\n")
```

→ 19. (a) 행렬:

$\begin{bmatrix} 3 & 2 & 5 & 6 \\ 6 & 5 & 6 & 7 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 2 & 5 \end{bmatrix}$

19. (a) 행렬식: 11.99999999999995

19. (b) 행렬:

$\begin{bmatrix} 0 & 5 & 2 & 3 & -1 \\ -5 & 0 & 6 & 7 & 2 \\ -2 & -6 & 0 & 4 & 8 \\ -3 & -7 & -4 & 0 & 1 \\ 1 & -2 & -8 & -1 & 0 \end{bmatrix}$

19. (b) 행렬식: -6.714628852932932e-13

19. (c) 행렬:

$\begin{bmatrix} 5 & 6 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 1 & 2 & 2 & 1 \\ 3 & 1 & 3 & 7 \end{bmatrix}$

19. (c) 행렬식: 186.9999999999999

19. (d) 행렬:

$\begin{bmatrix} 4 & 5 & 2 & 0 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & 2 & 6 \\ 0 & 1 & 1 & 5 \end{bmatrix}$

19. (d) 행렬식: -112.0000000000006

19. (e) 행렬:

$\begin{bmatrix} 4 & 2 & 0 & 4 \\ 1 & 5 & 2 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

19. (e) 행렬식: 36.0

27. 수반행렬을 이용하여 다음 행렬의 역행렬을 구하라. (np.linalg.inv() 사용)

```
1 # 27. (a)
2 A_27a = np.array([
3     [2, 5, 5],
4     [-1, -1, 0],
5     [2, 4, 3]
6 ])
7 # 행렬식 확인 (역행렬 존재 여부)
8 det_27a = np.linalg.det(A_27a)
9 print(f"27. (a) 행렬 A:\n{A_27a}")
10 print(f"27. (a) 행렬식(det): {det_27a}")
11 if np.isclose(det_27a, 0): # 행렬식이 0에 가까운지 확인
12     print("→ 행렬식이 0이므로 역행렬이 존재하지 않습니다 (특이 행렬).")
13 else:
14     A_inv_27a = np.linalg.inv(A_27a)
15     print(f"27. (a) 역행렬 A⁻¹:\n{A_inv_27a}\n")
16
17 # 27. (b)
18 A_27b = np.array([
19     [1, 0, 1],
20     [0, 1, 1],
21     [2, 0, 1]
22 ])
23 det_27b = np.linalg.det(A_27b)
24 print(f"27. (b) 행렬 A:\n{A_27b}")
25 print(f"27. (b) 행렬식(det): {det_27b}")
26 if np.isclose(det_27b, 0):
27     print("→ 행렬식이 0이므로 역행렬이 존재하지 않습니다 (특이 행렬).")
28 else:
29     A_inv_27b = np.linalg.inv(A_27b)
30     print(f"27. (b) 역행렬 A⁻¹:\n{A_inv_27b}\n")
31
32 # 27. (c)
33 A_27c = np.array([
34     [2, 1, 2],
35     [3, 2, 2],
36     [1, 2, 3]
37 ])
38 det_27c = np.linalg.det(A_27c)
39 print(f"27. (c) 행렬 A:\n{A_27c}")
40 print(f"27. (c) 행렬식(det): {det_27c}")
41 if np.isclose(det_27c, 0):
42     print("→ 행렬식이 0이므로 역행렬이 존재하지 않습니다 (특이 행렬).")
43 else:
44     A_inv_27c = np.linalg.inv(A_27c)
45     print(f"27. (c) 역행렬 A⁻¹:\n{A_inv_27c}\n")
```

```
27. (a) 행렬 A:  
[[ 2  5  5]  
 [-1 -1  0]  
 [ 2  4  3]]  
27. (a) 행렬식(det): -1.0000000000000002  
27. (a) 역행렬 A-1:  
[[ 3. -5. -5.]  
 [-3.  4.  5.]  
 [ 2. -2. -3.]]  
  
27. (b) 행렬 A:  
[[1 0 1]  
 [0 1 1]  
 [2 0 1]]  
27. (b) 행렬식(det): -1.0  
27. (b) 역행렬 A-1:  
[[-1.  0.  1.]  
 [-2.  1.  1.]  
 [ 2.  0. -1.]]  
  
27. (c) 행렬 A:  
[[2 1 2]  
 [3 2 2]  
 [1 2 3]]  
27. (c) 행렬식(det): 5.000000000000001  
27. (c) 역행렬 A-1:  
[[ 0.4  0.2 -0.4]  
 [-1.4  0.8  0.4]  
 [ 0.8 -0.6  0.2]]
```