# 실습 보고서

**[ 실습번호: 03 ]**

**[ 실습제목: 4 장 Inverse Matrix ]**

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|---|---|
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## 가. 구글 코랩 이용해서 행렬 계산하기

# 1장 프로그래밍 실습의 문제 2

20237107 하태영

```python
import numpy as np

A = np.array([[1, 2, 3],   # 3x3 행렬 A 생성
              [4, 5, 6],
              [7, 8, 9]])

v = np.array([[1],   # 3x1 행렬인 벡터 v 생성
              [2],
              [3]])
print("A =", A)
print("v =", v)
```

```
A = [[1 2 3]
 [4 5 6]
 [7 8 9]]
v = [[1]
 [2]
 [3]]
```

# 2장 프로그래밍 실습의 문제 1

20237107 하태영

```python
import numpy as np

a = np.zeros((2, 3))   # 2x3 영행렬
print("a = ", a)

b = np.ones((2, 2))    # 모든 성분이 1인 2x2 행렬
print("b = ", b)

c = np.full((3, 2), 3) # 모든 성분이 3인 3x2 행렬
print("c =", c)

d = np.eye(2)          # 2x2 단위행렬
print("d = ", d)
```

```
a =  [[0. 0. 0.]
 [0. 0. 0.]]
b =  [[1. 1.]
 [1. 1.]]
c = [[3 3]
 [3 3]
 [3 3]]
d =  [[1. 0.]
 [0. 1.]]
```

# 3장 프로그래밍 실습의 문제 2

20237107 하태영

```python
import numpy as np

# 행렬 A를 출력하는 함수
def pprint(msg, A):
  print("---", msg, "---")
  (n, m) = A.shape
  for i in range(0, n):
    line = ""
    for j in range(0, m):
      line += "{0:.2f}".format(A[i,j]) + "\t"
    print(line)
  print("")

A = np.array([[1., 2.], [3., 4.]])
B = np.array([[2., 2.], [1., 3.]])
C = np.array([[4., 5., 6.], [7., 8., 9.]])
v = np.array([[10.], [20.]])

pprint("A+B", A+B) # 행렬의 합 A+B
pprint("A-B", A-B) # 행렬의 차 A-B

pprint("3*A", 3*A) # 행렬의 스칼라배 3A
pprint("2*v", 2*v) # 벡터의 스칼라배 2v

pprint("matmul(A,B)", np.matmul(A,B)) # 행렬의 곱 AB
pprint("matmul(A,C)", np.matmul(A,C)) # 행렬의 곱 AC
pprint("A*v", A*v) # 행렬과 벡터의 곱 Av

pprint("matrix_power(A, 2)", np.linalg.matrix_power(A, 2)) # 행렬의 거듭제곱 A2
pprint("matrix_power(A, 3)", np.linalg.matrix_power(A, 3)) # 행렬의 거듭제곱 A3

pprint("A*B", A*B) # 행렬의 성분별 곱셈 A*B
pprint("A/B", A/B) # 행렬의 성분별 나눗셈 A/B
pprint("A**2 == A*A", A**2) # 행렬의 성분별 거듭제곱 A**2

pprint("A.T", A.T) # 행렬의 전치 AT
pprint("v.T", v.T) # 벡터의 전치 vT

M = np.diag([1, 2, 3]) # 대각행렬 diag(1,2,3) 생성
pprint("diag(1,2,3) =", M)

D11 = np.array([[1, 2], [3, 4]])
D12 = np.array([[5], [6]])
D21 = np.array([[7, 7]])
D22 = np.array([[8]])
D = np.block([[D11, D12], [D21, D22]]) # 블록행렬 D 생성
pprint("block matrix", D)
```

```
--- A+B ---
3.00    4.00
4.00    7.00

--- A-B ---
-1.00   0.00
2.00    1.00

--- 3*A ---
3.00    6.00
9.00    12.00

--- 2*v ---
20.00
40.00

--- matmul(A,B) ---
4.00    8.00
10.00   18.00

--- matmul(A,C) ---
18.00   21.00   24.00
40.00   47.00   54.00

--- A*v ---
10.00   20.00
60.00   80.00

--- matrix_power(A, 2) ---
7.00    10.00
15.00   22.00

--- matrix_power(A, 3) ---
37.00   54.00
81.00   118.00
```

```
--- A*B ---
2.00    4.00
3.00    12.00

--- A/B ---
0.50    1.00
3.00    1.33

--- A**2 == A*A ---
1.00    4.00
9.00    16.00

--- A.T ---
1.00    3.00
2.00    4.00

--- v.T ---
10.00   20.00

--- diag(1,2,3) = ---
1.00    0.00    0.00
0.00    2.00    0.00
0.00    0.00    3.00

--- block matrix ---
1.00    2.00    5.00
3.00    4.00    6.00
7.00    7.00    8.00
```

## 나. 3 장 연습문제 풀기

3장 연습문제

22. 행렬 $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$의 역행렬이 존재하기 위해서 k는 어떤 값이어야 하는지 설명하라.

$ad-bc \neq 0 \rightarrow 2\cdot k - 3\cdot 5 \neq 0 \rightarrow 2k \neq 15 \rightarrow k \neq \frac{15}{2}$

∴ k는 7.5가 아닌 모든 수 $\neq 7.5$

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23. 다음 문장이 참인지 거짓인지 판단하고, 거짓인 경우 그 이유를 설명하라

(a) 행렬 A가 가역이면 $(A^T)^{-1} = (A^{-1})^T$이다.

정리 3-8 (4) $A^k$은 가역이고, $(A^{-1})^k = (A^k)^{-1}$이다 (여기서 k는 0 이상의 정수이다.) ∴ 참

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(b) 행렬 A가 반대칭 행렬이면 $A = A^T$이다.

정의 3-10 정방행렬 A가 $A^T = -A$를 만족하면, A를 반대칭행렬이라 한다. ∴ 거짓

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(c) 정방행렬 A는 대칭행렬과 반대칭행렬의 합으로 나타낼 수 있다.

정리 3-10 (2) $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$이다. 즉 정방행렬 A는 대칭행렬인 $\frac{1}{2}(A+A^T)$와 반대칭행렬인 $\frac{1}{2}(A-A^T)$의 합으로 나타낼 수 있다. ∴ 참

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(d) $m \times n$ 행렬 A와 $n \times p$ 행렬 B의 곱 AB는 A의 열벡터 $a_i$와 B의 행벡터 $b_j^T$를 이용하여 $AB = a_1b_1^T + a_2b_2^T + \cdots + a_nb_n^T$로 표현할 수 있다. ∴ 참

$m=3,\ n=2,\ p=1$ $\quad AB = a_1b_1^T + a_2b_2^T = (1\ 4\ 6)\times 1 + (2\ 5\ 7)\times 2$

$A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}_{3\times 2}$ $\quad = (1\ 4\ 6) + (4\ 10\ 14) = (5\ 14\ 20)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad ||$

$B = (1\ 2)_{2\times 1}$ $\quad \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}(1\ 2) = (1\cdot 1+2\cdot 2\quad 4\cdot 1+5\cdot 2\quad 6\cdot 1+7\cdot 2) = (5\ 14\ 20)$

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(e) 하삼각행렬과 하삼각행렬의 곱은 하삼각행렬이다. ∴ 참

정리 3-14. (2) 하삼각행렬과 하삼각행렬의 곱은 하삼각행렬이다.

## 나. 3 장 복습 및 연습문제 풀기

3장 연습문제

25. (a) $\begin{pmatrix}1&0&0\\0&2&0\\0&0&4\end{pmatrix}\begin{pmatrix}1&2&3\\2&5&6\\1&3&8\end{pmatrix}=\begin{pmatrix}1&2&3\\4&10&12\\4&12&32\end{pmatrix}$

(b) $\begin{pmatrix}1&2&3\\2&5&6\\1&3&8\end{pmatrix}\begin{pmatrix}1&0&0\\0&2&0\\0&0&4\end{pmatrix}=\begin{pmatrix}1&4&12\\2&10&24\\1&6&32\end{pmatrix}$

(c) $I\cdot A=A=\begin{pmatrix}1&2&3\\2&5&6\\1&3&8\end{pmatrix}$

(d) $\begin{pmatrix}1&0&0&0\\0&3&0&0\\0&0&5&0\\0&0&0&2\end{pmatrix}\begin{pmatrix}-1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}-1&0&0&0\\0&6&0&0\\0&0&15&0\\0&0&0&2\end{pmatrix}$

27. $UV=\begin{pmatrix}2\\4\\1\end{pmatrix}\begin{pmatrix}1&3&2\end{pmatrix}=2\cdot1+4\cdot3+1\cdot2=2+12+2=16$

29. $A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$

$$=\frac{1}{2}\left(\begin{bmatrix}2&1&3\\4&5&2\\6&3&4\end{bmatrix}+\begin{bmatrix}2&4&6\\1&5&3\\3&2&4\end{bmatrix}\right)+\frac{1}{2}\left(\begin{bmatrix}2&1&3\\4&5&2\\6&3&4\end{bmatrix}-\begin{bmatrix}2&4&6\\1&5&3\\3&2&4\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}4&5&9\\5&10&5\\9&5&8\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&-3&-3\\3&0&-1\\3&1&0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}4&2&6\\8&10&4\\12&6&8\end{bmatrix}=\begin{bmatrix}2&1&3\\4&5&2\\6&3&4\end{bmatrix}$$

35. (a) $\begin{pmatrix}0&I\\I&0\end{pmatrix}\begin{pmatrix}B_{11}&B_{12}\\B_{21}&B_{22}\end{pmatrix}=\begin{pmatrix}I\cdot B_{21}&I\cdot B_{22}\\I\cdot B_{11}&I\cdot B_{12}\end{pmatrix}=I\cdot\begin{pmatrix}B_{21}&B_{22}\\B_{11}&B_{12}\end{pmatrix}=\begin{pmatrix}3&1&1&1\\3&2&1&2\\1&1&1&1\\1&2&1&1\end{pmatrix}$

(b) $\begin{pmatrix}C&0\\0&C\end{pmatrix}\begin{pmatrix}B_{11}&B_{12}\\B_{21}&B_{22}\end{pmatrix}=\begin{pmatrix}C\cdot B_{11}&C\cdot B_{12}\\C\cdot B_{21}&C\cdot B_{22}\end{pmatrix}=\begin{pmatrix}1&1&1&1\\0&1&0&0\\3&1&1&1\\0&1&0&1\end{pmatrix}$

$C\cdot B_{11}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}=\begin{pmatrix}1&1\\0&1\end{pmatrix}$ $C\cdot B_{21}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}3&1\\3&2\end{pmatrix}=\begin{pmatrix}3&1\\0&1\end{pmatrix}$

$C\cdot B_{12}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\1&1\end{pmatrix}=\begin{pmatrix}1&1\\0&0\end{pmatrix}$ $C\cdot B_{22}=\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}=\begin{pmatrix}1&1\\0&1\end{pmatrix}$

(c) $\begin{pmatrix}D&I\\I&E\end{pmatrix}\begin{pmatrix}B_{11}&B_{12}\\B_{21}&B_{22}\end{pmatrix}=\begin{pmatrix}D\cdot B_{11}+I\cdot B_{21}&D\cdot B_{12}+I\cdot B_{22}\\I\cdot B_{11}+E\cdot B_{21}&I\cdot B_{12}+E\cdot B_{22}\end{pmatrix}=\begin{pmatrix}\begin{pmatrix}2&2\\2&4\end{pmatrix}+\begin{pmatrix}3&1\\3&2\end{pmatrix}&\begin{pmatrix}2&2\\2&2\end{pmatrix}+\begin{pmatrix}1&1\\1&2\end{pmatrix}\\\begin{pmatrix}1&1\\1&2\end{pmatrix}+\begin{pmatrix}3&2\\3&1\end{pmatrix}&\begin{pmatrix}1&1\\1&1\end{pmatrix}+\begin{pmatrix}1&2\\1&1\end{pmatrix}\end{pmatrix}$

$D=2\cdot I$

$E\cdot B_{21}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}3&1\\3&2\end{pmatrix}=\begin{pmatrix}3&2\\3&1\end{pmatrix}$

$E\cdot B_{22}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}=\begin{pmatrix}1&2\\1&1\end{pmatrix}$

$$=\begin{pmatrix}5&3&3&3\\5&6&3&4\\4&3&2&3\\4&3&2&2\end{pmatrix}$$

# 나. 4 장 연습문제 풀기

4장 연습문제

2. (a) $R_2 \leftarrow 2R_1 + R_2$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $R_1 \leftrightarrow R_2$ $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (c) $R_2 \leftarrow 5R_2$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (d) $R_3 \leftarrow -R_1 + R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

---

3. (a) $R_4 \leftarrow 3R_1 + R_4$ (b) $R_3 \leftarrow 5R_3$ (c) $R_1 \leftrightarrow R_2$ (d) $R_1 \leftarrow -2R_3 + R_1$

---

4. (a)

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_3 + R_1} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_1 + R_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow -R_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

---

(b)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow \frac{1}{5}R_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## 나. 4 장 연습문제 풀기

4. (c)

$$\left(\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&3&0&1\end{matrix}\right)\left(\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}\right)\xrightarrow{R_4 \leftarrow R_4-3R_2}\left(\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}\right)\left(\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&-3&0&1\end{matrix}\right)$$

$$A=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&3&0&1\end{pmatrix}\qquad A^{-1}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&-3&0&1\end{pmatrix}$$

---

(d)

$$\begin{pmatrix}1&0&-2\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\xrightarrow{R_1 \leftarrow 2R_3+R_1}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&2\\0&1&0\\0&0&1\end{pmatrix}$$

$$A=\begin{pmatrix}1&0&-2\\0&1&0\\0&0&1\end{pmatrix}\qquad A^{-1}=\begin{pmatrix}1&0&2\\0&1&0\\0&0&1\end{pmatrix}$$

---

7. (a)

$$\frac{1}{\underbrace{(8\cdot4)}_{32}-\underbrace{(6\cdot5)}_{30}}\begin{pmatrix}4&-6\\-5&8\end{pmatrix}=\frac{1}{2}\begin{pmatrix}4&-6\\-5&8\end{pmatrix}=\begin{pmatrix}2&-3\\-\frac{5}{2}&16\end{pmatrix}$$

---

(b)

$$\frac{1}{\underbrace{(3\cdot5)}_{15}-\underbrace{(2\cdot8)}_{16}}\begin{pmatrix}5&-2\\-8&3\end{pmatrix}=-1\cdot\begin{pmatrix}5&-2\\-8&3\end{pmatrix}=\begin{pmatrix}-5&2\\8&-3\end{pmatrix}$$

---

(c)

$$\frac{1}{\underbrace{(7\cdot(-3))}_{-21}-\underbrace{(3\cdot(-6))}_{-(-18)}}\begin{pmatrix}-3&-3\\6&7\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}-3&-3\\6&7\end{pmatrix}=\begin{pmatrix}1&1\\-2&-\frac{7}{2}\end{pmatrix}$$

---

(d)

$$\frac{1}{\underbrace{(2\cdot(-6))}_{-12}-\underbrace{((-4)\cdot4)}_{-(-16)}}\begin{pmatrix}-6&4\\-4&2\end{pmatrix}=\frac{1}{4}\begin{pmatrix}-6&4\\-4&2\end{pmatrix}=\begin{pmatrix}-\frac{3}{2}&1\\-1&\frac{1}{2}\end{pmatrix}$$

---

8 (a)

$$\frac{1}{\underbrace{((-1)\cdot(-4))-((-2)\cdot(-3))}_{4\ -6}}\begin{pmatrix}-4&2\\3&1\end{pmatrix}=-\frac{1}{2}\begin{pmatrix}-4&2\\3&1\end{pmatrix}=\begin{pmatrix}2&-1\\-\frac{3}{2}&-\frac{1}{2}\end{pmatrix}$$

---

(b)

$$\frac{1}{\underbrace{(3\cdot2)-(4\cdot1)}_{6\ -\ 4}}\begin{pmatrix}2&-4\\-1&3\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2&-4\\-1&3\end{pmatrix}=\begin{pmatrix}1&-2\\-\frac{1}{2}&\frac{3}{2}\end{pmatrix}$$

## 나. 4 장 연습문제 풀기

8. (c)

$$\begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 1 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} \quad \boxed{A^{-1} = \text{존재하지 않는다.}}$$

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(d)

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 1 & 5 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - 4R_2} \begin{pmatrix} 1 & 1 & 5 \\ 1 & 0 & 3 \\ 0 & -3 & -4 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 5 \\ 0 & -1 & -2 \\ 0 & -3 & -4 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -4 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - 3R_2} \begin{pmatrix} 1 & 1 & 5 \\ 0 & -1 & -2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & -4 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_3} \begin{pmatrix} 1 & 1 & 5 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 2 & -4 & 1 \\ 3 & -4 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & 5 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & -3 & 0 \\ 2 & -4 & 1 \\ 3 & -4 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow -R_2} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & -3 & 0 \\ -2 & 4 & -1 \\ 3 & -4 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow \frac{1}{2}R_3} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & -3 & 0 \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - 5R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{9}{2} & 7 & -\frac{5}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix} \quad \boxed{A^{-1} = \begin{pmatrix} -\frac{9}{2} & 7 & -\frac{5}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix}}$$

## 나. 4 장 연습문제 풀기

9. (a)

$$\begin{pmatrix}8&6\\5&4\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}2\\-1\end{pmatrix} \rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix}=\underset{2\times2}{\begin{pmatrix}2&-3\\-\frac{5}{2}&4\end{pmatrix}}\underset{2\times1}{\begin{pmatrix}2\\-1\end{pmatrix}}=\underset{2\times1}{\begin{pmatrix}4+3\\-5-4\end{pmatrix}}=\begin{pmatrix}7\\-9\end{pmatrix}$$

$$\begin{pmatrix}8&6\\5&4\end{pmatrix}^{-1}=\frac{1}{\underset{32}{(8\cdot4)}-\underset{30}{(6\cdot5)}}\begin{pmatrix}4&-6\\-5&8\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}4&-6\\-5&8\end{pmatrix}=\begin{pmatrix}2&-3\\-\frac{5}{2}&4\end{pmatrix}$$

$$\boxed{\therefore x_1=7,\ x_2=-9}$$

---

(b)

$$\begin{pmatrix}7&3\\-6&-3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-9\\4\end{pmatrix} \rightarrow \begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}1&1\\-2&-\frac{7}{3}\end{pmatrix}\begin{pmatrix}-9\\4\end{pmatrix}=\begin{pmatrix}-9+4\\\underset{\frac{54}{3}}{18}-\frac{28}{3}\end{pmatrix}=\begin{pmatrix}-5\\\frac{26}{3}\end{pmatrix}$$

$$\begin{pmatrix}7&3\\-6&-3\end{pmatrix}^{-1}=\frac{1}{\underset{-21}{(7\cdot(-3))}-\underset{+18}{(3\cdot(-6))}}\begin{pmatrix}-3&-3\\6&7\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}-3&-3\\6&7\end{pmatrix}=\begin{pmatrix}1&1\\-2&-\frac{7}{3}\end{pmatrix}$$

$$\boxed{\therefore x_1=-5,\ x_2=\frac{26}{3}}$$

## 나. 4 장 연습문제 풀기

12. (a)

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \leftarrow R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 8 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 8 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \leftarrow \frac{1}{8}R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{8} \end{array}\right)$$

$$\xrightarrow{R_2 \leftarrow R_2 + 3R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & \frac{3}{8} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{8} \end{array}\right) \xrightarrow{R_1 \leftarrow R_1 - 3R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -\frac{9}{8} \\ 0 & 1 & 0 & -2 & 1 & \frac{3}{8} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{8} \end{array}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 & -3 & -\frac{9}{8} \\ -2 & 1 & \frac{3}{8} \\ 0 & 0 & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 7-9+\frac{9}{8} \\ -2+3-\frac{3}{8} \\ -\frac{1}{8} \end{pmatrix} = \begin{pmatrix} \frac{-16+9}{8} \\ \frac{1-3}{8} \\ -\frac{1}{8} \end{pmatrix} = \begin{pmatrix} -\frac{7}{8} \\ -\frac{1}{4} \\ -\frac{1}{8} \end{pmatrix}$$

$$\boxed{\therefore x_1 = -\frac{7}{8},\ x_2 = -\frac{1}{4},\ x_3 = -\frac{1}{8}}$$

## 나. 4 장 연습문제 풀기

12 (b)

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ 2 & 1 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$$\left(\begin{matrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ 2 & 1 & -6 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{matrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & 1 & -6 \end{matrix}\right)\left(\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}\right)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{matrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 2 & 1 & -6 \end{matrix}\right)\left(\begin{matrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{matrix}\right) \xrightarrow{R_3 \leftarrow R_3 - 2R_1} \left(\begin{matrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -3 & -8 \end{matrix}\right)\left(\begin{matrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -2 & 1 \end{matrix}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + 2R_2} \left(\begin{matrix} 1 & 0 & -5 \\ 0 & -1 & -3 \\ 0 & -3 & -8 \end{matrix}\right)\left(\begin{matrix} 2 & -3 & 0 \\ 1 & -2 & 0 \\ 0 & -2 & 1 \end{matrix}\right) \xrightarrow{R_3 \leftarrow R_3 - 3R_2} \left(\begin{matrix} 1 & 0 & -5 \\ 0 & -1 & -3 \\ 0 & 0 & 1 \end{matrix}\right)\left(\begin{matrix} 2 & -3 & 0 \\ 1 & -2 & 0 \\ -3 & 7 & 1 \end{matrix}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + 5R_3} \left(\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & -3 \\ 0 & 0 & 1 \end{matrix}\right)\left(\begin{matrix} -13 & 32 & 5 \\ 1 & -2 & 0 \\ -3 & 7 & 1 \end{matrix}\right) \xrightarrow{R_2 \leftarrow R_2 + 3R_3} \left(\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)\left(\begin{matrix} -13 & 32 & 5 \\ -8 & 19 & 3 \\ -3 & 7 & 1 \end{matrix}\right)$$

$$\xrightarrow{R_2 \leftarrow -R_2} \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)\left(\begin{matrix} -13 & 32 & 5 \\ 8 & -19 & -3 \\ -3 & 7 & 1 \end{matrix}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -13 & 32 & 5 \\ 8 & -19 & -3 \\ -3 & 7 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} ((-13)\cdot 2) + (32\cdot(-1)) + 5\cdot 4 \\ (8\cdot 2) + ((-19)\cdot(-1)) + ((-3)\cdot 4) \\ ((-3)\cdot 2) + ((-1)\cdot 7) + (1\cdot 4) \end{pmatrix} = \begin{pmatrix} -26-32+20 \\ 16+19-12 \\ -6-7+4 \end{pmatrix} = \begin{pmatrix} -38 \\ 23 \\ -9 \end{pmatrix}$$

$-58+20=-38$

$-13$, $9$    $35-12$

$$\boxed{\therefore x_1 = -38,\ x_2 = 23,\ x_3 = -9}$$

## 나. 4 장 연습문제 풀기

4장 연습문제

23. (a) $A=\left(\begin{array}{cc|cc} 5 & 5 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ \hline 0 & 0 & 6 & 4 \\ 0 & 0 & 4 & 2 \end{array}\right) = \begin{pmatrix} A_{11} & O \\ O & A_{22} \end{pmatrix}$ $\quad A^{-1} = \begin{pmatrix} A_{11}^{-1} & O \\ O & A_{22}^{-1} \end{pmatrix} = \boxed{\begin{pmatrix} \frac{2}{5} & -1 & 0 & 0 \\ -\frac{1}{5} & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 1 & -\frac{3}{2} \end{pmatrix}}$

$$A_{11}^{-1} = \frac{1}{10-5}\begin{pmatrix} 2 & -5 \\ -1 & 5 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & -5 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & -1 \\ -\frac{1}{5} & 1 \end{pmatrix}$$

$$A_{22}^{-1} = \frac{1}{12-16}\begin{pmatrix} 2 & -4 \\ -4 & 6 \end{pmatrix} = \frac{1}{-4}\begin{pmatrix} 2 & -4 \\ -4 & 6 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 1 \\ 1 & -\frac{3}{2} \end{pmatrix}$$

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(b) $A=\left(\begin{array}{cc|cc} 2 & 5 & 5 & 2 \\ 1 & 3 & -7 & -3 \\ \hline 0 & 0 & 3 & 1 \\ 0 & 0 & 11 & 4 \end{array}\right)$ $\quad A^{-1} = \begin{pmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ O & A_{22}^{-1} \end{pmatrix} = \boxed{\begin{pmatrix} -3 & 5 & -31 & 13 \\ 1 & -2 & 12 & -5 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -11 & 3 \end{pmatrix}}$

$$A_{11}^{-1} = \frac{1}{6-5}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix} \quad A_{22}^{-1} = \frac{1}{12-11}\begin{pmatrix} 4 & -1 \\ -11 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -11 & 3 \end{pmatrix}$$

$$-A_{11}^{-1}A_{12} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ -7 & -3 \end{pmatrix} = \begin{pmatrix} 15+35 & 6+15 \\ -5-14 & -2-6 \end{pmatrix} = \begin{pmatrix} 50 & 21 \\ -19 & -8 \end{pmatrix}$$

$$-A_{11}^{-1}A_{12}A_{22}^{-1} = \begin{pmatrix} 50 & 21 \\ -19 & -8 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ -11 & 3 \end{pmatrix} = \begin{pmatrix} 200-231 & -50+63 \\ -76+88 & 19-24 \end{pmatrix}$$

$$\begin{array}{r} \overset{3}{1}9 \\ \times\ 4 \\ \hline -76 \end{array} \quad \begin{array}{r} 88 \\ -76 \\ \hline 12 \end{array} \qquad = \begin{pmatrix} -31 & 13 \\ 12 & -5 \end{pmatrix}$$