실습 보고서

[실습번호: 11]
[실습제목: 8 장 연습 문제 풀기]

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가. 8장 연습문제

(1) 연습문제

2. $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$, $a=\begin{pmatrix}1\\1\end{pmatrix}$, $b=\begin{pmatrix}2\\1\end{pmatrix}$

$Aa=\begin{pmatrix}1&4\\2&3\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1+4\\2+3\end{pmatrix}=\begin{pmatrix}5\\5\end{pmatrix}=5a$

$Ab=\begin{pmatrix}1&4\\2&3\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2+4\\4+3\end{pmatrix}=\begin{pmatrix}6\\7\end{pmatrix}$

$\boxed{\text{a는 스칼라배이므로 고유벡터이다.}}$

$\boxed{\text{b는 스칼라배가 아니므로 고유벡터가 아니다.}}$

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3. $A=\begin{pmatrix}3&0\\2&2\end{pmatrix}$, $a=\begin{pmatrix}1\\1\end{pmatrix}$, $b=\begin{pmatrix}0\\1\end{pmatrix}$

$Aa=\begin{pmatrix}3&0\\2&2\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}3+0\\2+2\end{pmatrix}=\begin{pmatrix}3\\4\end{pmatrix}$

$Ab=\begin{pmatrix}3&0\\2&2\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}=2b$

$\boxed{\text{a는 스칼라배가 아니므로 고유벡터가 아니다.}}$

$\boxed{\text{b는 스칼라배이므로 고유벡터이다.}}$

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4. (a) $A=\begin{pmatrix}2&1\\-1&0\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}-\begin{pmatrix}2&1\\-1&0\end{pmatrix}=\begin{pmatrix}\lambda-2&-1\\1&\lambda\end{pmatrix}$

$\det(\lambda I-A)=\lambda(\lambda-2)+1=\lambda^2-2\lambda+1$

$\boxed{\therefore \text{특성다항식}: \lambda^2-2\lambda+1}$

(b) $B=\begin{pmatrix}2&27&0\\0&4&40\\0&3&30\end{pmatrix}$ $\quad \lambda I-B=\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}-\begin{pmatrix}2&27&0\\0&4&40\\0&3&30\end{pmatrix}=\begin{pmatrix}\lambda-2&-27&0\\0&\lambda-4&-40\\0&-3&\lambda-30\end{pmatrix}$

$\det(\lambda I-B)=\begin{vmatrix}\lambda-2&-27&0\\0&\lambda-4&-40\\0&-3&\lambda-30\end{vmatrix}\begin{matrix}\lambda-2&-27\\0&\lambda-4\\0&-3\end{matrix}$

$=(\lambda-2)(\lambda-4)(\lambda-30)+\cancel{(-27)\cdot(-40)\cdot 0}+\cancel{0\cdot 0\cdot(-3)}$

$-\cancel{(0\cdot(\lambda-4)\cdot 0)}-((\lambda-2)\cdot(-40)\cdot(-3))-\cancel{((-27)\cdot 0\cdot(\lambda-30))}$

$=(\lambda-2)(\lambda-4)(\lambda-30)-(120(\lambda-2))$

$=(\lambda^2-6\lambda+8)(\lambda-30)-120\lambda+240$

$=\lambda^3-\underline{6\lambda^2}+\underline{8\lambda}-\underline{30\lambda^2}+\underline{180\lambda}-\cancel{240}-\underline{120\lambda}+\cancel{240}$

$=\lambda^3-36\lambda^2+68\lambda$

$\boxed{\therefore \text{특성다항식}: \lambda^3-36\lambda^2+68\lambda}$

가. 8장 연습문제

(1) 연습문제

5. (a) $A=\begin{pmatrix}3&3\\3&1\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}-\begin{pmatrix}3&3\\3&1\end{pmatrix}=\begin{pmatrix}\lambda-3&-3\\-3&\lambda-1\end{pmatrix}$

$$\det(\lambda I-A)=(\lambda-3)(\lambda-1)-9=\lambda^2-4\lambda+3-9=\lambda^2-4\lambda-6=0$$

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{16+24}}{2}=\frac{4\pm2\sqrt{10}}{2}=2\pm\sqrt{10}$$

$\boxed{\therefore \text{고유값}: 2+\sqrt{10},\ 2-\sqrt{10}}$

(b) $A=\begin{pmatrix}1&5\\6&2\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}-\begin{pmatrix}1&5\\6&2\end{pmatrix}=\begin{pmatrix}\lambda-1&-5\\-6&\lambda-2\end{pmatrix}$

$$\det(\lambda I-A)=(\lambda-1)(\lambda-2)-30=\lambda^2-3\lambda+2-30=\lambda^2-3\lambda-28=0\quad(\lambda-7)(\lambda+4)=0\quad\lambda=-4,7$$

$\boxed{\therefore \text{고유값}: -4,\ 7}$

(c) $A=\begin{pmatrix}1&2&1\\6&-1&0\\-1&-2&-1\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}-\begin{pmatrix}1&2&1\\6&-1&0\\-1&-2&-1\end{pmatrix}=\begin{pmatrix}\lambda-1&-2&-1\\-6&\lambda+1&0\\1&2&\lambda+1\end{pmatrix}$

$$\det(\lambda I-A)=\begin{pmatrix}\lambda-1&-2&-1\\-6&\lambda+1&0\\1&2&\lambda+1\end{pmatrix}\begin{matrix}\lambda-1&-2\\-6&\lambda+1\\1&2\end{matrix}$$

$=(\lambda-1)(\lambda+1)(\lambda+1)+\cancel{(-2)\cdot0\cdot1}+(-1)\cdot(-6)\cdot2$

$-\left((-1)\cdot(\lambda+1)\cdot1+\cancel{(\lambda-1)\cdot0\cdot2}+(-2)\cdot(-6)\cdot(\lambda+1)\right)$

$=(\lambda^2-1)(\lambda+1)+12-(-\lambda-1+12\lambda+12)$

$=\lambda^3+\lambda^2\cancel{-\lambda}\cancel{-1}+\cancel{12}+\cancel{\lambda}+\cancel{1}-12\lambda-\cancel{12}$

$=\lambda^3+\lambda^2-12\lambda=\lambda(\lambda^2+\lambda-12)=\lambda(\lambda+4)(\lambda-3)=0\quad\lambda=0,3,-4$

$\boxed{\therefore \text{고유값}: 0,\ 3,\ -4}$

(d) $A=\begin{pmatrix}1&2&3\\1&2&3\\1&2&3\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}-\begin{pmatrix}1&2&3\\1&2&3\\1&2&3\end{pmatrix}=\begin{pmatrix}\lambda-1&-2&-3\\-1&\lambda-2&-3\\-1&-2&\lambda-3\end{pmatrix}$

$$\det(\lambda I-A)=\begin{pmatrix}\lambda-1&-2&-3\\-1&\lambda-2&-3\\-1&-2&\lambda-3\end{pmatrix}\begin{matrix}\lambda-1&-2\\-1&\lambda-2\\-1&-2\end{matrix}\qquad (x+A)(x+B)(x+C)=x^3+(A+B+C)x^2+(AB+BC+CA)x+ABC$$

$=(\lambda-1)(\lambda-2)(\lambda-3)+(-2)\cdot(-3)\cdot(-1)+(-3)\cdot(-1)\cdot(-2)$

$-\left((-3)\cdot(\lambda-2)\cdot(-1)+(\lambda-1)\cdot(-3)\cdot(-2)+(-2)\cdot(-1)\cdot(\lambda-3)\right)$

$=\lambda^3-6x^2+(2+6+3)x-6-6-6-(3\lambda-6+6\lambda-6+2\lambda-6)$

$=\lambda^3-6x^2+\cancel{11x}-\cancel{18}-\cancel{3\lambda}+\cancel{6}-\cancel{6\lambda}+\cancel{6}-\cancel{2\lambda}+\cancel{6}$

$=\lambda^3-6x^2=\lambda^2(\lambda-6)=0\quad\lambda=0,6$

$\boxed{\therefore \text{고유값}: 0,\ 6}$

가. 8장 연습문제

(1) 연습문제

6. (a) $A=\begin{pmatrix}0&1\\-2&-3\end{pmatrix}$ $\lambda I-A=\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}-\begin{pmatrix}0&1\\-2&-3\end{pmatrix}=\begin{pmatrix}\lambda&-1\\2&\lambda+3\end{pmatrix}$

$\det(\lambda I-A)=\lambda(\lambda+3)+2=\lambda^2+3\lambda+2=(\lambda+2)(\lambda+1)=0$ $\lambda=-2,-1$

$\lambda=-1 \to -I-A=\left(\begin{array}{cc|c}-1&-1&0\\2&2&0\end{array}\right)\xrightarrow{R_2\leftarrow R_2+2R_1}\left(\begin{array}{cc|c}-1&-1&0\\0&0&0\end{array}\right)$ $-x_1-x_2=0$ $-x_1=x_2$

$\boxed{\lambda=-1\text{일때, 고유벡터는 }\begin{pmatrix}-1\\1\end{pmatrix}\text{이다.}}$

$\lambda=-2 \to -2I-A=\left(\begin{array}{cc|c}-2&-1&0\\2&1&0\end{array}\right)\xrightarrow{R_2\leftarrow R_2+R_1}\left(\begin{array}{cc|c}-2&-1&0\\0&0&0\end{array}\right)$ $-2x_1-x_2=0$ $-2x_1=x_2$

$\boxed{\lambda=-2\text{일때, 고유벡터는 }\begin{pmatrix}-2\\1\end{pmatrix}\text{이다.}}$

(c) $A=\begin{pmatrix}1&2&2\\0&2&1\\-1&2&2\end{pmatrix}$ $\lambda I-A=\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}-\begin{pmatrix}1&2&2\\0&2&1\\-1&2&2\end{pmatrix}=\begin{pmatrix}\lambda-1&-2&-2\\0&\lambda-2&-1\\1&-2&\lambda-2\end{pmatrix}$

$\det(\lambda I-A)=\begin{pmatrix}\lambda-1&-2&-2\\0&\lambda-2&-1\\1&-2&\lambda-2\end{pmatrix}\begin{matrix}\lambda-1&-2\\0&\lambda-2\\1&-2\end{matrix}$

$=(\lambda-1)(\lambda-2)(\lambda-2)+(-2)\cdot(-1)\cdot 1+\cancel{(-2)\cdot 0\cdot(-2)}$

$-\left((-2)\cdot(\lambda-2)\cdot 1+(\lambda-1)\cdot(-1)\cdot(-2)+\cancel{(-2)\cdot 0\cdot(\lambda-2)}\right)$

$=\lambda^3+(-1-2-2)\lambda^2+(2+4+2)\lambda+(-4)+2-(-2\lambda+4+2\lambda-2)$

$=\lambda^3-5\lambda^2+8\lambda\cancel{-2}+\cancel{2\lambda}-4-\cancel{2\lambda}+\cancel{2}=\lambda^3-5\lambda^2+8\lambda-4$

$$\begin{array}{r|rrrr}1&1&-5&8&-4\\&&1&-4&4\\\hline&1&-4&4&0\end{array}$$

$\lambda^2-4\lambda+4=(\lambda-2)^2$

$(\lambda-1)(\lambda-2)^2=0 \to \lambda=1,2$

$\lambda=1\to I-A=\left(\begin{array}{ccc|c}0&-2&-2&0\\0&-1&-1&0\\1&-2&-1&0\end{array}\right)$ $-2y-2z=0,\ -y-z=0,\ x-2y-z=0$

$y=-z$ $x+2z-z=0\to x+z=0\to x=-z$

$x=-z,\ y=-z,\ z=z$ $(z=1)\to$ $x=-1,\ y=-1,\ z=1$

$\boxed{\lambda=1\text{일 때 고유벡터는 }\begin{pmatrix}-1\\-1\\1\end{pmatrix}\text{이다.}}$

$\lambda=2\to 2I-A=\left(\begin{array}{ccc|c}1&-2&-2&0\\0&0&-1&0\\1&-2&0&0\end{array}\right)$ $x-2y-2z=0,\ -z=0,\ x-2y=0$

$x-2y=0$ $x=2y$, $y=y$ (자유변수), $z=0$

$y=1$ $\to$ $x=2,\ y=1,\ z=0$

$\boxed{\lambda=2\text{일 때 고유벡터 }\begin{pmatrix}2\\1\\0\end{pmatrix}\text{이다.}}$

가. 8장 연습문제

(1) 연습문제

6.(d) $A=\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix}$ $\quad \lambda I-A=\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}-\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix}=\begin{pmatrix}\lambda-1&0&1\\-1&\lambda-2&-1\\-2&-2&\lambda-3\end{pmatrix}$

$$\det(\lambda I-A)=\begin{pmatrix}\lambda-1&0&1\\-1&\lambda-2&-1\\-2&-2&\lambda-3\end{pmatrix}\begin{matrix}\lambda-1&0\\-1&\lambda-2\\-2&-2\end{matrix}$$

$$=(\lambda-1)(\lambda-2)(\lambda-3)+\cancel{0\cdot(-1)\cdot(-2)}+1\cdot(-1)\cdot(-2)$$
$$-\left(1\cdot(\lambda-2)\cdot(-2)+(\lambda-1)\cdot(-1)\cdot(-2)+\cancel{0\cdot(-1)\cdot(\lambda-3)}\right)$$
$$=\lambda^3-6\lambda^2+(2+6+3)\lambda-6+2-(\cancel{-2\lambda}+4+\cancel{2\lambda}-2)$$
$$=\lambda^3-6\lambda^2+11\lambda-4-2=\lambda^3-6\lambda^2+11\lambda-6=0$$

$$\begin{array}{r|rrrr}1&1&-6&11&-6\\&&1&-5&6\\\hline&1&-5&6&\boxed{0}\end{array}$$

$(\lambda-1)(\lambda^2-5\lambda+6)=0$

$(\lambda-1)(\lambda-2)(\lambda-3)=0 \quad \lambda=1,2,3$

$\lambda=1 \to I-A=\left(\begin{array}{ccc|c}0&0&1&0\\-1&-1&-1&0\\-2&-2&-2&0\end{array}\right)\xrightarrow{R_3\leftarrow R_3-2R_2}\left(\begin{array}{ccc|c}0&0&1&0\\-1&-1&-1&0\\0&0&0&0\end{array}\right)$

$z=0,\ -x-y-z=0 \quad -x-y=0 \quad -x=y$

$\boxed{\lambda=1\text{일 때, 고유벡터는 }\begin{pmatrix}-1\\1\\0\end{pmatrix}\text{이다.}}$

$\lambda=2\to 2I-A=\left(\begin{array}{ccc|c}1&0&1&0\\-1&0&-1&0\\-2&-2&-1&0\end{array}\right)\xrightarrow{R_2\leftarrow R_2+R_1}\left(\begin{array}{ccc|c}1&0&1&0\\0&0&0&0\\-2&-2&-1&0\end{array}\right)$

$x+z=0\to x=-z$

$-2x-2y-z=0\to 2z-2y-z=0\to z=2y$

$x=-z,\ y=y,\ z=2y \qquad y=1\to z=2,\ x=-2$

$\boxed{\lambda=2\text{일 때, 고유벡터는 }\begin{pmatrix}-2\\1\\2\end{pmatrix}\text{이다.}}$

$\lambda=3\to 3I-A=\left(\begin{array}{ccc|c}2&0&1&0\\-1&1&-1&0\\-2&-2&0&0\end{array}\right)$

$2x+z=0 \to z=-2x$

$-x+y-z=0\to x+y=0\to x=-y$

$-2x-2y=0\to x=-y$

$x=-y,\ y=y,\ z=-2x$

$y=1\to x=-1,\ z=2$

$\boxed{\lambda=3\text{일 때, 고유벡터는 }\begin{pmatrix}-1\\1\\2\end{pmatrix}\text{이다.}}$

가. 8장 연습문제

(1) 연습문제

14.(b) $A=\begin{pmatrix}1&3&4\\4&2&5\\2&3&3\end{pmatrix}$ $\quad 1+2+3=6$ $\quad$ $\boxed{\therefore \text{고유값의 합} : 6}$

15.(b) $A=\begin{pmatrix}1&2&9\\12&11&2\\0&0&4\end{pmatrix}$ $\quad \det(A)=4\times(-1)^{3+3}\times\det\begin{pmatrix}1&2\\12&11\end{pmatrix}=4\cdot(\overset{-13}{11-24})=-52$

$\det(\lambda I-A)=\begin{pmatrix}\lambda-1&-2&-9\\-12&\lambda-11&-2\\0&0&\lambda-4\end{pmatrix}=(\lambda-4)\times\det\begin{pmatrix}\lambda-1&-2\\-12&\lambda-11\end{pmatrix}=(\lambda-4)\times(\lambda-1)(\lambda-11)-24$

$\lambda^2-12\lambda+11-24$

$(\lambda-4)(\lambda^2-12\lambda\underset{-13}{-13})\rightarrow(\lambda-4)(\lambda+1)(\lambda-13)=0$

$\lambda=-1,4,13$

$\boxed{\det(A)=-52,\quad \lambda=-1,4,13}$

16(b) $A=\begin{pmatrix}6&-3&5\\-1&4&-5\\-3&3&-4\end{pmatrix}$

$\det(\lambda I-A)=\begin{pmatrix}\lambda-6&3&-5\\1&\lambda-4&5\\3&-3&\lambda+4\end{pmatrix}$

$=(\lambda-6)\cdot(-1)^{1+1}\cdot\det\begin{pmatrix}\lambda-4&5\\-3&\lambda+4\end{pmatrix}+3\cdot(-1)^{1+2}\cdot\det\begin{pmatrix}1&5\\3&\lambda+4\end{pmatrix}+(-5)\cdot(-1)^{1+3}\cdot\det\begin{pmatrix}1&\lambda-4\\3&-3\end{pmatrix}$

$\lambda^2-16+15$ $\qquad -3(\lambda-11)$ $\qquad -5(-3\lambda+9)$ $\qquad -3-3\lambda+12$

$-3\lambda+9$

$(\lambda-6)(\lambda^2-1)=\lambda^3-\lambda-6\lambda^2+6$ $\qquad -3\lambda+33$ $\qquad 15\lambda-45$

$=\lambda^3-6\lambda^2+11\lambda-6$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$(\lambda-1)(\lambda^2-5\lambda+6)=0$

$\begin{matrix}-2\\-3\end{matrix}$

$(\lambda-1)(\lambda-2)(\lambda-3)=0$

$\lambda=1,2,3$ $\qquad \boxed{\lambda^3=1,8,27}$

가. 8장 연습문제

(1) 연습문제

17. (b) $A=\begin{pmatrix}4&2&2\\2&4&2\\2&2&4\end{pmatrix}$

$$\det(\lambda I-A)=\begin{pmatrix}\lambda-4&-2&-2\\-2&\lambda-4&-2\\-2&-2&\lambda-4\end{pmatrix}$$

$$=(\lambda-4)\cdot(-1)^{1+1}\cdot\det\begin{pmatrix}\lambda-4&-2\\-2&\lambda-4\end{pmatrix}+(-2)\cdot(-1)^{1+2}\cdot\det\begin{pmatrix}-2&-2\\-2&\lambda-4\end{pmatrix}+(-2)\cdot(-1)^{1+3}\cdot\det\begin{pmatrix}-2&\lambda-4\\-2&-2\end{pmatrix}$$

$(\lambda-4)(\lambda-4)(\lambda-4)-4(\lambda-4)$ $\quad 2(-2\lambda+8-4)$ $\quad -2(4+2\lambda-8)$

$\lambda^3-12\lambda^2+\underbrace{(16+16+16)}_{48}\lambda-64-4\lambda+16-4\lambda+8+8-4\lambda$

$\lambda^3-12\lambda^2+36\lambda-32$

$$\begin{array}{c|cccc} & & 32 & 16 & \\ 2 & 1 & -12 & 36 & -32 \\ & & 2 & -20 & 32 \\ \hline & 1 & -10 & 16 & \boxed{0}\end{array}$$

$(\lambda-2)(\lambda^2-10\lambda+16)=(\lambda-2)(\lambda-2)(\lambda-8)=0$

-2
-8

$\lambda=2,\ 2,\ 8$

$\boxed{\therefore \text{고유치의 역행렬 : } \frac{1}{2},\ \frac{1}{2},\ \frac{1}{8}}$

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23. $\lambda^3-4\lambda^2-4\lambda+16$

$$\begin{array}{c|cccc} 4 & 1 & -4 & -4 & 16 \\ & & 4 & 0 & -16 \\ \hline & 1 & 0 & -4 & \boxed{0}\end{array}$$

$(\lambda-4)(\lambda^2-4)$

$(\lambda+2)(\lambda-2)(\lambda-4)$

$\lambda=-2,\ 2,\ 4$

$\boxed{\det(A)=-2\cdot2\cdot4=-16}$

가. 8장 연습문제

(1) 연습문제

24. 케일리 해밀턴 정리 : $A^3-(trA)A^2+(S_2)A-(\det A)I=0 \rightarrow A^3-5A^2+7A-3I=0$

S_2 : 주대각선 2행렬식의 합

$$A=\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix} \qquad trA = 7+(-1)+(-1)=5$$

$$S_2 = \det\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} + \det\begin{pmatrix} 7 & -2 \\ 6 & -1 \end{pmatrix} + \det\begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix} = 1-4-7+12-7+12=7$$

$$\det(A) = 7\cdot(-1)^{1+1}\cdot \underbrace{\det\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}}_{1-4} + 2\cdot(-1)^{1+2}\cdot \underbrace{\det\begin{pmatrix} -6 & 2 \\ 6 & -1 \end{pmatrix}}_{6-12} + (-2)\cdot(-1)^{1+3}\cdot \underbrace{\det\begin{pmatrix} -6 & -1 \\ 6 & 2 \end{pmatrix}}_{-12+6}$$

$$= -21+12+12=3$$

$$A^3-5A^2+7A-3I=0 \rightarrow 3I = A^3-5A^2+7A$$

$$3A^{-1} = A^2-5A+7I$$

$$A^{-1} = \frac{1}{3}(A^2-5A+7I)$$

$$A^2 = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 49-12-12 & 14-2-4 & -14+4+2 \\ -42+6+12 & -12+1+4 & 12-2-2 \\ 42-12-6 & 12-2-2 & -12+4+1 \end{pmatrix} = \begin{pmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{pmatrix}$$

$$5A = 5\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 35 & 10 & -10 \\ -30 & -5 & 10 \\ 30 & 10 & -5 \end{pmatrix}$$

$$A^2-5A+7I = \begin{pmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{pmatrix} - \begin{pmatrix} 35 & 10 & -10 \\ -30 & -5 & 10 \\ 30 & 10 & -5 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 25-35+7 & 8-10+0 & -8+10+0 \\ -24+30+0 & -7+5+7 & 8-10+0 \\ 24-30+0 & 8-10+0 & -7+5+7 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{pmatrix}$$

$$\boxed{A^{-1} = \frac{1}{3}\begin{pmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{pmatrix}}$$

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27. $$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 & -1 \\ -2 & \lambda-2 & 0 \\ -8 & 0 & \lambda-3 \end{vmatrix} = (\lambda-1)\cdot(-1)^{1+1}\cdot\det\begin{pmatrix} \lambda-2 & 0 \\ 0 & \lambda-3 \end{pmatrix} + (-1)\cdot(-1)^{1+3}\cdot\det\begin{pmatrix} -2 & \lambda-2 \\ -8 & 0 \end{pmatrix}$$

$$= (\lambda-1)(\lambda-2)(\lambda-3) + (-1)\cdot(8\lambda-16)$$

$$= \lambda^3-6\lambda^2+(2+6+3)\lambda-6-8\lambda+16$$

$$= \lambda^3-6\lambda^2+3\lambda+10$$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 3 & 10 \\ & & 2 & -8 & -10 \\ \hline & 1 & -4 & -5 & \boxed{0} \end{array}$$

$$(\lambda-2)(\lambda^2-4\lambda-5) = (\lambda-2)(\lambda+1)(\lambda-5)$$

$$\lambda = -1, 2, 5$$

$3A^3-2A^2+A+4I$

$\lambda=-1 \rightarrow -3-2-1+4=-2$

$\lambda=2 \rightarrow 3\cdot2^3-2\cdot2^2+2+4 = 24-8+6=22$

$\lambda=5 \rightarrow 3\cdot5^3-2\cdot5^2+5+4 = 3\cdot125-50+9=375-41=334$

$$\boxed{\therefore \lambda = -2, 22, 334}$$

가. 8장 연습문제

(2) 구글 코랩 이용하여 eigenvalue, eigenvector 계산하기

5(a)

```python
import numpy as np

matA = np.array([[3, 3], [3, 1]])
w1, V1 = np.linalg.eig(matA)
print(w1)
print(V1)
```

```
[ 5.16227766 -1.16227766]
[[ 0.81124219 -0.58471028]
 [ 0.58471028  0.81124219]]
```

5(b)

```python
matB = np.array([[1, 5], [6, 2]])
w2, V2 = np.linalg.eig(matB)
print(w2)
print(V2)
```

```
[-4.  7.]
[[-0.70710678 -0.6401844 ]
 [ 0.70710678 -0.76822128]]
```

5(c)

```python
matC = np.array([[1, 2, 1], [6, -1, 0], [-1, -2, -1]])
w3, V3 = np.linalg.eig(matC)
print(w3)
print(V3)
```

```
[-4.00000000e+00  3.00000000e+00  9.61673584e-17]
[[ 0.40824829 -0.48507125 -0.0696733 ]
 [-0.81649658 -0.72760688 -0.41803981]
 [-0.40824829  0.48507125  0.90575292]]
```

5(d)

```python
matD = np.array([[1, 2, 3], [1, 2, 3], [1, 2, 3]])
w4, V4 = np.linalg.eig(matD)
print(w4)
print(V4)
```

```
[ 6.00000000e+00 -4.58953843e-16 -3.85359781e-17]
[[-0.57735027  0.89869292 -0.37043843]
 [-0.57735027 -0.43851903 -0.71112288]
 [-0.57735027 -0.00721829  0.5975614 ]]
```

가. 8장 연습문제

(2) 구글 코랩 이용하여 eigenvalue, eigenvector 계산하기

6(a)

```python
matA = np.array([[0, 1], [-2, -3]])
w1, V1 = np.linalg.eig(matA)
print(w1)
print(V1)
```

```
[-1. -2.]
[[ 0.70710678 -0.4472136 ]
 [-0.70710678  0.89442719]]
```

6(b)

```python
matB = np.array([[1, -1], [2, 4]])
w2, V2 = np.linalg.eig(matB)
print(w2)
print(V2)
```

```
[2. 3.]
[[-0.70710678  0.4472136 ]
 [ 0.70710678 -0.89442719]]
```

6(c)

```python
matC = np.array([[1, 2, 2], [0, 2, 1], [-1, 2, 2]])
w3, V3 = np.linalg.eig(matC)
print(w3)
print(V3)
```

```
[2.00000004 1.99999996 1.        ]
[[ 8.94427191e-01 -8.94427191e-01  5.77350269e-01]
 [ 4.47213595e-01 -4.47213595e-01  5.77350269e-01]
 [ 1.57560595e-08  1.57560598e-08 -5.77350269e-01]]
```

6(d)

```python
matD = np.array([[1, 0, -1], [1, 2, 1], [2, 2, 3]])
w4, V4 = np.linalg.eig(matD)
print(w4)
print(V4)
```

```
[2. 3. 1.]
[[-6.66666667e-01 -4.08248290e-01  7.07106781e-01]
 [ 3.33333333e-01  4.08248290e-01 -7.07106781e-01]
 [ 6.66666667e-01  8.16496581e-01  1.75541673e-16]]
```