실습 보고서

[실습번호: 09]

[실습제목: 6 장 벡터(6.6 절) 및 7 장 선형변환]

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가. 6 장 연습문제 풀기

(1) 연습문제 풀기

6장 연습문제

59. $f(x_1, x_2) = 3x_1^2 + 2x_2^2 - 4x_1x_2$ $\qquad$ $\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right) = (6x_1 - 4x_2, 4x_2 - 4x_1)$

$\frac{\partial f}{\partial x_1} = 6x_1 - 4x_2$, $\frac{\partial f}{\partial x_2} = 4x_2 - 4x_1$ $\qquad$ $\nabla f(1,2) = (\underset{6-8}{6\cdot 1 - 4\cdot 2}, \underset{8-4}{4\cdot 2 - 4\cdot 1}) = \boxed{(-2, 4)}$

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60. $F(x_1, x_2) = \begin{pmatrix} 3x_1^2 + 2x_1x_2 \\ 4x_1x_2^2 + x_2^2 \end{pmatrix}$ $\qquad$ $J_F = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}, & \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2}, & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 6x_1 + 2x_2 & 4x_2^2 \\ 2x_1 & 8x_1x_2 + 2x_2 \end{pmatrix}$

$\frac{\partial f_1}{\partial x_1} = 6x_1 + 2x_2$, $\frac{\partial f_2}{\partial x_1} = 4x_2^2$ $\qquad$ $J_F(1,2) = \begin{pmatrix} 6\cdot 1 + 2\cdot 2 & 4\cdot 2^2 \\ 2\cdot 1 & 8\cdot 1\cdot 2 + 2\cdot 2 \end{pmatrix} = \boxed{\begin{pmatrix} 10 & 16 \\ 2 & 20 \end{pmatrix}}$

$\frac{\partial f_1}{\partial x_2} = 2x_1$, $\frac{\partial f_2}{\partial x_2} = 8x_1x_2 + 2x_2$

가. 6 장 연습문제 풀기

(2)프로그래밍 실습

2번

```
import numpy as np

def angle2vectors(v, w):  # 두 벡터의 사잇각 계산
  vnorm = np.linalg.norm(v)
  wnorm = np.linalg.norm(w)
  vwdot = np.dot(v.T, w)
  angle = np.arctan(vwdot/(vnorm*wnorm))*360/np.pi
  return angle

def orthProj(u, x):
  xu_dot = np.dot(x.T, u)
  uu_dot = np.dot(u.T, u)
  projux = (xu_dot/uu_dot)*u
  return projux

A = np.array([[2], [4], [1]])
B = np.array([[1], [-1], [3]])
angle = angle2vectors(A, B)
projAB = orthProj(B, A)
print("A와 B의 사잇각 : ", angle)
print("A의 B 위로의 정사영 : \n", projAB)
```

```
A와 B의 사잇각 :  [[7.52871961]]
A의 B 위로의 정사영 :
 [[ 0.09090909]
 [-0.09090909]
 [ 0.27272727]]
```

3번

```python
import numpy as np

def tripleProduct(u, v, w):  # 스칼라 삼중적 u * (v x w) 계산
  M = np.zeros((3,3))
  M[0:] = u
  M[1:] = v
  M[2:] = w
  val = np.linalg.det(M) # 행벡터가 u, v, w인 행렬의 행렬식 계산
  return val

A = np.array([1, 2, 3])
B = np.array([0, 5, 2])
C = np.array([2, 2, 4])
D = np.array([2, 4, 1])
u = B-A
v = C-A
w = D-A
val = tripleProduct(u, v, w)
print("부피 : ", np.absolute(val))
```

```
부피 :  9.000000000000002
```

4번

```python
import numpy as np

def distPt2Pl(A, W, P):
  num = np.dot((P-A).T, W)
  deno = np.linalg.norm(W)
  val = np.absolute(num)/deno
  return val

A = np.array([2, 3, 4])
W = np.array([1, 2, 3])
P = np.array([0, 1, 2])
print("거리 : ", distPt2Pl(A, W, P))
```

```
거리 :  3.2071349029490928
```

나 7장 연습문제 풀기

(1) 연습문제 풀기

7장 연습문제

3(a) $L(x_1,x_2)=(x_1+x_2,\ 2x_2)$

$L: R^2 \to R^2$ 사상, $u=(u_1,u_2)$, $v=(v_1,v_2)$

1. 가산성 확인

$u+v=(u_1+v_1,\ u_2+v_2)$

$L(u+v)=L(u_1+v_1,\ u_2+v_2)=((u_1+v_1)+(u_2+v_2),\ 2(u_2+v_2))$

$=(u_1+u_2+v_1+v_2,\ 2u_2+2v_2)$

$L(u)+L(v)=(u_1+u_2,\ 2u_2)+(v_1+v_2,\ 2v_2)$

$=(u_1+u_2+v_1+v_2,\ 2u_2+2v_2)$

$L(u+v)=L(u)+L(v)$ 가산성 성립

2. 동차성 확인

$cu=(cu_1,\ cv_2)$

$L(cu)=L(cu_1,cu_2)=(cu_1+cu_2,\ 2(cu_2))$

$=(c(u_1+u_2),\ c(2u_2))$

$cL(u)=c(u_1+u_2,\ 2u_2)=(c(u_1+u_2),\ c(2u_2))$

$L(cu)=cL(u)$ 동차성 성립

$\boxed{\therefore L\text{은 선형변환입니다.}}$

3.(c) $L(x_1,x_2,x_3)=(2x_1+x_2,\ x_2+3x_3,\ x_1+4x_3)$

$L: R^3 \to R^3$ 사상, 각 성분이 입력 변수들의 상수항이 없는 일차 결합으로 표현되므로, 선형변환 입니다.

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$\boxed{\therefore L\text{은 선형변환입니다.}}$

3.(e)

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$L: R^3 \to R^2$ 사상

$A=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \end{pmatrix}$에 의한 곱셈 $L(x)=Ax$ 형태로 주어져 있습니다.

$L(u+v)=A(u+v)=Au+Av=L(u)+L(v)$ (가산성)

$L(cu)=A(cu)=c(Au)=c(Lu)$ (동차성)

$\boxed{\therefore L\text{은 선형변환입니다.}}$

7장 연습문제

5.(a) $L(x_1, x_2) = (-3x_1, 2x_2)$

$\rightarrow A(L(e_1), L(e_2)) \rightarrow e_1 = (1,0),\ e_2 = (0,1)$

$L(1,0) = (-3\cdot 1, 2\cdot 0) = (-3, 0)$ $\quad L(0,1) = (-3\cdot 0, 2\cdot 1) = (0, 2)$

$$\boxed{\therefore A = \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}}$$

(b) $L(x_1, x_2) = (x_1 + x_2, 3x_1 - 4x_2)$

$\rightarrow A(L(e_1), L(e_2)) \rightarrow e_1 = (1,0),\ e_2 = (0,1)$

$L(1,0) = (1+0, 3\cdot 1 - 4\cdot 0) = (1, 3)$

$L(0,1) = (0+1, 3\cdot 0 - 4\cdot 1) = (1, 4)$

$$\boxed{\therefore A = \begin{pmatrix} 1 & 1 \\ 3 & -4 \end{pmatrix}}$$

(c) $L(x_1, x_2, x_3) = (2x_1 + x_2 + x_3,\ 2x_2 - 4x_3)$

$\rightarrow A(L(e_1), L(e_2), L(e_3)) \rightarrow e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)$

$L(1,0,0) = (2\cdot 1 + 0 + 0,\ 2\cdot 0 - 4\cdot 0) = (2, 0)$

$L(0,1,0) = (2\cdot 0 + 1 + 0,\ 2\cdot 1 - 4\cdot 0) = (1, 2)$

$L(0,0,1) = (2\cdot 0 + 0 + 0,\ 2\cdot 0 - 4\cdot 1) = (0, -4)$

$$\boxed{\therefore A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & -4 \end{pmatrix}}$$

(d) $L(x_1, x_2, x_3) = (2x_1 + x_2 + x_3,\ 2x_2 - 4x_3)$

$L(1,0,0) = (2\cdot 1 + 0 + 0,\ 2\cdot 0 - 4\cdot 0) = (2, 0)$

$L(0,1,0) = (2\cdot 0 + 1 + 0,\ 2\cdot 1 - 4\cdot 0) = (1, 2)$

$L(0,0,1) = (2\cdot 0 + 0 + 1,\ 2\cdot 0 - 4\cdot 1) = (1, -4)$

$$\boxed{\therefore A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & -4 \end{pmatrix}}$$

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6. (a) $L(1,2,3) \rightarrow \underbrace{\begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & -1 \end{pmatrix}}_{2\times 3} \underbrace{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}_{3\times 1} = (2+4+3,\ 3+2-3) = \boxed{(9, 2)}$

(b) $L(1,0,-1) \rightarrow \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = (2+0-1,\ 3+0+1) = \boxed{(1, 4)}$

(c) $L(2,3,1) \rightarrow \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = (4+6+1,\ 6+3-1) = \boxed{(11, 8)}$

(d) $L(x, y, z) \rightarrow \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boxed{(2x+2y+z,\ 3x+y-z)}$

7장 연습문제

7 (a) $\boxed{\begin{pmatrix} 2 & -4 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 3 \end{pmatrix}}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1-4x_2 \\ x_1-x_3 \\ -x_2+3x_3 \end{pmatrix}$ (b) $\boxed{\begin{pmatrix} 3 & -2 \\ 1 & 4 \\ 0 & 1 \end{pmatrix}}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1-2x_2 \\ x_1+4x_2 \\ x_2 \end{pmatrix}$

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10. $t_x=2.\ t_y=3 \rightarrow T=\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}}$

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13. 회전변환

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \rightarrow \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

반사변환

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \boxed{\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}}$$

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19. $L_1(x,y)=(2x+y,\ x+2y),\ L_2(x,y)=(-x+y,\ 2y)$

$u_1=2x+y,\ u_2=x+2y \quad L_2(u_1,u_2)=(-u_1+u_2,\ 2u_2)$

$L_2(L_1(x,y))=(-2x-y+x+2y,\ 2x+4y)$

$\boxed{\therefore L_2\circ L_1(x,y) = (-x+y,\ 2x+4y)}$

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20. $L_2\circ L_1$의 표준행렬 $=BA$

$$BA=\begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} -2+2 & -3+8 \\ 6+1 & 9+4 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 5 \\ 7 & 13 \end{pmatrix}}$$

7장 연습문제

21. (a) 1. 크기 (norm)

$$\|a_1\|^2 = \left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = \frac{9}{9} = 1$$

$$\|a_2\|^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{9}{9} = 1$$

$$\|a_3\|^2 = \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 = \frac{4}{9} + \frac{1}{9} + \frac{4}{9} = \frac{9}{9} = 1$$

2. 직교성

$$a_1 \cdot a_2 = \frac{2}{3} \cdot \frac{1}{3} + -\frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} - \frac{4}{9} + \frac{2}{9} = 0$$

$$a_2 \cdot a_3 = \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \left(-\frac{1}{3}\right) + \frac{2}{3} \cdot \left(-\frac{2}{3}\right) = -\frac{4}{9}$$

직교가 성립하지 않으므로 노름보존 선형연산자가 아닙니다.

(b) 1. 크기

$$\|a_1\| = 1^2 + 0^2 + 0^2 = 1$$

$$\|a_2\| = 0^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$

$$\|a_3\| = 0^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

2. 직교성

$$a_1 \cdot a_2 = 1 \cdot 0 + 0 \cdot \frac{\sqrt{3}}{2} + 0 \cdot \left(-\frac{1}{2}\right) = 0$$

$$a_2 \cdot a_3 = 0 \cdot 0 + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} = 0$$

$$a_3 \cdot a_1 = 0 \cdot 1 + \frac{1}{2} \cdot 0 + \frac{\sqrt{3}}{2} \cdot 0 = 0$$

∴ A는 노름보존 선형연산자이다.

7장 연습문제

29. (a) $\begin{pmatrix} 1 & 2 & 4 & 4 \\ \color{blue}-3 & \color{blue}-6 & \color{blue}-12 & \color{blue}-12 \\ 3 & 4 & 8 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 2 & 4 & 4 \\ 0 & -2 & -4 & -12 \end{pmatrix}$ $\boxed{\therefore \operatorname{rank}(A) = 2}$

(b) $\begin{pmatrix} 1 & 2 & 3 \\ \color{blue}-2 & \color{blue}-4 & \color{blue}-6 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ \color{blue}-3 & \color{blue}-6 & \color{blue}-9 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - 3R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -2 & -2 \\ \color{blue}-4 & \color{blue}-8 & \color{blue}-12 \\ 4 & 5 & 9 \end{pmatrix}$

$\xrightarrow{R_4 \leftarrow R_4 - 4R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ & \color{blue}2 & \color{blue}2 \\ 0 & -2 & -2 \\ 0 & -3 & -3 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ & \color{blue}3 & \color{blue}3 \\ 0 & -3 & -3 \end{pmatrix} \xrightarrow{R_4 \leftarrow R_4 - 3R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\boxed{\therefore \operatorname{rank}(A) = 2}$

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30. (a) $\begin{pmatrix} 1 & 2 & 1 & 5 \\ 2 & 4 & -3 & 0 \\ -3 & 1 & 2 & -1 \\ \color{blue}-1 & \color{blue}-2 & \color{blue}-1 & \color{blue}-5 \\ 1 & 2 & -1 & 1 \end{pmatrix} \xrightarrow{R_4 \leftarrow R_4 - R_1} \begin{pmatrix} 1 & 2 & 1 & 5 \\ \color{blue}-2 & \color{blue}-4 & \color{blue}-2 & \color{blue}-10 \\ 2 & 4 & -3 & 0 \\ -3 & 1 & 2 & -1 \\ 0 & 0 & -2 & -4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 1 & 5 \\ 0 & 0 & -5 & -10 \\ \color{blue}3 & \color{blue}6 & \color{blue}3 & \color{blue}15 \\ -3 & 1 & 2 & -1 \\ 0 & 0 & -2 & -4 \end{pmatrix}$

$\xrightarrow{R_3 \leftarrow R_3 + 3R_1} \begin{pmatrix} 1 & 2 & 1 & 5 \\ 0 & 0 & -5 & -10 \\ 0 & 7 & 5 & 14 \\ 0 & 0 & -\overset{\color{blue}1}{\cancel{2}} & -\overset{\color{blue}2}{\cancel{4}} \end{pmatrix} \xrightarrow{R_4 \leftarrow -\frac{1}{2}R_4} \begin{pmatrix} 1 & 2 & 1 & 5 \\ 0 & 0 & -5 & -10 \\ \color{blue}0 & \color{blue}0 & \color{blue}5 & \color{blue}10 \\ 0 & 7 & 5 & 14 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

$\xrightarrow{R_2 \leftarrow R_2 + 5R_4} \begin{pmatrix} 1 & 2 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & 5 & 14 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{R_2 \leftrightarrow R_4} \begin{pmatrix} 1 & 2 & 1 & 5 \\ 0 & 7 & 5 & 14 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\operatorname{rank}(A) = 3$

$\operatorname{nullity}(A) = 4 - 3 = 1$

$\boxed{\therefore \operatorname{rank}(A) = 3,\ \operatorname{nullity}(A) = 1}$

(b) $\begin{pmatrix} 1 & -2 & 1 \\ \color{blue}-1 & \color{blue}2 & \color{blue}-1 \\ 1 & -1 & 3 \\ 1 & 1 & 7 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ \color{blue}-1 & \color{blue}2 & \color{blue}-1 \\ 1 & 1 & 7 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ & \color{blue}-3 & \color{blue}-6 \\ 0 & 3 & 6 \end{pmatrix}$

$\xrightarrow{R_3 \leftarrow R_3 - 3R_2} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$\operatorname{rank}(A) = 2$

$\operatorname{nullity}(A) = 3 - 2 = 1$

$\boxed{\therefore \operatorname{rank}(A) = 2,\ \operatorname{nullity}(A) = 1}$

7장 연습문제

31 (a) $A=\begin{pmatrix} 1 & 2 & -6 \\ -2 & -4 & 12 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2+2R_1} \begin{pmatrix} 1 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix}$, 열공간의 기저: $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\operatorname{rank}(A)=1$

$x_1+2x_2-6x_3=0 \rightarrow x_1=-2x_2+6x_3$, x_1: 기본 변수, x_2, x_3: 자유 변수 $x_2=s,\ x_3=t$

$x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} -2s+6t \\ s \\ t \end{pmatrix}=s\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}+t\begin{pmatrix} 6 \\ 0 \\ 1 \end{pmatrix} \rightarrow$ 영공간의 기저: $\left\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 1 \end{pmatrix}\right\}$, $\operatorname{nullity}(A)=2$

$\boxed{\therefore \text{열공간의 기저: } \begin{pmatrix} 1 \\ -2 \end{pmatrix},\ \text{영공간의 기저: } \left\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 1 \end{pmatrix}\right\}}$

(b) $A=\begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2-R_1} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 열공간의 기저: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\operatorname{rank}(A)=1$

$x_1+x_2=0 \rightarrow x_1=-x_2$, x_1: 기본 변수, x_2, x_3: 자유 변수 $x_2=s,\ x_3=t$

$x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} -s \\ s \\ t \end{pmatrix}=s\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}+t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ 영공간의 기저: $\left\{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$

$\boxed{\therefore \text{열공간의 기저: } \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ \text{영공간의 기저: } \left\{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}}$

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33 (a) $\left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 0 & -9 & -6 \\ -3 & 1 & 6 & 3 \\ 2 & -2 & -1 & -1 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2+3R_1} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ -2 & 0 & 6 & 4 \\ 2 & -2 & -1 & -1 \end{array}\right) \xrightarrow{R_3 \leftarrow R_3-2R_1} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ & 2 & -6 & -6 \\ 0 & -2 & 5 & 3 \end{array}\right)$

$\xrightarrow{R_3 \leftarrow R_3+2R_2} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -1 & -3 \end{array}\right) \xrightarrow{R_3 \leftarrow -R_3} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ & & 3 & 9 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2+3R_3} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ & & 3 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right)$

$\xrightarrow{R_1 \leftarrow R_1+3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{array}\right)$ $x_1=7,\ x_2=6,\ x_3=3$ $\boxed{\therefore \text{해는 유일하다.}}$

7장 연습문제

33 (b)

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 1 & -3 & -4 \\ \overset{-2}{2} & \overset{4}{-5} & \overset{-6}{6} & \overset{12}{-5} \end{array}\right) \xrightarrow{R_3 \leftarrow R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 1 & -3 & -4 \\ 0 & \overset{-1}{-1} & 0 & \overset{7}{7} \end{array}\right)$$

$$\xrightarrow{R_2 \leftarrow R_2 + R_3} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 0 & -3 & 3 \\ 0 & -1 & 0 & 7 \end{array}\right) \xrightarrow[R_3 \leftarrow -R_3]{R_2 \leftarrow -\frac{1}{3}R_2} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -7 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -6 \\ 0 & \overset{2}{1} & 0 & \overset{-14}{-7} \\ 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow{R_1 \leftarrow R_1 + 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 3 & -20 \\ 0 & 1 & \overset{-3}{0} & \overset{3}{-7} \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -1 \end{array}\right) \qquad x_1 = -17,\ x_2 = -7,\ x_3 = -1 \qquad \boxed{\therefore x\text{는 유일하다.}}$$

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36. $A = \begin{pmatrix} 2 & -2 & 4 \\ 1 & -1 & 2 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ $\boxed{\text{열공간 기저: } \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$

x_1 = 기본 변수. x_2, x_3 : 자유 변수 $x_2 = s,\ x_3 = t.$

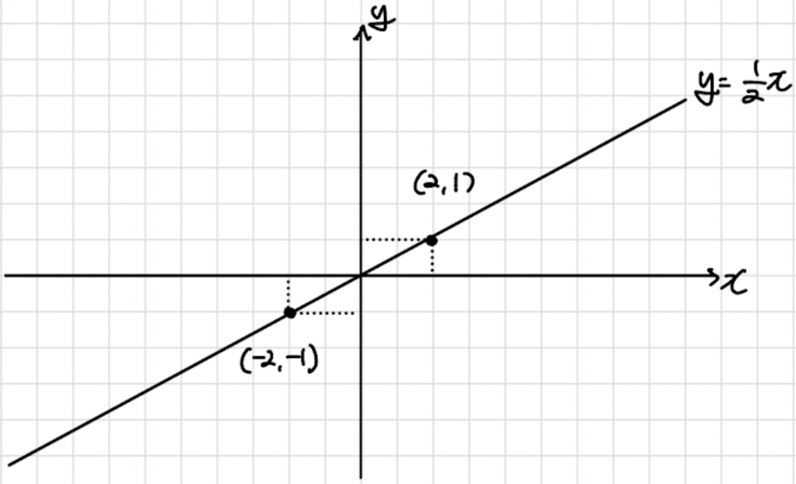
$2x_1 - 2x_2 + 4x_3 = 0 \rightarrow 2x_1 = 2x_2 - 4x_3 \rightarrow x_1 = x_2 - 2x_3$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} s - 2t \\ s \\ t \end{pmatrix} = s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \qquad \boxed{\text{영공간의 기저: } \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right\}}$$

7장 연습문제

36.

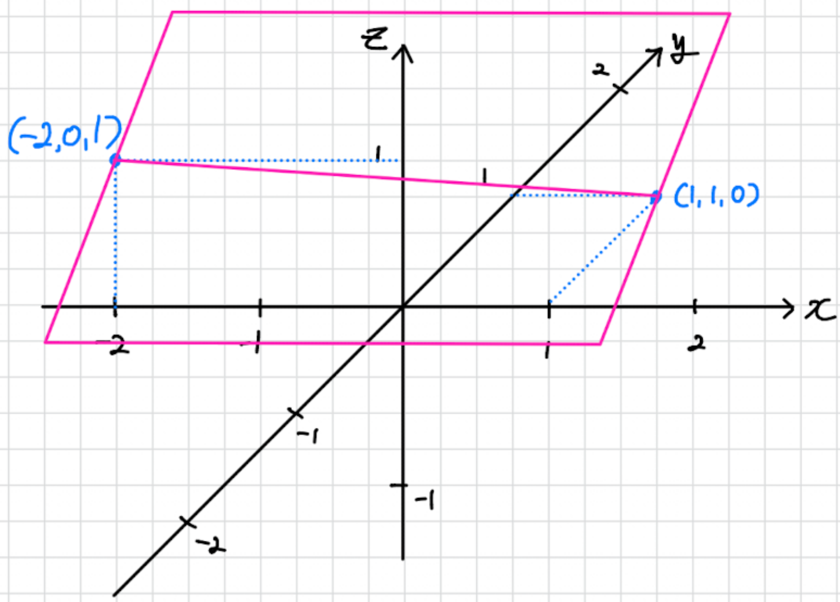
열공간 기저: $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = c\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2c \\ c \end{pmatrix} \rightarrow \begin{matrix} x=2c \\ y=c \end{matrix} \rightarrow x=2y \rightarrow y=\frac{1}{2}x$



영공간의 기저: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\begin{array}{rrrr} & 1 & 1 & 0 \\ \times & -2 & 0 & 1 \\ \hline & 1 & -1 & 2 \end{array} \rightarrow x-y+2z=0$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$



7장 연습문제

41.

$$\left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 4 & 0 & 2 & 2 \\ 0 & 6 & -3 & 4 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 0 & -4 & 4 & 0 \\ 0 & 6 & -3 & 4 \end{array}\right) \xrightarrow{R_2 \leftarrow -\frac{1}{4}R_2} \left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 6 & -3 & 4 \end{array}\right)$$

$$\xrightarrow{R_3 \leftarrow R_3 - 6R_2} \left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 4 \end{array}\right) \xrightarrow{R_3 \leftarrow \frac{1}{3}R_3} \left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right)$$

$$\xrightarrow{R_2 \leftarrow R_2 + R_3} \left(\begin{array}{ccc|c} 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right) \xrightarrow{R_1 \leftarrow R_1 - 2R_2} \left(\begin{array}{ccc|c} 2 & 0 & -1 & -\frac{5}{3} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right) \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{6} \\ 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right)$$

$\boxed{\therefore \text{연립방정식의 해는 유일하다}}$