Dubins Orienteering Problem

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***Abstract*—In this letter, we address the orienteering problem (OP) for curvature constrained vehicle. For a given set of target locations, each with associated reward, the OP stands to find a tour from a prescribed starting location to a given ending location such that it maximizes collected rewards while the tour length is within a given travel budget constraint. The addressed generalization of the Euclidean OP is called the Dubins Orienteering Problem (DOP) in which the reward collecting tour has to satisfy the limited turning radius of the Dubins vehicle. The DOP consists not only of selecting the most valuable targets and determination of the optimal sequence to visit them, but it also involves the determination of the vehicle's heading angle at each target location. The proposed solution is based on the Variable neighborhood search technique, and its feasibility is supported by an empirical evaluation in existing OP benchmarks. Moreover, an experimental verification in a real practical scenario further demonstrates the necessity of the proposed direct solution of the Dubins Orienteering Problem.**

***Index Terms*—Aerial systems: applications, motion and path planning, nonholonomic motion planning.**

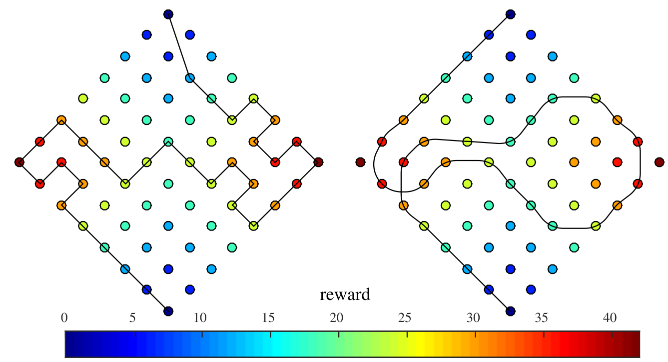

Fig. 1. Solutions of the Dubins Orienteering Problem on Set 64 for the budget $T_{\max} = 50$ and different turning radii ρ. For $\rho = 0$, the problem becomes the ordinary EOP with the sum of the collected rewards $R = 900$ (on left), while for $\rho = 1.3$ the problem has to be directly solved as the DOP to satisfy $T_{\max}$ and the collected reward is $R = 714$ (on right). In both cases, the constructed path lengths are maximally 0.2 bellow the allowed $T_{\max}$.

I. Introduction

IN THIS letter, we study a generalization of the Orienteering Problem (OP) [1] for curvature-constrained vehicles. The problem is called the Dubins Orienteering Problem (DOP), and its objective is to maximize the total collected rewards by visiting a subset of the given target locations by Dubins vehicle [2] while the length of the collecting tour does not exceed a given travel budget. The proposed generalization of the existing OP with Euclidean distance [3], further denoted as the Euclidean OP (EOP), is motivated by data collection scenarios with Unmanned Aerial Vehicles (UAVs) that can be modeled as the non-holonomic Dubins vehicle [4].

The Orienteering Problem can be considered as a variant of the Traveling Salesman Problem (TSP). In contrast to the TSP, in which the goal is to minimize the tour length to visit all the targets, the OP objective is to maximize the total sum of the collected rewards while the reward collecting tour does not exceed the specified travel budget. Thus, the OP is more suitable formulation for cases where visiting all the targets is unfeasible with the given travel budget.

In the EOP, the distance between the target locations corresponds to the length of the straight line segment connecting them and the objective is to select a maximal reward subset of target locations for which the length of the path visiting them is shorter or equal to the predefined maximal total path length.

Although the objective in the DOP is similar to the EOP, i.e., to maximize the collected reward within the given travel budget, the final reward collecting path has to satisfy the limited curvature constraint, as shown in Fig. 1, and thus the final path consists of a sequence of optimal Dubins maneuvers [2] connecting the selected target locations. Therefore, a solution of the DOP requires determining particular heading angles at the target locations to minimize the length of Dubins maneuvers between the targets. Regarding computational complexity, the DOP can be considered as more challenging than the EOP as changing only one heading angle or target location in the reward collecting path usually enforces the change of all heading angles of nearby connected target locations.

A variant of the TSP with Dubins maneuvers [5] is known as the Dubins Traveling Salesman Problem (DTSP) [6]. Contrary to the DTSP which aims to minimize the total travel cost, the DOP allows to address the limited travel budget, and thus respects a practical deployment of UAVs with limited operational time. Therefore, we propose to directly solve the DOP, and our proposed solution is based on the Variable Neighborhood Search (VNS) metaheuristic for combinatorial optimization [7], which has been deployed to the OP in [8].

The letter is organized as follows. An overview of related work on the EOP and DTSP is presented in the next section.

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In Section III, the proposed DOP is formally introduced. Section IV presents the proposed direct solution of the DOP. Evaluation results together with the report on the method experimental deployment in a real-scale outdoor scenario are presented in Section V. Section VI concludes the letter.

II. Related Work

The introduced Dubins Orienteering Problem (DOP) builds on the existing approaches for the Euclidean Orienteering Problem (EOP) [3] and Dubins Traveling Salesman Problem (DTSP) [9]. Therefore an overview of the existing approaches is presented in this section. Both the EOP [10] and DTSP [6] can be used for planning UAV missions; however, the EOP produces unfeasible paths for the Dubins vehicle, and the DTSP does not respect the travel budget.

The Euclidean OP has been studied since 1984 when Tsiligirides proposed two heuristics [11]. The first S-algorithm uses Monte Carlo method for picking the best solution from a large number of randomly generated paths with probabilities based on the reward per additional distance to the target location. The second D-algorithm uses a method for vehicle-scheduling with one depot by Wren and Holiday [12]. Tsiligirides further proposed a route-improvement algorithm that improves an initial route by using target insertion, target exchange, and 2-Opt operations [11].

A Four-Phase heuristic for the OP [1] uses insertion, improvement, and deletion phases to iteratively improve the path. In the insertion phase, new target locations are introduced to the path while using additional reward per distance and relaxed budget constraint. The second phase is based on 2-Opt and 3-Opt improvement operations. The deletion phase removes a target location with the minimal reward per distance and continues to the first phase with decreased relaxation of the travel budget. The fourth phase is the maximal insertion, and it follows after the iteration of previous three phases is terminated.

Chao *et al.* (1996) proposed a fast and effective heuristic for the OP in [13]. The heuristic considers only the target locations inside an ellipse with the foci in origin and ending locations with the major axis length equal to the travel budget. Using the most distant target locations from foci, a number of paths are generated during initialization with a greedy algorithm. The highest reward path $path_{op}$ is then improved by the Two-point exchange, i.e., by one-point move and 2-Opt operations, that systematically exchanges the target locations between $path_{op}$ and set of alternative paths $path_{nop}$ formed from unused target locations.

The Variable Neighborhood Search (VNS) metaheuristic [7] has been used to solve OP by Sevkli *et al.* (2006) [8]. This VNS-based method utilizes a predefined neighborhood structure, namely target insert/exchange and path insert/exchange operations. Using these four structures, the VNS algorithm iteratively performs *shake* and *local search* procedures. During the *shake* procedure, the currently best achieved solution is randomly changed to escape from a local minimum. In the *local search* procedure, the changed solution is searched within a smaller neighborhood structure to obtain a possibly better solution than the current best one.

Regarding the DTSP, the most relevant methods are the sampling based approaches [9], [14], [15] that allow a combinatorial optimization by using a discrete set of possible headings at the target locations. The DTSP stands to determine the minimum length path to visit all the target locations and satisfies the minimum curvature constraint of Dubins vehicle. The sampling based methods use a uniform sampling of the vehicle heading angle at each target location. The problem is then considered as the Generalized Asymmetric TSP that is further transformed and solved as the Asymmetric TSP (ATSP) [16], e.g., using Lin-Kernighan algorithm [17].

The closest existing problem formulation to the proposed DOP is the OP for kinodynamic vehicles outlined in [18]. Their solution of the Stochastic TSP and OP for kinodynamic vehicle is based on dividing the configuration space into cells with an equal volume, and merging the cells with no or few target locations into larger ones. In the TSP, the vehicle traverses each cell and collects the target locations inside by making small deviations from a fixed path that goes through all cells. For the OP, the vehicle selects a TSP sub-path with the highest reward. Even though the algorithm provides a possible approach to the DOP, it is useful mainly for the stochastic version of the OP where the target locations are randomly placed. In such a case, the algorithm provides an approximation of the optimal trajectory with a high probability. Moreover, the algorithm does not directly maximize the collected reward as the herein proposed method; it rather selects a part of the TSP path with the maximal reward and length below or equal to the budget.

The proposed solution of the introduced Dubins Orienteering Problem (DOP) is based on the VNS technique already deployed for the Euclidean OP in [8], which is actually one of the best performing methods for the EOP. The developed algorithm for the DOP is therefore compared with existing approaches for the EOP proposed by Chao *et al.* [13], Four-phase heuristic [1], and the original VNS-based method [8]. This comparison is done for the existing datasets by Tsiligirides [11] and two problem instances by Chao *et al.* [13]. Further experimental evaluation is performed for a practical scenario with a real UAV, see Section V.

III. Problem Statement

The motivation for the proposed Dubins Orienteering Problem (DOP) is in data collection scenarios for multirotor Unmanned Aerial Vehicles (UAVs) with limited operational time, where each of the target location requested to be visited has assigned a particular reward value, and the vehicle needs to follow a curvature-constrained path. The proposed solution can be however applied to any Dubins vehicle such as the fixed wings UAVs [19] or even the Ackermann vehicles. Hence, the objective is to find a data collection path for the Dubins vehicle that maximizes the sum of the collected rewards R such that the path length does not exceed the specified maximal travel budget $T_{\max}$. The existing Euclidean OP [3] cannot be directly used in such scenarios as it produces unfeasible paths for the considered Dubins vehicle model and thus, it may lead to miss some of the target locations or violation of the budget constraint. The proposed DOP is a generalization of the Euclidean OP, and therefore, the EOP is formally introduced in the next section followed by its generalization for the Dubins vehicle in Section III-B.

A. Euclidean Orienteering Problem (EOP)

Having a set of target locations $S = \{s_1, \dots, s_n\}$, the Orienteering Problem seeks to find a maximal reward subset $S_k \subseteq S$ and a path visiting S_k such that its length is limited by the given $T_{\max}$. The origin and ending locations are given and denoted as s_1 and s_n. The subset selection in the problem, which determines the collected reward, is similar to the NP-hard Knapsack problem. The problem is also related to the NP-hard Traveling Salesman Problem (TSP) in finding a minimal-length path on S_k.

Each considered target location s_i is defined by its position denoted as $s_i \in \mathbb{R}^2$ (for simplicity and better readability) and its reward r_i. We assume that the reward of the origin and ending locations are zero $r_1 = r_n = 0$ and strictly positive for all other locations, i.e., $r_i > 0$ for $1 < i < n$. The EOP includes determination of k target locations defining the subset S_k and a sequence to their visits that can be described as a permutation $\Sigma = (\sigma_1, \dots, \sigma_k)$, where $1 \leq \sigma_i \leq n$, $\sigma_i \neq \sigma_j$ for $i \neq j$ and $\sigma_1 = 1$, $\sigma_k = n$. For the Euclidean distance $\mathcal{L}_e(s_{\sigma_i}, s_{\sigma_j})$ between two locations s_{σ_i} and s_{σ_j}, the EOP can be formulated as the optimization problem:

$$
\begin{aligned}
&\underset{k, S_k, \Sigma}{\text{maximize}}\ R = \sum_{i=1}^{k} r_{\sigma_i} \\
&\text{subject to} \sum_{i=2}^{k} \mathcal{L}_e(s_{\sigma_{i-1}}, s_{\sigma_i}) \leq T_{\max} \\
&\sigma_1 = 1, \sigma_k = n\,.
\end{aligned}
\tag{1}
$$

B. Dubins Orienteering Problem (DOP)

The Dubins Orienteering Problem (DOP) is a generalization of the OP for the Dubins vehicle model to determine a feasible path over selected target locations S_k. The state of the Dubins vehicle $q = (x, y, \theta)$ consists of its position in plane $(x, y) \in \mathbb{R}^2$ and its heading $\theta \in \mathbb{S}^1$, i.e., $q \in SE(2)$. One of the specifics of this non-holonomic vehicle model is the minimal turning radius ρ that influences the length of the shortest path between two states. The kinematic model of Dubins vehicle with a constant forward velocity v and a control input u can be described as:

$$
\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}, u \in [-1, 1]\,. \tag{2}
$$

In [2], Dubins proved that for the model (2) the shortest path between two states consists only of straight line arc (S-segment) and arcs with the curvature ρ (turning left denoted as L-segment or right as the R-segment) and the optimal path is one of six possible maneuvers $\{LSL, LSR, RSL, RSR, LRL, RLR\}$ that are further denoted as Dubins maneuvers. For any two states q_i and q_j the Dubins maneuver together with its length $\mathcal{L}_d(q_i, q_j)$ can be determined analytically [2]; however, regarding the studied DOP, we need to determine the particular headings θ_i and θ_j of the vehicle at corresponding locations s_i and s_j, respectively.

Hence, each target location s_i is considered as the state $q_i = (s_i, \theta_i)$ in the DOP and in addition to the determination of the subset S_k of the k locations and the permutation $\Sigma = (\sigma_1, \dots, \sigma_k)$, the DOP intends to find the corresponding heading angles $\Theta = (\theta_{\sigma_1}, \dots, \theta_{\sigma_k})$. The Dubins Orienteering Problem for the model (2) can be then formulated as the optimization problem:

$$
\begin{aligned}
&\underset{k, S_k, \Sigma, \Theta}{\text{maximize}}\ R = \sum_{i=1}^{k} r_{\sigma_i} \\
&\text{subject to} \sum_{i=2}^{k} \mathcal{L}_d(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{\max} \\
&\sigma_1 = 1, \sigma_k = n\,.
\end{aligned}
\tag{3}
$$

In contrast to the Euclidean OP, the introduced DOP considers the Dubins vehicle model, and the path is constructed using the Dubins maneuvers between the adjacent target locations (states). Notice that the optimization problem (3) is not only over all possible subsets and respective permutations of the target locations (k, S_k, Σ), but also over all possible heading angles Θ at the target locations. This makes the problem computationally challenging as the already NP-hard EOP is extended to optimize over heading angles.

IV. Proposed Approach for the DOP

The proposed algorithm to solve the introduced Dubins Orienteering Problem (DOP) is based on the Variable Neighborhood Search (VNS) [7], which has been already deployed to the EOP in [8]. In contrast to the EOP, the DOP has to consider the heading angle at the target locations, which requires a new formulation of the solution search method. Therefore a brief overview of the VNS and the used approach for dealing with heading angles is provided prior detail description of the proposed VNS-based solution for the DOP.

The VNS is a metaheuristic proposed by Hansen and Mladenovic [7] for combinatorial optimization. The method operates on an initially defined Neighborhood structures $N(l_1, \dots, l_{\max})$, where l denotes the maximal distance between two solutions in the neighborhood. In the OP, the distance l is the number of different target locations inside the solution vector $(q_{\sigma_1}, \dots, q_{\sigma_n})$. A set $N_l(x)$ contains all solutions in l distant neighborhood of the solution x. Particular Neighborhood structure is then expressed by an operation that changes the given solution within the desired distance.

Two main procedures are used in the VNS to search the solution space starting from an initial solution. In the *shaking* procedure, the incumbent solution x is randomly moved to different solution x' within the neighborhood. This is used to get farther from the current best solution which may be only a local minimum. Afterward, the *local search* procedure systematically searches for the best solution in the neighborhood of the solution x'. The solution from the *local search* becomes a new incumbent

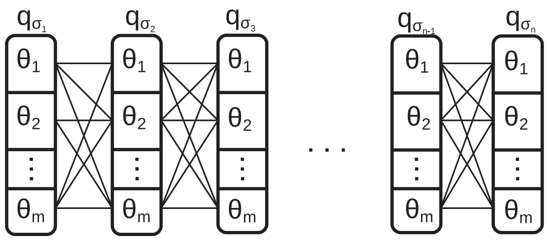

Fig. 2. Search graph of the DOP with m uniformly sampled heading angles at each target location q_{σ_i}, $0 \leq i \leq n$. For a particular selected sequence of the target locations $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ a graph search over all heading values is performed to obtain headings providing the minimal path length.

solution if it improves the current best solution. The procedures continue until stopping criterion is met, which is either a number of iterations, CPU time or maximal time between improvements.

For solving the DOP, we used the Randomized Variable Neighborhood Search (RVNS) variant of the VNS [7]. The RVNS algorithm uses a randomized *local search* procedure instead of the systematic approach used in the regular VNS. The randomized variant of the *local search* tries, during a predefined number of iterations, to randomly change the solution x' inside the Neighborhood structure to improve the solution by collecting more rewards. As it is shown for the EOP with the VNS [8], the RVNS is faster than the regular VNS and generates solutions that achieve the same rewards.

In the VNS, the Dubins Orienteering Problem is represented by a solution vector $(q_{\sigma_1}, \ldots, q_{\sigma_k}, q_{\sigma_{k+1}}, \ldots, q_{\sigma_n})$, where the first k target locations $(q_{\sigma_1}, \ldots, q_{\sigma_k})$ are within the budget constraint limit $\sum_{i=2}^{k} \mathcal{L}_d(q_{\sigma_{i-1}}, q_{\sigma_i}) \leq T_{\max}$, $\sigma_1 = 1$ and $\sigma_k = n$. The remaining vector $(q_{\sigma_{k+1}}, \ldots, q_{\sigma_n})$ consists of all other target locations that are above the budget.

Each state q_{σ_i} consists of the location $s_{\sigma i}$ and particular heading θ_{σ_i} that is selected from uniformly sampled angles from the interval $\theta \in \langle 0, 2\pi)$ into m samples $(\theta_1, \ldots, \theta_m)$.

The main difference of the proposed VNS-based DOP algorithm, compared to the existing variant for the EOP [8], is the determination of particular heading θ_{σ_i} at each target location. For a given number of samples m and a sequence of targets, the algorithm finds the shortest path by trying all possible combinations of sampled headings. The utilized search graph of the VNS DOP for a sequence of target locations $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ is visualized in Fig. 2. A graph search is used to determine particular sequence of heading samples that produces path with the minimal length. A dynamic programming technique is utilized to store distances from the origin q_{σ_1} and ending q_{σ_k} locations to simplify further target location insertion/deletion.

In the proposed VNS method for the DOP, we utilize only a subset of reachable locations S_r such that $q_i \in S_r \Leftrightarrow (\mathcal{L}_d(q_1, q_i) + \mathcal{L}_d(q_i, q_n)) \leq T_{\max}$ for any combination of sampled heading angles $(\theta_1, \theta_i, \theta_n)$. This selects all target locations that are reachable by the Dubins vehicle within the travel budget.

The initial solution x required for the VNS technique is generated using a greedy algorithm. For an initial zero reward Dubins path from q_1 to q_n, we iteratively add a new target location from S_r that minimizes additional distance per target reward as long as the length of the whole path is below $T_{\max}$.

Algorithm 1: Variable Neighborhood Search for the DOP.

Input : S – set of target locations
Input : T_{max} – maximal allowed budget
Input : m – number of heading values for each target
Output: P – found data collecting path

$S_r \leftarrow$ getReachableLocations(S)
$P \leftarrow$ createInitialPath(S_r,T_{max}) `// greedy`
while *stopping condition is not met* **do**
  $l \leftarrow 1$
  while $l \leq l_{max}$ **do**
    $P' \leftarrow$ shake(P, l)
    $P'' \leftarrow$ localSearch(P', l)
    if $\mathcal{L}_d(P'') \leq T_{max}$ ***and*** $R(P'') > R(P)$ **then**
      $P \leftarrow P''$
      $l \leftarrow 1$
    else
      $l \leftarrow l + 1$

After an initial path P is found, the proposed VNS-based algorithm uses the following neighborhood structures in *shaking* and *local search* procedures to obtain solutions with higher rewards. The randomized *shaking* uses the structures:

1) *Path Move:* uses a randomly selected path $(q_{\sigma_i}, \ldots, q_{\sigma_j})$, where $1 < i < j < n$, from the solution vector $(q_{\sigma_1}, \ldots, q_{\sigma_n})$, and moves it to a randomly selected position $\sigma_o < i$ or $\sigma_o > j$. For the purpose of the VNS, this operation represents neighborhood $l = 1$ despite the fact that the number of different target locations is usually larger than one.
2) *Path Exchange* also uses a randomly selected path $(q_{\sigma_i}, \ldots, q_{\sigma_j})$ from the solution vector, but exchange the path with a second random non-overlapping path $(q_{\sigma_o}, \ldots, q_{\sigma_p})$. The path exchange operation represents the neighborhood $l = 2$.

The *local search* procedureemploys different and much closer neighborhoods. Unlike the *shaking*, the *local search* procedure uses an iterative search in the particular neighborhood such that it tries numerous operations on the same solution. For the RVNS, the *local search* tries random operations for a number of times that is equal to the square of the number of the target locations. This ensures that the neighborhood of solution x' from *shaking* is searched more deeply for local optima than in the *shaking* procedure. The procedure uses the following neighborhood structures.

1) *One Point Move:* corresponds to the $l = 1$ neighborhood in which only one randomly selected target is moved to a different position within the solution vector.
2) *One Point Exchange:* is a farther neighborhood $l = 2$ and it uses two randomly selected distinct targets from the solution vector and exchanges their positions.

In all four presented neighborhood structures, the operations also search through all sampled heading angles as described above, to minimize the solution path length for a particular sequence of the target locations.

The proposed VNS-based algorithm for the DOP is summarized in Algorithm 1. For brevity, the rewards collected by a path $P = P(k, S_k, \Sigma, \Theta)$ is $R(P) = \sum_{i=1}^{k} r_{\sigma_i}, \sigma_i \in \Sigma$, and the path length is $\mathcal{L}_d(P) = \sum_{i=2}^{k} \mathcal{L}_d(q_{\sigma_{i-1}}, q_{\sigma_i})$.

TABLE I
ABBREVIATION RELATED WITH THE RESULTS

Set 1, **Set 2**, **Set 3**	Test instances created by Tsiligirides [11].
Set 64, **Set 66**	Test instances proposed by Chao [13].
4Phase	Four-Phase heuristic for EOP by Ramesh *et al.* [1].
Chao	Fast and effective heuristic by Chao *et al.* [13].
VNS	VNS-based algorithm by Sevkli *et al.* [8].
VNS DOP	Proposed Dubins Orienteering VNS method.

The VNS-based DOP algorithm uses only two neighborhood structures for both *shaking* and *local search*, which means that the maximal neighborhood distance is $l_{\max} = 2$. An extension of the neighborhood structures to $l_{\max} > 2$ is possible by concurrently moving more than two target locations in the *local search*.

Evaluation results of the proposed DOP method are presented in the next section together with the comparison to the existing algorithms for the EOP. Besides, results from the real practical experiments with UAV are presented as well.

V. Results

The proposed method for the Dubins Orienteering Problem (DOP) has been evaluated on five existing datasets from the literature [20] and also in real data collection scenario with Unmanned Aerial Vehicle (UAV). Using the existing datasets, the proposed VNS-based method is compared with existing Euclidean Orienteering Problem (EOP) approaches as to the best of our knowledge there is no existing solution for the introduced DOP. The maximal achieved rewards for particular non-zero turning radii are presented alongside to show the influence of increasing turning radius on the collected reward. Furthermore, the real experiment with hexarotor UAV is presented. The results show practical applicability of the introduced DOP and the proposed VNS-based method for robotic data collection planning.

A. Results on Datasets and Existing EOP Approaches

A comparison of the proposed DOP method with solutions for the EOP, namely with the heuristic proposed by Chao *et al.* [13], 4-phase heuristic by Ramesh *et al.* [1], and VNS-based algorithm by Sevkli *et al.* [8] has been performed. Abbreviation of the methods and used existing benchmarks are in Table I. Results for the proposed method are presented for multiple representative turning radii $\rho \in \{0, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3\}$, where $\rho = 0$ is a solution of the EOP.

The utilized VNS is a stochastic procedure, and therefore, each scenario has been solved 10 times for each budget $T_{\max}$. The results were computed using a single core of Intel i7 3.4 GHz CPU. A single sample of the heading angle $m = 1$ has been used for the DOP problems with $\rho = 0$ as the heading angle does not influence the distance between the target locations. For $\rho > 0$, equidistant sampling of the heading angle into $m = 16$ values has been utilized. The stopping criterion is the maximal number of 10 000 iterations with the maximal 3 000 iterations without improvement.

Results for Tsiligirides datasets **Set 1**, **Set 2**, **Set 3** are presented in Tables II, III and IV, respectively. Tables V and VI

TABLE II
RESULTS COMPARISON FOR SET 1

$\mathbf{T}_{\max}$	Chao	4Phase	Proposed VNS-based DOP						
			$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.1$	$\rho = 1.3$
5	10	10	10	10	10	0	0	0	0
10	15	15	15	15	15	15	15	15	15
15	45	45	45	45	45	45	45	40	35
20	65	65	65	65	60	60	60	60	50
25	90	90	90	90	85	85	85	85	75
30	110	110	110	110	110	105	105	105	95
35	135	135	135	135	135	130	130	120	120
40	155	150	155	155	155	150	145	140	140
46	175	175	175	175	175	170	170	165	160
50	190	180	190	185	185	185	175	175	165
55	205	205	205	200	200	195	195	185	185
60	225	225	220	220	220	215	215	205	205
65	240	240	240	240	235	235	235	225	220
70	260	260	260	260	255	250	250	240	235
73	265	265	265	265	265	260	260	250	240
75	270	275	270	270	265	265	260	255	245
80	280	280	280	280	275	275	270	265	255
85	285	285	285	285	285	280	275	270	265

TABLE III
RESULTS COMPARISON FOR SET 2

$\mathbf{T}_{\max}$	Chao	4Phase	Proposed VNS-based DOP						
			$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.1$	$\rho = 1.3$
15	120	120	120	120	120	115	115	115	95
20	200	200	200	190	190	180	175	165	135
23	210	210	210	205	200	200	200	200	160
25	230	230	230	230	220	220	205	200	165
27	230	230	230	230	230	230	230	220	180
30	265	260	265	260	255	255	240	230	225
32	300	300	300	290	290	275	275	260	240
35	320	320	310	320	315	310	300	285	285
38	360	385	360	350	345	340	330	325	310
40	395	395	395	385	375	375	365	355	335
45	450	450	450	440	440	420	410	395	370

TABLE IV
RESULTS COMPARISON FOR SET 3

$\mathbf{T}_{\max}$	Chao	4Phase	Proposed VNS-based DOP						
			$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.1$	$\rho = 1.3$
15	170	170	170	170	160	160	160	150	140
20	200	200	200	190	190	180	180	180	180
25	260	260	260	260	260	260	250	250	230
30	320	320	320	320	320	320	320	310	300
35	390	390	390	380	380	360	380	380	360
40	430	430	430	430	420	420	420	420	400
45	470	470	470	470	460	450	450	450	440
50	520	520	520	520	510	510	470	500	490
55	550	550	550	550	540	540	530	530	520
60	580	580	580	580	570	560	560	560	550
65	610	610	610	610	600	590	590	590	580
70	640	640	640	640	630	610	610	600	610
75	670	670	670	670	650	650	640	630	630
80	710	710	700	700	690	680	680	670	670
85	740	740	740	730	730	700	700	710	710
90	770	770	770	760	760	740	750	710	730
95	790	790	790	790	780	780	770	760	750
100	800	800	800	800	790	790	790	780	770
105	800	800	800	800	800	800	800	800	790
110	800	800	800	800	800	800	800	800	800

TABLE V
RESULTS COMPARISON FOR SET 64

T_{max}	Chao	VNS	Proposed VNS-based DOP						
			$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.1$	$\rho = 1.3$
15	96	96	96	96	96	96	96	96	96
20	294	294	294	294	294	294	252	252	252
25	390	390	390	390	384	366	360	300	300
30	474	474	474	468	468	468	468	408	390
35	570	576	576	570	570	564	546	498	492
40	714	714	714	696	696	690	672	582	582
45	816	816	816	798	792	780	756	642	636
50	900	900	900	888	882	876	834	708	714
55	984	984	984	978	960	960	924	804	786
60	1044	1062	1062	1044	1026	1026	1008	834	834
65	1116	1116	1116	1098	1098	1098	1080	918	900
70	1176	1188	1188	1170	1170	1134	1134	990	960
75	1224	1236	1236	1230	1206	1194	1194	1038	1014
80	1272	1272	1272	1272	1260	1254	1224	1074	1080

TABLE VI
RESULTS COMPARISON FOR SET 66

T_{max}	Chao	VNS	Proposed VNS-based DOP						
			$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$	$\rho = 1.1$	$\rho = 1.3$
5	10	10	10	10	10	10	0	0	0
10	40	40	40	40	40	40	40	40	40
15	120	120	120	100	100	100	100	95	95
20	195	205	205	205	200	195	195	195	170
25	290	290	280	290	280	280	280	275	260
30	400	400	400	400	380	370	370	370	370
35	460	465	465	465	465	460	455	450	445
40	575	575	575	570	570	570	545	540	535
45	650	650	650	645	650	650	645	640	640
50	730	730	730	725	725	710	710	695	690
55	825	825	825	825	825	800	820	795	790
60	915	915	915	895	895	895	890	890	860
65	980	980	980	980	930	925	950	945	945
70	1070	1070	1070	1065	1030	1070	1070	1070	1035
75	1140	1140	1140	1140	1120	1110	1080	1085	1090
80	1215	1215	1215	1195	1190	1170	1175	1165	1155
85	1270	1270	1270	1270	1245	1260	1245	1235	1200
90	1340	1340	1340	1320	1320	1305	1295	1295	1295
95	1380	1395	1395	1395	1390	1370	1370	1360	1320
100	1435	1465	1465	1445	1445	1435	1420	1420	1390
105	1510	1520	1520	1495	1505	1495	1485	1470	1445
110	1550	1560	1550	1550	1550	1545	1545	1530	1505
115	1595	1595	1590	1580	1580	1580	1575	1555	1550
120	1635	1635	1625	1625	1625	1610	1600	1595	1575
125	1655	1670	1670	1655	1655	1645	1640	1640	1620
130	1680	1680	1680	1680	1675	1675	1670	1670	1655

show results for Chao datasets **Set 64** and **Set 66**. The presented results are the maximal achieved collected rewards R from the all 10 runs for the particular problem and budget.

Presented results show that the proposed VNS-based DOP algorithm provides competitive results to the existing EOP approaches for the turning radius $\rho = 0$. Nevertheless in some test instances for $\rho = 0$ the DOP does not provide the best known results due to the fact that the most rewarded solutions are in terms of number of different nodes very far from the previously found result with a slightly lower budget. However, the proposed algorithm solves the DOP, which is not possible by existing methods for the EOP. For increasing ρ, the collected reward decreases for almost all problem instances. This indicates that increasing turning radius results in longer paths, and thus solutions provided by the EOP approaches would violate the budget constraint for Dubins vehicle. The computational time to find the maximal achieved rewards and the number of iterations needed to obtain the solutions of EOP and DOP using the proposed VNS-based algorithm are shown in Fig. 3.

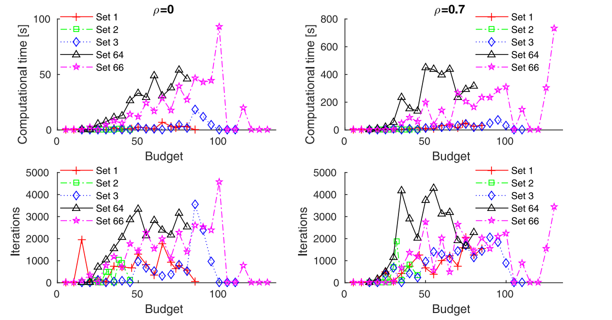

Fig. 3. Computational time and number of iterations for the EOP (DOP with $\rho = 0$), on the left, and the DOP with $\rho = 0.7$, on the right.

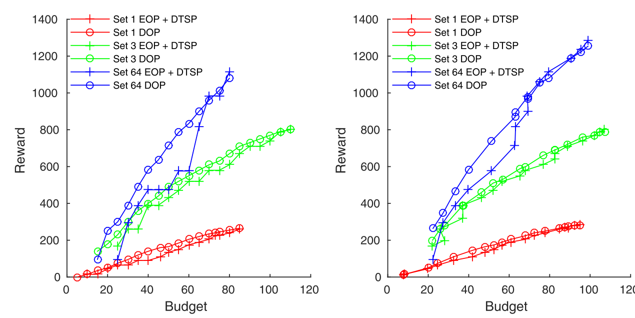

Fig. 4. A comparison of the proposed DOP algorithm with the subset selection by the EOP and finding the path as a solution of the DTSP. The same parameters $m = 16$ and $\rho = 1.3$ are used in both cases. Two strategies are considered for the EOP+DTSP approach. The results on the left are obtained for an iterative decrease of the budget for the EOP such that the solution of the DTSP meets the original travel budget. For the results on the right, the length of the path obtained by the EOP+DTSP is considered as a new travel budget in the proposed DOP algorithm.

A further comparison of the proposed direct solution of the DOP with existing approaches for the EOP is based on a straightforward combination of solving the EOP and Dubins Traveling Salesman Problem (DTSP). This naive approach is based on finding the subset of target locations S_k, with the highest collected reward, by solving the EOP. The sampling-based solution of the DTSP [9] is then used to find the data collecting path for the subset S_k with respect to the sampling of the heading angle m. The results are shown in Fig. 4, where the plot on the left shows that by using a smaller budget for the EOP and afterward the found S_k in the DTSP leads (in most cases) to lower rewards than a direct solution of the DOP. On the other hand, the right plot in Fig. 4 shows that in most cases (especially for lower budgets) the rewards collected by the solution of the DOP is higher. These results support suitability of the proposed algorithm for the introduced Dubins Orienteering Problem. Hence, it is not beneficial to solve the DOP by a separate selection of the target locations, e.g., by solving the EOP, and consecutive path planning for the Dubins vehicle. Solving the EOP may provide equally rewarded

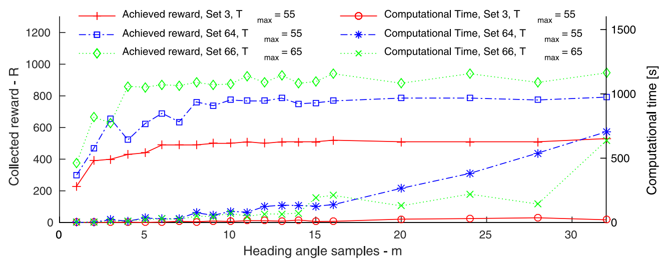

Fig. 5. Sums of collected rewards R and computational time for different heading sample rate m with $\rho = 1.2$.

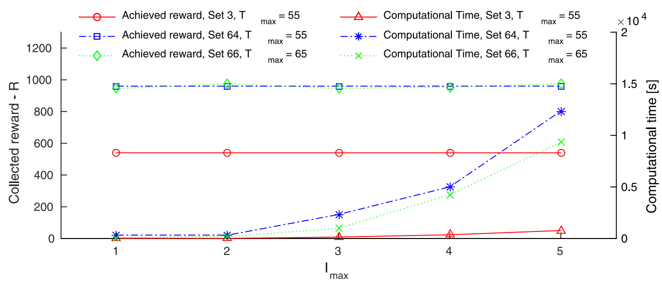

Fig. 6. Computational time and collected rewards of the proposed VNS-based DOP algorithm for increasing value of the maximal neighborhood distance $l_{\max}$ for particular problems.

paths with multiple different subsets of the target locations. However, some of the subsets can be connectable in the consequent DTSP respecting the budget constraint, but some may not.

The proposed VNS DOP algorithm uses m sampled heading values at each target location. A particular number m influences the path length and higher number of samples may almost always produce shorter paths and thus, a high reward collected for a given travel budget $T_{\max}$. An influence of m on the sum of the collected rewards R and the computational time on m for the selected problems is shown in Fig. 5. The results show that R tends to increase until $m = 12$. This is caused by the fact that the main objective of the DOP optimization is the sum of collected rewards R and the path length is not important as far as it is shorter than $T_{\max}$.

The proposed VNS-based DOP uses the maximal neighborhood distance $l_{\max} = 2$ but the value can be increased by concurrently moving $l_{\max} > 2$ target locations in the *local search* procedure. Fig. 6 with the computational times and collected reward for different $l_{\max}$ shows that the solution convergence is slower for increased $l_{\max}$ because a single iteration lasts longer and the randomized RVNS algorithm does not benefit from the enlarged neighborhood distance.

B. Real Experiments

The proposed method has been experimentally evaluated in the real data collection scenario with a hexarotor UAV.[1] The UAV is requested to visually inspect as many high rewarded target locations as possible during the length-limited flight. The

[1] We refer to http://mrs.felk.cvut.cz/icra17dop for more information about the experiment.

Fig. 7. Hexarotor UAV during the visual data collection of the colored target object with displayed reward.

considered scenario consists of 20 target locations, where a particular colored object with marked reward is located. The objects are placed in the area of approximately 100×50 m large. Fig. 7 shows the colored target object with the used hexarotor UAV, originally developed for multi-robot applications [21]. The considered travel budget is $T_{\max} = 150$ m for which the UAV has to visit the locations of the objects and maximize the collected reward.

Although the hexarotor UAV can drive through a path from the Euclidean OP, in certain cases, it is then required to decelerate during the sharp turns. Therefore, the hexarotor UAV modeled as the Dubins vehicle with a smooth path over the target locations allow using constant speed trajectories. Moreover, the Dubins model respects the real constraints of the UAV such as the maximal speed and acceleration. This allows to the used onboard trajectory controller [22] to precisely navigate through the trajectory without missing the target location which can happen for the path produced by solving the related EOP.

The crucial parameter of the Dubins vehicle is the minimal turning radius ρ that is computed from the desired constant velocity v_c and the maximal acceleration of the UAV $a_{\max}$. The equation of circular motion with constant speed $\rho = v_c^2 / a_{\max}$ is used to get the radius, which produces the maximal allowed acceleration during the circular parts of the path. The constant velocity $v_c = 4$ m.s^{-1} and the maximal allowed acceleration $a_{\max} = 2.6$ m.s^{-2} has been used and the considered turning radius ρ is $\rho = 6.15$ m.

Paths found by the proposed DOP algorithm for the turning radius $\rho = 0$ and $\rho = 6.15$ m are shown in Fig. 8. A solution is found within a second using the same parameters as in Section V-A. The particular rewards of the found solutions are $R = 71$ and $R = 65$, for $\rho = 0$ and $\rho = 6.15$ m respectively, with total path lenghts of 149.0 m and 148.4 m. Although the solution for $\rho = 0$ provides a higher reward, the path is not feasible for the constant speed motion, and the onboard controller has to violate the planned path. This causes cutting of sharp turns to fulfill the schedule of the plan as it is shown for the "`EOP path traveled by UAV`" curve in Fig. 8. On the other hand, a solution of the DOP with $R = 65$ respects the maximal acceleration with the desired constant speed of the vehicle and all target objects have been successfully captured.

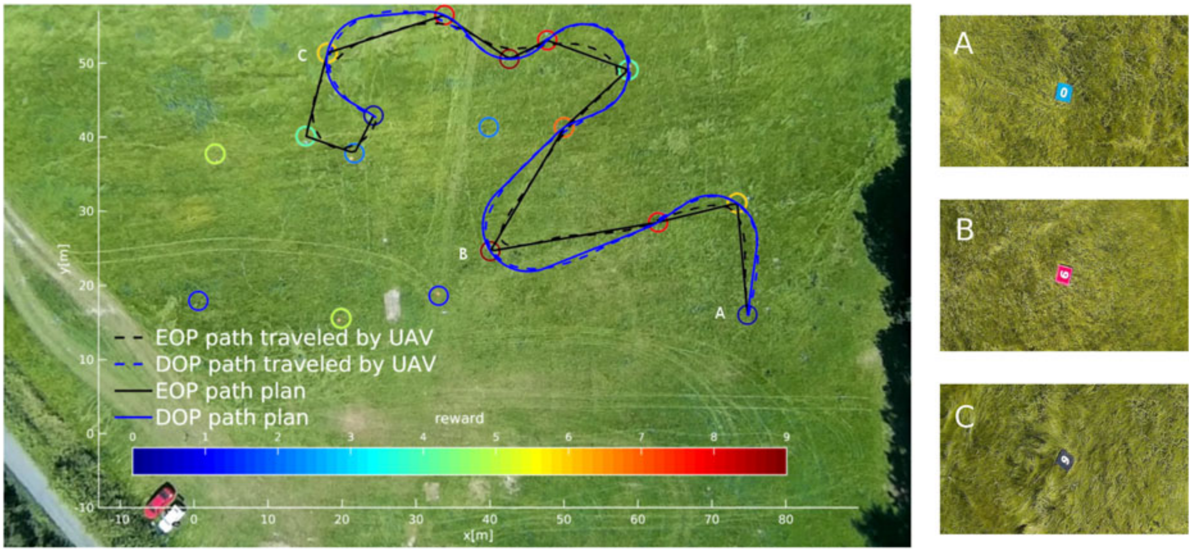

Fig. 8. Plots of the UAV position for solutions of the data collection scenario as the DOP and EOP.

VI. Conclusion

This letter introduces a generalization of the Orienteering Problem to the Dubins vehicle that is called the Dubins Orienteering Problem (DOP). We propose a novel Variable Neighborhood Search (VNS) based method for solving this challenging problem. A sampling based approach is used to search for an appropriate sequence of heading angles at the target locations. The presented results indicate that for zero turning radius, the proposed DOP solver is competitive to existing methods for the Euclidean OP. Results for non-zero turning radius show that the collected reward decreases with the increasing radius. Moreover, the presented results demonstrate that a solution of the DOP as a combination of the Euclidean OP and consecutive Dubins Traveling Salesman Problem is not plausible. We also show that the sampling based approach to heading angles is viable as the prime objective of the DOP is to maximize the collected reward and a higher number of samples does not necessarily increase the quality of solution (the collected reward). Finally, results from the real deployment of the proposed approach further demonstrate a necessity of the proposed direct solution of the Dubins Orienteering Problem. For future work, we intend to investigate the OP for other more complex maneuvers such as splines, and to extend the DOP for possible data collection within proximity of the target locations.

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