Basic RL.4

Judy Tutorial

Recall Q-learning:

Value Update

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left[r + \gamma \max_{a'} Q(s',a') - Q(s,a) \right]$$

$$\pi(a|s) \leftarrow \mathrm{argmax}_a Q(s,a)$$

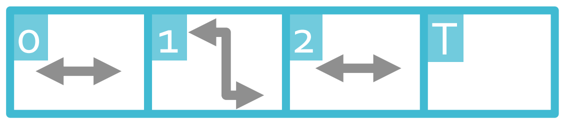
Policy Update for sampling (s,a,r,s')

Problems w/ Value-based methods

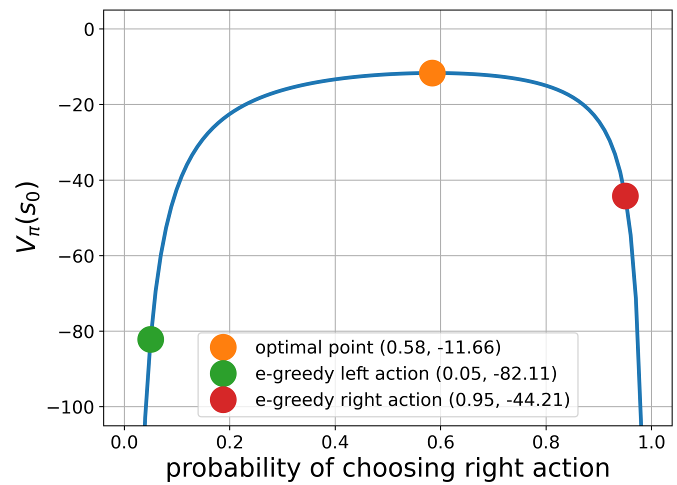
- If state and action spaces are large:

$\pi(a|s) \leftarrow \text{argmax}_a Q(s,a)$ becomes impossible

- There is no natural way for value-based method to find a stochastic optimal policy



-1 reward at all states



A parameterized policy:

$$\pi(a|s;\boldsymbol{\theta})$$

parameters

updated using gradient ascent:

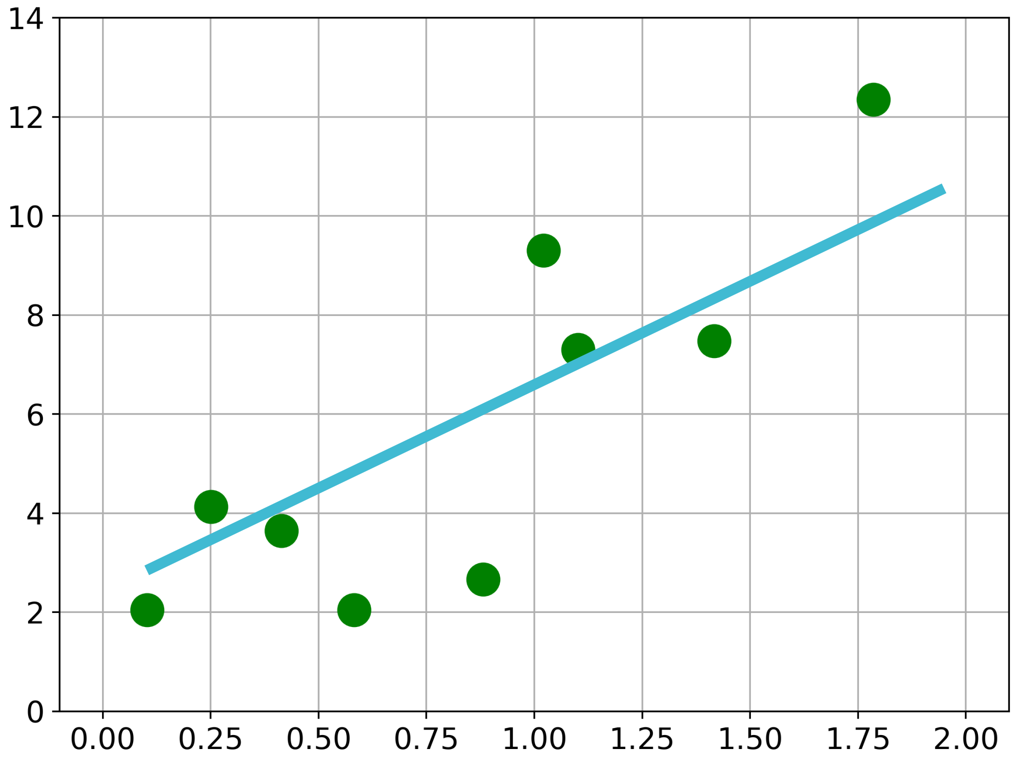
$$\boldsymbol{\theta}_{t+1} \leftarrow \boldsymbol{\theta}_t + \alpha \widehat{\nabla \boldsymbol{J}(\boldsymbol{\theta}_t)}$$

where $\boldsymbol{J}(\boldsymbol{\theta})$ is a scalar performance measure

To understand what are $J(\boldsymbol{\theta})$ and its **gradient**

Let's start with some basic concepts and examples in machine learning

Linear Regression



- Model the pattern of the dots
- Use the model to predict any new dots

Generalizability!

Element-wise

Hypothesis function

$$h(\boldsymbol{\theta}) = \theta_1 x + \theta_0 = \boxed{\boldsymbol{\theta}^{\mathrm{T}} X}$$

inputs

dot product

parameters/weights

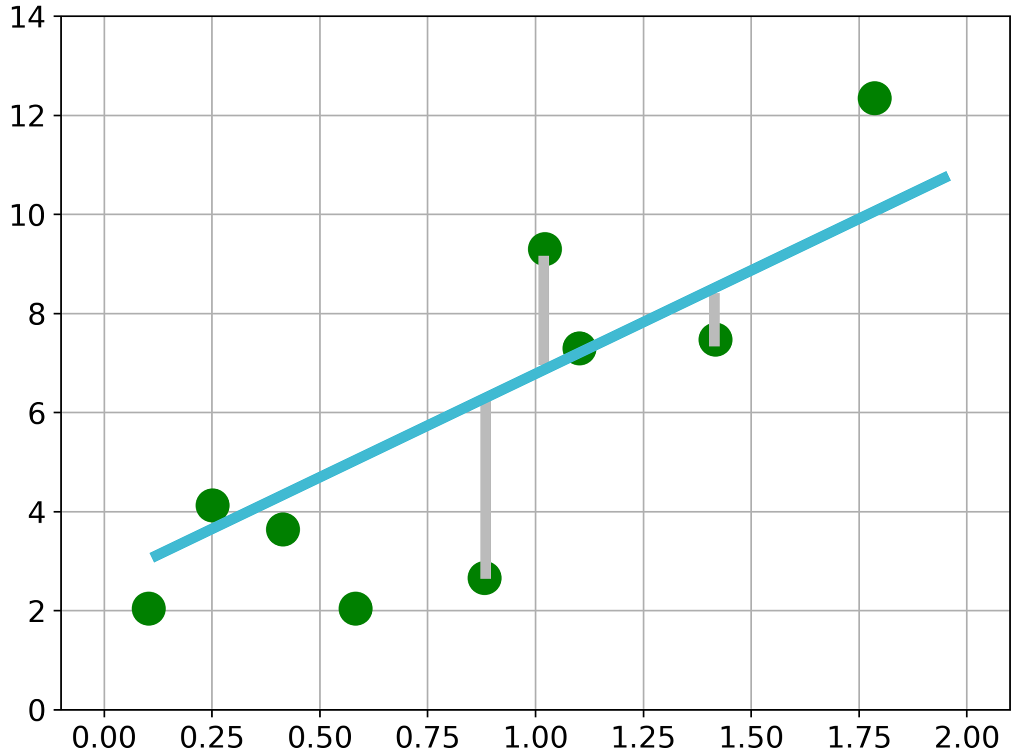
$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}, X = \begin{bmatrix} 1 \\ x \end{bmatrix}$$

Objective function

$$J(\boldsymbol{\theta}) = \frac{1}{2N} \sum_{i=1}^{N} \left[h(\boldsymbol{\theta})^i - y^i \right]^2$$

x	y
0.103	2.048
0.251	4.13
0.414	3.646
0.582	2.043
0.882	2.66
1.022	9.298
1.102	7.297
1.416	7.472
1.786	12.347
1.793	14.444

Objective



$$J(\boldsymbol{\theta}) = \frac{1}{2N}\sum_{i=1}^{N}\Big[\underbrace{h(\boldsymbol{\theta})^{i} - y^{i}}_{\text{error}}\Big]^2$$

Vector-wise

Hypothesis function

$$h(\boldsymbol{\theta})_{1\times 10} = \boldsymbol{\theta}^{T}_{2\times 1}\boldsymbol{X}_{2\times 10}$$

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \qquad \boldsymbol{X}^T = \begin{bmatrix} 1.0 , 0.103 \\ 1.0 , 0.251 \\ 1.0 , 0.414 \\ 1.0 , 0.582 \\ 1.0 , 0.882 \\ 1.0 , 1.022 \\ 1.0 , 1.102 \\ 1.0 , 1.416 \\ 1.0 , 1.786 \\ 1.0 , 1.793 \end{bmatrix}$$

Objective function

$$J(\boldsymbol{\theta}) = \frac{1}{2N}\|\boldsymbol{h}_{1\times 10} - \boldsymbol{Y}_{1\times 10}\|^2$$

$$\boldsymbol{Y}^T = \begin{bmatrix} 2.048 \\ 4.13 \\ 3.646 \\ 2.043 \\ 2.66 \\ 9.298 \\ 7.297 \\ 7.472 \\ 12.347 \\ 14.444 \end{bmatrix}$$

Objective function

$$J(\boldsymbol{\theta}) = \frac{1}{2N}\sum_{i=1}^{N}\left[h(\boldsymbol{\theta})^i - y^i\right]^2 = \frac{1}{2N}\|\boldsymbol{h}(\boldsymbol{\theta}) - \boldsymbol{Y}\|^2$$

Gradient

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \nabla_{\boldsymbol{\theta}} \frac{1}{2N}\sum_{i=1}^{N}\left[h(\boldsymbol{\theta})^i - y^i\right]^2$$

Element-wise

$$= \frac{1}{2N}\cdot 2\sum_{i=1}^{N}\left[h(\boldsymbol{\theta})^i - y^i\right]\nabla_{\boldsymbol{\theta}} h(\boldsymbol{\theta})^i$$

Need to take gradient w.r.t θ_j one by one

$$= \frac{1}{N}\boldsymbol{X}\left[\boldsymbol{h}(\boldsymbol{\theta}) - \boldsymbol{Y}\right]^T$$

Vector-wise

Gradient Descent

$$\boldsymbol{\theta}_{t+1} \leftarrow \boldsymbol{\theta}_t - \alpha\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$$

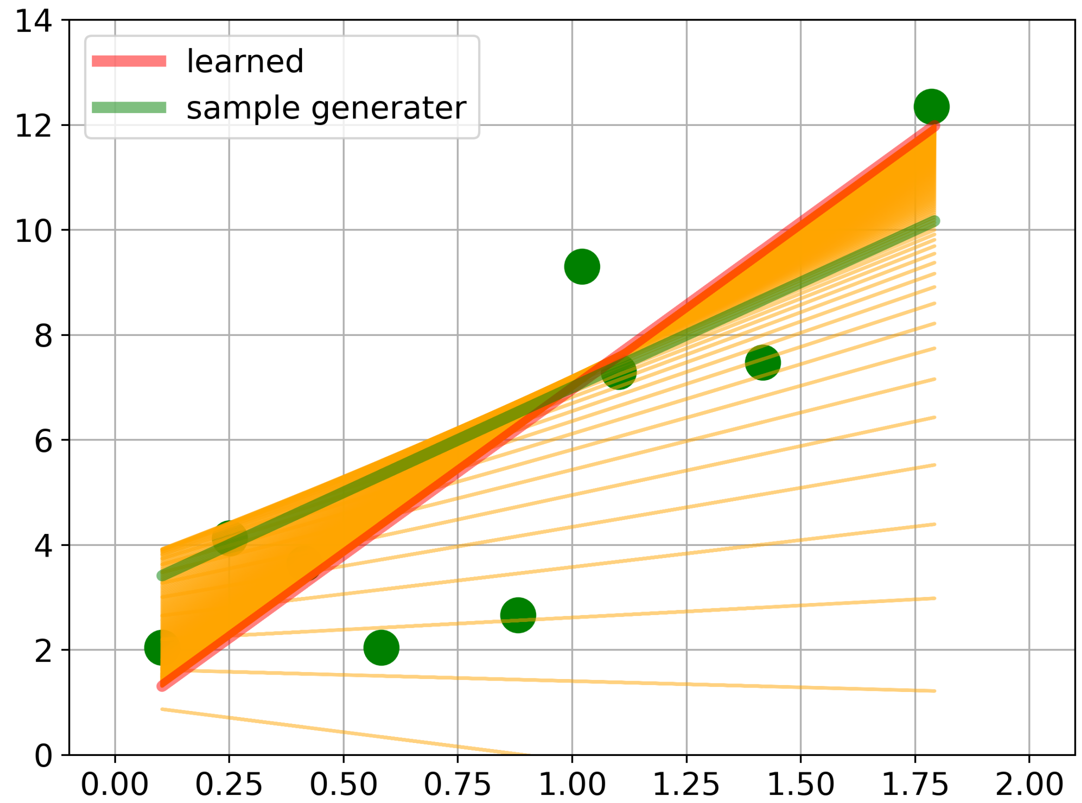
in python

```python
np.random.seed(3)
x=2*np.random.rand(10)
y=3+4*x+np.random.randn(10)*2.5

X=np.ones((2,10))
X[1,:]=x

theta=np.random.randn(2,1)
lr=0.01
N=200

for i in range(N):

    h=np.dot(theta.T,X)
    grad=np.dot(X,(h-y.reshape(1,10)).T)
    theta=theta-lr*grad
```

X - (2,10)
theta - (2,1)
h=theta.T*X - (1,10)<-(1,2)(2,10)
grad=X*(h-Y).T - (2,1)<-(2,10)(10,1)
theta-=lr*grad - (2,1)

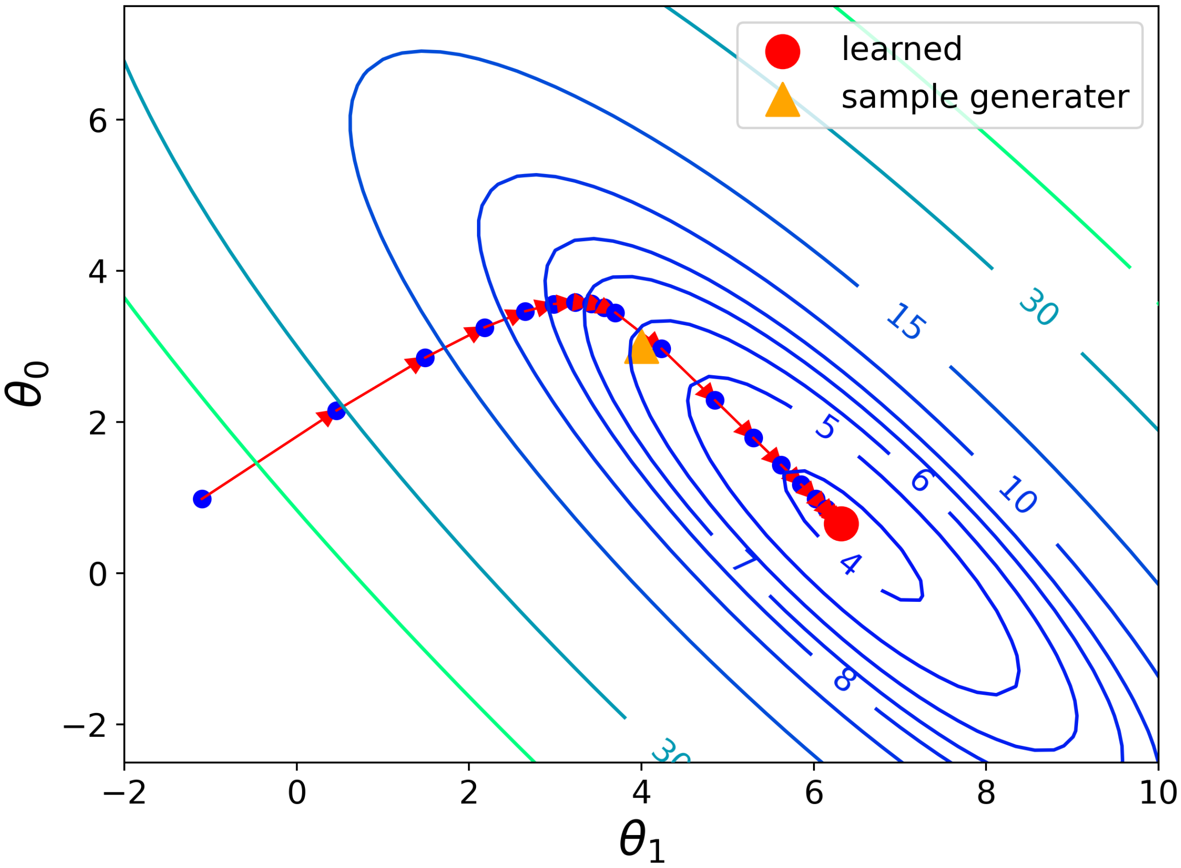
Data shape is important in implementation!

learning behavior



$$\boldsymbol{\theta}_0 = \begin{bmatrix} 0.982 \\ -1.1 \end{bmatrix} \rightarrow \boldsymbol{\theta}_f = \begin{bmatrix} 0.658 \\ 6.317 \end{bmatrix} \quad ? \quad \boldsymbol{\theta}_f ? = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Gradient is the direction of steepest descent



Back to our policy-based RL

$$\boldsymbol{J(\theta)}, \pi(a|s; \boldsymbol{\theta}), \widehat{\nabla \boldsymbol{J}(\boldsymbol{\theta_t})}$$

How do we define these items?