Machine Learning I (NEU)

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Exercise 1. To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Solution
$P(Hansen|tested) = \frac{P(tested|Hansen)P(Hansen)}{P(tested)}$

$$= \frac{P(tested|Hansen)P(Hansen)}{P(tested|Hansen)P(Hansen) + P(tested|noHansen)P(noHansen)}$$

$$= \frac{0.98 \cdot 0.05}{0.98 \cdot 0.05 + 0.03 \cdot (1 - 0.05)} = 0.6322$$

Exercise 2. Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution:
1. Univariate normal distribution.
2. (Optional) Multivariate normal distribution.

Solution
We have:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

w.r.t μ as mean and σ as standard deviation.

Prove that $\int_{-\infty}^{\infty} f(x)dx = 1$*:*

$$\int_{-\infty}^{\infty} f(x)dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} (1)$$

$$Let t = \frac{(x-\mu)}{\sqrt{2}\sigma} => dt = \frac{dx}{\sigma\sqrt{2\pi}} => dx = \sqrt{2}\sigma \cdot dt$$

Therefore,

$$(1) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2}\sigma dt = \frac{1}{\sigma\sqrt{2\pi}} \cdot \sqrt{2}\sigma\sqrt{\pi} = 1$$

$$(since \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi})$$

Prove mean $= \mu$

$$\begin{aligned} E[f(x)] &= \int_{-\infty}^{\infty} x \times \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \times \int_{-\infty}^{\infty} x \times e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx \\ &= \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} \times \int_{-\infty}^{\infty} \left(\sqrt{2}\sigma t + \mu\right) \times e^{-t^2} dt \\ &= \frac{1}{\sqrt{\pi}} \times \left(\sqrt{2}\sigma \times \int_{-\infty}^{\infty} t \times e^{-t^2} dt + \mu \int_{-\infty}^{\infty} e^{-t^2} dt\right) \\ &= \frac{1}{\sqrt{\pi}} \times \left(\sqrt{2}\sigma \times \left[\frac{-1}{2} \times e^{-t^2}\right] + \mu \times \sqrt{\pi}\right) \\ &= \frac{\mu\sqrt{\pi}}{\sqrt{\pi}} \\ &= \mu \end{aligned} \tag{1}$$

Prove Standard deviation $= \sigma^2$ *(I would send the PNG file because I could not find the corresponding formula in Latex)*