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BEE 14-C

413970

Question #1

a) $x(t) = e^{-3t}$

Energy signal

$$E = \int_0^\infty |x(t)|^2 = \int_0^\infty |e^{-3t}|^2 dt = \left| \frac{e^{-6t}}{-6} \right|_0^\infty = \frac{1}{6} < \infty$$

b) $x(t) = \cos^2 w_0 t$

. Periodic function

. Power signal

$$\cos^4 w_0 t = \left[\frac{1+\cos 2(w_0 t)}{2} \right]^2$$

$$= \frac{1 + 2\cos 2w_0 t + \cos^2 2(w_0 t)}{4}$$

$$= \frac{3}{8} + \frac{1}{2}\cos 2w_0 t + \frac{1}{8}\cos 4w_0 t$$

$$P = \frac{1}{T}\int_0^T \cos^4 w_0 t \, dt = \frac{1}{T}\left| \frac{3}{8}t + \frac{\sin 2w_0 t}{4w_0} + \frac{1}{32}\frac{\sin 4w_0 t}{w_0} \right|_{-T}$$

$$= \frac{3}{8} \quad \text{(Power Signal)}$$

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c) $x(t) = u(t)$

$$E = \int_0^\infty |u(t)|^2 dt = \int_0^\infty 1\, dt = \infty$$

$$P = \frac{1}{T}\int_{-T}^{T} |u(t)|^2 dt = \frac{1}{T}\int_{-T}^{T} 1\, dt = 1$$

- Infinit ~~power~~ Energy   Power signal
- Non zero, finite power

d) $x(t) = \frac{1}{2}(\cos \omega t + 1)$ $\frac{-\pi}{\omega} \leq t \leq \frac{\pi}{\omega}$

$$E = \frac{1}{2}\int_{\frac{-\pi}{\omega}}^{\frac{\pi}{\omega}} (\cos \omega t + 1) dt = \frac{1}{2}\left[\frac{\sin \omega t}{\omega} + t\right]_{-\pi/\omega}^{\pi/\omega}$$

$$= \frac{1}{2}\left[\frac{\sin \pi + \sin \pi}{\omega} + \frac{2\pi}{3}\right] = \frac{1}{2}\left[\frac{2\sin \pi}{\omega} + \frac{2\pi}{3}\right]$$

Energy Signal $= \frac{\sin \pi}{\omega} + \frac{\pi}{3}$

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e) $g_1(t) = 3\cos[\cos(t)] + \cos(5t - \frac{2\pi}{3}) + \cos(8t + \frac{2\pi}{3})$

. Periodic so energy will be finite.

$A\cos(\omega t + \phi) \Rightarrow P = \frac{A^2}{2}$

$P_1 = 3\cos(t) = \frac{9}{2}$, $P_2 = \cos(5t - \frac{2\pi}{3}) = \frac{1}{2}$

$P_3 = \cos(8t + \frac{2\pi}{3}) = \frac{1}{2}$

$P = 5.5$ Power Signal

Question #2

$$g(t) = \frac{4a}{t^2 + a^2}$$

$$\frac{1}{a^2 + t^2} = \frac{\pi}{a} e^{-a|\omega|} \Rightarrow \frac{4a}{a^2 + t^2} = 4a\frac{\pi}{a} e^{-a|\omega|}$$

$$G(\omega) = \frac{4a}{t^2 + a^2} = 4\pi e^{-a|\omega|}$$

→ By Parseval:

$$E = \int_{-\infty}^{0} e^{-2a|\omega|} d\omega + \int_{0}^{\infty} e^{-2a|\omega|} = \frac{1}{a}$$

$$E_g = \frac{16\pi^2}{a}$$

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92% of signal Energy Eg:

$$0.92\,E_g = 0.92 \times \frac{16\pi^2}{a}$$

$$0.92 \times \frac{16\pi^2}{a} = 16\pi^2 \int_{-B}^{B} e^{-2a|w|}\,dw$$

$$\frac{0.92}{a} = \left| \frac{-1}{2a} e^{-2aw} \right|_{-B}^{B}$$

$$\frac{0.92}{a} = \frac{-1}{2a} + \frac{1}{2a} e^{-2aB} - \frac{1}{2a} e^{-2aB} + \frac{1}{2a}$$

$$= \frac{1-e^{-2aB}}{2a} - \frac{e^{-2aB}}{2a}$$

$$= \frac{1-e^{-2aB} - e^{-2aB} + 1}{2a}$$

$$= \frac{1-e^{-2aB}}{a}$$

$$\frac{0.92}{a} = \frac{1-e^{-2aB}}{a}, \quad e^{-2aB} = 0.08$$

$$\boxed{B = \frac{1.2628}{a}}$$

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$$\frac{0.92}{a} = \frac{1-e^{-2aB}}{a}, \quad e^{-2aB} = 0.08$$

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Question #3

We cannot describe this function as a signal as it is of random nature.

$$R_g(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g(t)g(t-\tau)\,dt$$

. Let N bits (pulses) during this interval T so that $T = NT_b$ & $T\to\infty$ $N\to\infty$, thus:

$$R_g(\tau) = \lim_{N\to\infty} \frac{1}{NT_b}\int_{-NT_b/2}^{NT_b/2} g(t)g(t-\tau)\,dt$$

Total area is $N\left(\frac{T_b}{2} - \tau\right)$

$$R_g(\tau) = \lim_{N\to\infty} \frac{1}{NT_b}\left[N\left(\frac{T_b}{2} - \tau\right)\right] = \frac{1}{2}\left(1 - \frac{2\tau}{T_b}\right)$$

$$R_g(\tau) = \frac{1}{2}\left(1 - \frac{2|\tau|}{T_b}\right) \qquad |\tau| < \frac{T_b}{2}$$

$$R_g(\tau) = 0 \qquad |\tau| > \frac{T_b}{2}$$

Auto correlation factor $= \frac{1}{2}\Delta(\tau/T_b)$

$$S_g(\omega) = \frac{T_b}{4}\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right)$$

90% are of spectrum is contained.

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Question #4

Let $T = N T_b$. On average, there are $\frac{N}{2}$ pulses in the waveform of duration T.

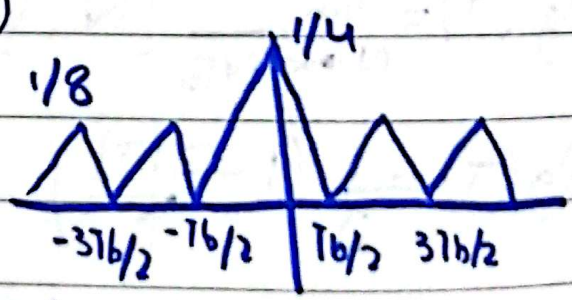
$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(\tau)x(\tau-\alpha)\,d\tau$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{|\tau|}{T_b}\right)$$

1) $\frac{T_b}{2} \leq |\tau| \leq T_b$, no overlap b/w pulses $R_x(\tau)=0$.

2) Overlap at $T_b \leq |\tau| \leq \frac{3T_b}{2}$

$R_x(\tau) = R_1(\tau) + R_2(\tau)$



$$S_x(\omega) = \frac{T_b}{16}\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right) + S_2(\omega)$$

$S_2(\omega)$ is the fourier transform of periodic triangle function

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$$R_x(\tau) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 \tau}, \quad \omega_0 = \frac{2\pi}{T_0}$$

$$D_n = \frac{1}{16} \operatorname{sinc}^2\left(\frac{n\pi}{2}\right)$$

$$S_2(\omega) = \frac{\pi}{8} \sum_{n=-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{n\pi}{2}\right) \delta(\omega - n\omega_b)$$

$$\omega_b = \frac{2\pi}{T_b}$$

$$S_x(\omega) =$$

$$\frac{T_b}{16} \operatorname{sinc}^2\left(\frac{\omega T_b}{4}\right) + \frac{\pi}{8} \sum_{n=-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{n\pi}{2}\right) \delta(\omega - n\omega_b)$$