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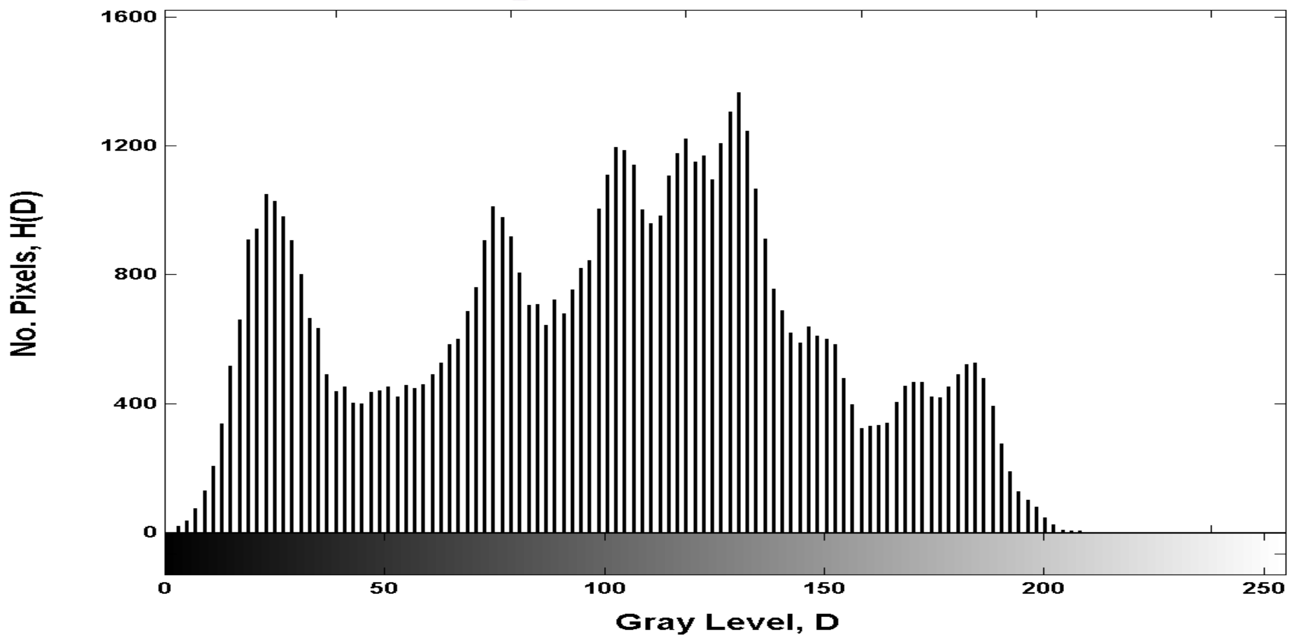
Histograma

María Patricia Trujillo

Histograma

- Cuenta el numero de veces que aparece un valor de intensidad (profundidad o amplitud)



P2 - PGM grey scale image, stored in ASCII, one value per pixel

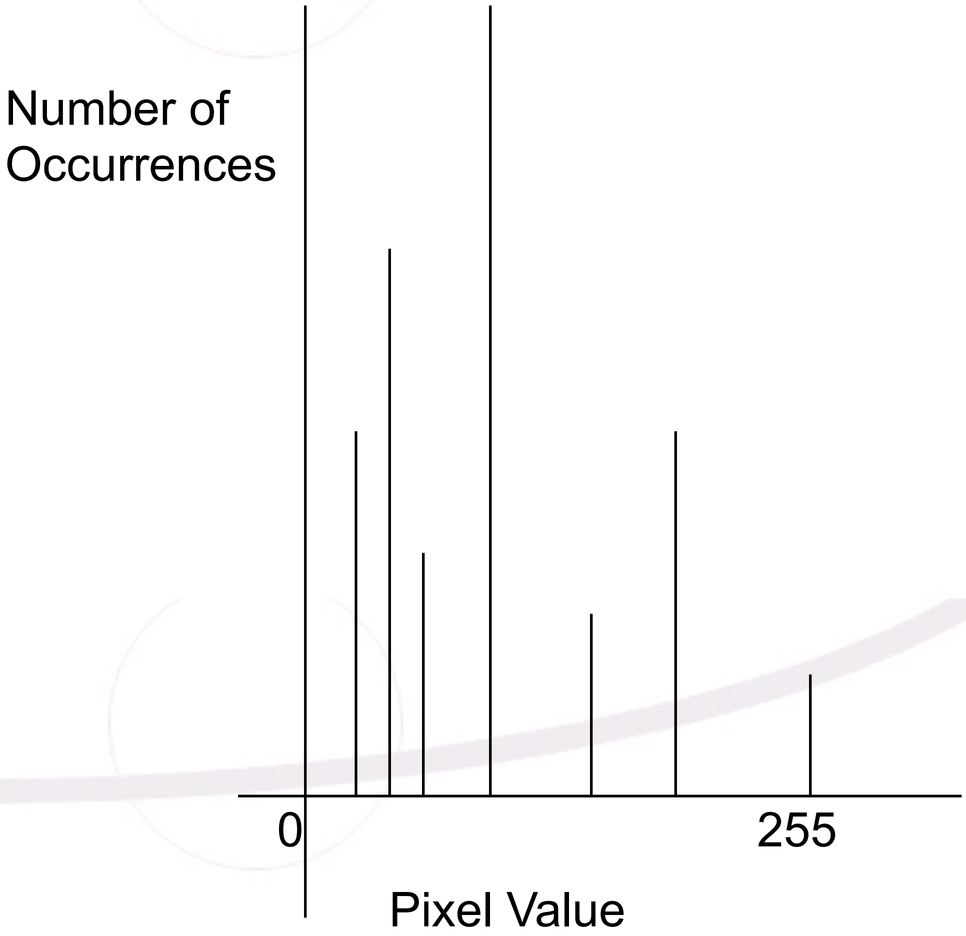
```
P2
16 8
8
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 3 3 3 3 0 0 7 7 7 7 0 0 1 1 1
0 3 0 0 0 0 0 7 0 0 0 0 0 1 0 0
0 3 3 3 0 0 0 7 7 7 0 0 0 1 1 1
0 3 0 0 0 0 0 7 0 0 0 0 0 1 0 0
0 3 0 0 0 0 0 7 7 7 7 0 0 1 1 1
0 0 0 0 4 4 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 1 2 3 3 0 0 1
```

86
14
1
12
2
0
0
13
0

Frequencies

Grey-Scale Histograms

- Number of pixels at each grey-level or in a range of grey levels
- Plot of frequencies of grey levels as function of pixel value
- Relative frequencies are equivalent to the Probability Density Function (pdf)

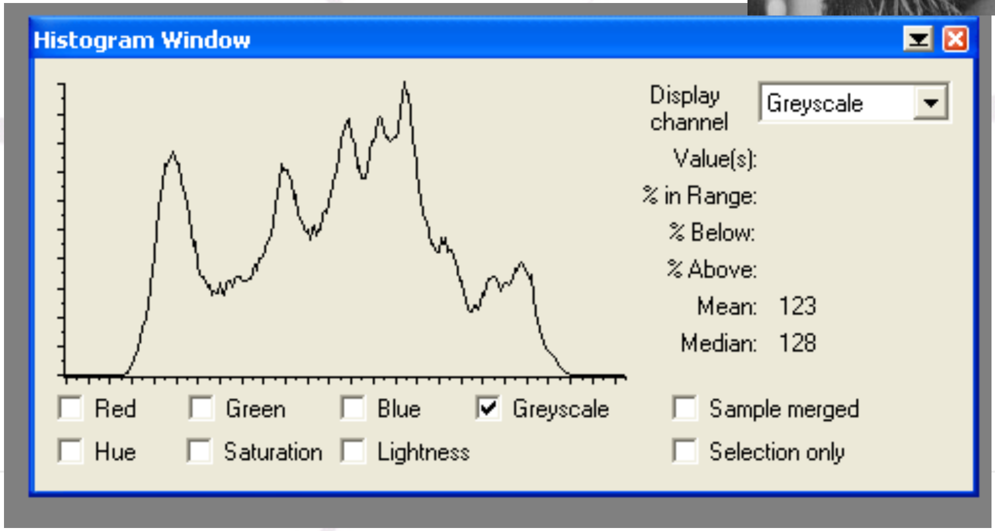


Histogram Properties

- Histogram is a many-to-one mapping
- Different images can have the same Histogram
- Consequently: Non invertible mapping

Histogram Properties

- Histogram is invariant under certain geometric image operations
  - Rotation
  - Scaling
  - Flip
  - Mirroring
  - Skew
  - etc



Geometric Transformations

- Translation

  $$x' = x + tx$$
  $$y' = y + ty$$
  $$z' = z + tz$$

- Scaling changes

  $$x' = x * sx$$
  $$y' = y * sy$$
  $$z' = z * sz$$

- Rotation

  $$x' = x * \cos(a) - y * \sin(a)$$
  $$y' = x * \sin(a) + y * \cos(a)$$
  $$z' = z$$

- Reflection

  $$x' = -x$$
  $$y' = y$$
  $$z' = z$$

Rotation

Flip

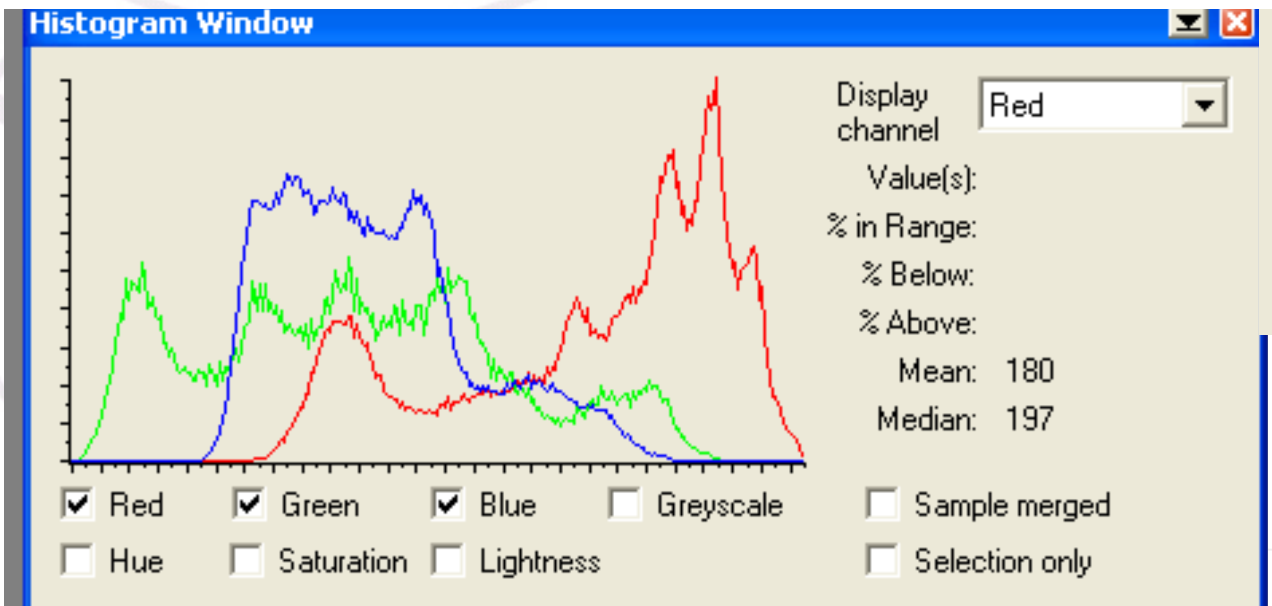
Mirroring

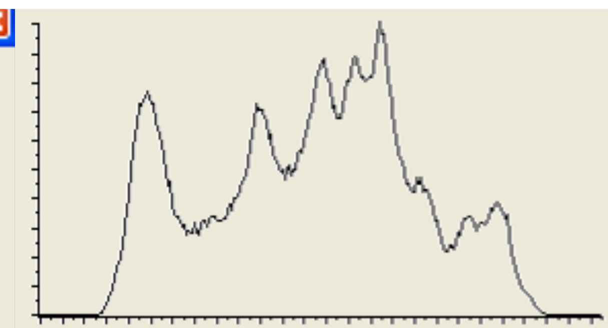
Color Histograms

red

green

blue





Greyscale

How are the histograms of these two images?

Do these pictures have identical colour distribution?

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Conclusions

- Histograms encapsulate a lot of information about images
- It gives the grey level colour distribution
- It condense the grey level information into sequences
- Can be used to change image contrast and grey level range
- It does not tell us anything about the spatial information
- Two totally different images my have the same histogram

Applications: Digitization

- Histogram gives a quick indication as to whether or not the image cover the whole brightness range of digitizer
- Histogram are a powerful tool for image segmentation

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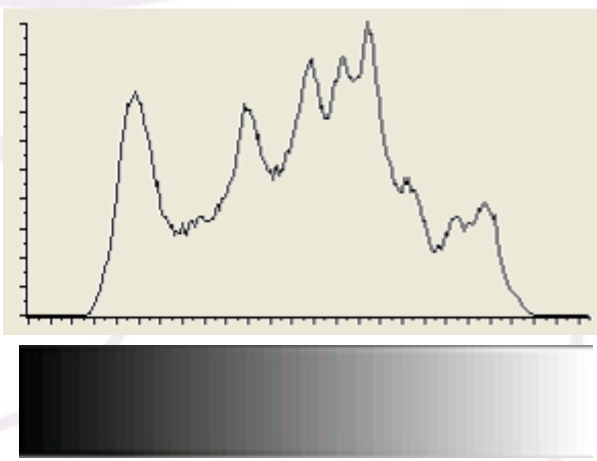
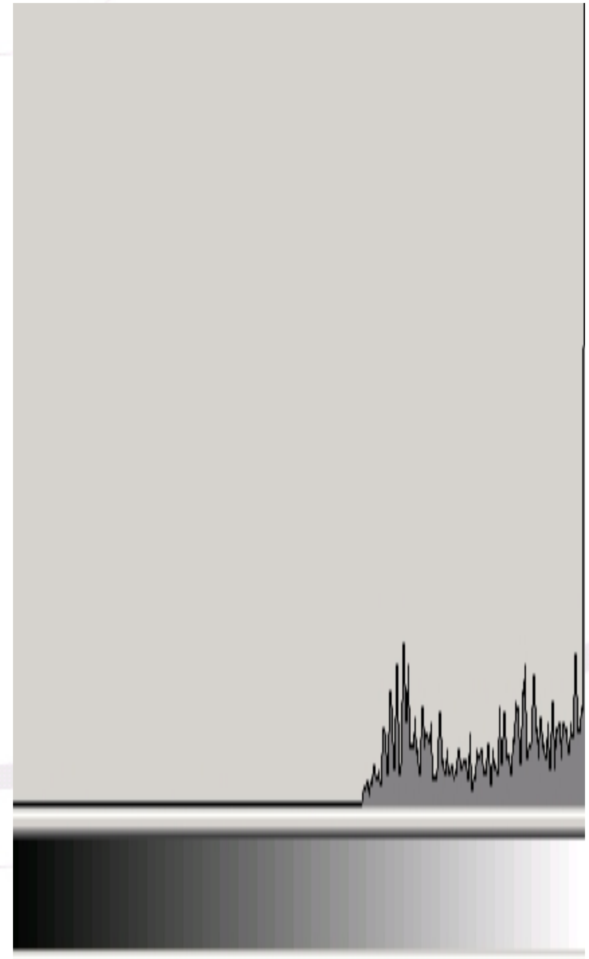
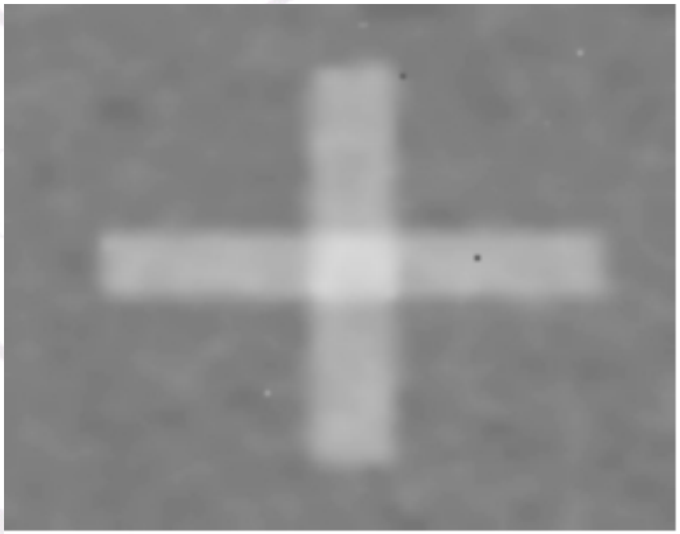
Adding 128 and Clipping

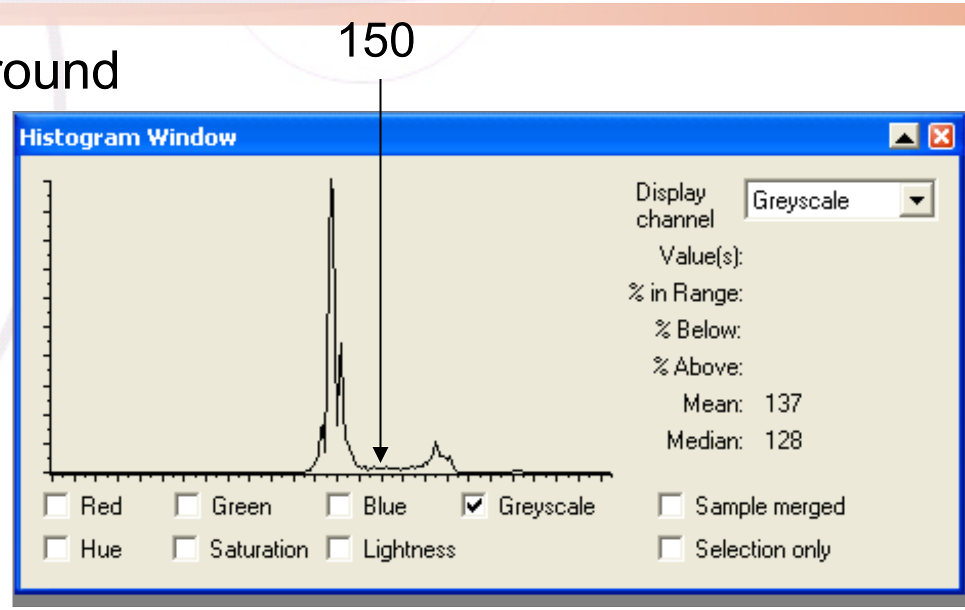




Application: Thresholding

- Separate objects from background
- Segmentation: Later …

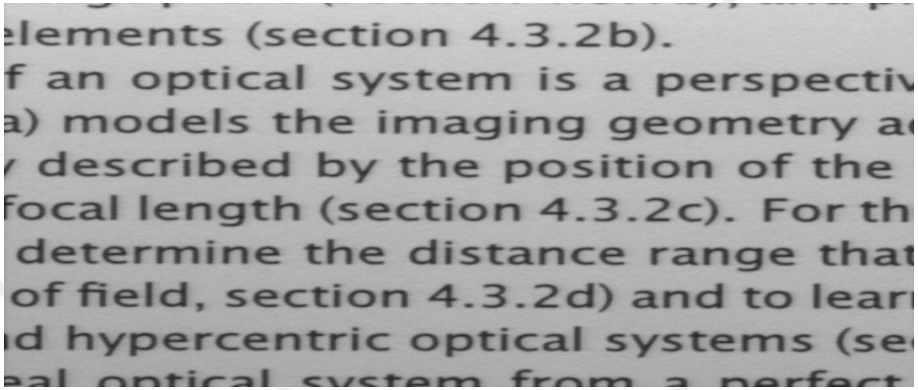


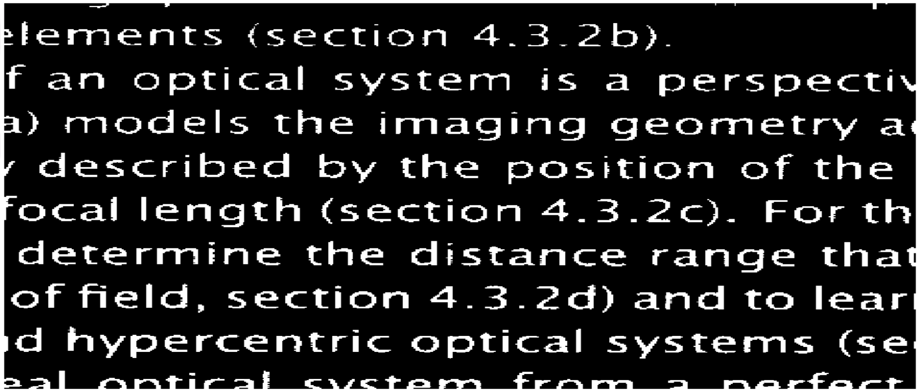


Threshold=150

Application: Thresholding

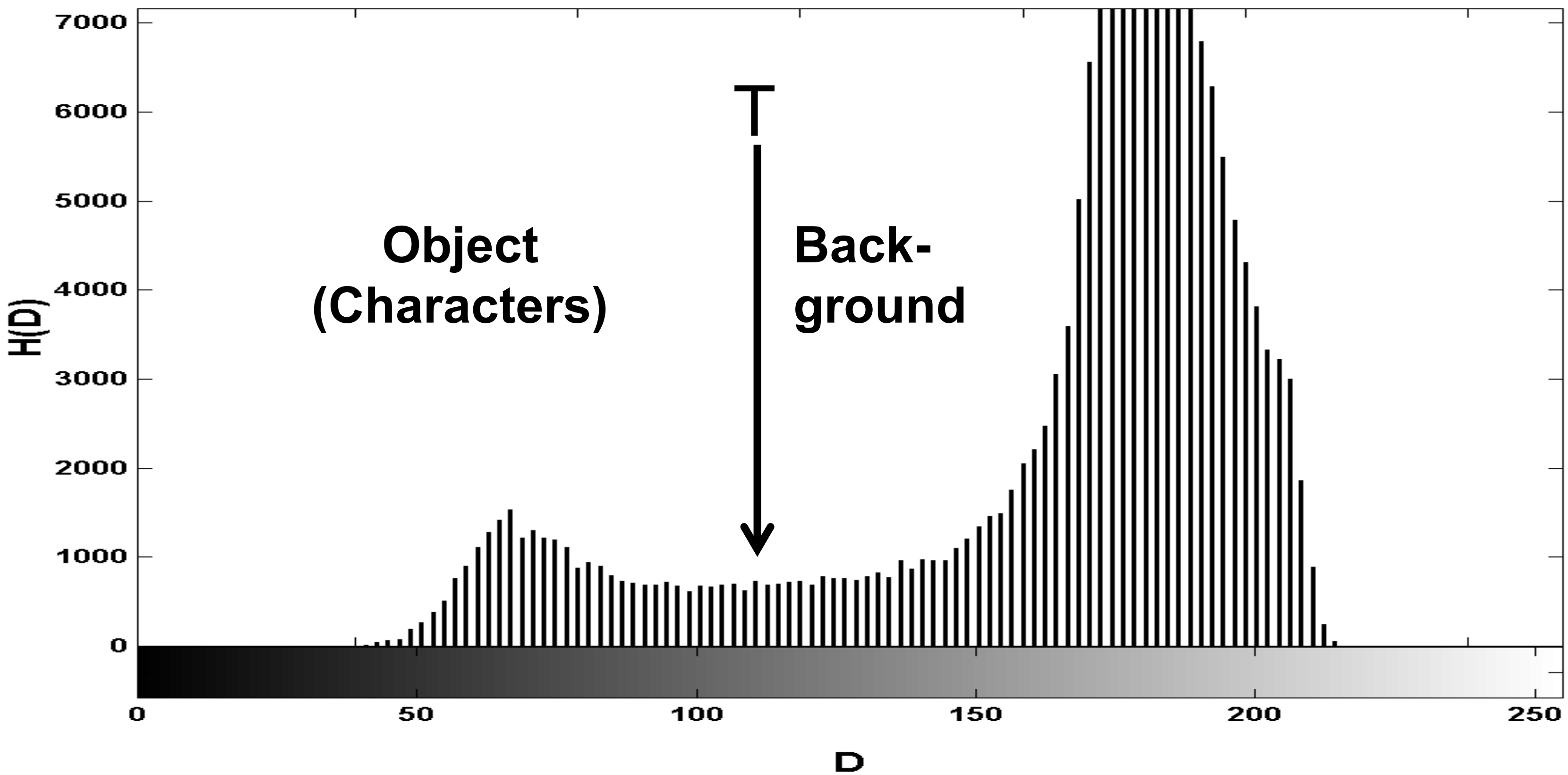
- Separate *contrasting* objects (in this case, letters) from background
- Segmentation: Later …





Application: Thresholding

- For an object on a contrasting background, histogram is <u>bimodal</u> (i.e., contains two peaks)
- Threshold gray level T separates object from background



Application: Thresholding

Dark objects are segmented or separated from a light background. Theresholding leads to a binary image by setting all pixels with gray levels less than or equal to T to 1 and others to 0

What happen if the object is bright and the background dark?

Application: Thresholding

The total area A of dark objects on a contrasting light background can be computed from the histogram $H(D)$ and the threshold T.

How?

What are the units of area?

If the threshold T is selected at the minimum, then variations in T will have minimal effect on the estimated area of the objects:

$$A = \sum_{D=0}^{T} H(D)$$

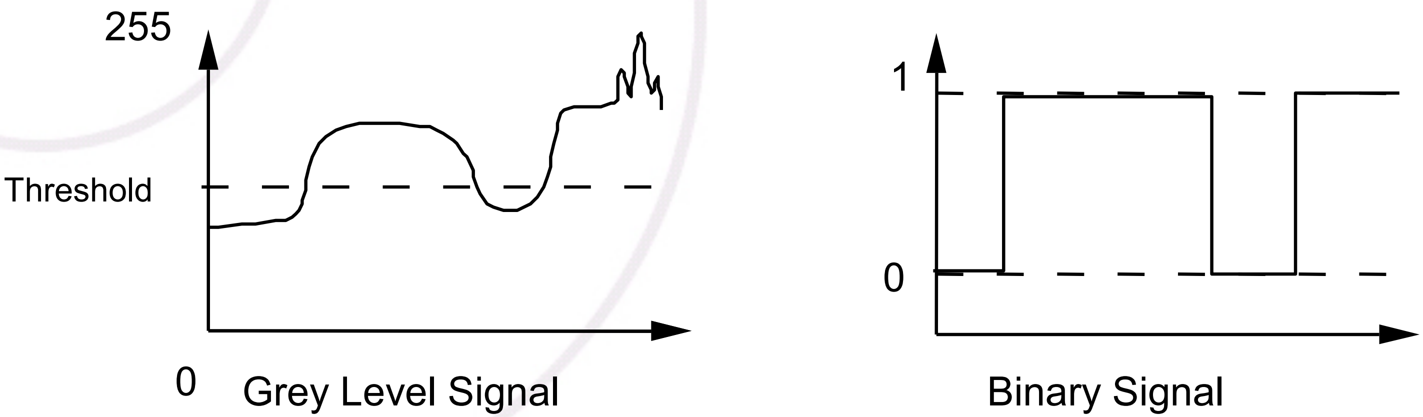
because $H(D)$ takes small values at the minima

What are the units of area? Think about pixels …

Ejercicio

- Supongamos una escena con tres objetos de intensidades distintas sobre un fondo muy brillante. ¿Como sería el histograma correspondiente?
- ¿Cómo podríamos calcular en forma automática el umbral ideal para separar el fondo de los objetos en la imagen? ¿Dónde podría existir un problema?
- Si en el histograma existe un pico muy alto en el extremo superior derecho del histograma, que es lo que esto sugiere?

Thresholding



- Simplest method: if I(x,y) < T then *black* else *white*
- Implementation can be in hardware: LUT

- Issues
  - Choice of "best" threshold
  - Is image segmentable by this technique?

Dr. Panos Liatsis
Information and Biomedical Engineering Centre
p.liatsis@city.ac.uk

Intensity/amplitude thresholding

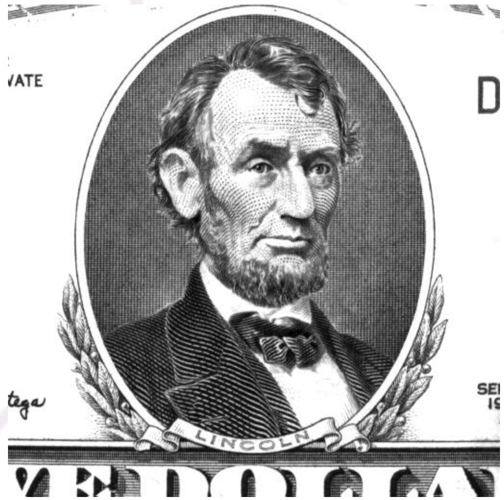
- If $I(x,y) > T$ $\quad I_t(x,y) = 1$
  $< T$ $\quad I_t(x,y) = 0$

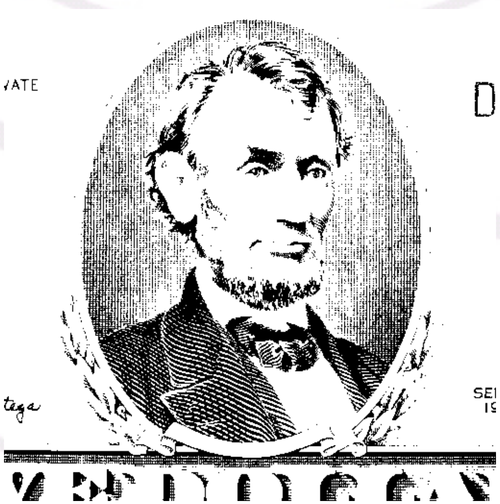
Requires:

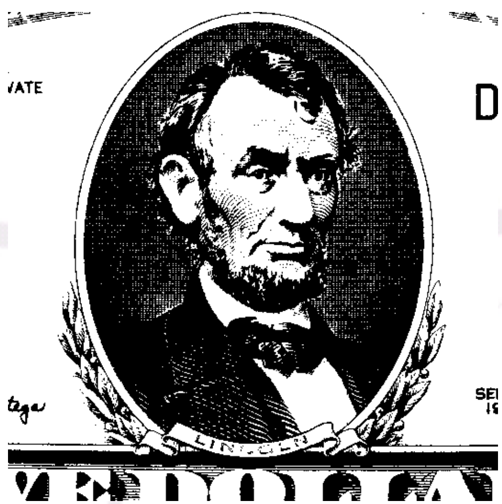
- constant intensity background
- constant intensity objects
- good contrast (difference between background and object intensity)

- Examples : printed text, silhouette objects

Thresholding







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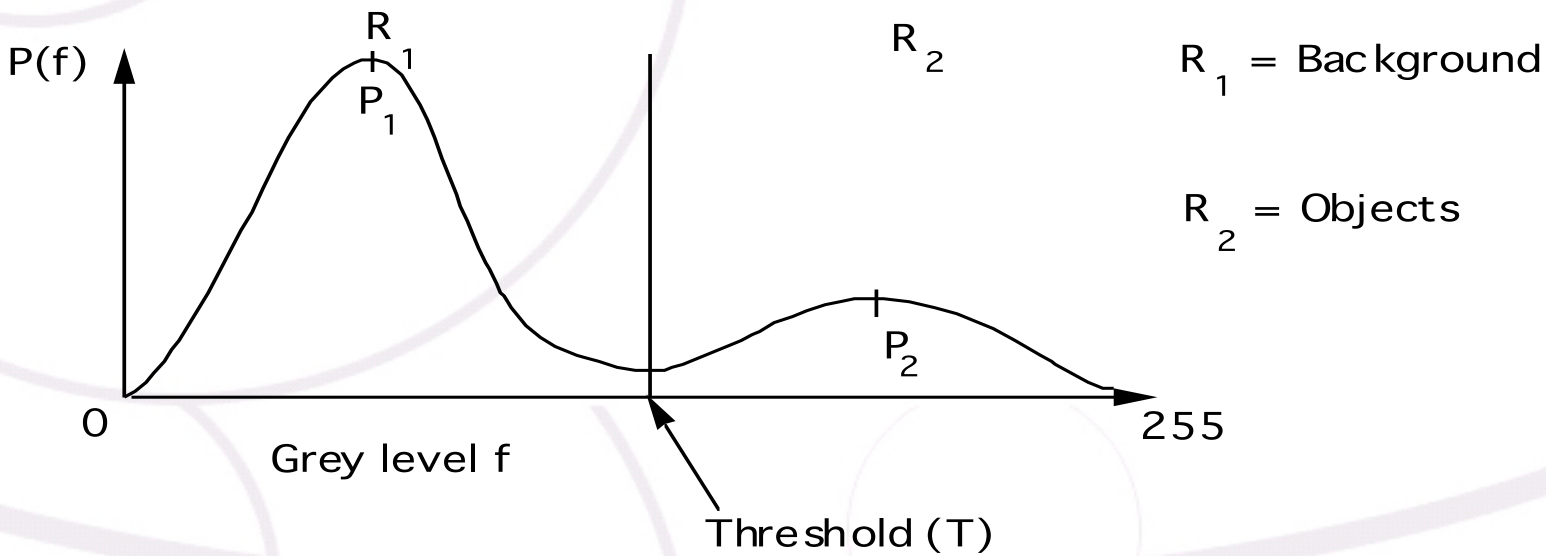
Information and Biomedical Engineering Centre
p.liatsis@city.ac.uk

Choice of Threshold

- Simple "try and see if it works"
- Histogram analysis



- Search for minimum between p_1 and p_2
- Fit 2nd order equation
  - differentiate → find minimum
- Smooth histogram and/or image first

Dr. Panos Liatsis
Information and Biomedical Engineering Centre
p.liatsis@city.ac.uk

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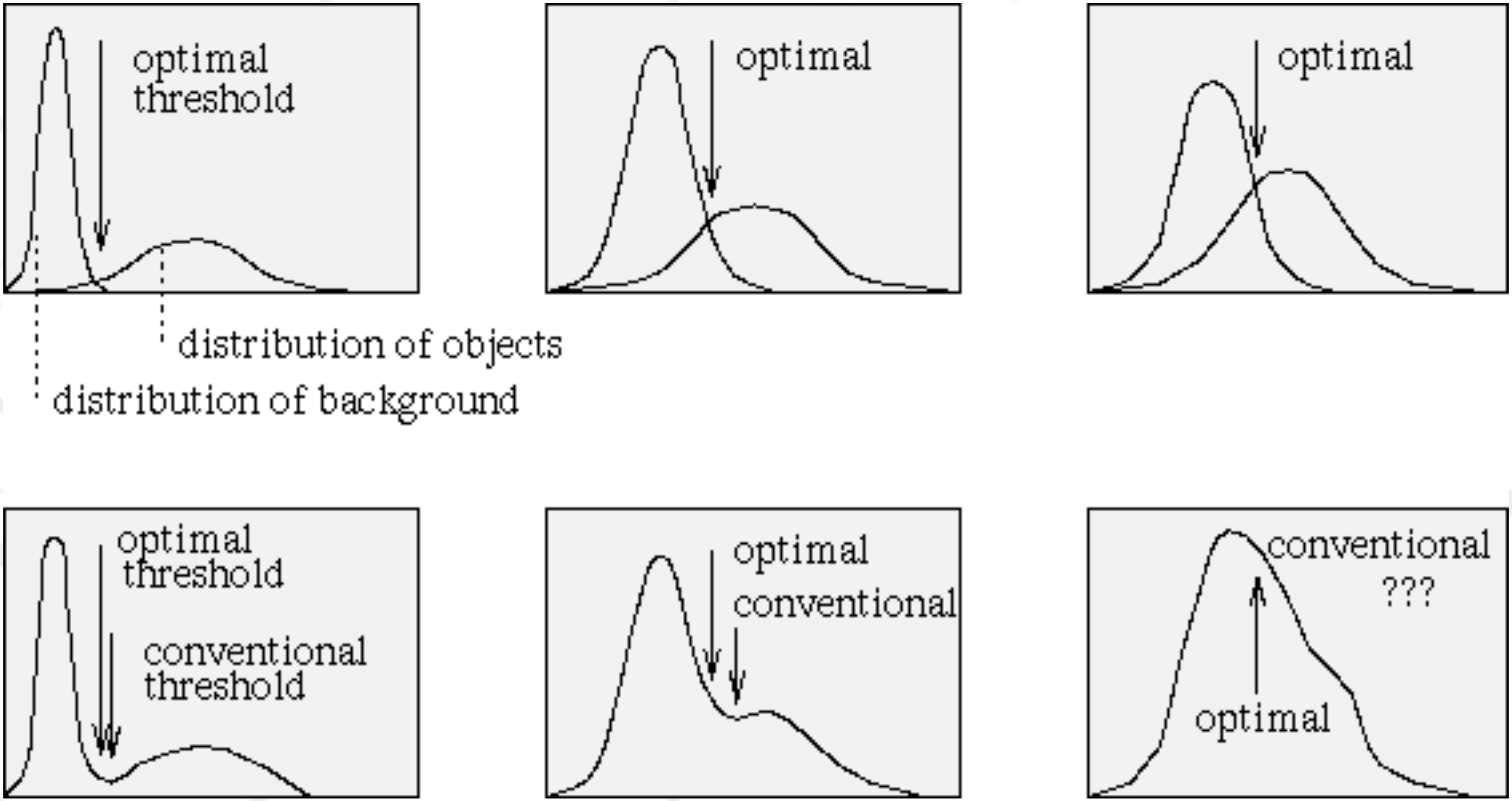
Threshold selection

- p-tile : known proportion of image is ‘object’
- mode : bi- or multi-modal histogram
  - detect minimum between the peaks (heuristic)
- gaussian fitting - intensity distributions from two groups considered ‘normally’ distributed
- common problem - spatially variable illumination
  - adaptive threshold => threshold computed separately in m sub-regions of image

Dr. Panos Liatsis
Information and Biomedical Engineering Centre
p.liatsis@city.ac.uk

Thresholding



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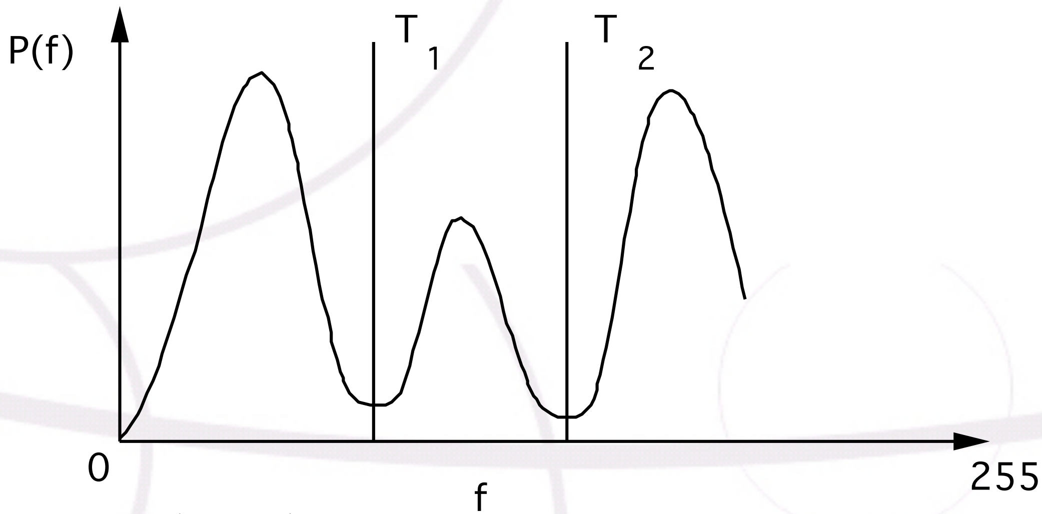
Information and Biomedical Engineering Centre
p.liatsis@city.ac.uk

Thresholding

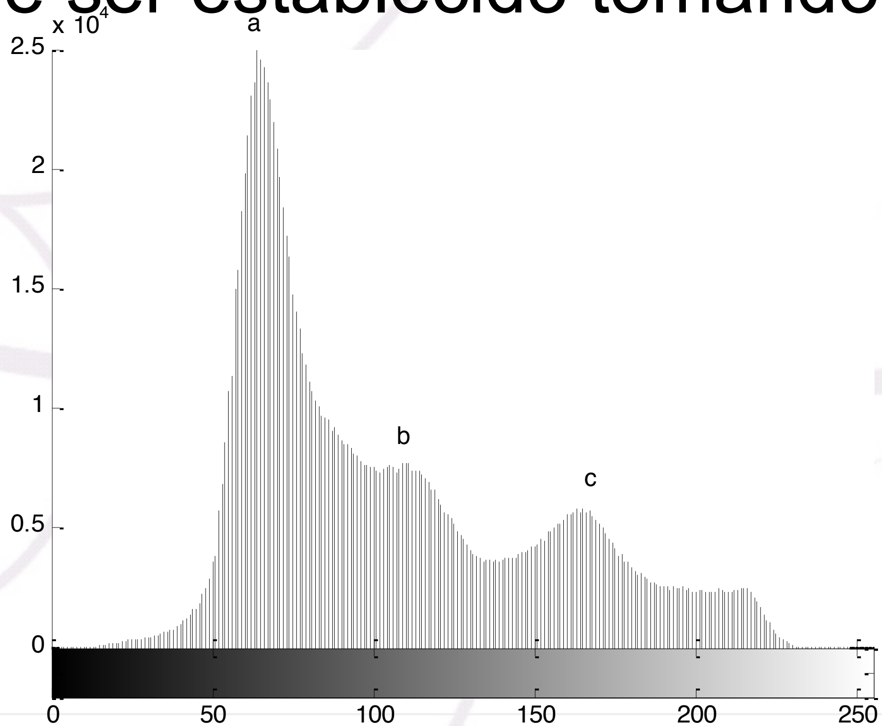
- Multi dimensional thresholding
  - Use thresholds on red, green, blue images for colour
  - Also hue, saturation, intensity for colour
  - Multi dimensional - infra-red, ultra-violet



- Multiple thresholds for one image
  - Two thresholds for three distinct regions

Método de Dos Picos

- El histograma tiene tres picos, *a*, *b* y *c*. El pico más alto corresponde al punto *a*, el segundo pico más alto es el punto *b*, pero el umbral debe ser establecido tomando el punto *c*



- El método de Dos Picos tiene en cuenta la distancia al pico máximo. Sea *h*(*k*) el pico más alto o máximo del histograma, el segundo pico más alto, incluyendo una restricción de distancia, está dado por:

$$h(j) = \max_{0 \le j \le 255}\left((j-k)^2 h(j)\right)$$

Método Isodata

El método sigue los siguientes pasos:

- Inicia dividiendo el histograma en dos partes, utilizando como umbral inicial el promedio de las intensidades,
- Calcula los promedios de cada subconjunto en que ha sido dividido el histograma, para calcular un nuevo umbral *T*

$$T = \frac{\mu_1 + \mu_2}{2}$$

Método de Otsu

- Otsu definió la variancia entre clases de una imagen umbralizada como

$$\sigma_c^2(T) = w_1\left(\mu_1 - \mu_c(T)\right)^2 + w_2\left(\mu_2 - \mu_c(T)\right)^2$$

donde $\mu_c(T)$ es el promedio ponderado de los promedios de cada grupo, μ_1 *y* μ_2*; y* w_1 *y* w_2 *son los valores de ponderación, tal que* $w_1+w_2=1$

- A partir de la premisa de la existencia de dos clases, fondo y objetos. Utilizando un umbral inicial, se definen dos distribuciones de probabilidad

$$p_1(i) = \frac{h(i)}{n_1}, 0 \leq i \leq T, \quad w_1 = \sum_{i=0}^{T} p(i), n_1 = \sum_{i=0}^{T} h(i)$$

$$p_2(i) = \frac{h(i)}{n_2}, T < i \leq 255, w_2 = \sum_{i=T+1}^{255} p(i), n_2 = \sum_{i=T+1}^{255} h(i)$$

- Los parámetros de la media de las distribuciones están dados por

$$\mu_1 = \sum_{i=0}^{T} \frac{i \times p_1(i)}{w_1(t)},$$

$$\mu_2 = \sum_{i=T+1}^{255} \frac{i \times p_2(i)}{w_2(t)}.$$

- El parámetro de posición para la distribución combinada esta dado por

$$\mu_c(T) = w_1\mu_1 + w_2\mu_2.$$

- La dispersión total esta dada por

$$\sigma_c^2(T) = w_1\left(\mu_1 - \mu_c(T)\right)^2 + w_2\left(\mu_2 - \mu_c(T)\right)^2$$

- Para una umbralización de dos niveles, Otsu verificó que el umbral óptimo se elige tal que la varianza sea máxima

$$T^* = \max_{0 \le T \le 255} \left(\sigma_c^2(T) \right)$$

Método de Entropía de Tsallis

- El método usa un umbral, T, inicial para el cálculo inicial de la entropía de Tsallis en cada grupo. El umbral se calcula recursivamente, con base en la regla seudoaditiva definida por Tsallis

$$S_q(T) = S_q^1(T) + S_q^2(T) + (1-q)S_q^1(T)S_q^2(T)$$

donde los valores $S_q{}^1(T)$ y $S_q{}^2(T)$ son los valores de entropía de Tsallis, calculada para cada grupo, y q es un parámetro tal que $q \in \Re$

- El umbral óptimo está dado por el valor que maximiza la regla seudoaditiva en (8), tal que

$$T_{optimo} = \underset{0 \le T \le 255}{\arg\max} \left(S_q(T) \right).$$

Método de Entropía de Renyi

- En el primer paso, se calcula el valor de intensidad promedio en el vecindario de cada pixel usando una ventana de *3x3*, el cual se define como $g(x, y)$
- Se crea un nuevo arreglo *G*, de dimensiones *NxM*, asignando en cada posición (x,y) la parte entera del promedio del vecindario de $I(x,y)$
- Posteriormente, se calcula la frecuencia conjunta de intensidad en *I* y valor promedio en *G*, para crear un histograma bidimensional

- El umbral es calculado como un vector T=(t,s), donde t representa un umbral de los niveles de grises, en I, y s representa un umbral para el nivel de gris promedio de las vecindades, en G
- El histograma bidimensional es dividido en cuatro cuadrantes. Se toman para el calculo los cuadrantes $[0,t]\times[0,s]$ y $[t+1,255]\times[\mathbf{s}+1, 255]$

(255,0) Primer Cuadrante	(0,0) Segundo Cuadrante
(255,255) Tercer Cuadrante	(0,255) Cuarto Cuadrante

- Cuadrantes del histograma bidimensional

- Esto se realiza suponiendo que en estos cuadrantes está contenida la información de los objetos y el fondo y están distribuidos independientemente
- De esta forma la imagen es segmentada en dos clases
- El umbral óptimo es el vector *(t*(α), s*(α))*, qué maximiza la función

$$(t^*(\alpha), s^*(\alpha)) = \arg\max[H_1^\alpha(t,s) + H_2^\alpha(t,s)]$$

- donde $H_1^{\alpha}(t,s)$ y $H_2^{\alpha}(t,s)$ son los valores de entropía de Renyi en cada clase. El valor de α se selecciona tal que $\alpha \geq 1$. Si α tiende a 1, la entropía Renyi, se convierte en una generalización de la entropía de Shannon

Método de Entropia de Tsallis- Havrda-Charvát

- Usando un principio similar al del método de entropía de Renyi, se calcula el histograma bidimensional. El umbral es calculado como un vector T=(t,s), donde t representa un umbral de los niveles de grises, en I, y s representa un umbral para el nivel de gris promedio de las vecindades, en G

- Se calcula la entropía de Tsallis-Havrda-Charvát con las frecuencias de los cuadrantes $[0,t]\times[0,s]$ y $[t+1,255]\times[\mathbf{s}+1, 255]$. Se asume que en estos cuadrantes está contenida la información de los objetos y el fondo y tienen distribuciones independientes
- El umbral óptimo es el vector $(t^*(\alpha), s^*(\alpha))$, qué maximiza la entropía de Tsallis-Havrda-Charvát, asociada a las distribuciones de los objetos y el fondo

- El umbral óptimo se calcula usando la regla seudoaditiva de Tsallis, definida en

$$\phi_\alpha(t,s) = H_1^\alpha(t,s) + H_2^\alpha(t,s) + (1-\alpha)\cdot H_1^\alpha(t,s)H_2^\alpha(t,s)$$

  donde $H_1^\alpha(t,s)$ y $H_2^\alpha(t,s)$ son los valores de entropía de Tsallis-Havrda-Charvát, calcula para cada clase

- Los umbrales óptimos maximizan la función en

$$(t^*(\alpha), s^*(\alpha)) = \arg\max[\phi_\alpha(t,s)]$$

Other Thresholding Algorithms

- Maximum Entropy Thresholding
- Niblack Thresholding
- Bernsen Thresholding
- Abutaleb Thresholding
- Sauvola Thresholding
- Moment-Preserving Thresholding
- Inner-class Variance
- Pun Thresholding

Visual perception

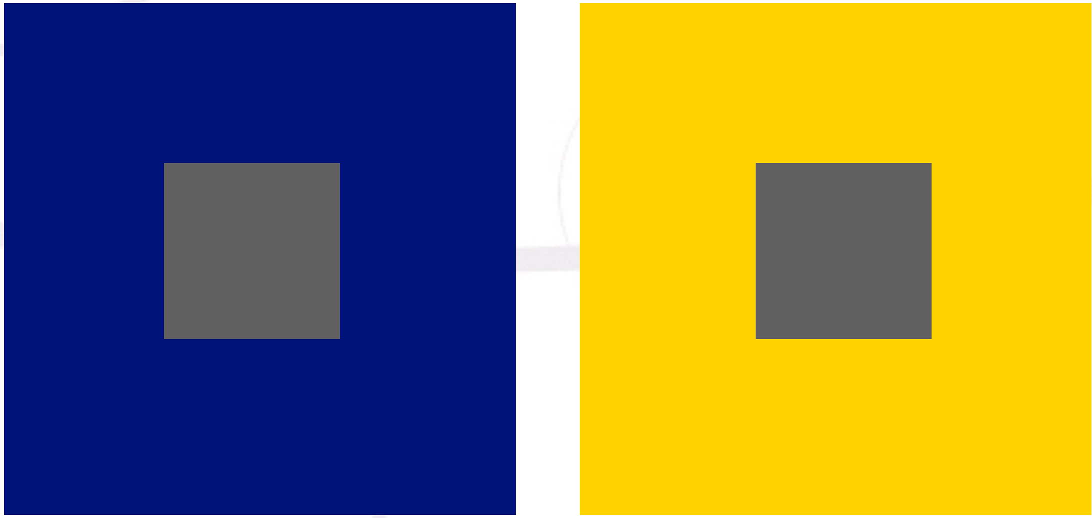
How we perceive color image information

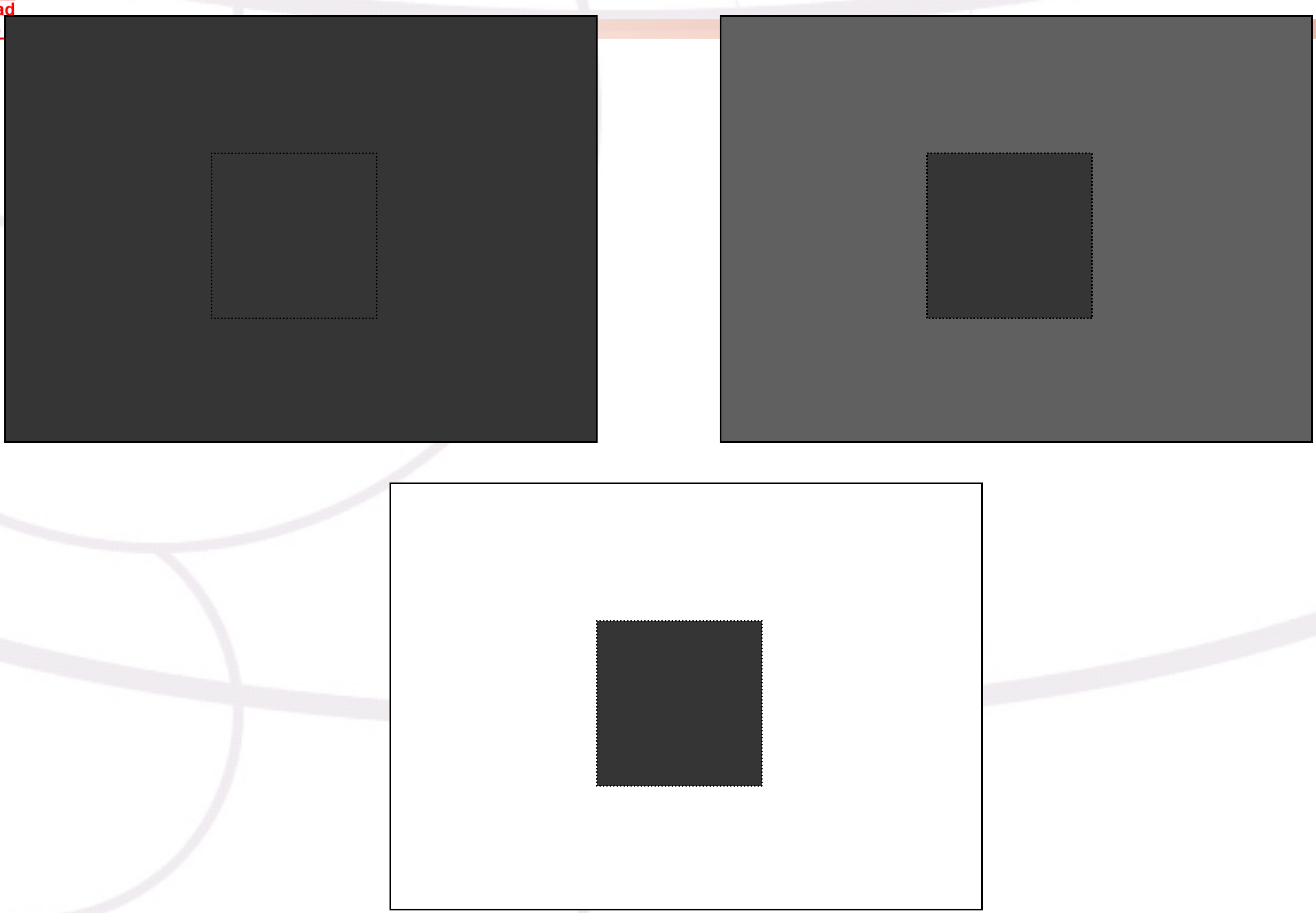
- Contrast: is the range from the darkest regions of the image to the lightest regions.
- The mathematical representation is:

$$Contrast = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Contrast

- Amount of difference between average gray level of an object and that of surroundings
- Intuitively, how vivid or washed-out an image appears
- **Example**: The grey squares are identical but the background contras is different





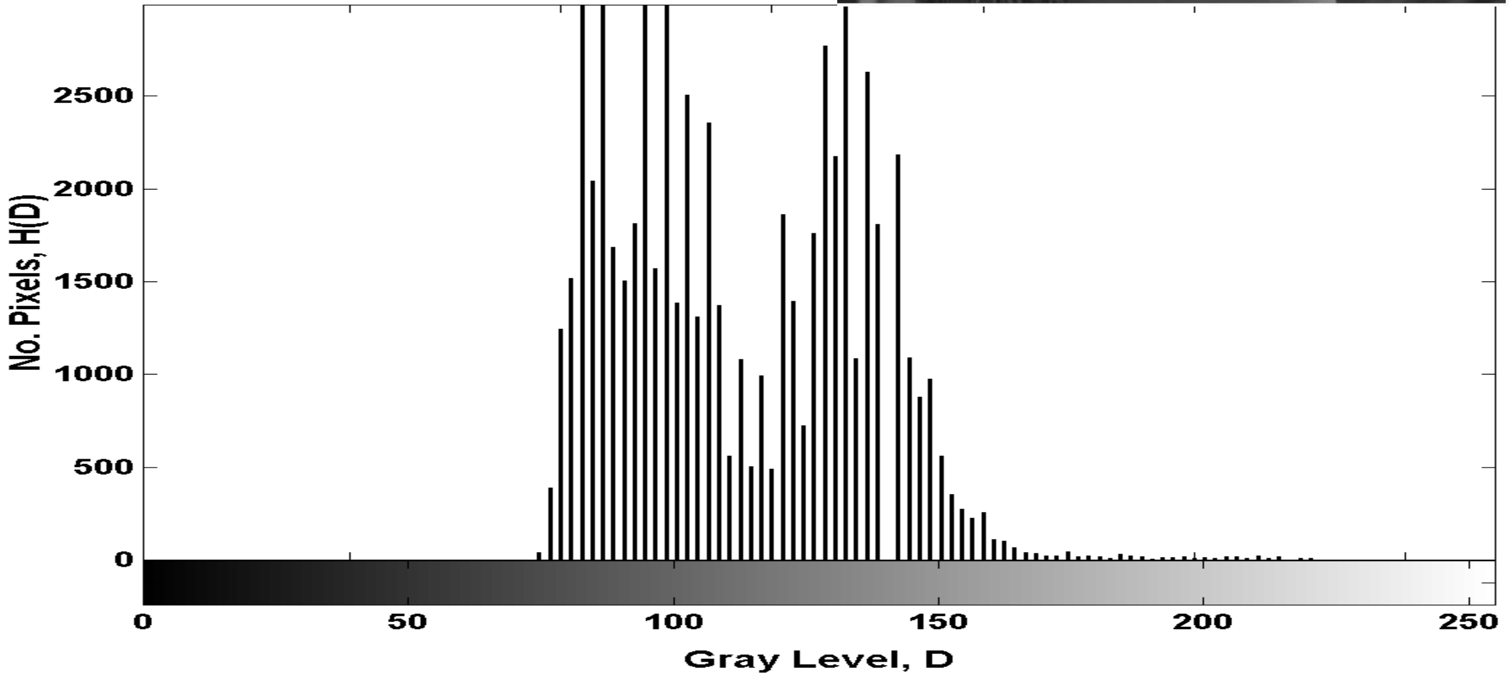
BRIGHTNESS Range

- Brightness span of an image’s gray scale

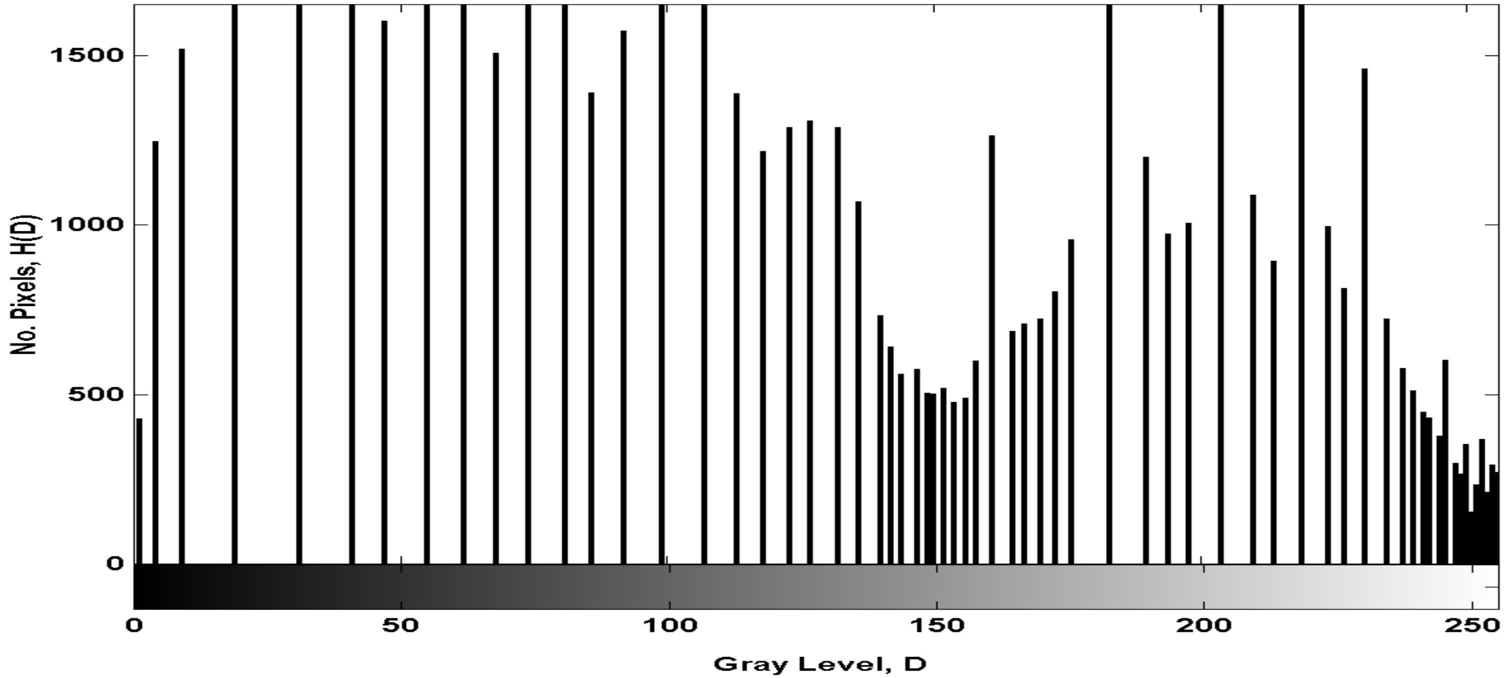
Examples

Low contrast and

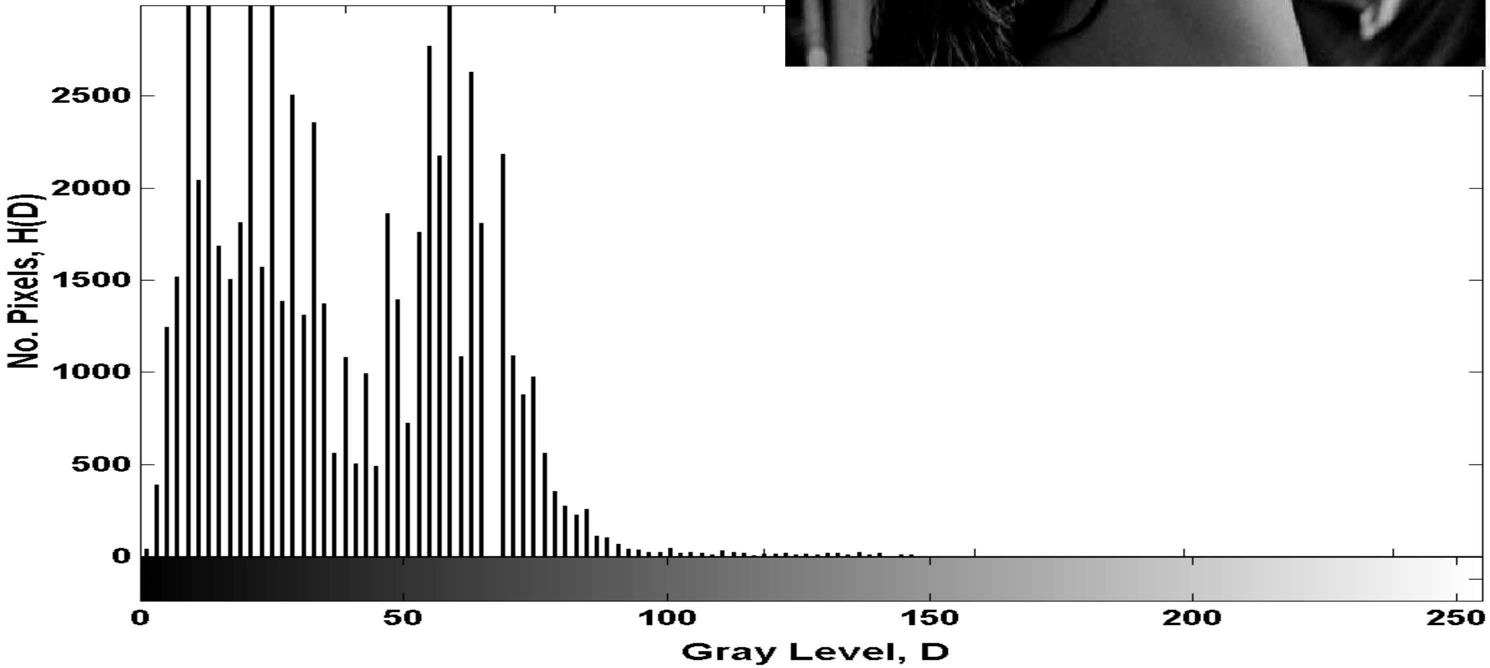
low Brightness range



Higher contrast & Brightness range



Dark image



No. Pixels, H(D)
2500
2000
1500
1000
500
0
0
50
100
150
200
250
Gray Level, D

Contrast: Gamma Correction Function

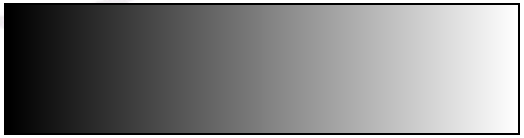
- The gamma correction function adjusts the brightness of an image
- Commonly used for compensating nonlinear responses in imaging sensors, displays and films

$$output(x) = x^{1/\gamma}$$

- γ=1.0 is null transform
- 0 < γ < 1.0 the result is logarithmic curves that brighten an image
- RGB monitors have gamma values of 1.4 to 2.8

In some problems,

intensity curves like this



are displayed like this

so, images are darker

The transformation with gamma=0.5 shrinks the dark side of the intensity curve, and so things that are darker appear brighter

Contrast stretching

- Contrast stretching is applied to an image to stretch a histogram to fill the full dynamic range of the image
- This technique works with images that have a Gaussian or near-Gaussian distribution

$$output(x) = \frac{x - I_{\min}}{I_{\max} - I_{\min}} \times 250$$

- Solarizing, used by digital artists, transforms an image according to the following formula

$$output(x) = \begin{cases} x & for\ x \le Threshold \\ 255 - x & for\ x > Threshold \end{cases}$$

- Parabola transformation

$$output(x) = 255 - 255\left(\frac{x}{128} - 1\right)^2$$

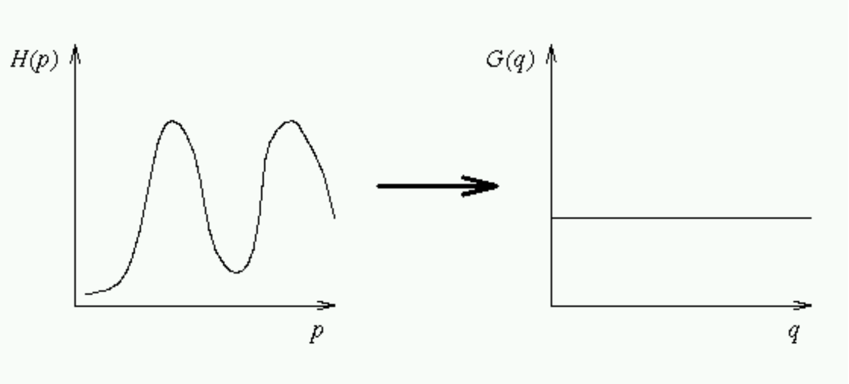
$$output(x) = 255\left(\frac{x}{128} - 1\right)^2$$

- End-in-search works well with images that have pixels of all possible intensity but have a pixel concentration in one part of the histogram
- The lower threshold, low, is the value of the histogram to where the lower percentage is reached up from the bottom
- The higher threshold, high, is the value of the histogram to where the lower percentage is reached down from the top

$$output(x) = \begin{cases} 0 \;\; for \;\; x \leq low \\ 255 \times \dfrac{x - low}{high - low} \quad for \quad low < x < high \\ 255 \quad for \quad x > high \end{cases}$$

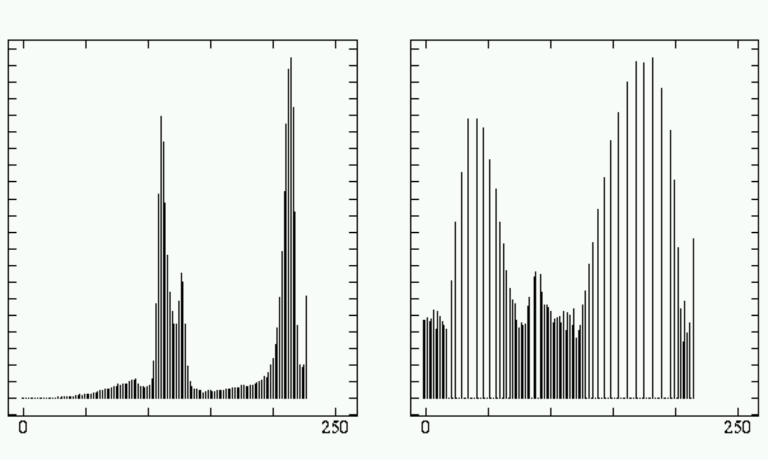
Histogram Equalization

- It improves contrast
- Its goal is to obtain a uniform histogram



Operations

1. Compute histogram
2. Calculate normalized sum of histogram
3. Transform input image to output image



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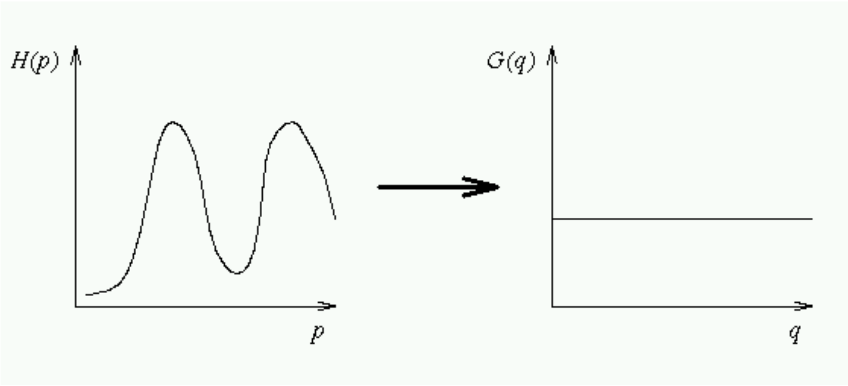
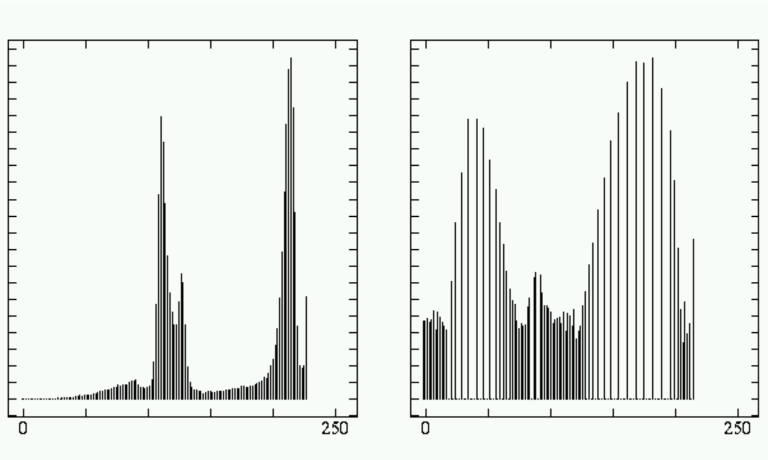




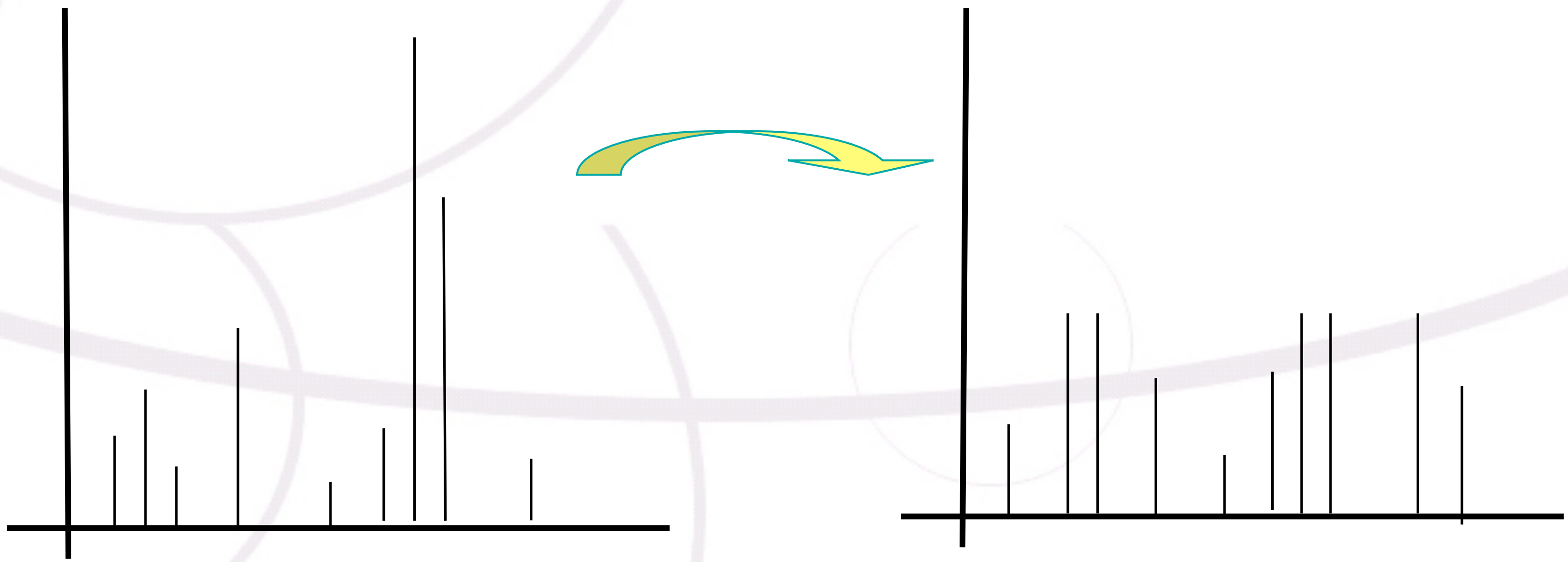
Histogram equalization

- Goal: To produce images with “flat histograms”
- Convert image with arbitrary histogram into an image with a uniform histogram
- Number of grey levels remains invariant

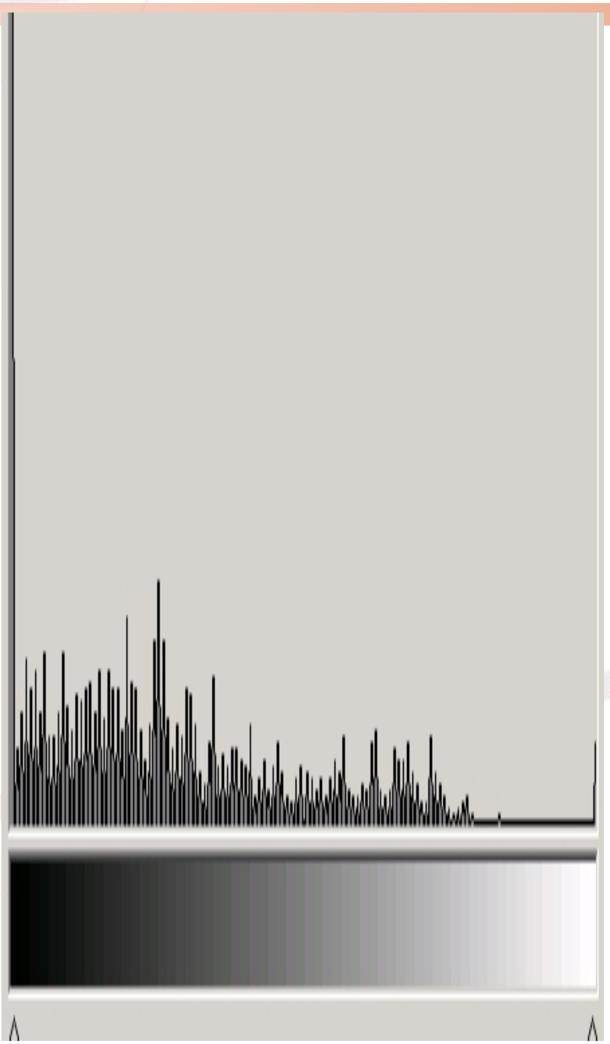
- **Application:**
- Automatization of contrast adjustments

Histogram Equalization

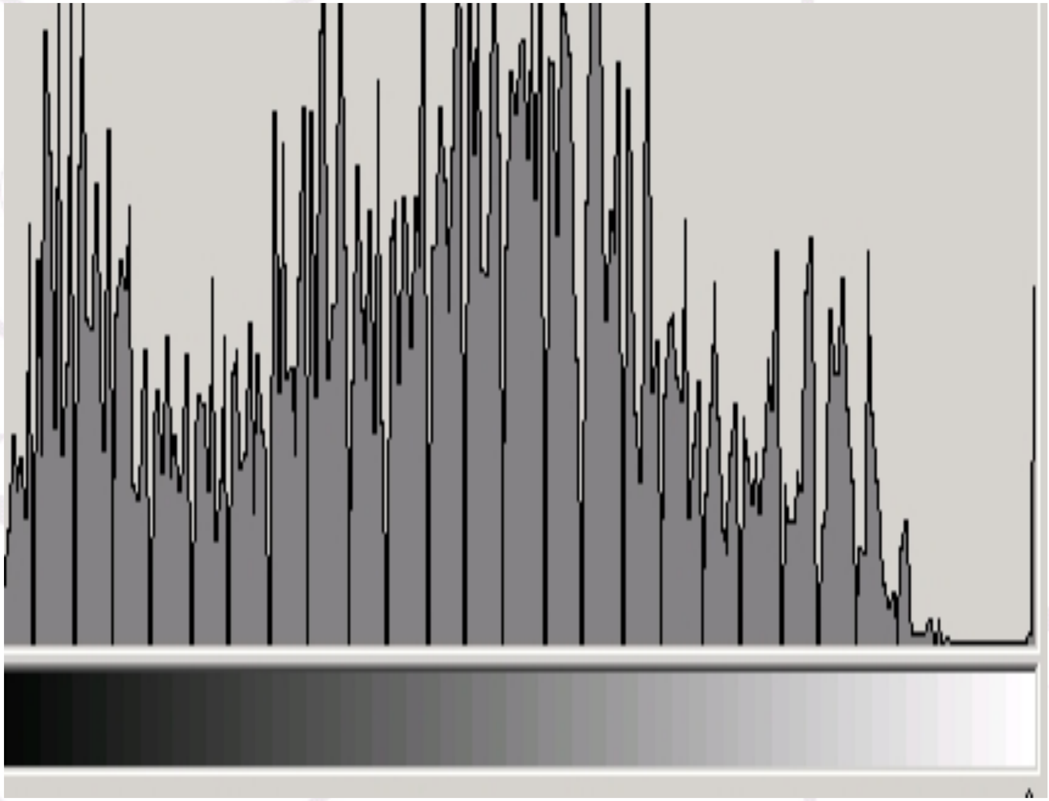
Maps the gray levels of an image into a target image with an approximate uniform distribution of grey levels



Histogram



Equalized Image



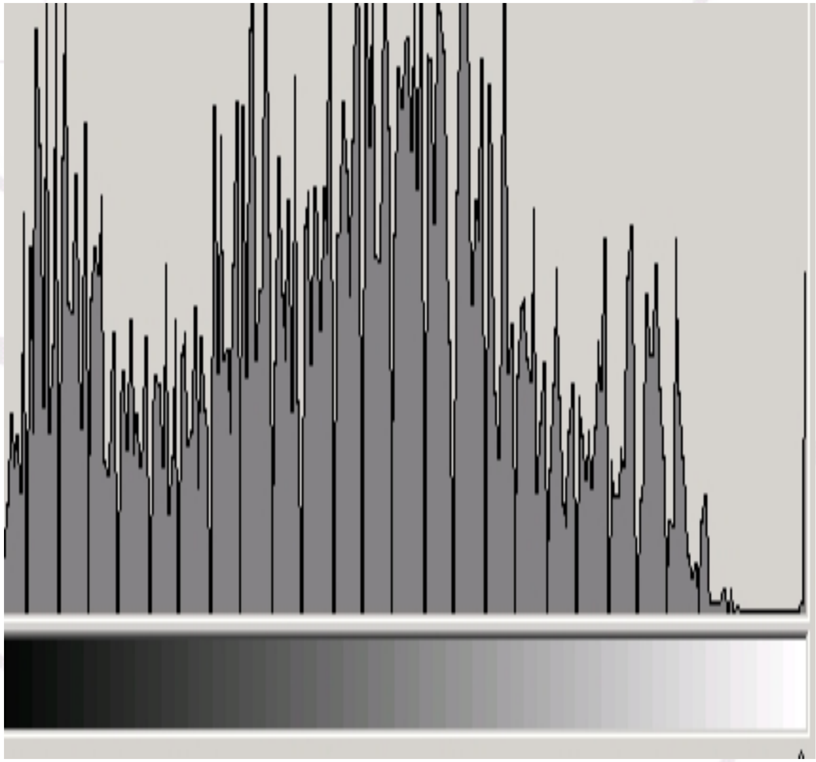
Notation

- Let D be the pixel value at a given sampling position. The same than $I(x,y)$.
- D_m The maximum grey level (usually, 255)
- $HD(i)$ Frequency of a pixel in an image
- $f(D)$ *M*apping from one image to another.
- $f^{-1}(D)$ Inverse mapping

Cumulative Distribution Function

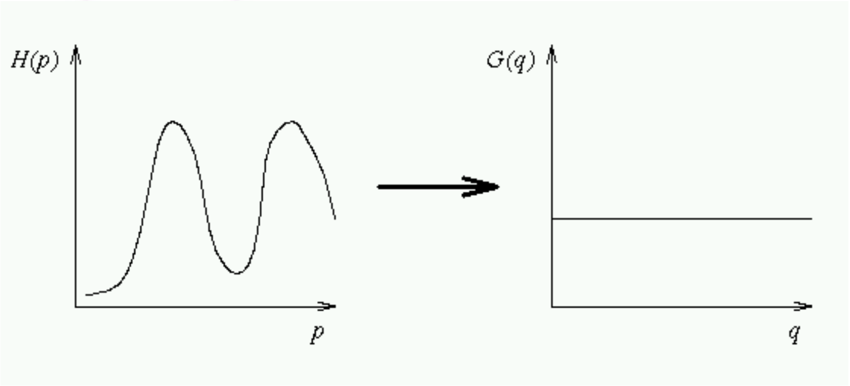
Given a continuous image A and its corresponding Histogram (Distribution Function) H_A, the CDF is given by:

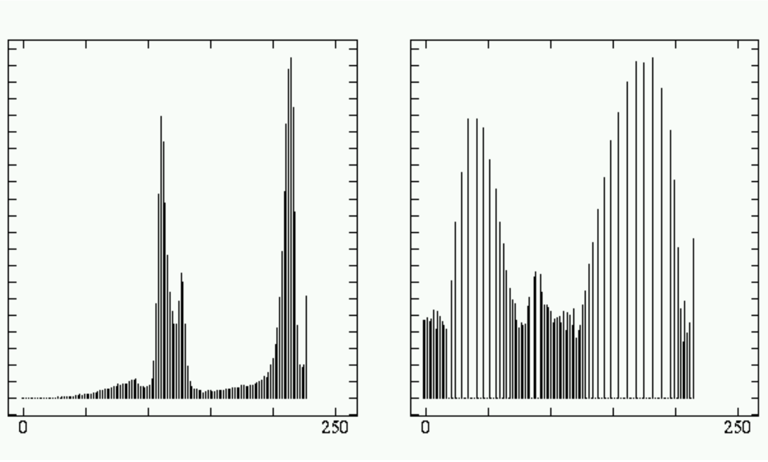
$$P_A(D_A) = \frac{1}{A_0}\int_0^{D_A} H_A(u)\,du$$



Ecualización del Histograma

1. Calcula el histograma
2. Calcula el histograma normalizado
3. Transforma la imagen de entrada





El histograma se aproxima a un histograma uniforme

Algorithm -Discrete Case-

Given an Image *A*:

1. Obtain histogram, $H_A(D_A)$, of A.

2. For each input gray level, D_A, compute the cumulative sum

$$\sum_{i=0}^{D_A} H_A(i)$$

3. For each D_A, scale sum by D_m / A_0.

4. Discretize $f(D_A)$.

5. Replace each gray level D_A in the input image by the corresponding value of $f(D_A)$.

Discrete Case Example

Let's consider a 4-bit image *A* with intensity range [0, 15]

D_A	$H_A(D_A)$
0	0
1	0
2	0
3	0
4	1674
5	22402
6	15481
7	8806
8	16087
9	4615
10	445
11	146
12	104
13	78
14	2
15	0

D_A	$H_A(D_A)$
0	0
1	0
2	0
3	0
4	1674
5	22402
6	15481
7	8806
8	16087
9	4615
10	445
11	146
12	104
13	78
14	2
15	0

$$\sum_{i=0}^{D_A} H_A(i)$$

$$f(D_A) = \frac{D_m}{A_0} Sum$$

$$A_0 = 69840$$

$$D_m = 15$$

Cumulative sum	$f(D_A)$	Discretized version of $f(D_A)$
0	0	0
0	0	0
0	0	0
0	0	0
1674	0.3595	0
24076	5.1710	5
39557	8.4959	8
48363	10.3872	10
64450	13.8424	14
69065	14.8335	15
69510	14.9291	15
69656	14.9605	15
69760	14.9828	15
69838	14.9996	15
69840	15.0000	15
69840	15.0000	15

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Replace each gray level D_A in the input image by the corresponding value of $f(D_A)$

D_A	$H_A(D_A)$	New D_A
0	0	**0**
1	0	**0**
2	0	**0**
3	0	**0**
4	1674	**0**
5	22402	**5**
6	15481	**8**
7	8806	**10**
8	16087	**14**
9	4615	**15**
10	445	**15**
11	146	**15**
12	104	**15**
13	78	**15**
14	2	**15**
15	0	**15**

Rescan the image and write output image with gray-levels g_p setting:

$$g_p = T\lfloor g_p \rfloor$$