Introduction

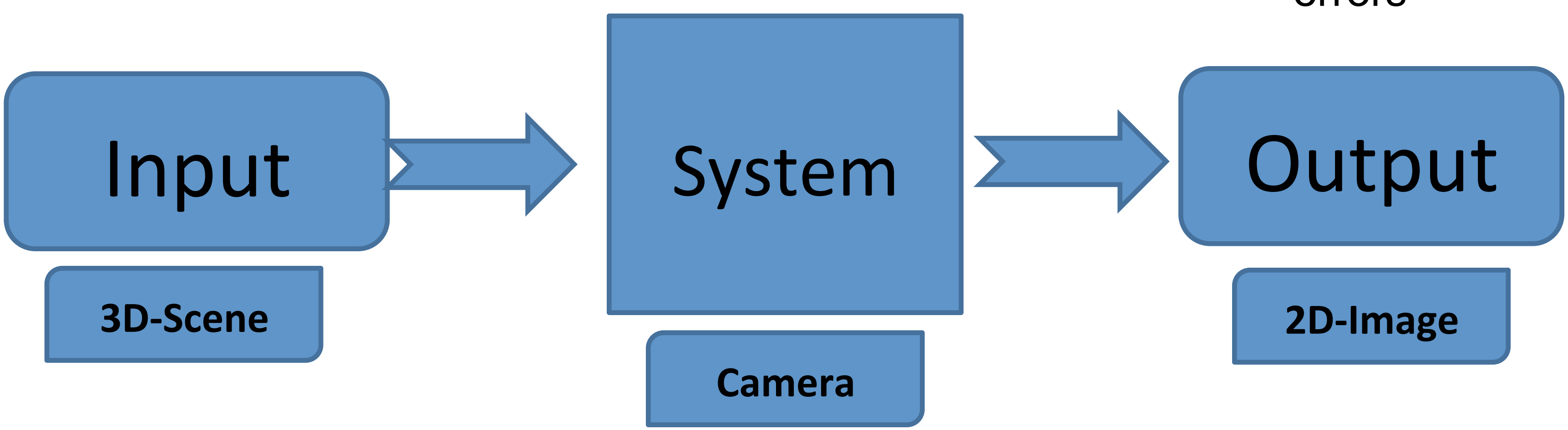
Inverse Problems

- Why Inverse Problems?

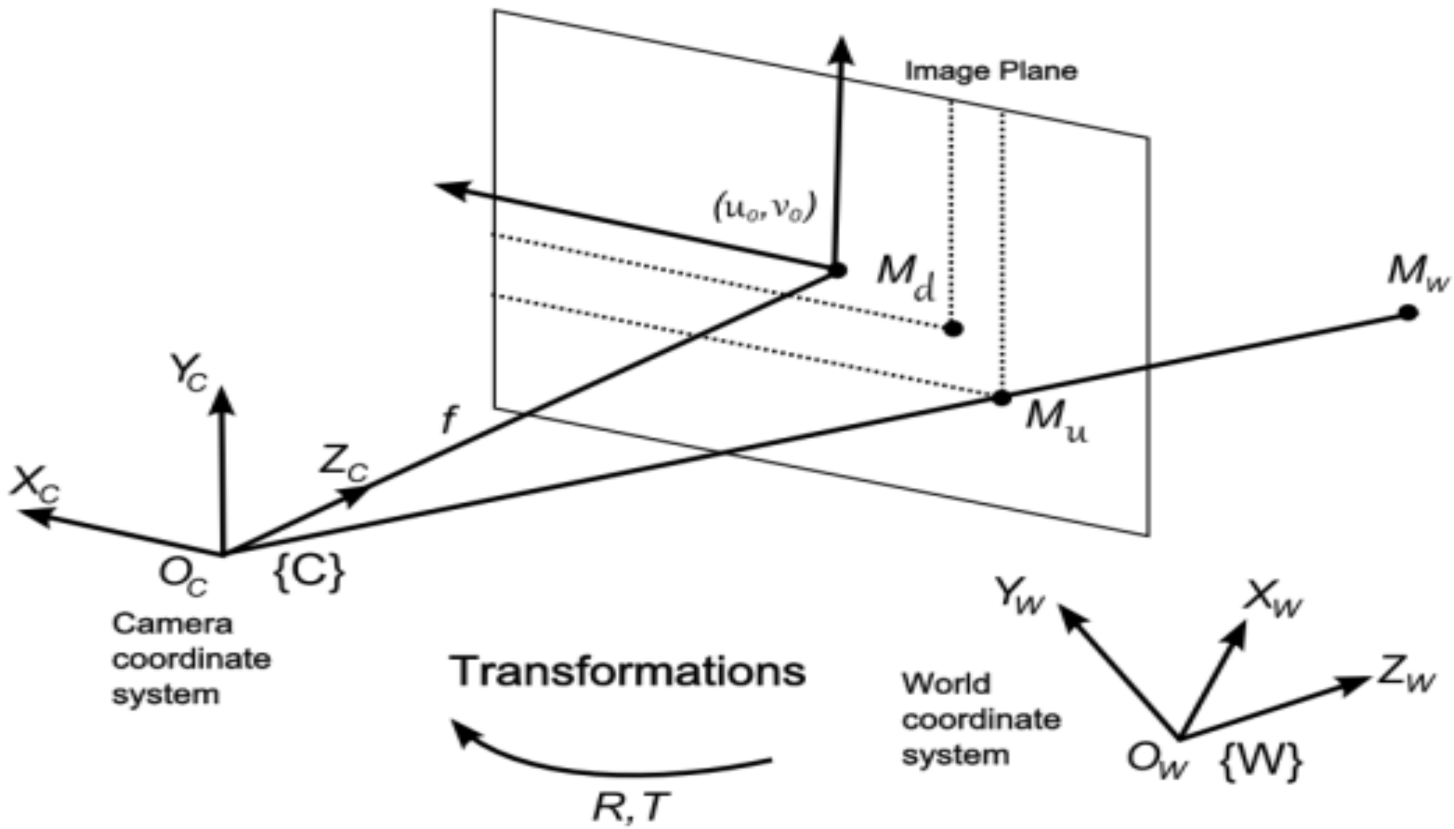
typically arise when one wants to compute information about input properties using output measurements

One of these is known

Known but with errors



Sistemas de Coordenadas



What is a Digital Image?

A discrete function from a 2D domain in a set of real positive numbers

Digital image: $$I : D \rightarrow \Re^{+}$$

Having values at discrete samples usually in a regular rectilinear grid

The function values represent gray levels, color channels, opacities, transparency or tissue density in an MR Images

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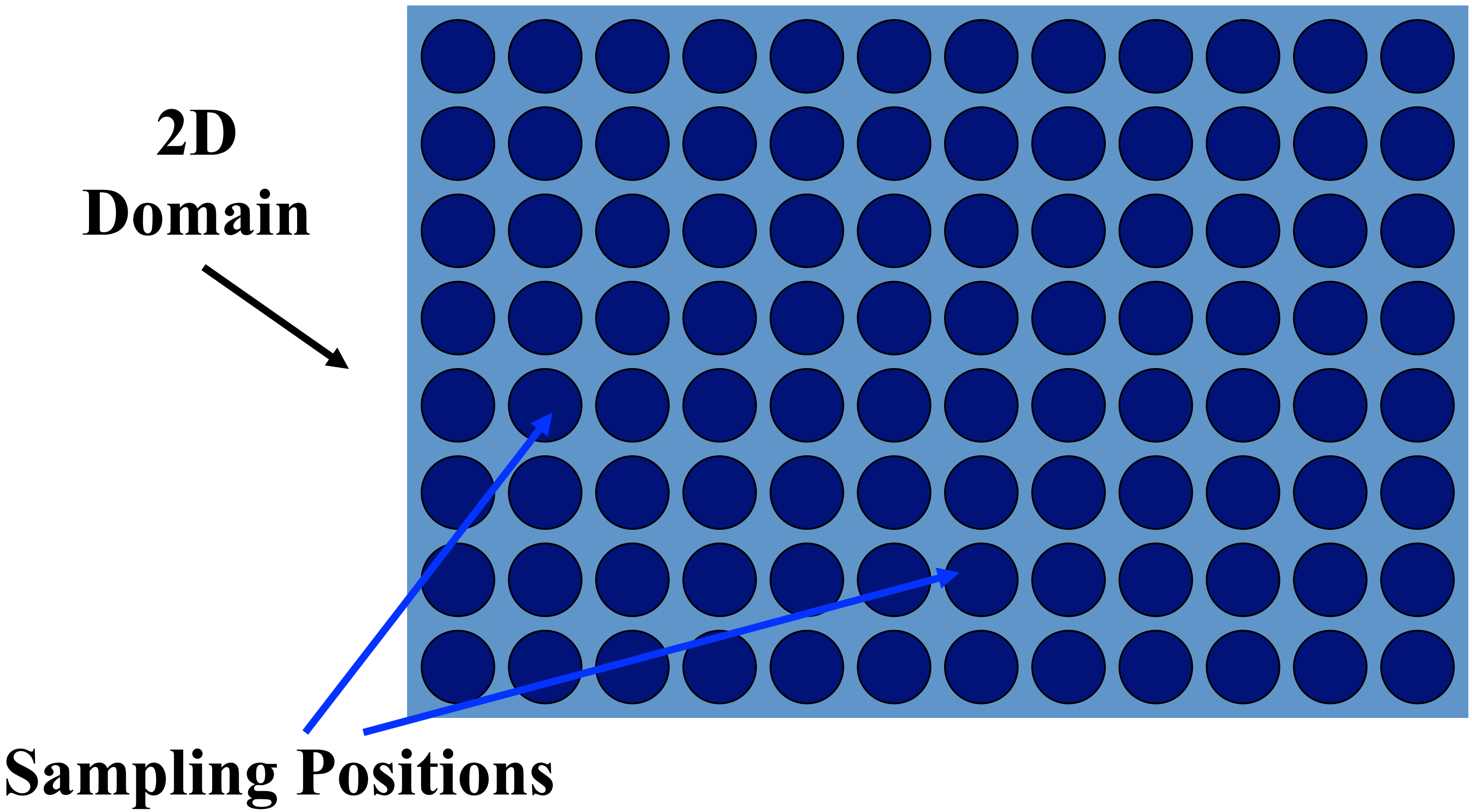
Sensores Activos

$$I : D \rightarrow \Re^{+}$$

- En rayos X representa densidad óptica
- En CT representa un coeficiente de atenuación de los tejidos
- En MRI representa una respuesta de los tejidos a una señal de resonancia magnética
- En US representa el eco de un ultrasonido

Leer: http://personal.telefonica.terra.es/web/radiologia/temas/tema5.htm

What does this mean?



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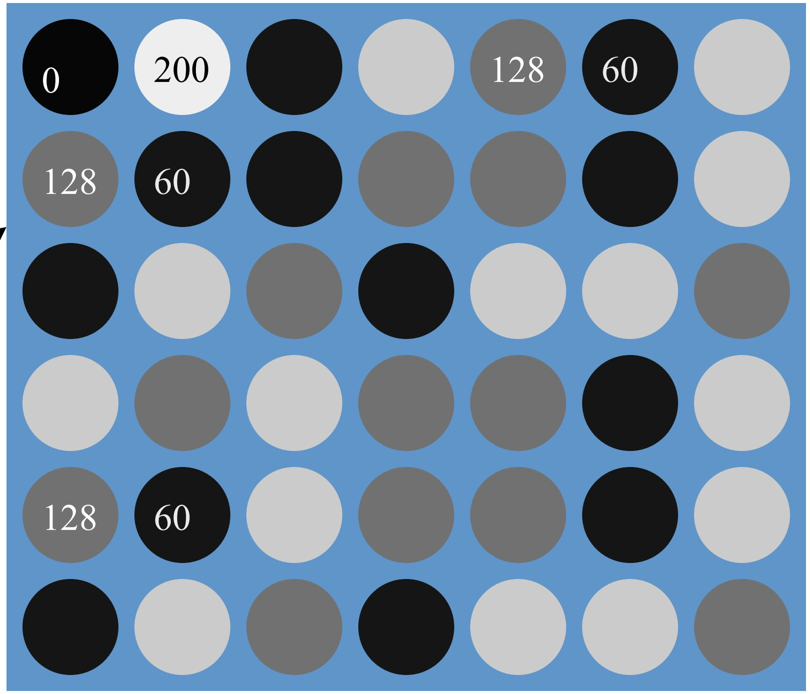
What does this mean?

Function Values at discrete grid points



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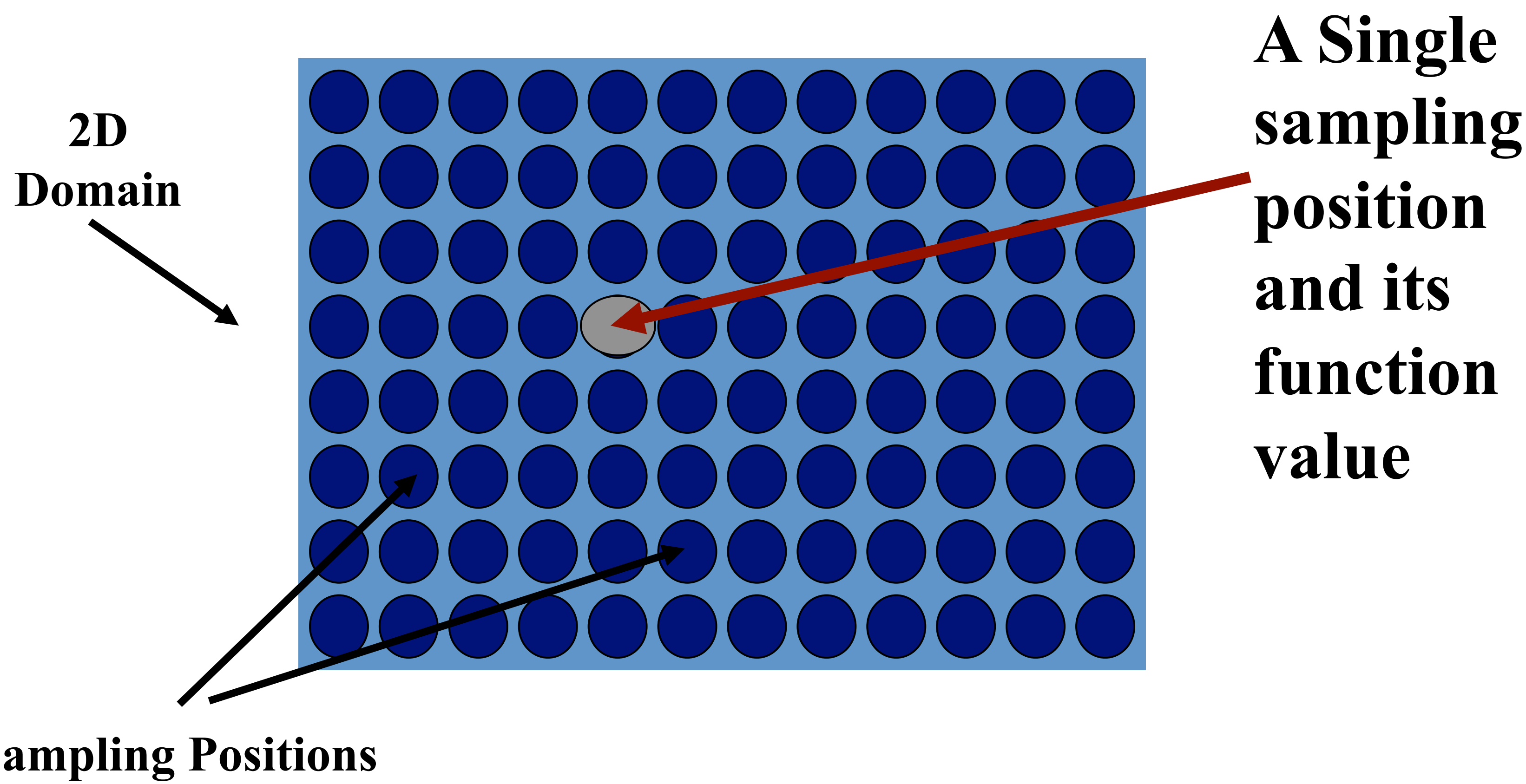
Pixels or Pels



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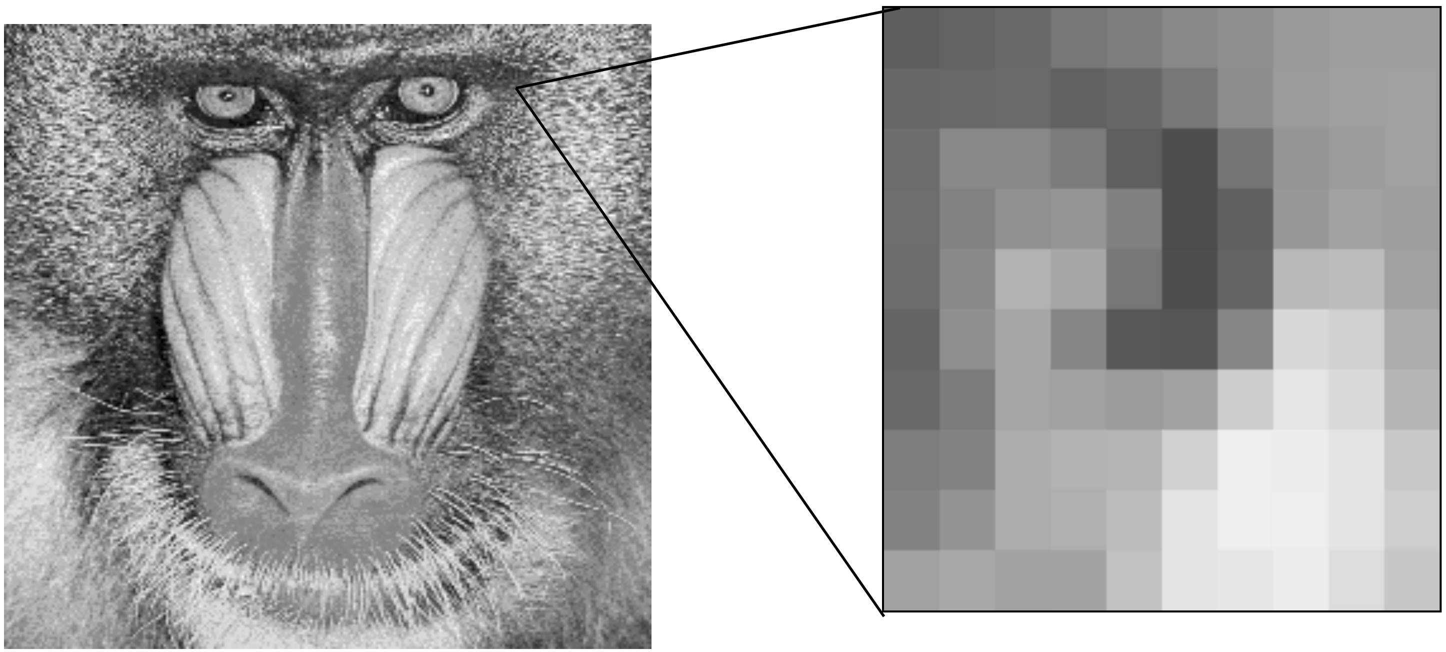
Digital Images (I)

- A digital image is a nD array of pixel values
- For example, in the 2D case the image data contains information of the graylevel at each position in the image



Magnifyed pixels at few sampling positions

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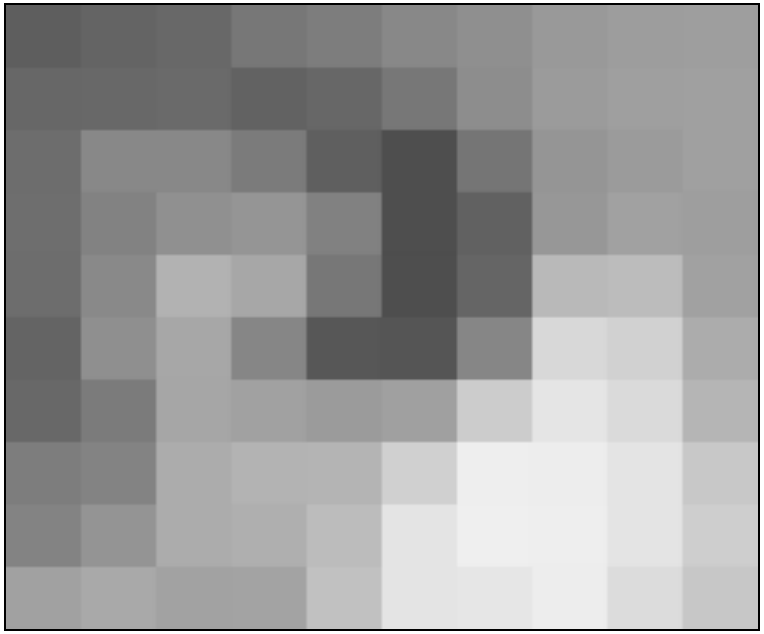
Digital Images (II)



Pixels

94	100	104	119	125	136	143	153	157	158
103	104	106	98	103	119	141	155	159	160
109	136	136	123	95	78	117	149	155	160
110	130	144	149	129	78	97	151	161	158
109	137	178	167	119	78	101	185	188	161
100	143	167	134	87	85	134	216	209	172
104	123	166	161	155	160	205	229	218	181
125	131	172	179	180	208	238	237	228	200
131	148	172	175	188	228	239	238	228	206
161	169	162	163	193	228	230	237	220	199

Corresponding array

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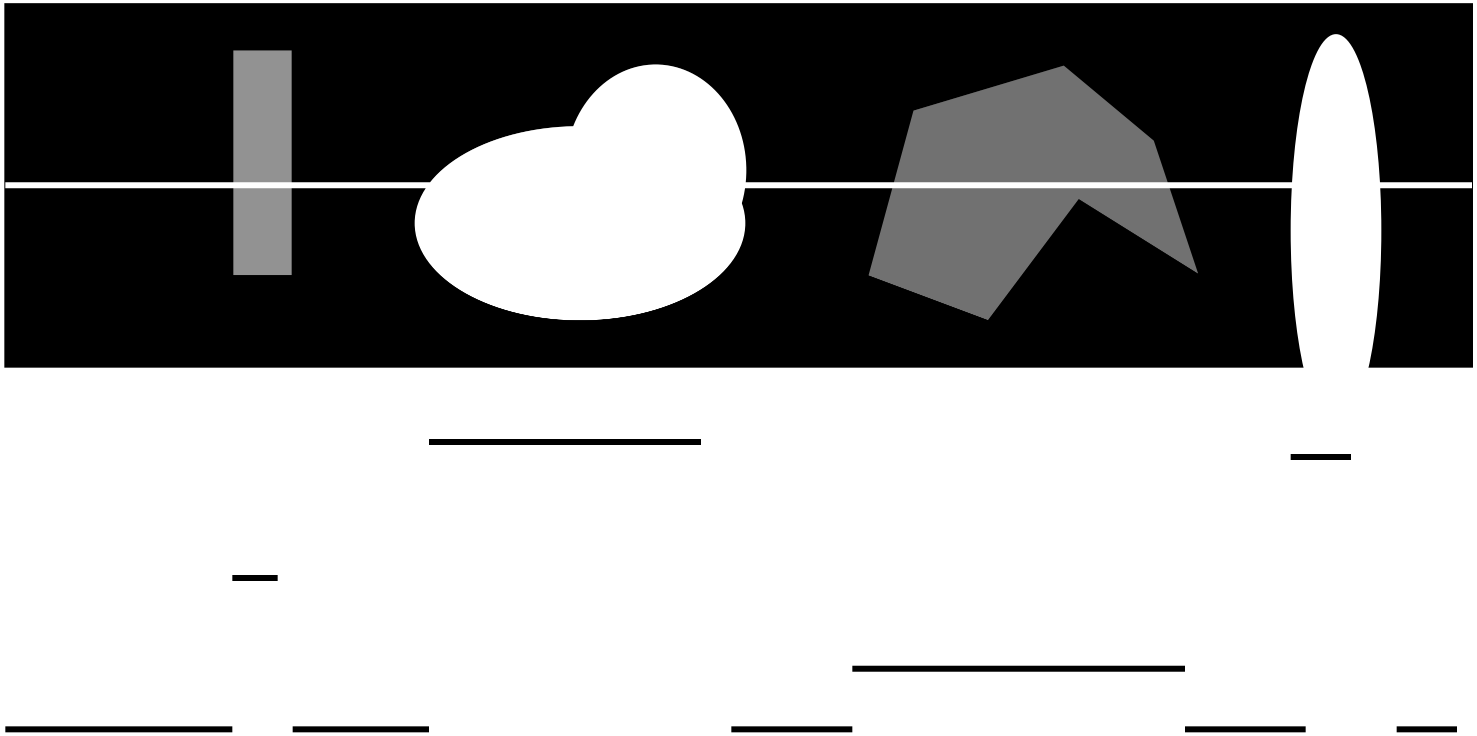
Muestreo y la Cuantización

- Una imagen digital es el resultado de efectuar, sobre una señal continua, un proceso de muestreo en dos direcciones perpendiculares

- Causas que producen perdida de información en la captura de imágenes son: la naturaleza discreta de los pixeles y el rango limitado de valores para representar la intensidad luminosa

Definition

- Converting the continuous 2D signal in a digital image by sampling per scanlines



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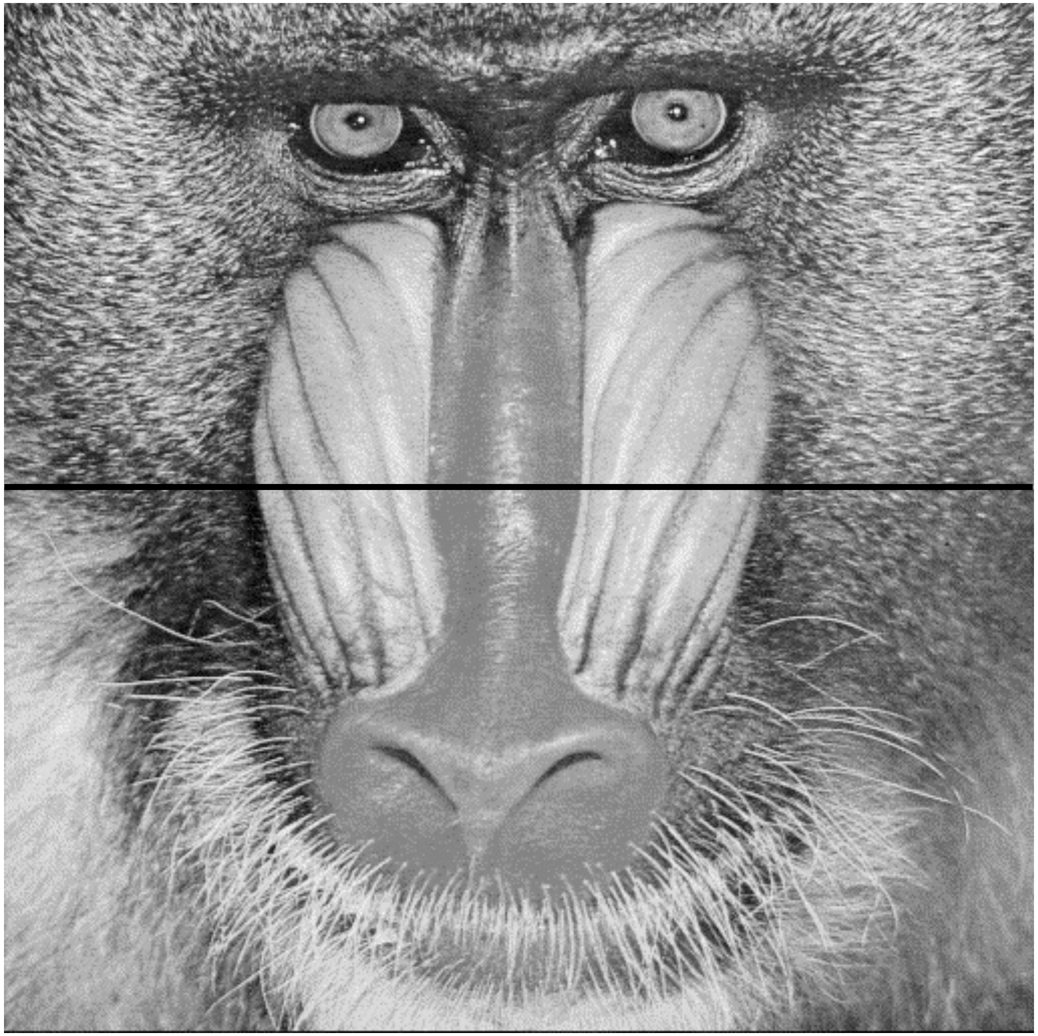
Example



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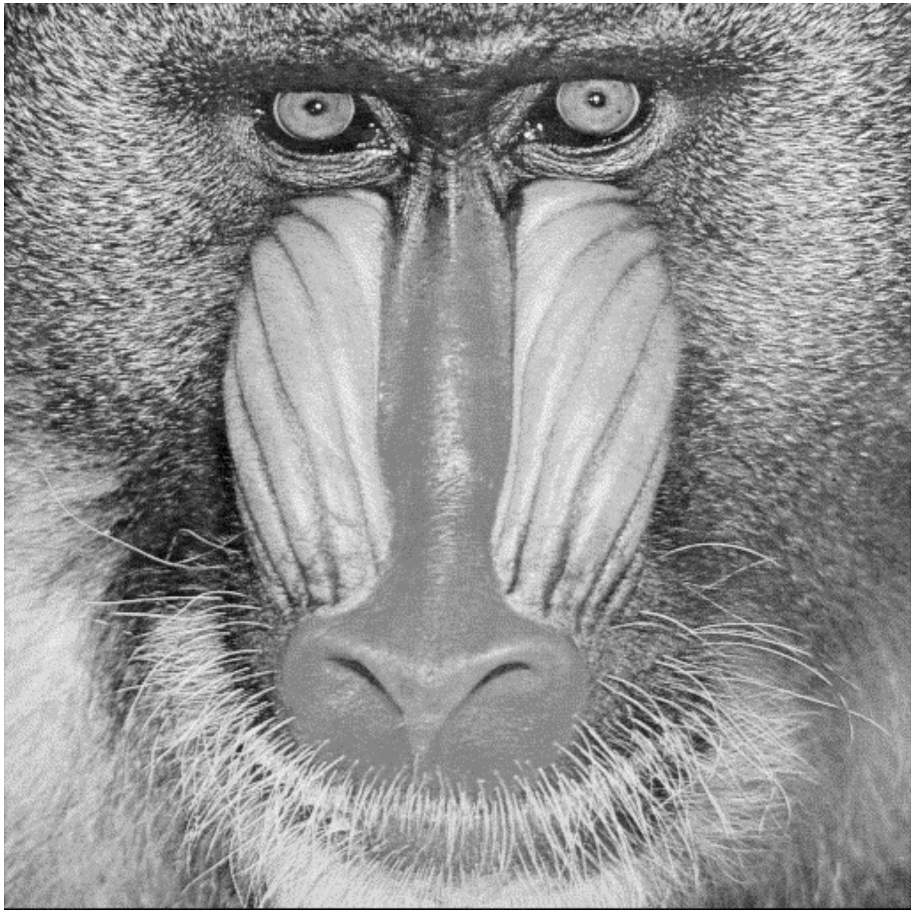
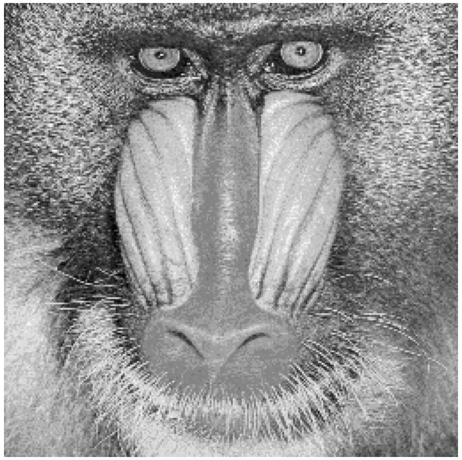
Muestreo

- El muestreo de una imagen tiene el efecto de reducir la resolución espacial
- Hay una perdida de información debido al aumento del paso de muestreo, así como el ruido que se va introduciendo en forma de patrones rectangulares sobre la imagen

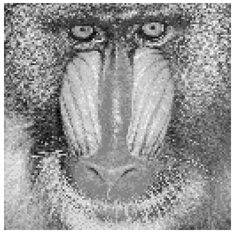
Muestreo



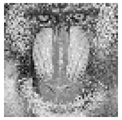
512



256

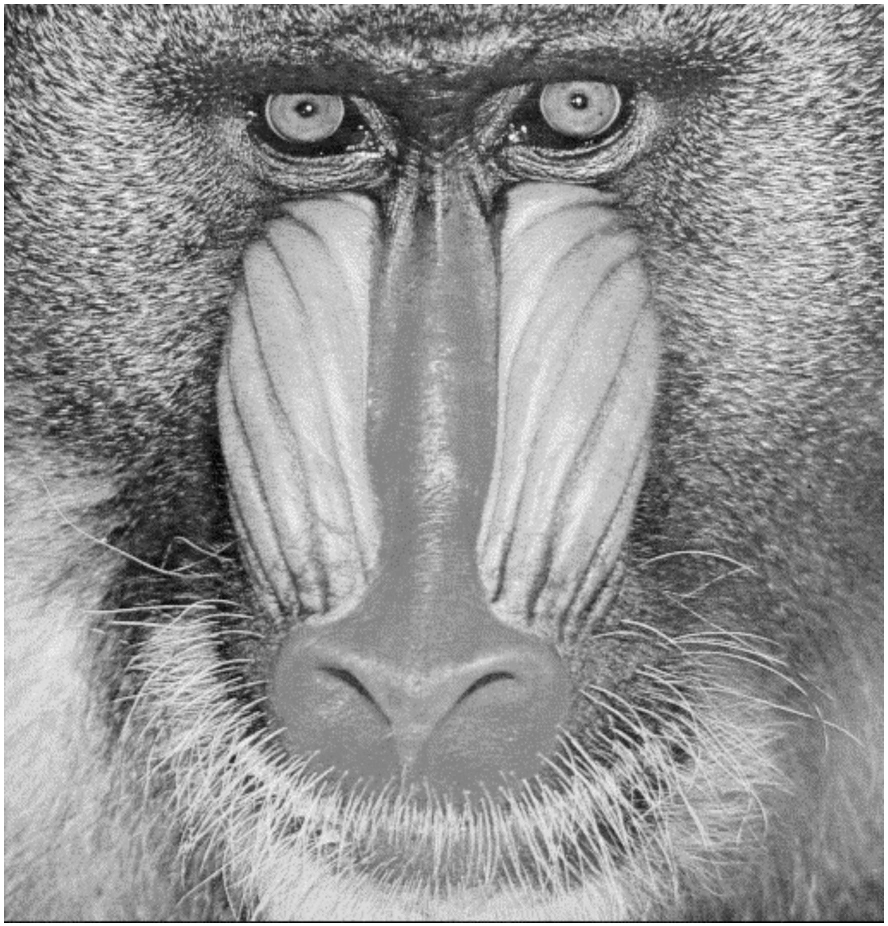


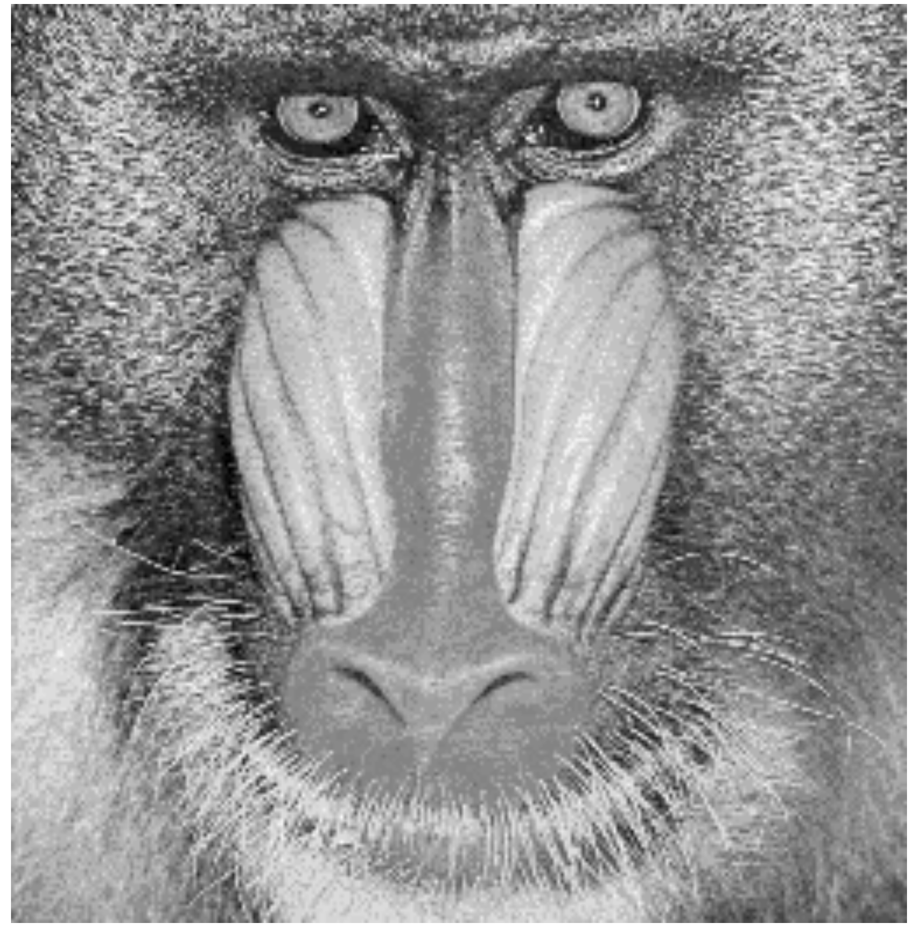
128



64

32





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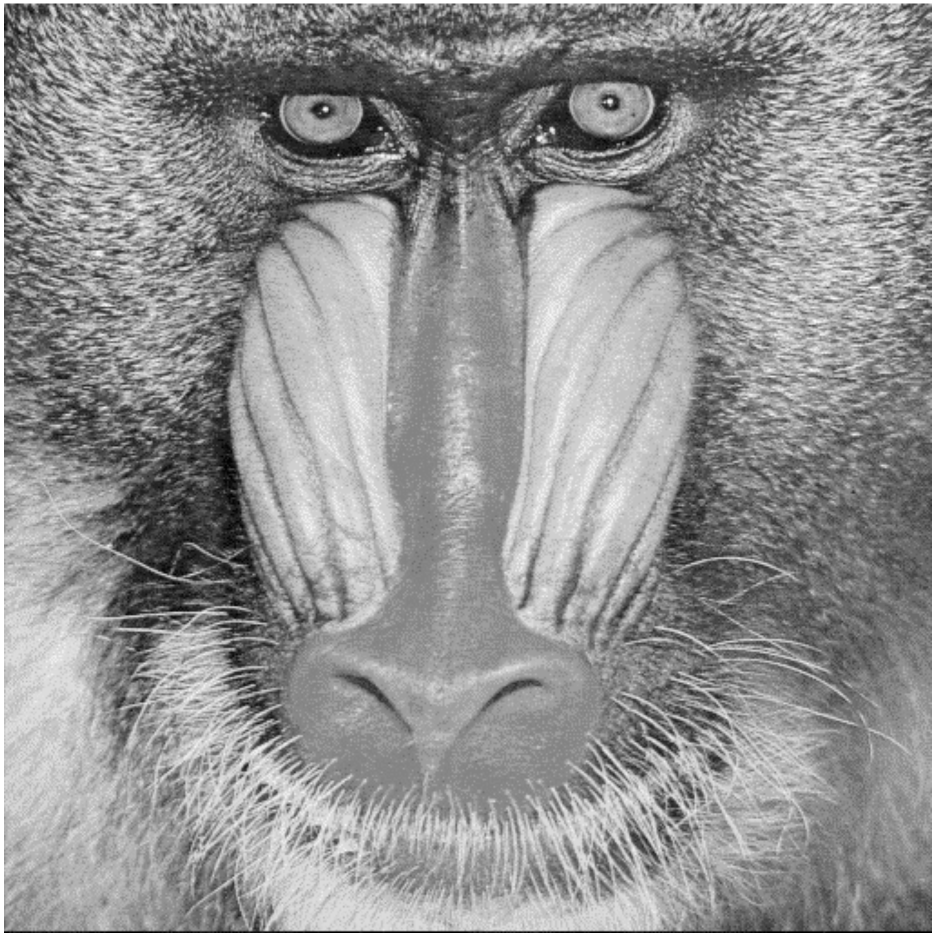

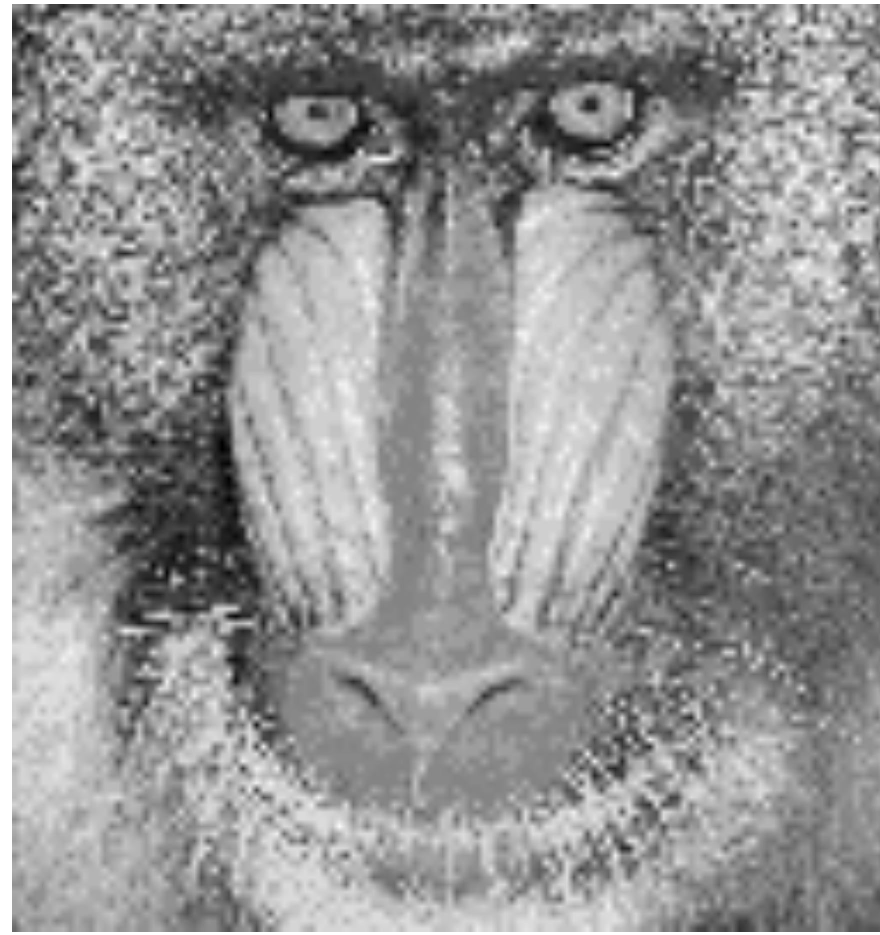

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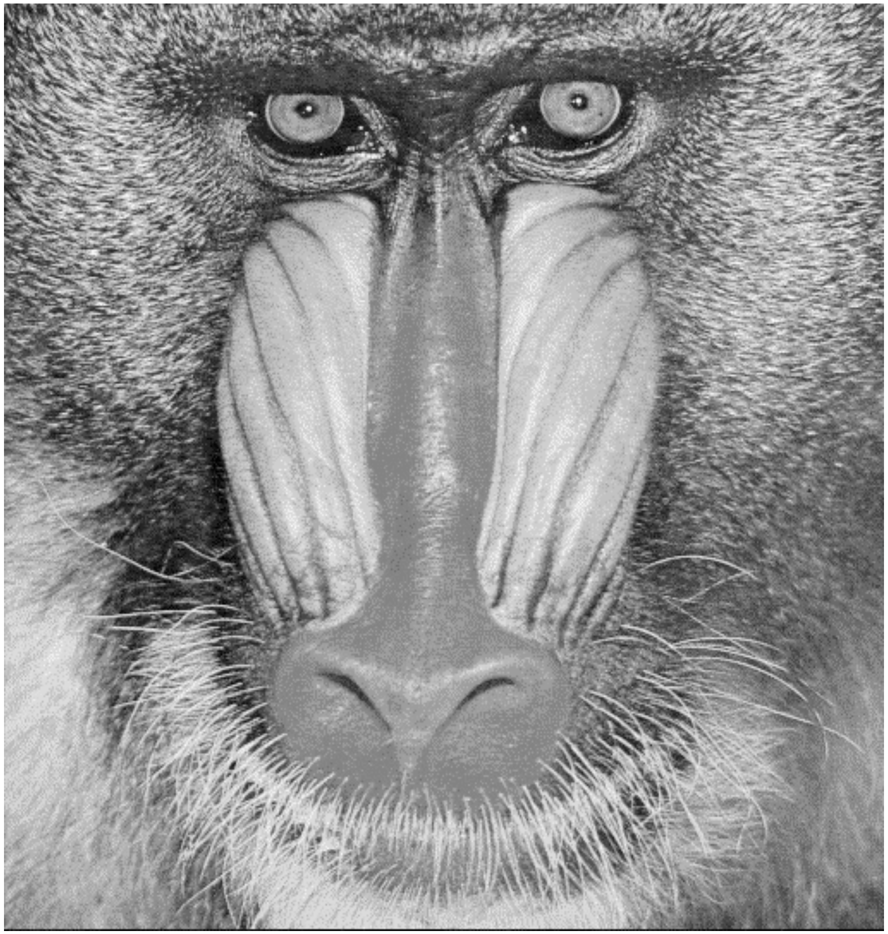

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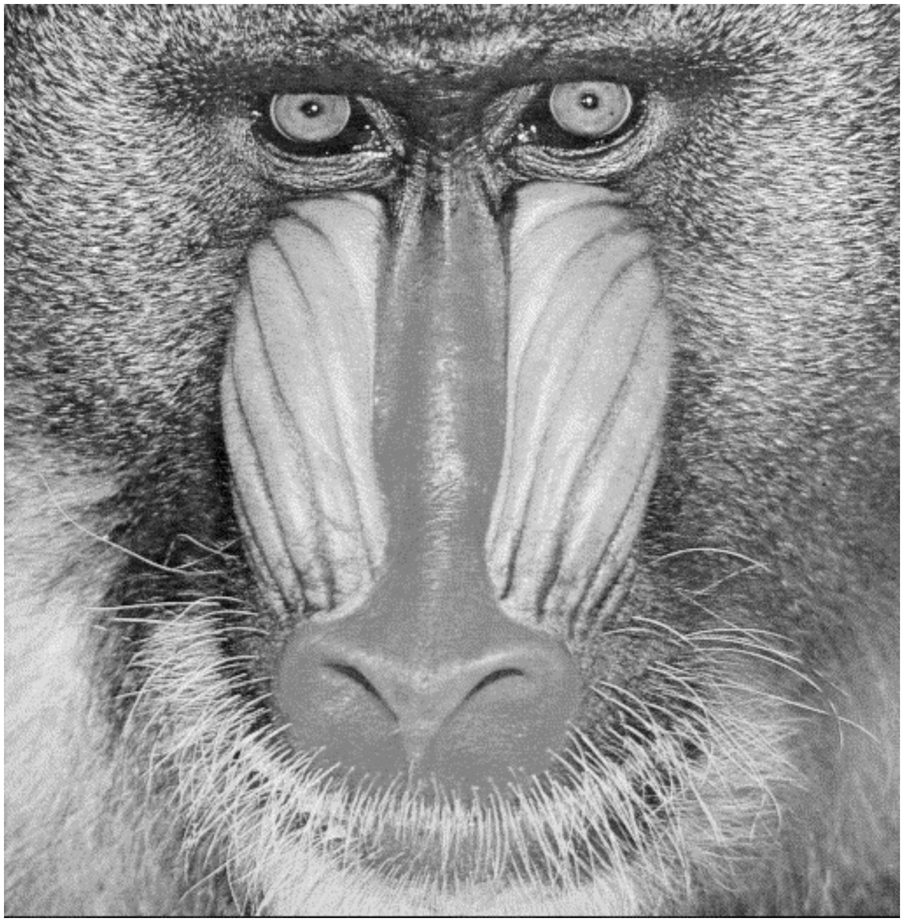

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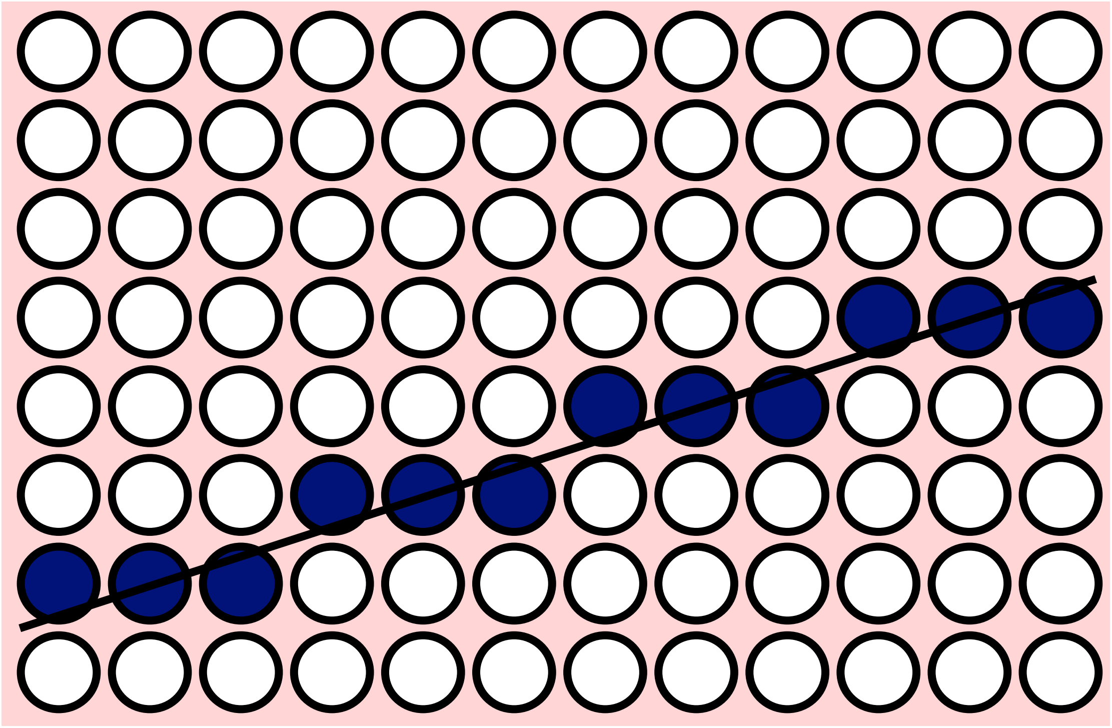
Sampling Effects

Representing a line with discrete pixel values leads to sampling error and loss of information



Standard midpoint line on a binary representation

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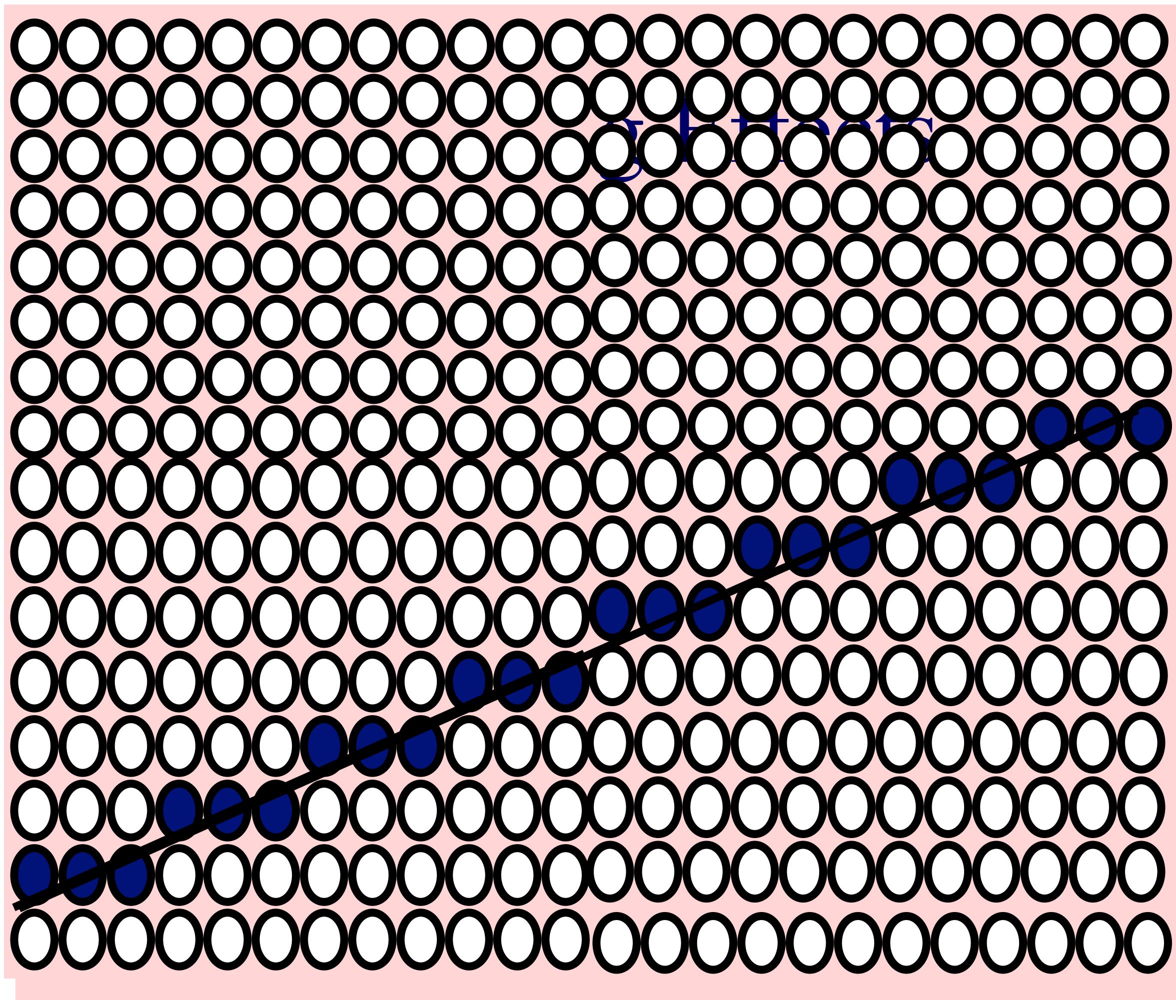
Same line with twice the linear resolution



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Sampling Effects

- Doubling resolution does not solve the problem
- It costs 4 times memory, bandwidth and scan conversion time!

- The problem can be alleviated using more grey-levels

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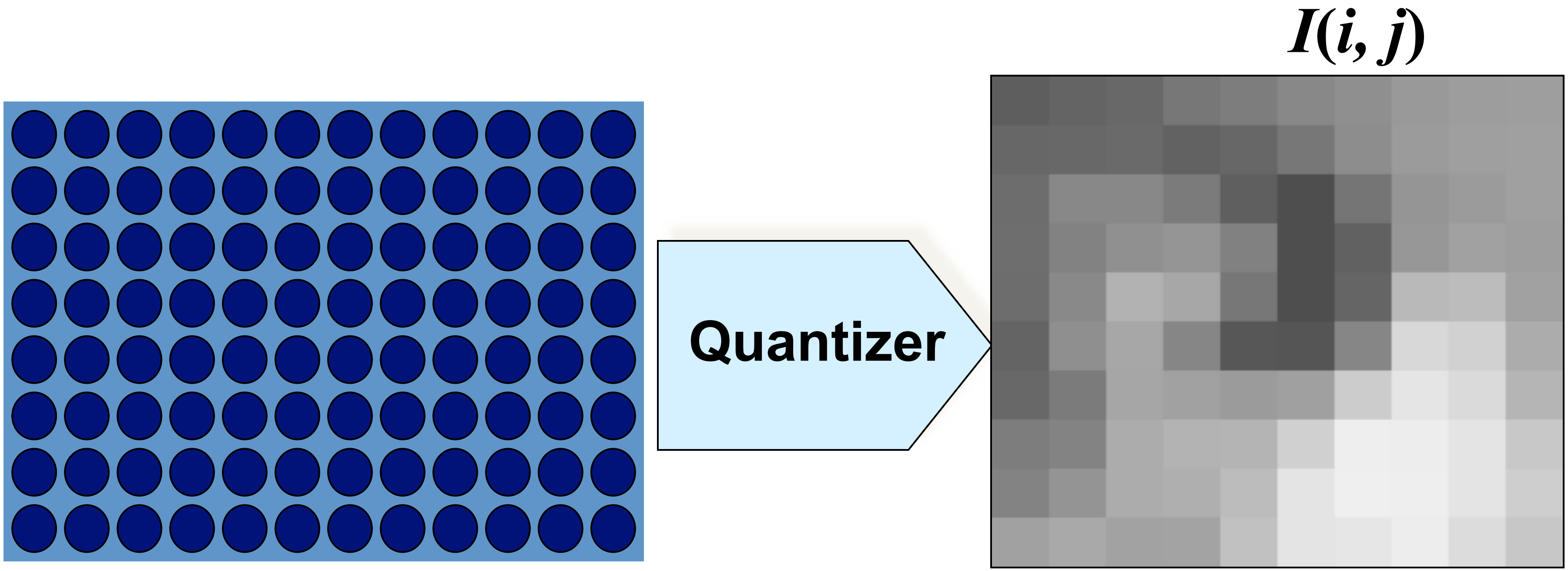
Cuantización

- El efecto de cuantización viene dado por la imposibilidad de tener un rango infinito de valores de medida para la intensidad de brillo de los pixeles
- Se usa un valor para codificar este valor lumínico, el rango de posibles valores esta dado por la cantidad de bits
- 0-255 (8bit), 0-1023 (10bit), 0-4095 (12-bit)

Quantization



Each element in the matrix is quantized, i.e, replaced by an integer

Quantized values are called gray levels

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Digital Image Visualization

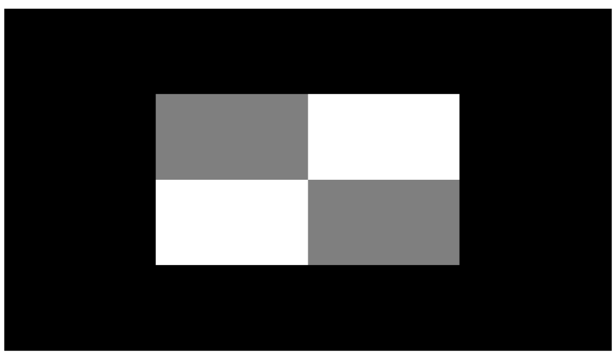
Usually each pixel in the image is shown by a single pixel on the screen

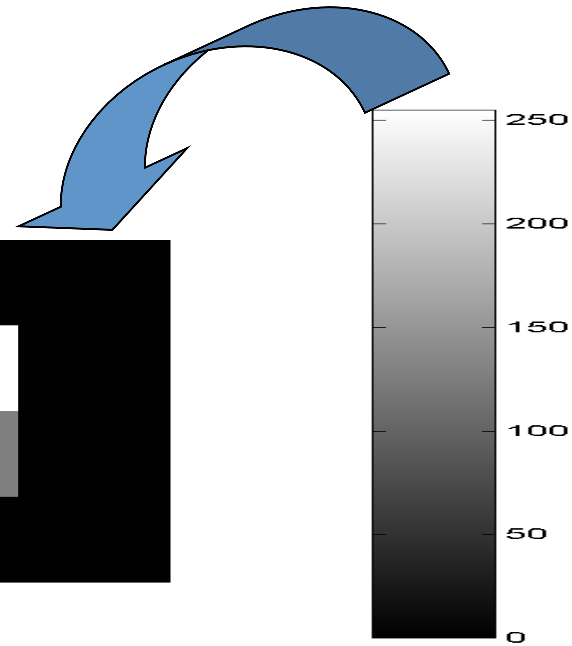
E.g., for L = 256 gray levels, 0 maps into black, 255 into white and values in between map linearly into various levels of gray

Image

0	0	0	0
0	127	255	0
0	255	127	0
0	0	0	0

Screen





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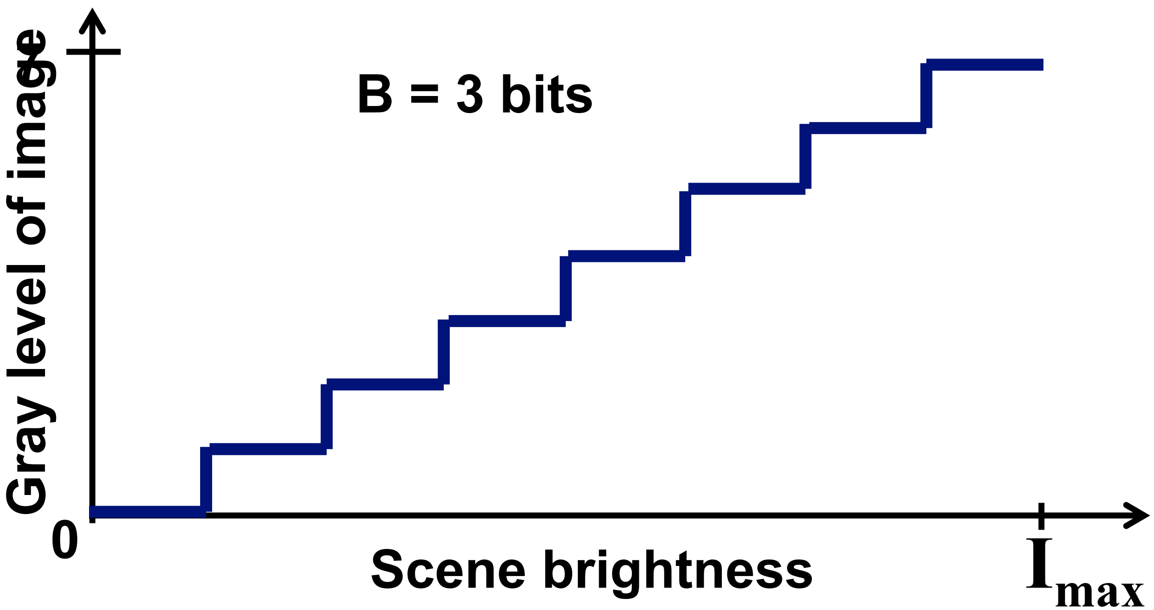
Intensity Resolution

- **Refers to how accurately a pixel's gray level represents the brightness of the corresponding point in the original scene**
- **During quantization, the brightness sampled at each point in the continuous-tone image is replaced by an integer value**



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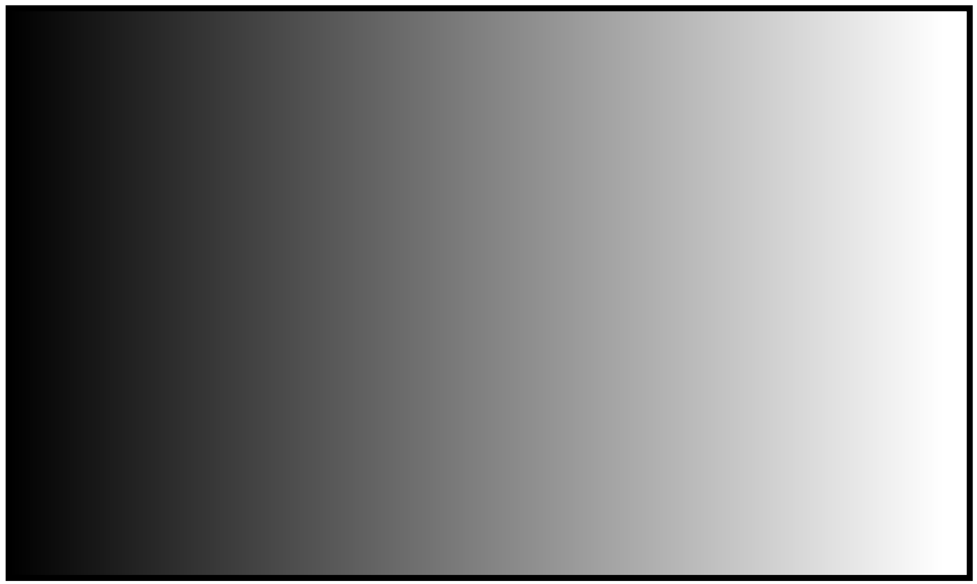
Intensity Resolution

Intensity resolution depends on the number of bits available



This figure shows a digital image quantized with 8 bits (256 gray levels)
The image appears continuous

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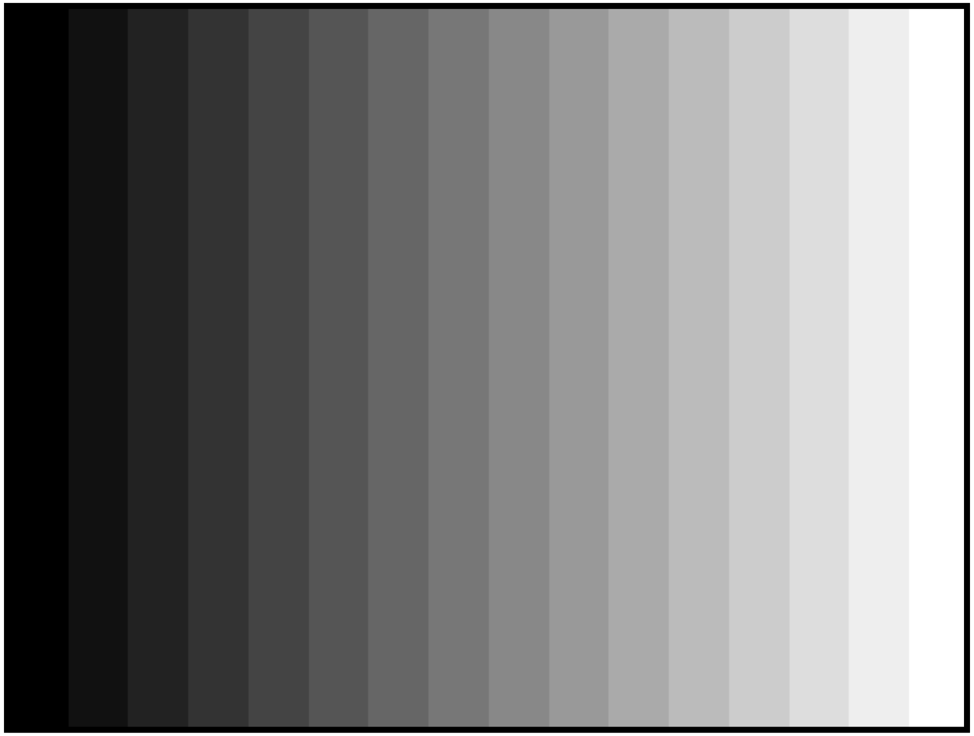
Intensity Resolution

The same image quantized with only 4 bits (16 gray levels)

Now the image brightness appears discontinuous



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Intensity Resolution

- **With fewer bits, we cannot accurately represent the gradual intensity variations in the original scene because a wider range of intensities in the original scene is mapped into a single gray level**

- **Generally, the more bits we have, the better the brightness resolution**

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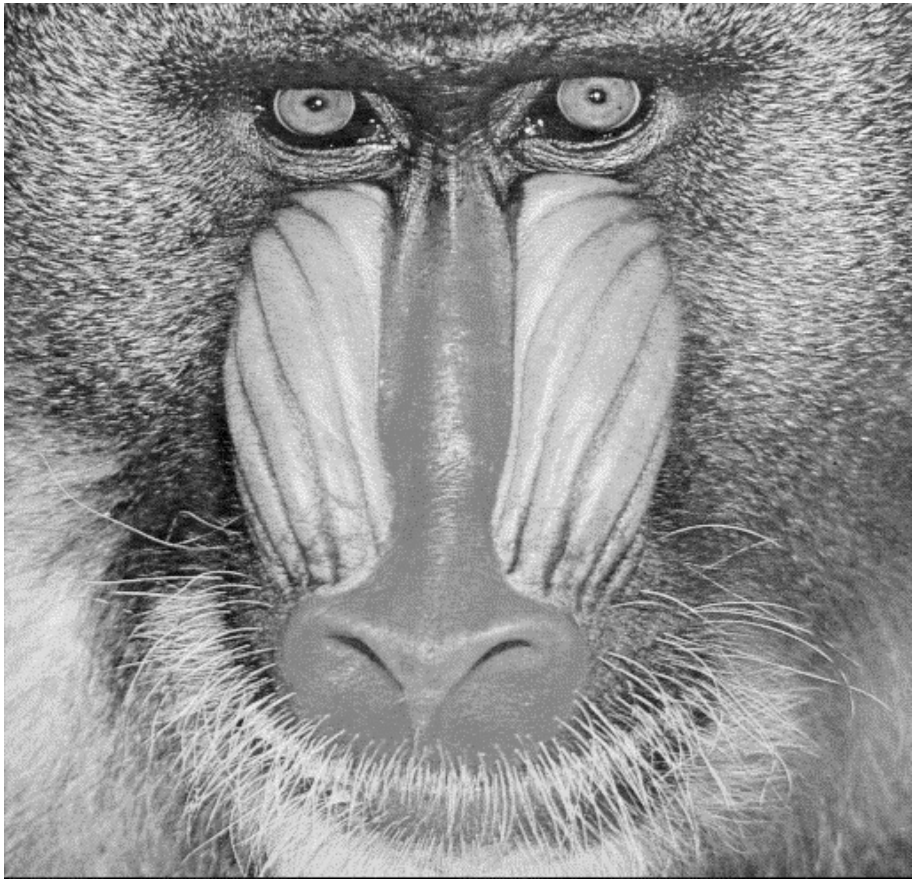
Grey-level quantization

256



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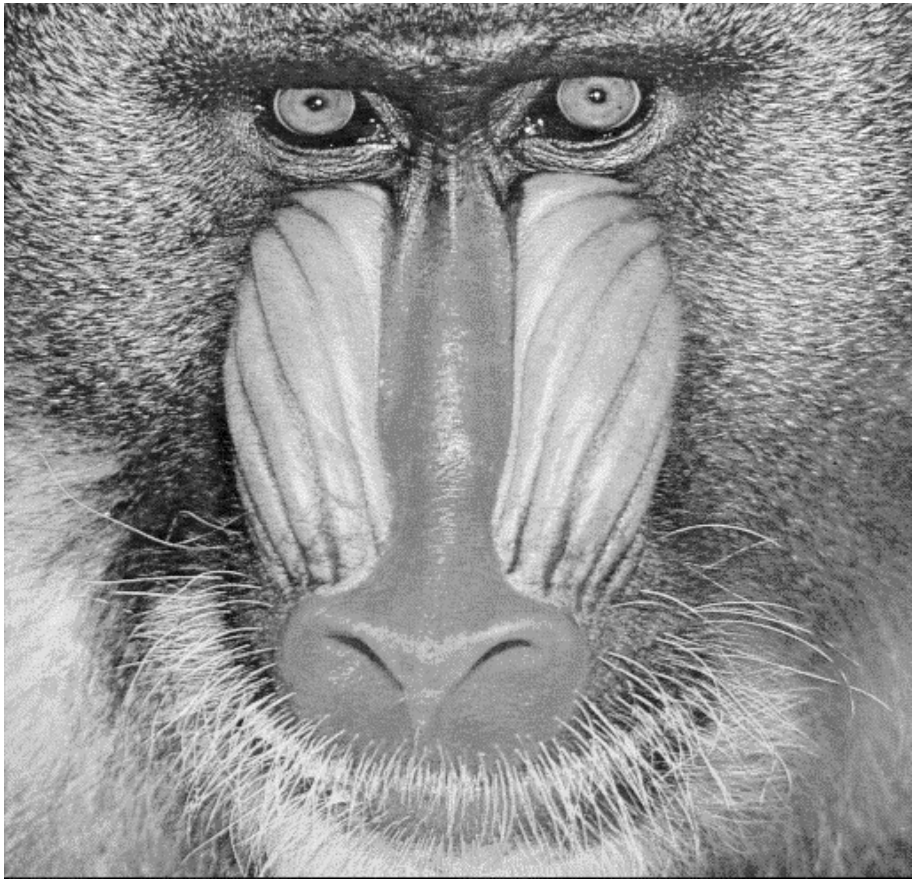
Grey-level quantization

32



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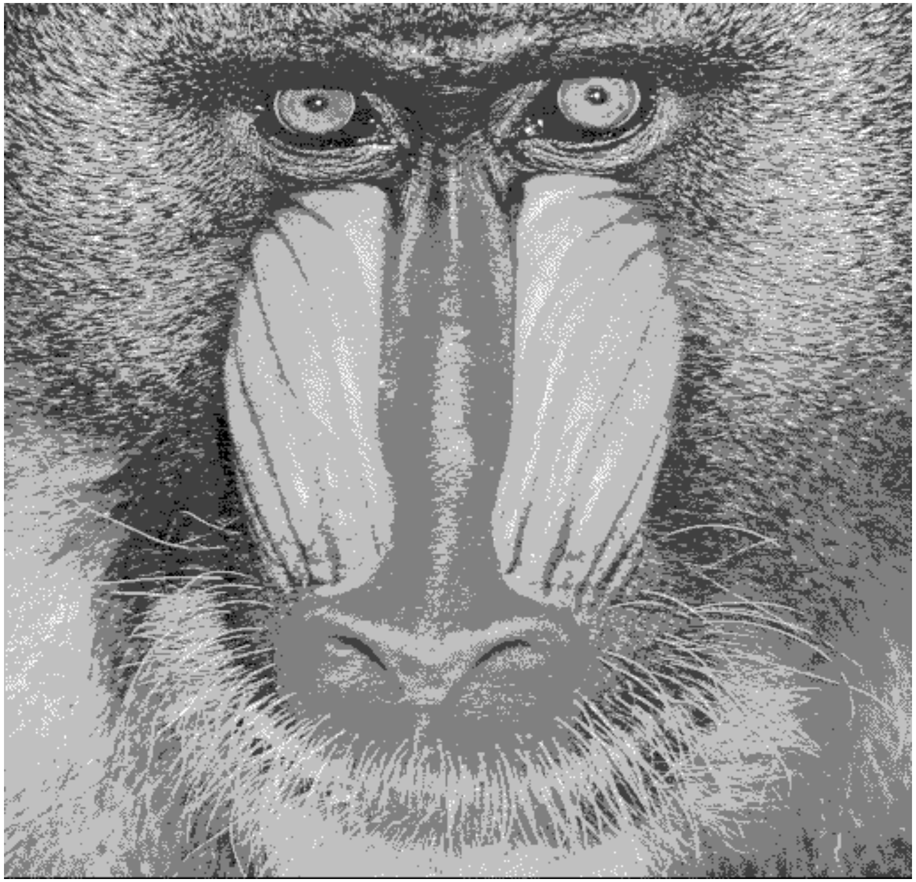
Grey-level quantization

8



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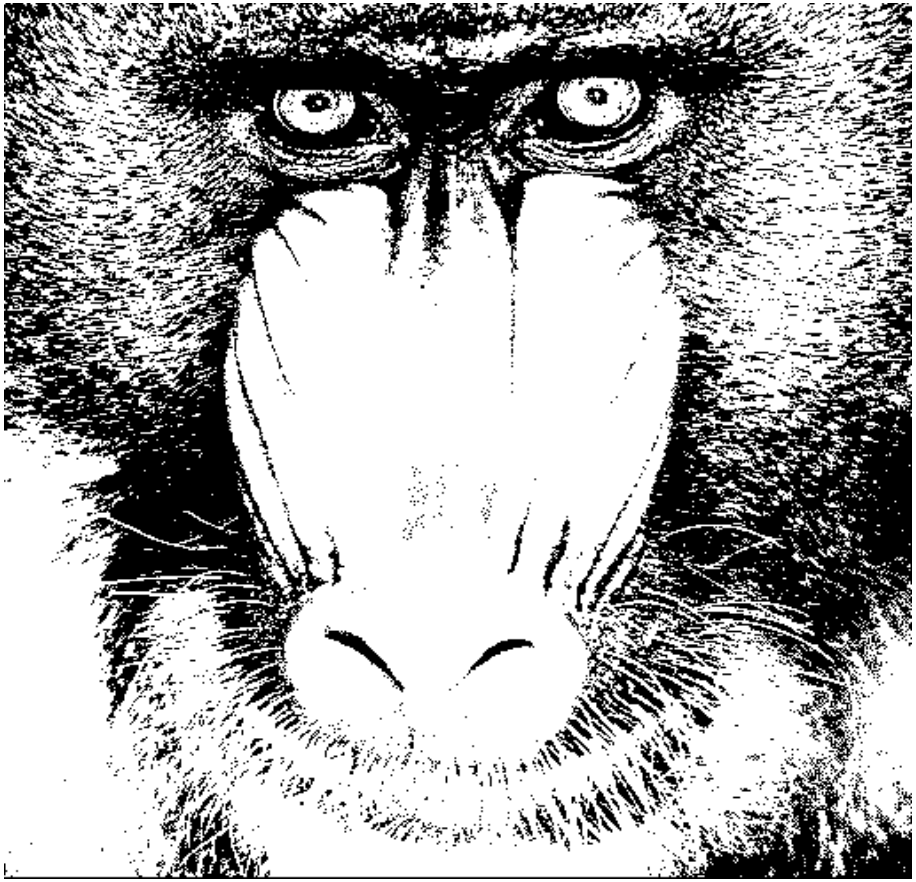
Grey-level quantization

2



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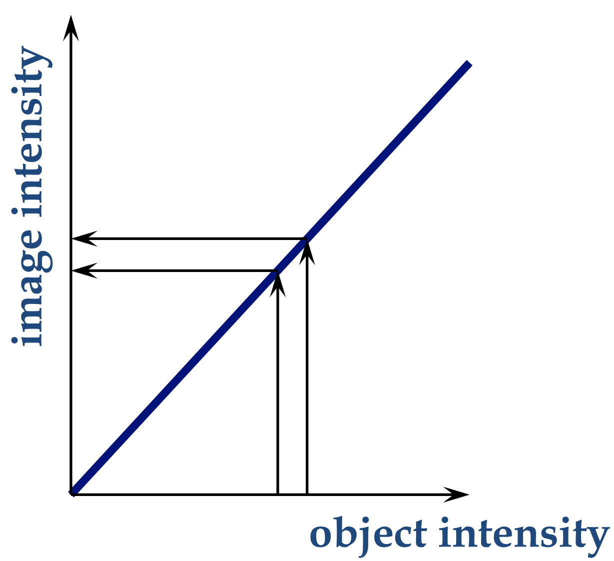
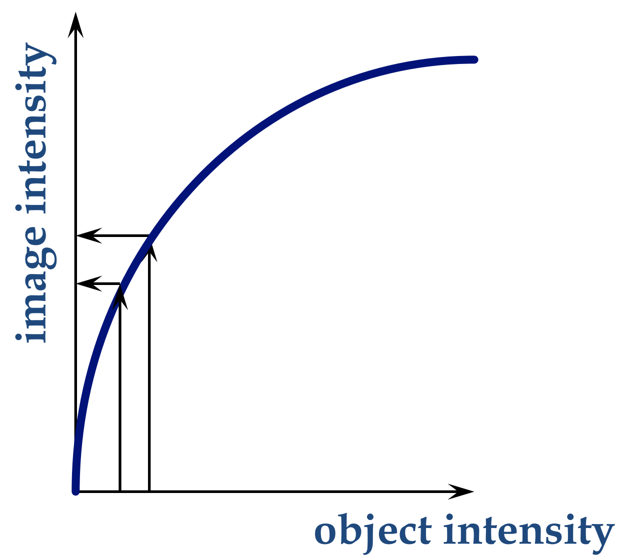
Quantization Methods

•Uniform or linear - intensity of object is lineary mapped to gray-levels of image
•Logarithmic - higher intensity resolution in darker areas (the human eye is logarithmic)





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Common Quantization Levels

I(i,j) is given by integer values [0-max], $max=2^n-1$

n=1	[0 - 1]	”binary image”
n=5	[0 - 31]	maximum the human eye can resolve (locally)
n=8	[0 - 255]	1 byte, very common
n=16	[0 - 65535]	common in research
n=24	[0 - $16.2*10^6$]	common in color images (i.e. 3*8 for RGB)

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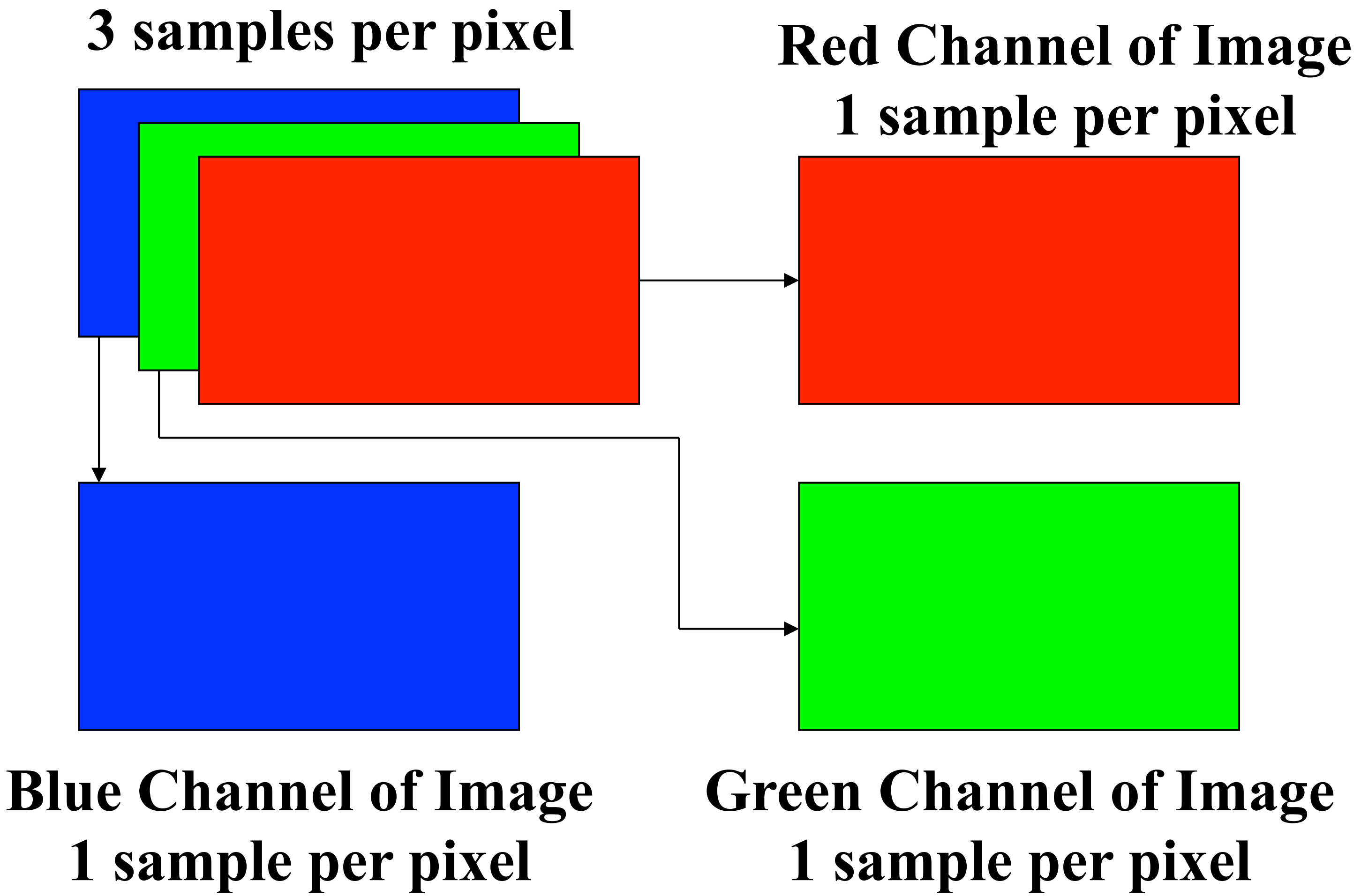
Channels in Colour Images



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Color Models

- **Hardware-oriented models**: not intuitive
  - RGB, used with color CRT monitors
  - YIQ, the broadcast TV color system
  - CMY (cyan, magenta, yellow) for color printing
  - CMYK (cyan, magenta, yellow, black) for color printing
- **User-oriented models**
  - HSV (hue, saturation, value) – also called HSB (hue, saturation, brightness)
  - HLS (hue, lightness, saturation)
  - The Munsell system
  - CIE Lab

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Transformaciones de Color

Conversión de espacios de color

- Los espacios de color más usados y fiables son aquellos establecidos por CIE, que se consideran estándares internacionales
- CIE relaciona entre si todos los colores perceptibles por el ojo humano y permite establecer los colores absolutos
- La conversión de espacio de color se realiza con una matriz de transformación

RGB a CIE XYZ

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} * \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

CIE XYZ a CIE L*a*b*

$L^* = 116 * (Y/Y_n)^{1/3} - 16$ para $Y/Y_n > 0.008856$

$L^* = 903.3 * Y/Y_n$ en otro caso

$a^* = 500 * (f(X/X_n) - f(Y/Y_n))$

$b^* = 200 * (f(Y/Y_n) - f(Z/Z_n))$

donde $f(t) = t^{1/3}$ para $t > 0.008856$

$f(t) = 7.787 * t + 16/116$ en otro caso

X_n, Y_n y Z_n son valores triestímulo del referentes al blanco

CIE XYZ   a CIE LUV

$L^* = 116 * (Y/Yn)^{1/3} - 16$
$u^* = 13L^* * (u' - u_n')$
$v^* = 13L^* * (v' - v_n')$

donde

$u' = 4X / (X + 15Y + 3Z) = 4x / (-2x + 12y + 3)$
$v' = 9Y / (X + 15Y + 3Z) = 9y / (-2x + 12y + 3)$
$u_n' = 0.2009$, $v_n' = 0.4610$

RGB a YCrCb

$$Y = 0.299R + 0.587G + 0.114B$$
$$Cb = (B - Y) / 1.772 + 0.5$$
$$Cr = (R - Y) / 1.402 + 0.5$$

RGB a HSV y HSL

M=max(R,G,B)
m=min(R,G,B)
C=M-m

$$H' = \begin{cases} indefinido, & si\ C = 0 \\ \dfrac{G-B}{C} mod 6, & si\ M = R \\ \dfrac{B-R}{C} + 2, & si\ M = G \\ \dfrac{R-G}{C} + 4, & si\ M = B \end{cases}$$

HSV

$$H = 60° \times H'$$

$$\mathrm{V} = \mathrm{M}$$

$$S = \begin{cases} 0 & si\ V = 0 \\ \dfrac{C}{V} & en\ otro\ caso. \end{cases}$$

HSL

$H = 60° \times H'$

$\mathrm{L} = 0.5\mathrm{xM} + 0.5\mathrm{xm}$

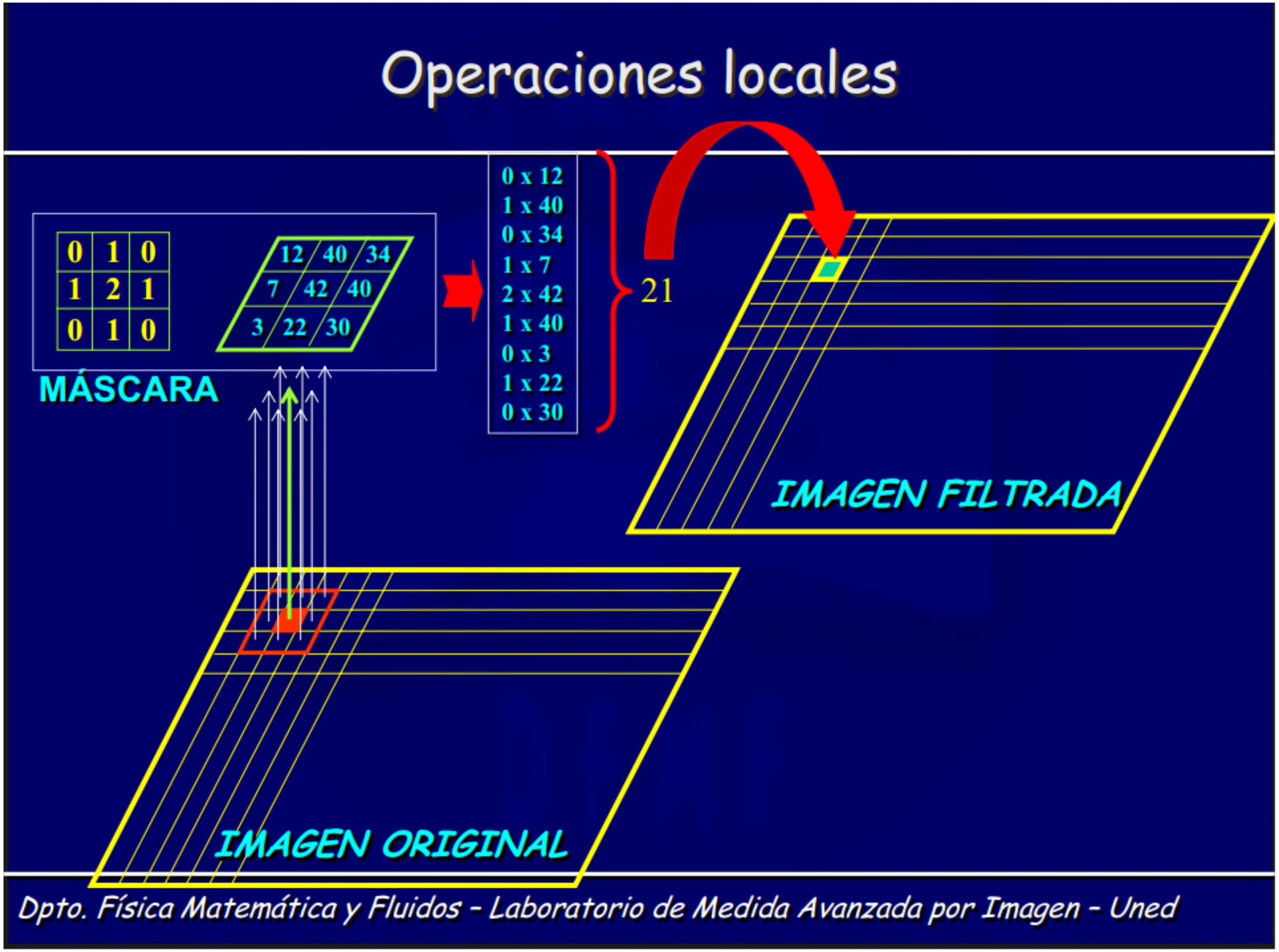
$$S = \begin{cases} 0 & si\ L \in \{0,1\} \\ \frac{C}{1-|2L-1|} & en\ otro\ caso \end{cases}.$$

Noise

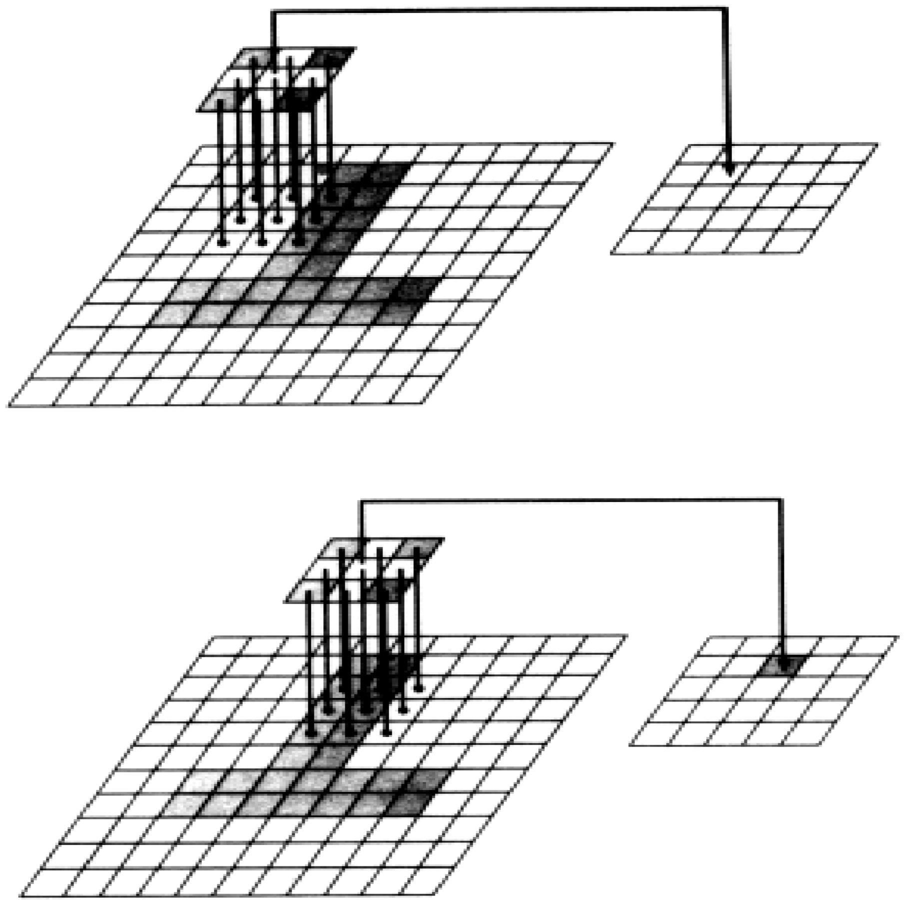
Low-pass Smoothing

- Reduces high-frequency noise
- Smooth image
  - Sigma Filter
  - Nagao-Matsuyama filter
  - Median Filter
  - Mean Filter
  - Gaussian Filter (after convolution)

Convolution



Noise removal



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131	148	172	175	188	228	239	238	228	206
161	169	162	163	193	228	230	237	220	199

Mean filter

- Need for normalization

  To conserve the total “energy” of the Image (sum of all greylevels)

  – Quick

  – Severe edge blurring

$$1/9 \text{ x } \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

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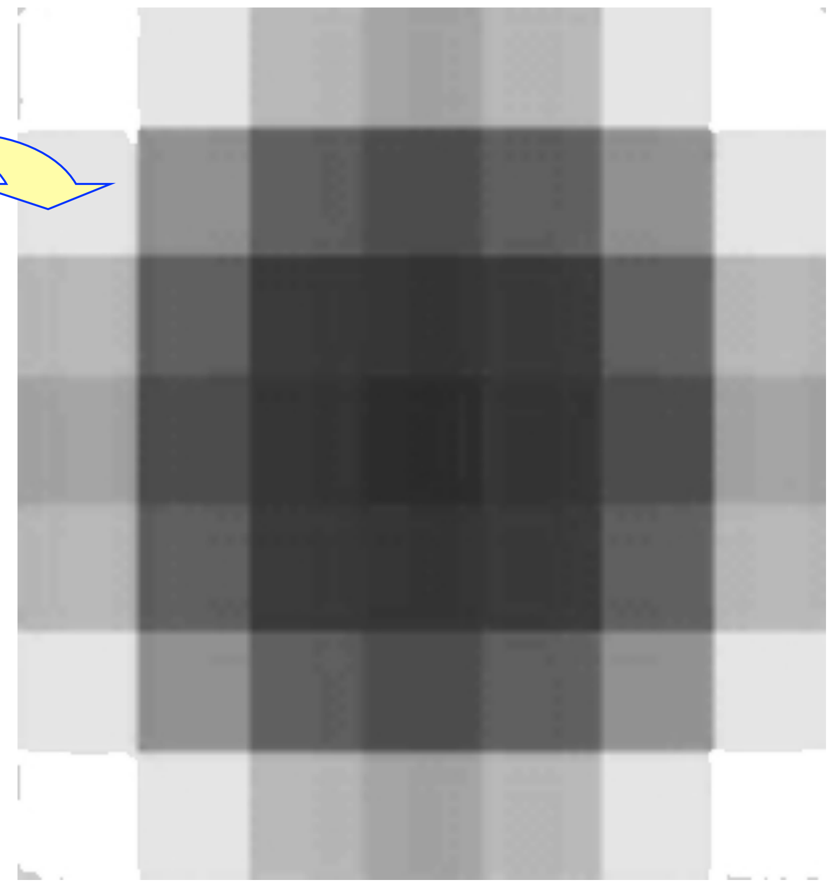
Gaussian Filters

- 2D Gaussian kernel

  (the darker, the higher the filter value, σ^2)



- The weighting values decrease proportional to the distance to the center
- The “decrease” exponential

2D Gaussian operator (filter)

$$G(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \times \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

$$= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

- The standard deviation is the only parameter of the Gaussian filter, it is proportional to the size of the neighborhood on which the filter operates

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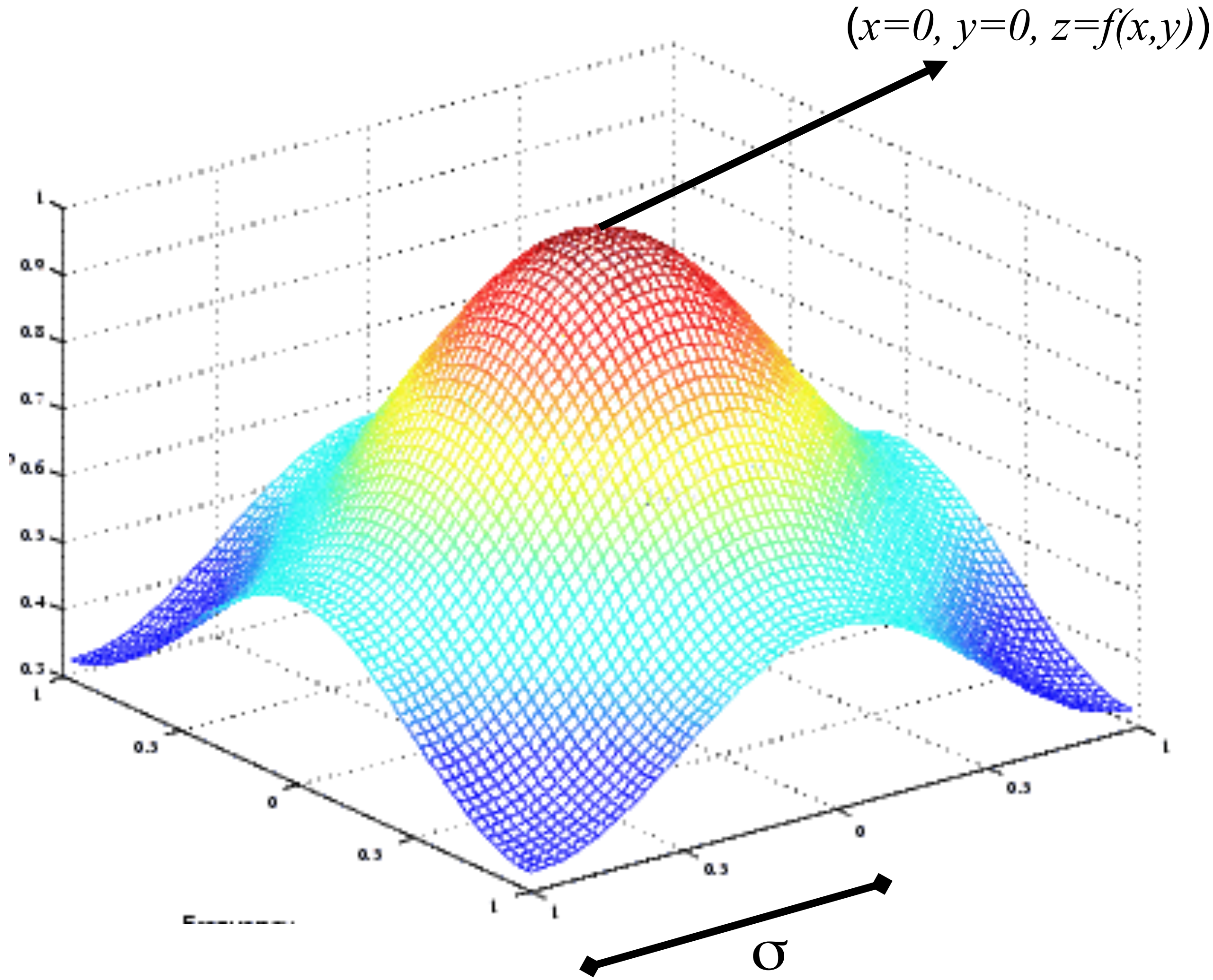

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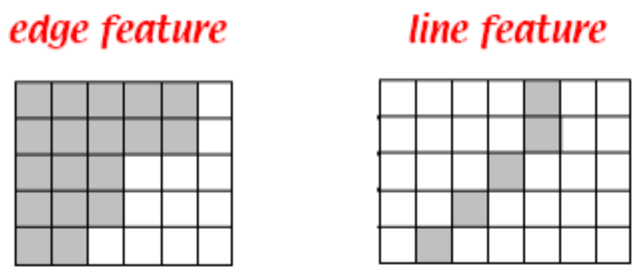
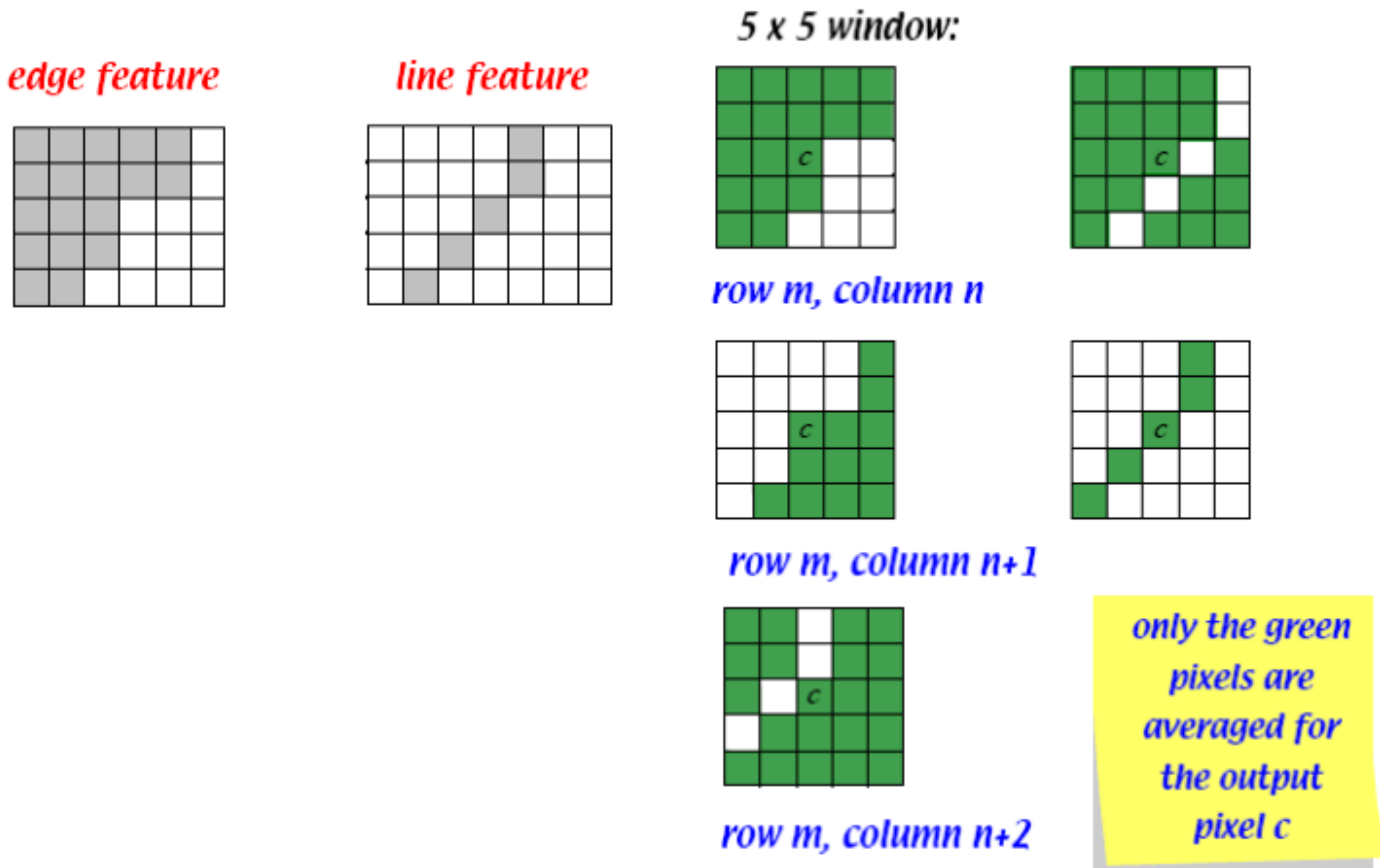
Sigma Filter

- Average selected pixels within moving window
- Average only those pixels that are within a threshold difference Δ from the DN of the centre pixel, DNc



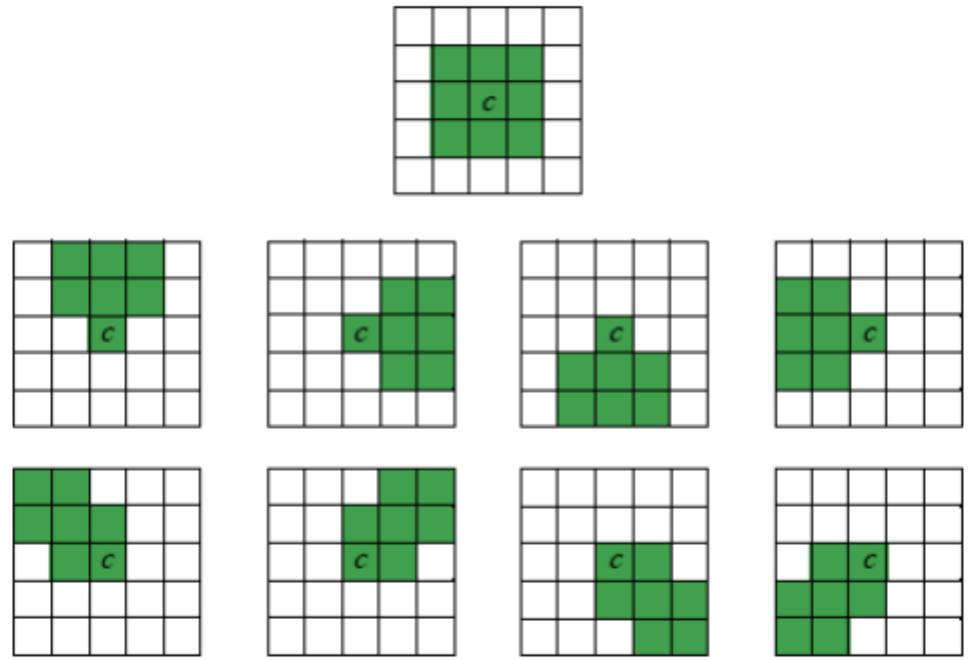
Input: the value at the centre pixel
Output: a new value at the centre pixel

Sigma Filter



Nagao-Matsuyama filter

- Calculate the variance of 9 subwindows within a 5 x 5 moving window
- Output pixel is the mean of the subwindow with the lowest variance



Impulse Noise

- Salt and pepper noise DN is "outlier" relative

Noise Cleaning (pixels)

$DN_{neighbors}$ = average DN (8-neighbors)

If $|DN_{test} - DN_{neighbors}| > \Delta$, $DN_{test} = DN_{neighbors}$

If $|DN_{test} - DN_{neighbors}| \leq \Delta$, $DN_{test} = DN_{test}$

Noise Cleaning (lines)

$DN_{neighbors}$ = average DN (2-neighbors above and below)

If $|DN_{test} - DN_{neighbors}| > \Delta$, $DN_{test} = DN_{neighbors}$

If $|DN_{test} - DN_{neighbors}| \leq \Delta$, $DN_{test} = DN_{test}$

Median Filter

- Separable 2D median filter
- 2D edges are preserved