Part a)

k	n
0	0
1	2
2	4
3	8
4	16
...	
2^{2k}	

$i = 2 \times 2,\ 2 \times 2 \times 2,\ 2 \times 2 \times 2 \times 2$

The pattern is that

$i = 2^{2k}$

$2^{2k} \geq n$

$2^k = \log_2 n$

Runtime $= \Theta = \log_2(\log_2(n))$

Part b) $k < i^3$

i	k less than
1	1
2	8
3	27

i	n=1	n=9
1	✓	X
2	✓	X
3	✓	✓
4	✓	X
5	✓	X

3, 6, 9

$27\ 6^3\ 9^3$

$$\sum_{i=1}^{n} \left(\Theta(1) + \sum_{k=1}^{\sqrt{n}} i^3 \right)$$

$$\sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{n} \sum_{k=1}^{\sqrt{n}} (k\sqrt{n})^3$$

$\sqrt{n} \rightarrow 1$

$2\sqrt{n} \rightarrow 2$

$3\sqrt{n} \rightarrow 3$

$i = k\sqrt{n}$

n $\sqrt{n}$

$$(\sqrt{n})^3 \sum_{k=1}^{\sqrt{n}} k^3 = \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2} \right)^2 \cdot \sqrt{n}^3 + \Theta(n)$$

$$= \frac{n^2 + 2\sqrt{n} + n}{4} \cdot n\sqrt{n} + n$$

Most significant

$$= \frac{n^3\sqrt{n} + 2n^2 + n^2\sqrt{n}}{4} + n$$

$\Theta(n^{3/2})$

Part c)

i	k	A(k)
1	1	

A(k) == 1

m = 1 m++
m = 2 m = 4

for // O(n)
  for // O(n)
    if // O(1)
      for // O(n)

$$T(n) = \sum_{i=1}^{n} \sum_{K=1}^{n} (\Theta(1) + \Theta(\sum_{m=1}^{n} \Theta(1)))$$

Worst case is when A[K] = i for all elements of k, so needs to occur n times

$$T(n) = \sum_{i=1}^{n} \sum_{K=1}^{n} \Theta(1) + n \sum_{m=1}^{n} \Theta(1)$$

$m = 2^x$

$$T(n) = \Theta(n^2) + \Theta(n \cdot \log(n))$$

Runtime: $\Theta(n^2)$

3. d) for (int i = 0, i < n, i++) runs n times

$$\sum_{i=1}^{n} \left(\Theta(1) + O\left(\sum_{j=1}^{10 \times \frac{3}{2}s} \Theta(1)\right)\right)$$

$i = 10 \times \frac{3}{2}s$

$n = 10 \times \frac{3}{2}s$

$s = \log_{\frac{3}{2}} \frac{n}{10}$

$$= \sum_{i=0}^{n} \Theta(1) + \sum i \sum_{j=1}^{10 \times \frac{3}{2}k} \Theta(1)$$

$$= \Theta(n) + \sum_{k=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \Theta\left(10 \times \frac{3}{2}k\right)$$

$$= \Theta(n) + 10\left(\frac{3}{2}\right)^{\log_{\frac{3}{2}} \frac{n}{10}}$$

$$= \Theta(n) + \Theta\left(10^{(n/10)}\right)$$

$$= \Theta(2n)$$

Runtime: $\Theta(n)$