Counting Problems

1. unusual   5/7 letters

$\{n, s, a, 1\}$

a	s	1	u	n	1 "u"
s	a	1	u	n	$\rightarrow$ $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 5!$
u	n	s	a	1	$= 120$

2 "u"'s

$\underline{u_2} \; \underline{u_1} \; \underline{n} \; \underline{a} \; \underline{1}$ $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 2!$

$u_1 \quad u_2 \; \underline{n} \; \underline{a} \; \underline{1}$ $\dfrac{5!}{2!} \cdot 4 = 240$

$\{u_1, u_2, u_3, u_4, u_5\}$

$\binom{4}{2} = 6$

3 "u"'s $\underline{5} \; \underline{4} \; \underline{3} \; \underline{2} \; \underline{1} = 5!$

$\underline{u_1} \; \underline{u_2} \; \underline{u_3} \; \underline{n} \; \underline{a}$ $\dfrac{5!}{3!} \cdot 6$

$\underline{u_2} \; \underline{u_1} \; \underline{u_3} \; \underline{n} \; \underline{a} - /3!$

$\underline{u_3} \; \underline{u_2} \; \underline{u_1} \; \underline{n} \; \underline{a}$ $= 120$

$120 + 240 + 120 =$

$\boxed{480}$

2. 5 cards   2 pairs

$\underset{\smile}{\underline{x} \quad \underline{x}} \quad \underset{\smile}{\underline{Y} \quad \underline{Y}} \quad \underline{z}$

random

13 values,   4 suits   Out of 44
choosing 2   choosing 2   cards choose 1

$$\binom{13}{2} \quad \times \quad \binom{4}{2}^2 \quad \times \quad \binom{44}{1}$$
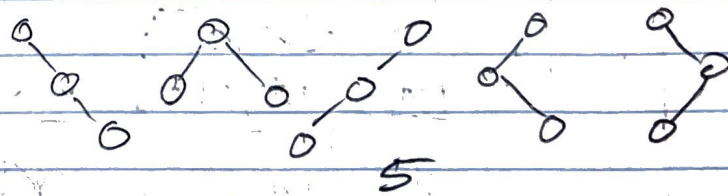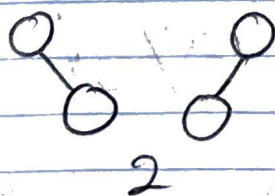
$$= \boxed{123,552}$$

couples/songs

3. $\binom{n+k-1}{k} = \binom{6+16-1}{16} = \binom{21}{16}$
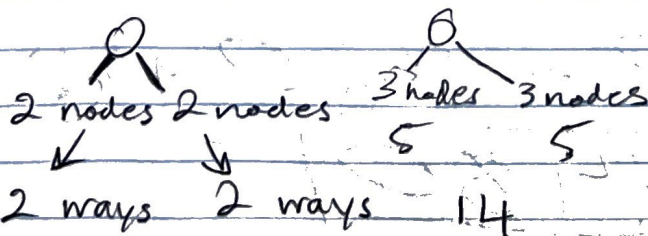
$\binom{6+15-1}{15} = \binom{20}{15}$

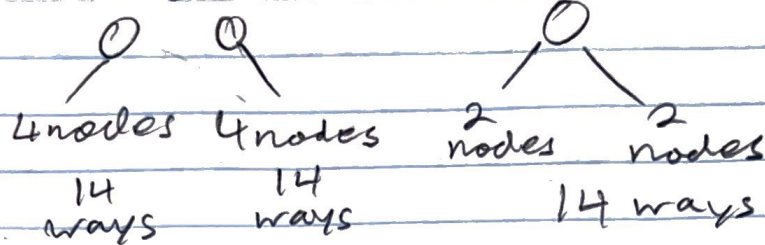$\binom{21}{16} + \binom{20}{15} = 20349 + 15504$

$$= \boxed{35853}$$

4. 2 nodes: 3 nodes:

2   5

4 nodes:

2 nodes 2 nodes   3 nodes 3 nodes
  ↙   ↘   5   5
2 ways   2 ways   14

5 nodes

O   Q   O

4 nodes 4 nodes   2 nodes   2 nodes

14 ways 14 ways   14 ways

$$2 \cdot 5 \cdot 14 = \boxed{420 \text{ ways}}$$

5.

Case 1	Case 2	10 friends
3 Nurses	4 nurses	1 nurse may or may not

Case 1 — 3 Nurses
1. 4 3 3
2. 4 4 2
3. 5 3 2
4. 5 4 1
5. 6 2 2
6. 6 3 1
7. 7 2 1
8. 8 1 1

Case 2 — 4 nurses
1. 2 2 2 4
2. 3 3 3 1
3. 3 3 2 2
4. 4 3 2 1
5. 4 4 1 1
6. 6 2 1 1
7. 5 3 1 1
8. 6 2 1 1
9. 7 1 2 1

$$\boxed{17 \text{ combinations}}$$