Task-Priority Control

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https://github.com/haakonbaa/task-priority-control-notes

Contents

1 1981 Hanafusa — Analysis and Control of Articulated Robot Arms with Redundancy

[1] H. Hanafusa *et al.*, "Analysis and Control of Articulated Robot Arms with Redundancy," *IFAC Proceedings Volumes*, vol. 14, no. 2, pp. 1927–1932, Aug. 1981, ISSN: 14746670. DOI: 10.1016/S1474-6670(17)63754-6. Accessed: Sep. 10, 2024. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S1474667017637546

2 1987 Nakamura — Task-Priority Based Redundancy Control of Robot Manipulators

[2] Y. Nakamura *et al.*, "Task-Priority Based Redundancy Control of Robot Manipulators," *The International Journal of Robotics Research*, vol. 6, no. 2, pp. 3–15, Jun. 1987, ISSN: 0278-3649, 1741-3176. DOI: 10.1177/027836498700600201. [Online]. Available: https://journals.sagepub.com/doi/10.1177/027836498700600201

2.1 abstract

Nakamura et al describes a scheme for control of reduntant manipulators. Tasks are defined with ordered priority and controll authority is given to tasks with the highest priority. Results are shown in simulation on a three-jointed robot mainupulator with two tasks, and on a physical 6-DOF manipulator.

2.2 summary

Let θ be a vector of joint coordinates. Define a task as

$$\boldsymbol{r}_i = \boldsymbol{f}_i(\boldsymbol{\theta}) \tag{1}$$

$$\implies \dot{\boldsymbol{r}}_{\boldsymbol{i}} = J_i(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{2}$$

$$\implies \ddot{\boldsymbol{r}}_{\boldsymbol{i}} = J_i(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \dot{J}_i(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{3}$$

Solutions $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$ to Equation 2 and Equation 3 satisfy

3 Khatib1987 — A unified approach for motion and force control of robot manipulators: The operational space formulation

[3] O. Khatib, "A unified approach for motion and force control of robot manipulators: The operational space formulation," *IEEE Journal on Robotics and Automation*, vol. 3, no. 1, pp. 43–53, Feb. 1987, ISSN: 0882-4967. DOI: `10.1109/jra.1987.1087068`. [Online]. Available: `http://ieeexplore.ieee.org/document/1087068/`

3.1 Summary

Consider the equations of motions for a manipulator in joint space.

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) = \Gamma \tag{4}$$

Let x denote the position of the end effector of a manipulator, then

$$\ddot{x} = J(q)\ddot{q} + h(q, \dot{q}) \tag{5}$$

Applying the joint space generalized forces $\Gamma = J^T F$,the equations of motion of the end effector can be written as

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) = J(q)^T F \tag{6}$$

$$\ddot{q} + A^{-1}(q)b(q, \dot{q}) + A^{-1}(q)g(q) = A^{-1}(q)J(q)^T F \tag{7}$$

$$J(q)\ddot{q} + h + J(q)A^{-1}(q)b(q, \dot{q}) + J(q)A^{-1}(q)g(q) = J(q)A^{-1}(q)J(q)^T F + h(q, \dot{q}) \tag{8}$$

$$\ddot{x} + J(q)A^{-1}(q)b(q, \dot{q}) + J(q)A^{-1}(q)g(q) = J(q)A^{-1}(q)J(q)^T F + h(q, \dot{q}) \tag{9}$$

$$\Lambda_r(q)\ddot{x} + \mu_r(q, \dot{q}) + p_r(q) = F \tag{10}$$

Having a desired force F_m^* accting on the end-effector, we apply the joint-space generalized forces

$$\Gamma = J^T(q)\Lambda_r(q)F_m^* + \bar{b}_r(q, \dot{q}) + g(q) \tag{11}$$

This is the joint space generalized

4 1997 Chiaverini — Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators

[4] S. Chiaverini, "Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators," *IEEE Transactions on Robotics and Automation*, vol. 13, no. 3, pp. 398–410, Jun. 1997, ISSN: 1042296X. DOI: `10.1109/70.585902`. Accessed: Sep. 10, 2024. [Online]. Available: `http://ieeexplore.ieee.org/document/585902/`

4.1 Abstract

In [4] several ways of dealing with kinematic and algorithmic singularities are presented. A new resolution thechnique is developed aimed at overcomming the effects of algorithmic singularities. The computational aspects of the method is discussed and a controller is applied to a seven-degree-of-freedom manipulator to demonstrate its effectiveness.

4.2 Summary

Consider the task

$$\dot{x}_E = J_E(q)\dot{q} \tag{12}$$

Solving for q we get

$$\dot{q} = J_E^+\dot{x}_E + (\mathbb{I} - J_E^+ J_E)\dot{q}_0 \tag{13}$$

for some arbitrary $\dot{q}_0$. Selecting the value of $\dot{q}_0$ that minimizes

$$\dot{x}_C - J_C\dot{q} \tag{14}$$

we get

$$\dot{q} = J_E^+\dot{x}_E + \left(J_C(\mathbb{I} - J_E^+ J_E)\right)^+ \left(\dot{x}_C - J_C J_E^+ \dot{x}_E\right) \tag{15}$$

Note that

$$\dot{q} = J_E^+\dot{x}_E + \left(\mathbb{I} - J_E^+ J_E\right) J_C^+ \dot{x}_C \tag{16}$$

Equation 16 reduces to Equation 15 when the constraint task is compatible with the end-effector task. For optimization purpuses, note that Equation 16 can be rewritten as

$$\dot{q} = J_E^+\left(\dot{x}_E - J_E\dot{q}_0\right) + J_C^+\dot{x}_C \tag{17}$$

When J_C and J_E are close to being rank defficient, the pseudo inverse tends to be ill conditioned and joint belocity discontinuities occure. To deal with this problem the author introduces more numerically stable alternatives to the pseudo inverse. Let J be an arbitrary matrix with singular value decomposition

$$J = U\Sigma V^T = \Sigma_{i=1}^r \sigma_i u_i v_i^T \tag{18}$$

note that if J is full rank, the pseudo inverse can be written as

$$J = \sum_{i=1}^r \frac{1}{\sigma_i} v_i u_i^T \tag{19}$$

4.2.1 damped least-squares inverse

The damped least-squares inverse is defined as

$$J^* = \sum_{i=1}^r \frac{\sigma_i}{\sigma_i^2 + \lambda^2} v_i u_i^T \tag{20}$$

It avoids illconditioned matrices and will be well defined even at singular points. The cost is that away from singular points the lambda terms will affect the velocity components. To adress this the author introduces the variable damped least-squares inverse.

4.2.2 Variable Damped least-squares inverse

The variable damped least-squares inverse is defined as the least-squares inverse, but the damping factor λ is 0 far from singularities and some other value close to the singularities. The author suggests the following damping λ

$$\lambda^2 = \begin{cases} 0 & \sigma_m \geq \epsilon \\ \left(1 - \left(\frac{\sigma_m}{\epsilon}\right)\right) \lambda^2_{max} & \sigma_m < \epsilon \end{cases} \tag{21}$$

resulting in (for one singular value equal to zero)

$$J^\circ = \frac{\sigma_m}{\sigma_m^2 + \lambda^2} v_m u_m^T + \sum_{i=1}^{m-1} \frac{1}{\sigma} v_i u_i^T \tag{22}$$

4.2.3 Low Isotropic Variable Damped Least-squares Inverse

choosing $\beta^2 << \lambda^2$, the low isitropic variable damped least-squares inverse becomes (for one singular value equal to zero.)

$$J^\diamondsuit = \frac{\sigma_m}{\sigma_m^2 + \beta^2 + \lambda^2} v_i u_i^T + \sum_{i=1}^{m-1} \frac{\sigma_i}{\sigma_i^2 + \beta^2} v_i u_i^T \tag{23}$$

5 2004 Khatib — Whole-Body Dynamic Behaviour and Control of Human-like Robots

[5] O. Khatib *et al.*, “WHOLE-BODY DYNAMIC BEHAVIOR AND CONTROL OF HUMAN-LIKE ROBOTS,” *International Journal of Humanoid Robotics*, vol. 01, no. 01, pp. 29–43, Mar. 2004, ISSN: 0219-8436, 1793-6942. DOI: `10.1142/S0219843604000058`. Accessed: Sep. 10, 2024. [Online]. Available: `https://www.worldscientific.com/doi/abs/10.1142/S0219843604000058`

5.1 Abstract

5.2 Summary

Consider the joint space dynamics of the robot

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) = \Gamma \tag{24}$$

And a task x_t assosiated with it a jacobian

$$\dot{x}_t = J_t(q)\dot{q} \tag{25}$$

The dynamically consistent generalized inverse [3] is

$$\bar{J}_t(q) = A^{-1} J_t^T (J_t A^{-1} J_t^T)^{-1} \tag{26}$$

6 2005 Sentis — Synthesis of Whole-Body Behaviours Through Hierarchical Control of Behavioural Primitives

[6] L. Sentis and O. Khatib, "SYNTHESIS OF WHOLE-BODY BEHAVIORS THROUGH HIERARCHICAL CONTROL OF BEHAVIORAL PRIMITIVES," *International Journal of Humanoid Robotics*, vol. 02, no. 04, pp. 505–518, Dec. 2005, ISSN: 0219-8436, 1793-6942. DOI: 10.1142/S0219843605000594. Accessed: Sep. 10, 2024. [Online]. Available: https://www.worldscientific.com/doi/abs/10.1142/S0219843605000594

6.1 Abstract

7 2018 Schmidt — Modeling of Articulated Underwater Robots for Simulation and Control

[7] H. M. Schmidt-Didlaukies *et al.*, "Modeling of Articulated Underwater Robots for Simulation and Control," in *2018 IEEE/OES Autonomous Underwater Vehicle Workshop (AUV)*, Porto, Portugal: IEEE, Nov. 2018, pp. 1–7, ISBN: 978-1-72810-253-5. DOI: 10.1109/AUV.2018.8729806. Accessed: Sep. 10, 2024. [Online]. Available: https://ieeexplore.ieee.org/document/8729806/

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[2] Y. Nakamura, H. Hanafusa, and T. Yoshikawa, "Task-Priority Based Redundancy Control of Robot Manipulators," *The International Journal of Robotics Research*, vol. 6, no. 2, pp. 3–15, Jun. 1987, ISSN: 0278-3649, 1741-3176. DOI: 10.1177/027836498700600201. [Online]. Available: https://journals.sagepub.com/doi/10.1177/027836498700600201.

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[5] O. Khatib, L. Sentis, J. Park, and J. Warren, "WHOLE-BODY DYNAMIC BEHAVIOR AND CONTROL OF HUMAN-LIKE ROBOTS," *International Journal of Humanoid Robotics*, vol. 01, no. 01, pp. 29–43, Mar. 2004, ISSN: 0219-8436, 1793-6942. DOI: 10.1142/S0219843604000058. Accessed: Sep. 10, 2024. [Online]. Available: https://www.worldscientific.com/doi/abs/10.1142/S0219843604000058.

[6] L. Sentis and O. Khatib, "SYNTHESIS OF WHOLE-BODY BEHAVIORS THROUGH HIERARCHICAL CONTROL OF BEHAVIORAL PRIMITIVES," *International Journal of Humanoid Robotics*, vol. 02, no. 04, pp. 505–518, Dec. 2005, ISSN: 0219-8436, 1793-6942. DOI: 10.1142/S0219843605000594. Accessed: Sep. 10, 2024. [Online]. Available: https://www.worldscientific.com/doi/abs/10.1142/S0219843605000594.

[7] H. M. Schmidt-Didlaukies, A. J. Sorensen, and K. Y. Pettersen, "Modeling of Articulated Underwater Robots for Simulation and Control," in *2018 IEEE/OES Autonomous Underwater Vehicle Workshop (AUV)*, Porto, Portugal: IEEE, Nov. 2018, pp. 1–7, ISBN: 978-1-72810-253-5. DOI: 10.1109/AUV.2018.8729806. Accessed: Sep. 10, 2024. [Online]. Available: https://ieeexplore.ieee.org/document/8729806/.

A Mathematics

A.1 KKT-Conditions

Consider the following optimization problem

$$\begin{aligned} &\min f(\boldsymbol{x}) \\ \text{s.t. } &\boldsymbol{g}(\boldsymbol{x}) \leq \mathbf{0} \\ \text{s.t. } &\boldsymbol{h}(\boldsymbol{x}) = \mathbf{0} \end{aligned} \tag{27}$$

Define

$$L(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\mu}^T \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\lambda}^T \boldsymbol{h}(\boldsymbol{x}) \tag{28}$$

Suppose $\boldsymbol{x}^*$ is a solution to Equation 27, then the following holds

$$\partial_x f(\boldsymbol{x}^*) = \mathbf{0}^T \qquad \text{stationarity} \tag{29}$$

$$\boldsymbol{g}(\boldsymbol{x}^*) \leq \mathbf{0} \qquad \text{primal feasibility} \tag{30}$$

$$\boldsymbol{g}(\boldsymbol{x}^*) = \mathbf{0} \qquad \text{primal feasibility} \tag{31}$$

$$\boldsymbol{\mu} \geq \mathbf{0} \qquad \text{dual feasibility} \tag{32}$$

$$\boldsymbol{\mu}^T \boldsymbol{g}(\boldsymbol{x}^*) = 0 \qquad \text{complementary slackness} \tag{33}$$

A.2 Generalized inverse

Consider a "fat" matrix J with full row rank. Let A be a symmetric positive definite matrix. The solution to

$$\begin{aligned} &\min_{\boldsymbol{x}} \boldsymbol{x}^T A \boldsymbol{x} \\ \text{s.t. } &\boldsymbol{y} = J\boldsymbol{x} \end{aligned} \tag{34}$$

is

$$\boldsymbol{x} = A^{-1} J^T \left(J A^{-1} J^T \right)^{-1} y \tag{35}$$

Proof: Using the KKT conditions

$$\begin{cases} A\boldsymbol{x} - J^T \lambda = \mathbf{0} \iff x = A^{-1} J^T \boldsymbol{\lambda} \\ \boldsymbol{y} - J\boldsymbol{x} = \mathbf{0} \end{cases} \tag{36}$$

$$\begin{cases} A\boldsymbol{x} - J^T \lambda = \mathbf{0} \\ \boldsymbol{y} - J A^{-1} J^T \boldsymbol{\lambda} = \mathbf{0} \iff \boldsymbol{\lambda} = \left(J A^{-1} J^T \right)^{-1} \boldsymbol{y} \end{cases} \tag{37}$$

$$\begin{cases} A\boldsymbol{x} - J^T \left(J A^{-1} J^T \right)^{-1} = \mathbf{0} \iff \boldsymbol{x} = A^{-1} J^T \left(J A^{-1} J^T \right)^{-1} \boldsymbol{y} \\ \boldsymbol{\lambda} = \left(J A^{-1} J^T \right)^{-1} \end{cases} \tag{38}$$

$$\begin{cases} \boldsymbol{x} = A^{-1} J^T \left(J A^{-1} J^T \right)^{-1} \boldsymbol{y} \\ \boldsymbol{\lambda} = \left(J A^{-1} J^T \right)^{-1} \end{cases} \tag{39}$$

A.3 Solution to under-determined system

consider the "fat" full row rank matrix A. The general solution of

$$\dot{r} = J(\theta)\dot{\theta} \tag{40}$$

is [2]

$$\dot{\theta} = J^+ \dot{r} + \left(\mathcal{I} - J^+ J \right) y \tag{41}$$

for some arbitary vector y. Here, $\mathcal{I} - J^+ J$ is the projection on to the nullspace of J operator.