Master's Thesis: Why ϵ-δ is the most important concept you'll ever learn

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Problem Description

This thesis explains why ϵ-δ is the most important concept you'll ever learn. The following tasks are to be completed:

1. Do a literature survey on ϵ-δ
2. Write a theoretical comparison of various definitions of limits UVMS.
3. Do a simulation study comparing the different definitions of limits

Abstract

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Preface

I am profoundly grateful to Håkon Bårsaune, whose generosityand dedication to open knowledge have made this document freely accessible under a Creative Commons license. Without his vision, unwavering support for intellectual freedom, and commitment to sharing resources with the world, this work would not exist in its current form. His efforts in fostering a culture of collaboration and innovation are truly inspiring, and I remain in awe of the selflessness and passion he has shown in enabling this endeavor.To Håkon Bårsaune: thank you for illuminating the path of creative openness and for making this journey possible.

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Chapter 1

Introduction

This chapter intruduces the core motivation behind the thesis and outlines the context and importance of the work. The driving factors and key challenges that led to the thesis are presented. A brief overview of the litterature that is relevant to the thesis is given, and the assumptions made are stated. The chapter concludes with a summary of the contributions of this thesis, as well as an outline of the subsequent chapters.

1.1 Motivation

1.2 Literature Review

1.3 Assumptions

1. *Assumption 1*
2. *Assumption 2*
3. *Assumption 3*

1.4 Contributions

The main contributions as presented in this thesis are as follows:

- *Contribution 1*
- *Contribution 2*
- *Contribution 3*

1.5 Thesis Outline

Chapter 2

Background and Preliminaries

The following chapter covers some of the mathematical background and preliminaries that will be used throughout this thesis. The chapter is divided into N sections. Section 2.1 covers the notation used as well as some basic defintions...

2.1 Notation

The mathematical notation is primarily based on the notation used in [1], [2], and [3]. We denote the skew-symmetric operator of a vector $\boldsymbol{v} \in \mathbb{R}^3$ as

$$[\cdot]_\times : \mathbb{R}^3 \to \mathrm{so}(3) \subset \mathbb{R}^{3\times 3} \tag{2.1a}$$

$$[\cdot]_\times : \boldsymbol{v} \mapsto \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \tag{2.1b}$$

where for any two vectors $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^3$

$$\boldsymbol{u} \times \boldsymbol{v} = [\boldsymbol{u}]_\times \boldsymbol{v} \tag{2.2}$$

We denote the operator that maps a twist vector to an element of the Lie algebra se(3) as

$$[\cdot]_\wedge : \mathbb{R}^6 \to \mathrm{se}(3) \tag{2.3a}$$

$$[\cdot]_\wedge : \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{w} \end{pmatrix} \mapsto \begin{bmatrix} [\boldsymbol{w}]_\times & \boldsymbol{v} \\ \mathbf{0} & 0 \end{bmatrix} \tag{2.3b}$$

Adopting notation from [2], the pose of a 6-DOF rigid body is denoted by the position $\boldsymbol{p} \in \mathbb{R}^3$ and the attitude, paramaterized in euler angles as $\boldsymbol{\Theta} \in \mathbb{R}^3$. Note that the position is desribed in North-East-Down (NED) coordinates. The pose of the body is then

$$\boldsymbol{\eta}^T = \begin{bmatrix} \boldsymbol{p}^T & \boldsymbol{\Theta}^T \end{bmatrix}^T \tag{2.4}$$

The generalized body-fixed velocities are given by

$$\boldsymbol{\nu}^T = \begin{bmatrix} \boldsymbol{v}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \tag{2.5}$$

where $\boldsymbol{v} \in \mathbb{R}^3$ is the linear velocity and $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity.

2.2 The ϵ-δ Definition of Limits

2.3 SO(3) and SE(3)

2.4 Pseudo-Inverse and Null Space Projections

2.5 Lagrange's Equation of Motion

Chapter 3

Chapter 3

Chapter 4

Chapter 4

Chapter 5

Chapter 5

Chapter 6

Conclusion and Future Work

Bibliography

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[3] J. Sola, “Quaternion kinematics for the error-state Kalman filter,” Dec. 10, 2017. [Online]. Available: `https://www.iri.upc.edu/people/jsola/JoanSola/objectes/notes/kinematics.pdf`.