

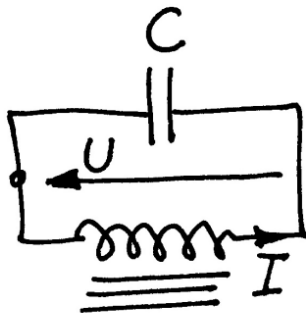


# Mästarprov 13: Strömkretsen

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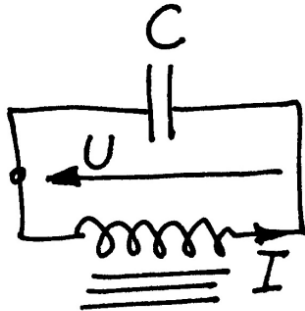


# LC-circuit



# LC-circuit

- What is an LC-circuit?







# Electronic article surveillance





# Project statement

- ▶ What is the current in the circuit?



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- ▶ How is the current affected by the voltage?



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- ▶ What is the current in the circuit?
- ▶ How is the current affected by the voltage?
- ▶ Prove  $E(t)$  is constant
- ▶ How big is the error?
- ▶ Fourier transform
- ▶ Voluntary part

## Mathematical formulation

$$L = \frac{L_0}{1 + I^2}$$

$$U = L \frac{dI}{dt}$$

$$I = -C \frac{dU}{dt} \quad t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

# Mathematical formulation

$$\frac{d^2 I}{dt^2} = \frac{2I}{1 + I^2} \left( \frac{dI}{dt} \right)^2 - \frac{I (1 + I^2)}{L_0 C}$$

## Mathematical formulation

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{1+y_1(t)^2}{L_0C} \end{bmatrix}$$

## Numerical Methods - Runge Kutta 4

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$K_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$K_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4) + O(h^5)$$



## Mathematical formulation

$$E(t) = U(t)^2 - \log(1 + I(t)^2)$$



# Mathematical formulation

$$\frac{dE}{dt} = \frac{d}{dt} (U(t)^2 - \log(1 + I(t)^2))$$

## Mathematical formulation

$$= \frac{d}{dt} \left( \left( \frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \log (1 + I(t)^2) \right)$$

## Mathematical formulation

$$= \frac{d}{dt} \left( \frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \frac{d}{dt} (\log (1 + I(t)^2))$$



# Mathematical formulation

$= \dots$

## Mathematical formulation

$$= 2 \left( \frac{\frac{dI}{dt} \left( 1 + I(t)^2 \left( \frac{d^2 I}{dt^2} \right) - 2I(t) \left( \frac{dI}{dt} \right)^2 \right)}{(1 + I(t)^2)^3} - \frac{I(t) \left( \frac{dI}{dt} \right)}{(1 + I(t)^2)} \right)$$

$$= 2 \left( \frac{\frac{dI}{dt}}{1 + I(t)^2} \right) \left( \frac{1 + I(t)^2 \left( \frac{d^2 I}{dt^2} \right) - 2I(t) \left( \frac{dI}{dt} \right)^2}{(1 + I(t)^2)^2} - \frac{I(t)}{1} \right)$$

## Mathematical formulation

$$\frac{dE}{dt} = 0 \quad \forall \quad t \in \mathbb{R} \quad \Longleftrightarrow \quad \frac{d}{dt}I(0) = 0 \quad \text{or} \quad I(0) = 0 \quad \because$$

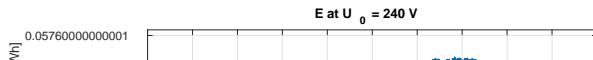
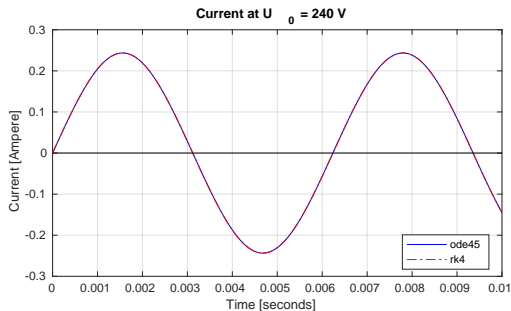
$$\frac{dE}{dt} = 0 \implies \frac{d^2 I}{dt^2} = \left( \frac{2I(t) \frac{dI}{dt}}{1 + I(t)^2} + \frac{4 \left( \frac{dI}{dt} \right)^3 I(t)}{(1 + I(t)^2)^3} \right) \frac{(1 + I(t)^2)^2}{2 \left( \frac{dI}{dt} \right)}$$

Which by definition is a second order autonomous ODE. Thus, If  $I = c$  is a specific solution, then its phase line is independent of the time at which initial conditions are applied. Hence

$$\frac{d^2 I}{dt^2} = 0, \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0$$

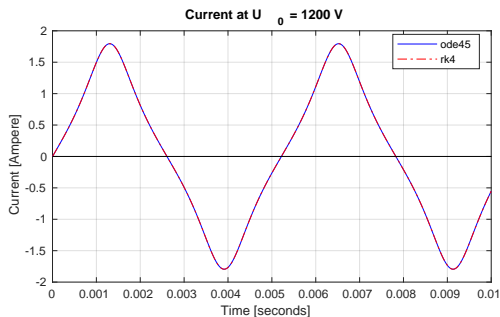
□

# Numerical Results





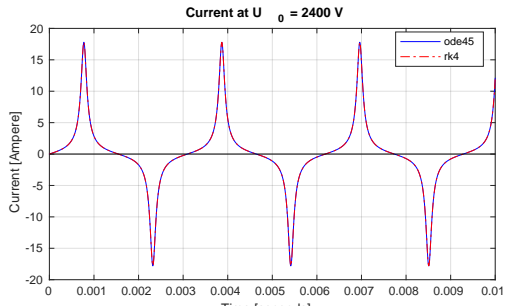
# Numerical Results



E at  $U_0 = 1200 \text{ V}$



# Numerical Results





# Fourier analysis

$$I(t) = a_1 \sin(\omega t) + a_2 \sin(2\omega t) + a_3 \sin(3\omega t) + \dots, \quad \omega = 2\pi/T$$

$$a_k = \frac{2}{T} \int_0^T I(t) \sin(k\omega t) dt, \quad k = 1, 2, 3, \dots$$



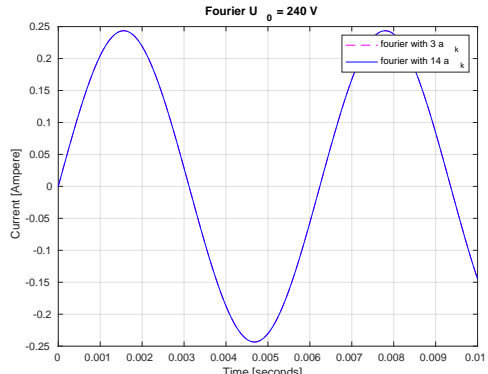
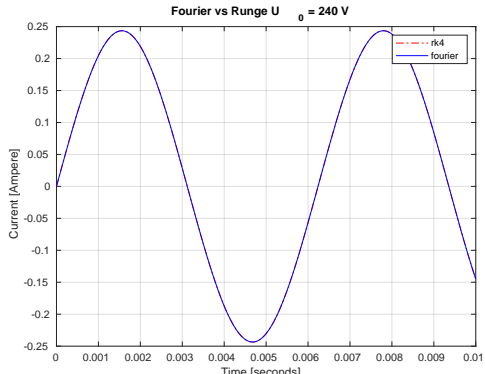
# Fourier analysis

- ▶ Trapezoidal integration for periodic functions

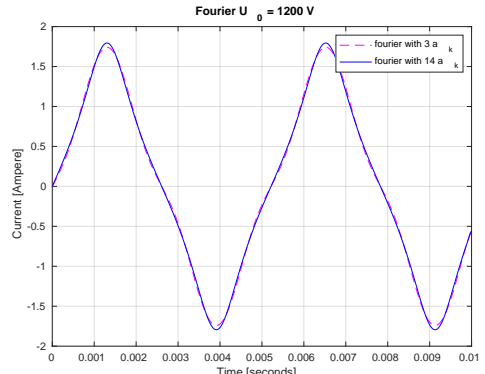
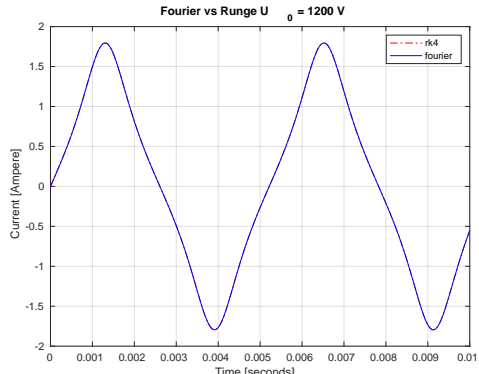
- Trapezoidal integration for periodic functions

$$I = T(h) - \frac{f'(b) - f'(a)}{12}h^2 + \frac{f'''(b) - f'''(a)}{720}h^4 - \frac{f^{(5)}(b) - f^{(5)}(a)}{30240}h^6 + O(h^8)$$

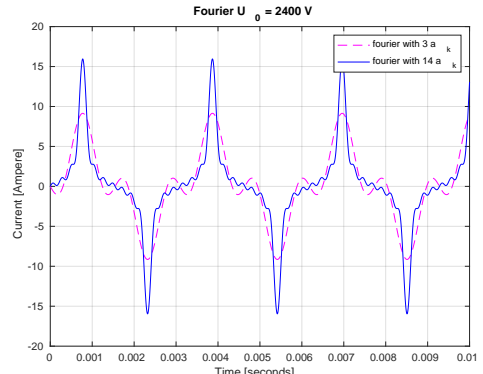
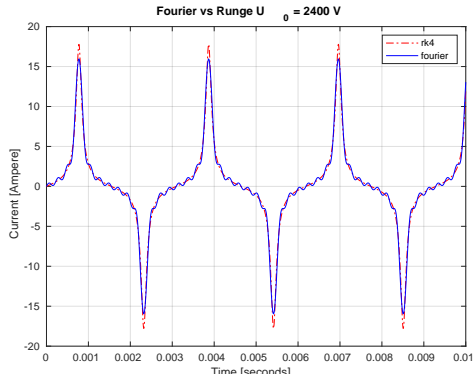
$$U_0 = 240V$$



$$U_0 = 1200V$$



$$U_0 = 2400V$$







# Voluntary part 1

$$I_{max} := 10$$



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$$I_{\alpha} = \max \{ \quad I(U_0^*, L_0) : U_0 \in ]240, 2400[, L_0 \in \mathbb{R} \quad \}$$



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$$g(U_0^*, L_0) = 0 = I_{max} - I_{\alpha}$$

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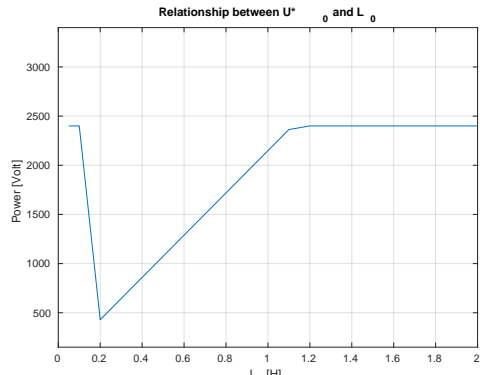
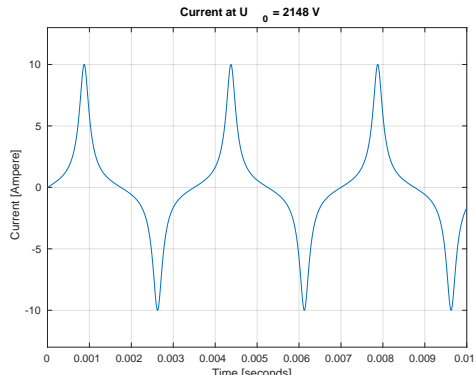
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$$g(U_0^*, L_0) = 0 = I_{max} - I_{\alpha}$$

- Bisection method

# Voluntary part 1

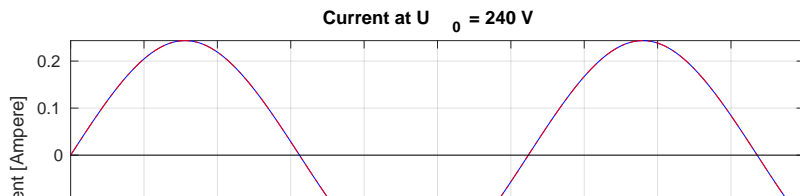


## Voluntary part 2

$$U = L \frac{dI}{dt} \quad I = -C \frac{dU}{dt} \quad L = \frac{L_0}{1 + I^2}$$

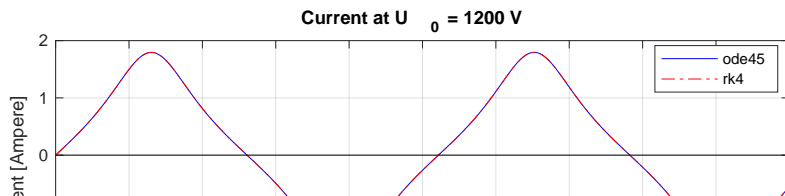
$$\tilde{y}' = \begin{bmatrix} \frac{dI}{dt} \\ \frac{dU}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1+I^2}{L_0} U \\ \left( \frac{dI}{dt} \right) \frac{1}{-C} \end{bmatrix}$$

## Voluntary part 2

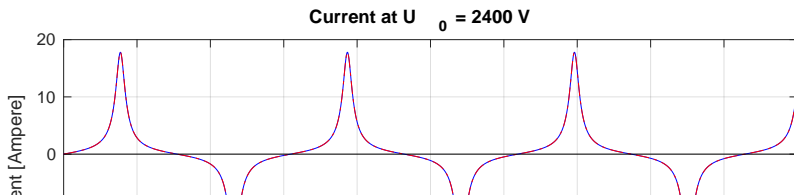




## Voluntary part 2



## Voluntary part 2





Thanks

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