Mästarprov 13: Strömkretsen

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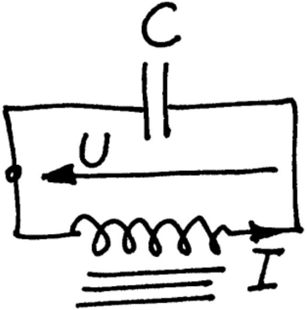
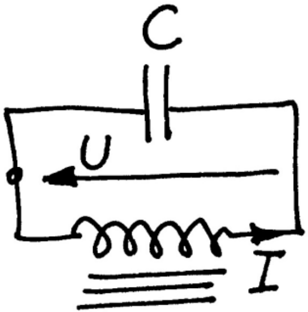
LC-circuit



LC-circuit

- What is an LC-circuit?



RFID

Electronic article surveillance

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Project statement

- What is the current in the circuit?

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- How is the current affected by the voltage?

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- Prove $E(t)$ is constant
- How big is the error?
- Fourier transform
- Voluntary part

Mathematical formulation

$$L = \frac{L_0}{1 + I^2} \qquad U = L\frac{dI}{dt}$$

$$I = -C\frac{dU}{dt} \qquad t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

Mathematical formulation

$$\frac{d^2 I}{dt^2} = \frac{2I}{1+I^2}\left(\frac{dI}{dt}\right)^2 - \frac{I\left(1+I^2\right)}{L_0 C}$$

Mathematical formulation

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{1+y_1(t)^2}{L_0C} \end{bmatrix}$$

Mathematical formulation

$$E(t) = U(t)^2 - log(1 + I(t)^2)$$

Mathematical formulation

$$\frac{dE}{dt} = \frac{d}{dt}\left(U(t)^2 - log(1 + I(t)^2)\right)$$

Mathematical formulation

$$= \frac{d}{dt}\left(\left(\frac{L_0\frac{dI}{dt}}{1+I(t)^2}\right)^2 - log\left(1+I(t)^2\right)\right)$$

Mathematical formulation

$$= \frac{d}{dt}\left(\frac{L_0\frac{dI}{dt}}{1+I(t)^2}\right)^2 - \frac{d}{dt}\left(log\left(1+I(t)^2\right)\right)$$

Mathematical formulation

$$= \dots$$

Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}\left(1 + I(t)^2\left(\frac{d^2I}{dt^2}\right) - 2I(t)\left(\frac{dI}{dt}\right)^2\right)}{\left(1 + I(t)^2\right)^3} - \frac{I(t)\left(\frac{dI}{dt}\right)}{\left(1 + I(t)^2\right)}\right)$$

Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}}{1+I(t)^2}\right)\left(\frac{1+I(t)^2\left(\frac{d^2I}{dt^2}\right)-2I(t)\left(\frac{dI}{dt}\right)^2}{\left(1+I(t)^2\right)^2}-\frac{I(t)}{1}\right)$$

Mathematical formulation

$$\frac{dE}{dt} = 0 \quad \forall \quad t \in \mathbb{R} \quad \iff \frac{d}{dt} I(0) = 0 \quad or \quad I(0) = 0 \quad \because$$

$$\frac{dE}{dt} = 0 \implies \frac{d^2 I}{dt^2} = \left(\frac{2I(t)\frac{dI}{dt}}{1 + I(t)^2} + \frac{4\left(\frac{dI}{dt}\right)^3 I(t)}{\left(1 + I(t)^2\right)^3} \right) \frac{\left(1 + I(t)^2\right)^2}{2\left(\frac{dI}{dt}\right)}$$

Which by definition is a second order autonomous ODE. Thus, If $I = c$ is a specific solution, then its phase line is indenpendent of the time at which initial conditions are applied. Hence

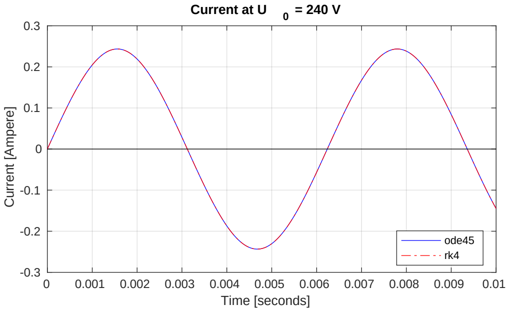
$$\frac{d^2 I}{dt^2} = 0, \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0$$

$\square$

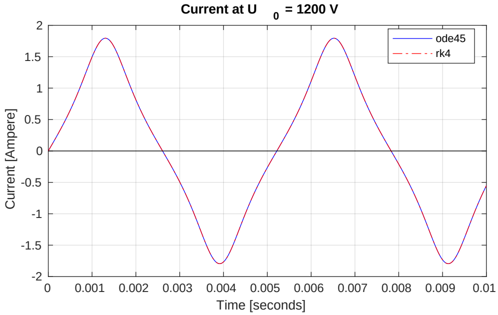
Numerical Methods - Runge Kutta 4

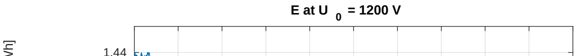
$$
\begin{aligned}
K_1 &= hf(x_n, y_n)\\
K_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})\\
K_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})\\
K_4 &= hf(x_n + h, y_n + k_3)\\
y_{n+1} &= y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) + O(h^5)
\end{aligned}
$$

Numerical Results

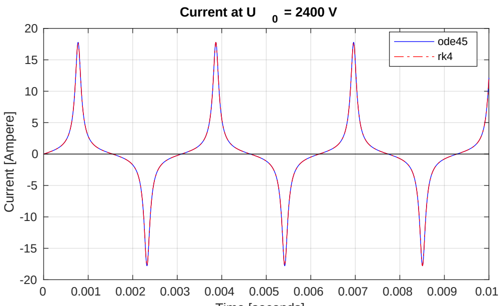


Numerical Results





Numerical Results



Fourier analysis

$$I(t) = a_1 sin(\omega t) + a_2 sin(2\omega t) + a_3 sin(3\omega t) + \cdots, \qquad \omega = 2\pi/T$$

$$a_k = \frac{2}{T}\int_0^T I(t) sin(k\omega t) dt, \quad k = 1, 2, 3....$$

Fourier analysis

- Trapezoidal integration for periodic functions

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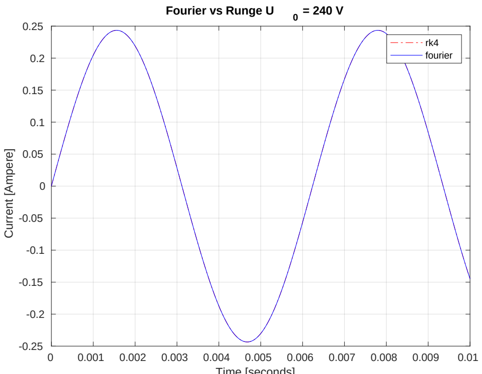
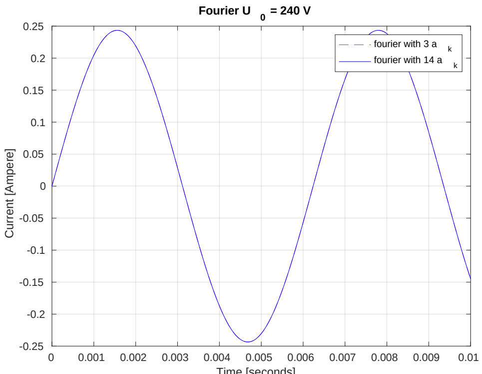
$$I = T(h) - \frac{f'(b) - f'(a)}{12}h^2 + \frac{f'''(b) - f'''(a)}{720}h^4 - \frac{f^{(5)}(b) - f^{(5)}(a)}{30240}h^6 + O(h^8)$$

$U_0 = 240V$

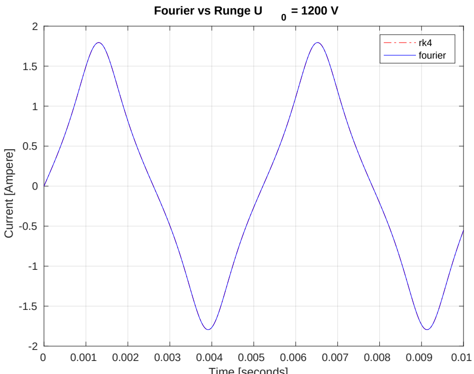


Fourier vs Runge U_0 = 240 V
rk4
fourier
Current [Ampere]
Time [seconds]

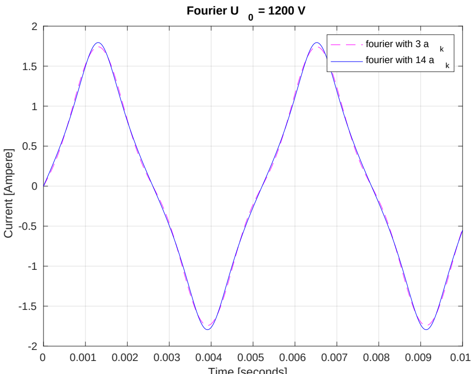


Fourier U_0 = 240 V
fourier with 3 a_k
fourier with 14 a_k
Current [Ampere]
Time [seconds]

$U_0 = 1200V$

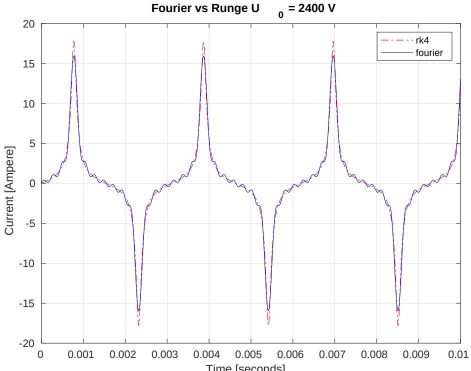


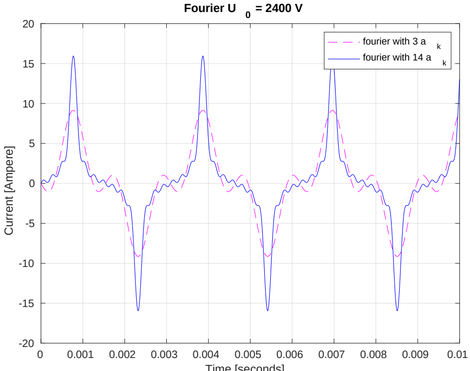
Fourier vs Runge U 0 = 1200 V
rk4
fourier
Current [Ampere]
Time [seconds]



Fourier U 0 = 1200 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]

$U_0 = 2400V$


Fourier vs Runge U 0 = 2400 V
rk4
fourier
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier U 0 = 2400 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

Voluntary part 1

$$I_{max} := 10$$

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$$I_\alpha = max\left\{ \quad I(U_0^*, L_0) : U_0 \in]240, 2400[, L_0 \in \mathbb{R} \quad \right\}$$

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$$g(U_0^*, L_0) = 0 = I_{max} - I_\alpha$$

Voluntary part 1

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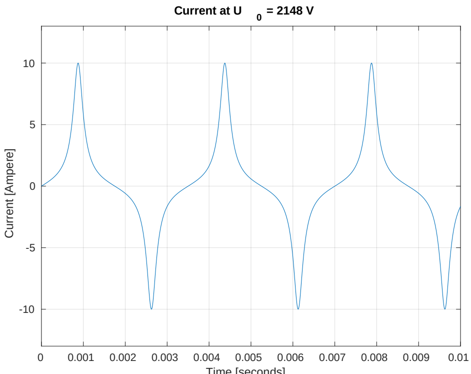
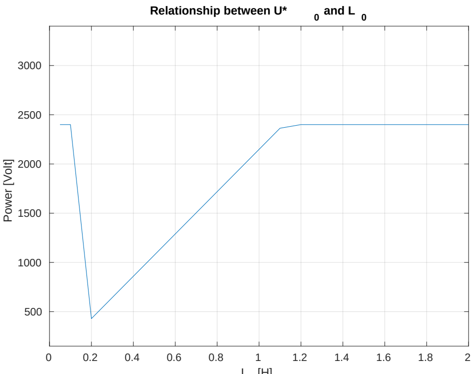
- Bisection method

Voluntary part 1





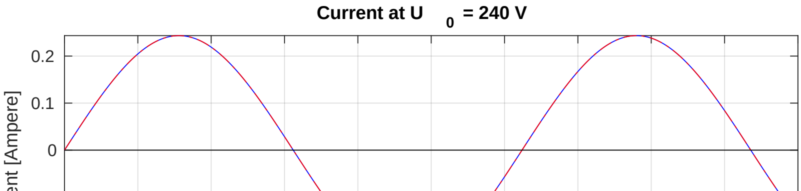
Voluntary part 2

$$U = L\frac{dI}{dt} \qquad I = -C\frac{dU}{dt} \qquad L = \frac{L_0}{1+I^2}$$

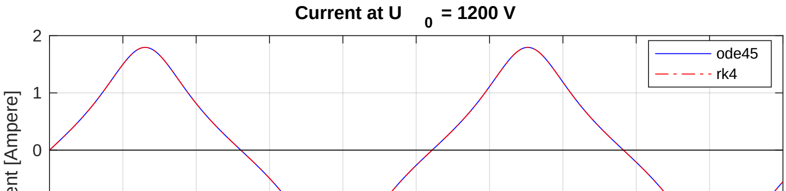
Voluntary part 2

$$\tilde{y}' = \begin{bmatrix} \frac{dI}{dt} \\ \\ \frac{dU}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1+I^2}{L_0} U \\ \\ \frac{\left(\frac{dI}{dt}\right)}{-C} \end{bmatrix}$$

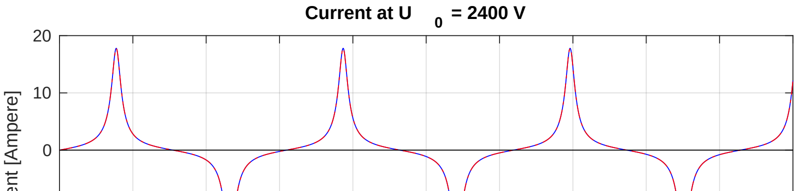
Voluntary part 2



Voluntary part 2



Voluntary part 2



Thanks

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