# SF1518/19 Mästarprov 13: Strömkretsen

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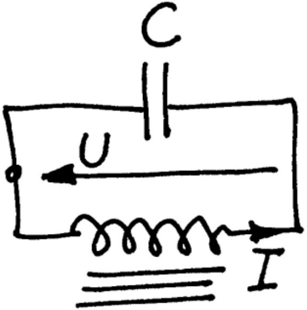
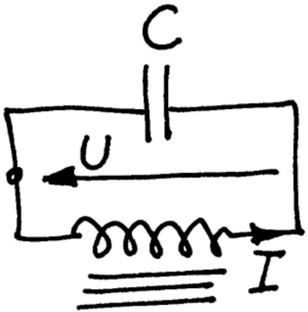
# LC-circuit



# LC-circuit

- What is an LC-circuit?



# RFID

# Electronic article surveillance

5 012345 678900

# Project statement

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- What is the current in the circuit?

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- How is the current affected by the voltage?

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# Project statement

- What is the current in the circuit?
- How is the current affected by the voltage?
- Prove $E(t)$ is constant
- How big is the error?
- Fourier transform
- Voluntary part

# Mathematical formulation

$$L = \frac{L_0}{1 + I^2} \qquad U = L\frac{dI}{dt}$$

$$I = -C\frac{dU}{dt} \qquad t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

# Mathematical formulation

$$\frac{d^2I}{dt^2} = \frac{2I}{1+I^2}\left(\frac{dI}{dt}\right)^2 - \frac{I\left(1+I^2\right)}{L_0C}$$

# Mathematical formulation

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{y_1(t)\left(1+y_1(t)^2\right)}{L_0 C} \end{bmatrix}$$

# Numerical Methods - Runge Kutta 4

$$
\begin{aligned}
K_1 &= hf(x_n, y_n) \\
K_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\
K_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \\
K_4 &= hf(x_n + h, y_n + k_3) \\
y_{n+1} &= y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) + O(h^5)
\end{aligned}
$$

# Mathematical formulation

$$E(t) = U(t)^2 - log(1 + I(t)^2)$$

# Mathematical formulation

$$\frac{dE}{dt} = \frac{d}{dt}\left(U(t)^2 - log(1 + I(t)^2)\right)$$

# Mathematical formulation

$$= \frac{d}{dt}\left(\left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2}\right)^2 - log\left(1 + I(t)^2\right)\right)$$

# Mathematical formulation

$$= \frac{d}{dt}\left(\frac{L_0 \frac{dI}{dt}}{1+I(t)^2}\right)^2 - \frac{d}{dt}\left(log\left(1+I(t)^2\right)\right)$$

# Mathematical formulation

$$= \ldots$$

# Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}\left(1+I(t)^2\left(\frac{d^2I}{dt^2}\right)-2I(t)\left(\frac{dI}{dt}\right)^2\right)}{\left(1+I(t)^2\right)^3}-\frac{I(t)\left(\frac{dI}{dt}\right)}{\left(1+I(t)^2\right)}\right)$$

## Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}}{1+I(t)^2}\right)\left(\frac{1+I(t)^2\left(\frac{d^2I}{dt^2}\right)-2I(t)\left(\frac{dI}{dt}\right)^2}{\left(1+I(t)^2\right)^2}-\frac{I(t)}{1}\right)$$

# Mathematical formulation

$$\frac{dE}{dt} = 0 \quad \forall \quad t \in \mathbb{R} \quad \iff \frac{d}{dt}I(0) = 0 \quad or \quad I(0) = 0 \quad \because$$

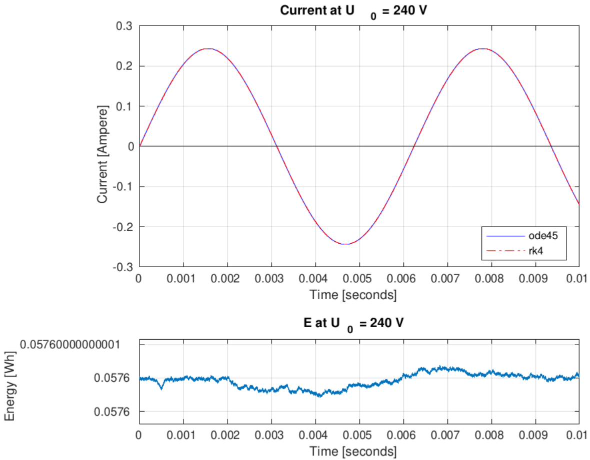
$$\frac{dE}{dt} = 0 \implies \frac{d^2 I}{dt^2} = \left( \frac{2I(t)\frac{dI}{dt}}{1 + I(t)^2} + \frac{4\left(\frac{dI}{dt}\right)^3 I(t)}{\left(1 + I(t)^2\right)^3} \right) \frac{\left(1 + I(t)^2\right)^2}{2\left(\frac{dI}{dt}\right)}$$

Which by definition is a second order autonomous ODE. Thus, If $I = c$ is a specific solution, then its phase line is indenpendent of the time at which initial conditions are applied. Hence

$$\frac{d^2 I}{dt^2} = 0, \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0$$

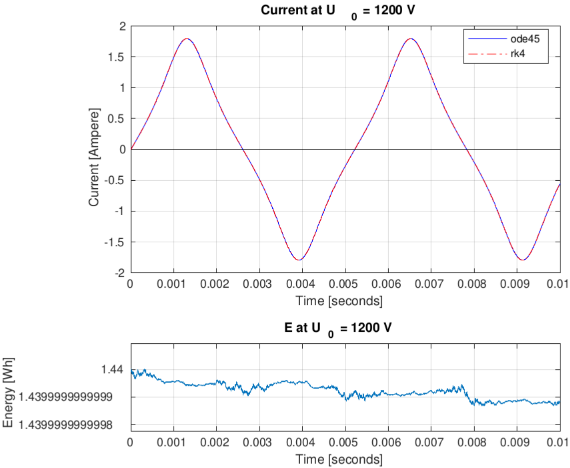
$\square$

# $U_0 = 240V$



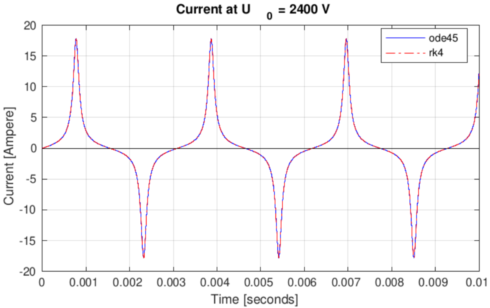

| Iteration | Period [seconds] | Error |
|---|---|---|
| 1 | 6.280000000000000e-03 | 2.000000000000000e-05 |
| 2 | 6.260000000000000e-03 | 1.000000000000000e-05 |
| 3 | 6.250000000000000e-03 | 5.000000000000000e-06 |
| 4 | 6.245000000000000e-03 | 5.000000000000000e-06 |
| 5 | 6.240000000000000e-03 | 1.250000000000000e-06 |
| 6 | 6.238750000000000e-03 | 0 |
| 7 | 6.238750000000000e-03 | 3.125000000000000e-07 |
| 8 | 6.238437500000000e-03 | 3.125000000000000e-07 |
| 9 | 6.238125000000000e-03 | 0 |
| 10 | 6.238125000000000e-03 | 3.906200000000000e-08 |
| 11 | 6.238085937500000e-03 | 3.906300000000000e-08 |
| 12 | 6.238046875000000e-03 | 9.765599999999999e-09 |
| 13 | 6.238037109375000e-03 | 4.882800000000000e-09 |
| 14 | 6.238032226562000e-03 | 0 |
| 15 | 6.238032226562000e-03 | 2.441400000000000e-09 |
| 16 | 6.238029785156000e-03 | 6.103500000000000e-10 |
| 17 | 6.238029174805000e-03 | NaN |

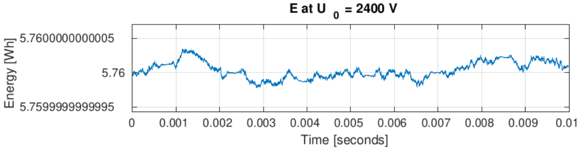
# $U_0 = 1200V$




| Iteration | Period [seconds] | Error |
|---|---|---|
| 1 | 5.240000000000000e-03 | 0 |
| 2 | 5.240000000000000e-03 | 2.000000000000005e-05 |
| 3 | 5.230000000000000e-03 | 0 |
| 4 | 5.220000000000000e-03 | 2.499999999999898e-06 |
| 5 | 5.220000000000000e-03 | 2.499999999999898e-06 |
| 6 | 5.218750000000000e-03 | 0 |
| 7 | 5.218125000000000e-03 | 6.250000000001914e-07 |
| 8 | 5.217500000000000e-03 | 0 |
| 9 | 5.217343750000000e-03 | 7.812500000007813e-08 |
| 10 | 5.217265625000000e-03 | 7.812499999964445e-08 |
| 11 | 5.217226562500000e-03 | 1.953125000012795e-08 |
| 12 | 5.217207031250000e-03 | 0 |
| 13 | 5.217207031250000e-03 | 9.765625000063977e-09 |
| 14 | 5.217197265625000e-03 | 2.441406000107393e-09 |
| 15 | 5.217194824219000e-03 | 1.220703000053697e-09 |
| 16 | 5.217193603516000e-03 | 0 |
| 17 | 5.217193603516000e-03 | NaN |

# $U_0 = 2400V$




| Iteration | Period [seconds] | Error |
|---:|---|---:|
| 1 | 3.120000000000000e-03 | 0 |
| 2 | 3.120000000000000e-03 | 2.000000000000005e-05 |
| 3 | 3.100000000000000e-03 | 0 |
| 4 | 3.100000000000000e-03 | 2.499999999999898e-06 |
| 5 | 3.097500000000000e-03 | 2.499999999999898e-06 |
| 6 | 3.095000000000000e-03 | 0 |
| 7 | 3.095000000000000e-03 | 6.250000000001914e-07 |
| 8 | 3.094375000000000e-03 | 0 |
| 9 | 3.094375000000000e-03 | 7.812500000007813e-08 |
| 1 | 3.094296875000000e-03 | 7.812499999964445e-08 |
| 1 | 3.094218750000000e-03 | 1.953125000012795e-08 |
| 1 | 3.094199218750000e-03 | 0 |
| 1 | 3.094199218750000e-03 | 9.765625000063977e-09 |
| 1 | 3.094189453125000e-03 | 2.441406000107393e-09 |
| 1 | 3.094187011719000e-03 | 1.220703000053697e-09 |
| 1 | 3.094185791016000e-03 | 0 |
| 1 | 3.094185791016000e-03 | NaN |

# Fourier analysis

$$I(t) = a_1 sin(\omega t) + a_2 sin(2\omega t) + a_3 sin(3\omega t) + \cdots, \qquad \omega = 2\pi/T$$

$$a_k = \frac{2}{T}\int_0^T I(t) sin(k\omega t) dt, \quad k = 1, 2, 3....$$

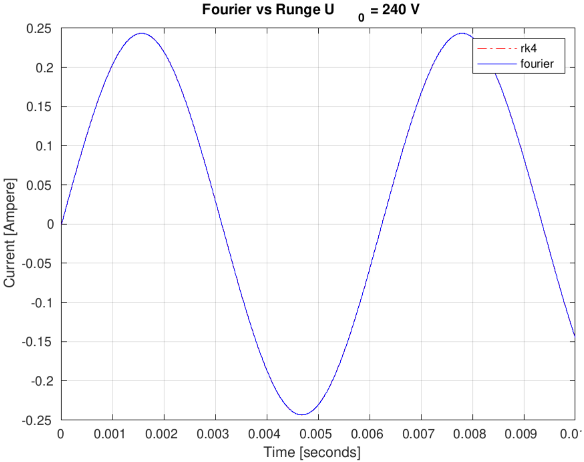
# Fourier analysis

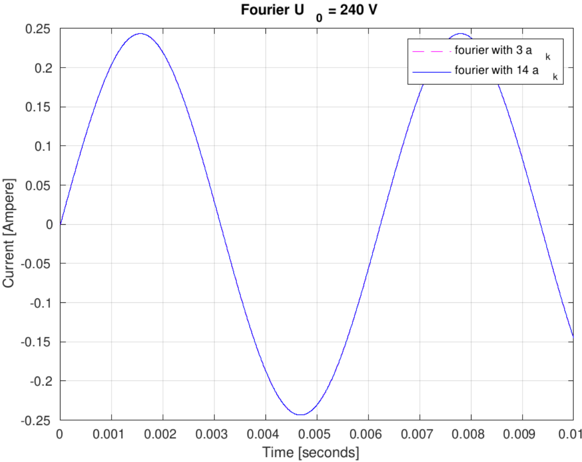
- Trapezoidal integration for periodic functions

$$T(h) = \frac{h}{2}\left(f_0 + f_m + 2\sum_{m-1}^{i=1} y_i\right)$$

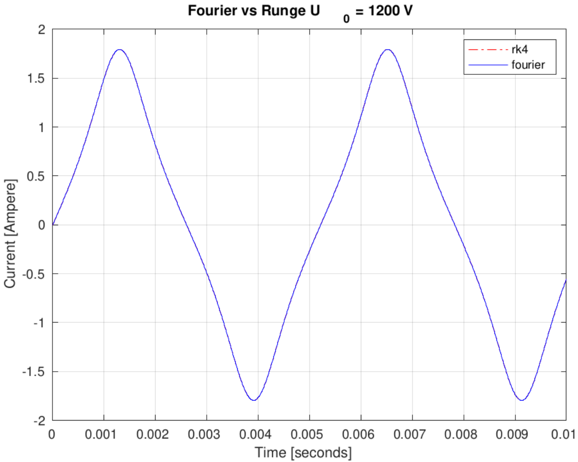
$$I = T(h) - \frac{f'(b) - f'(a)}{12}h^2 + \frac{f'''(b) - f'''(a)}{720}h^4 - \frac{f^{(5)}(b) - f^{(5)}(a)}{30240}h^6 + O(h^8)$$

## $U_0 = 240V$

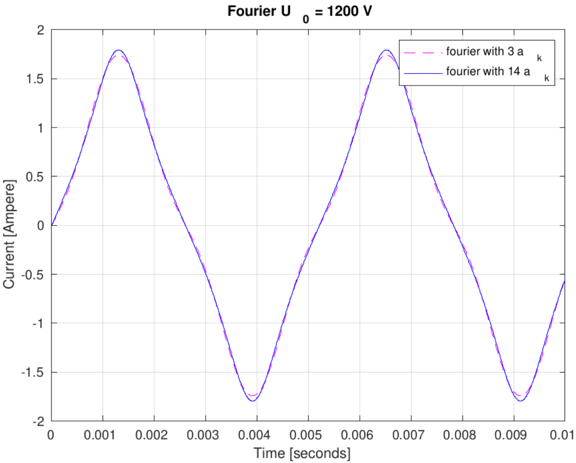

Fourier vs Runge U 0 = 240 V
rk4
fourier
Current [Ampere]
Time [seconds]
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009


Fourier U 0 = 240 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009

# $U_0 = 1200V$

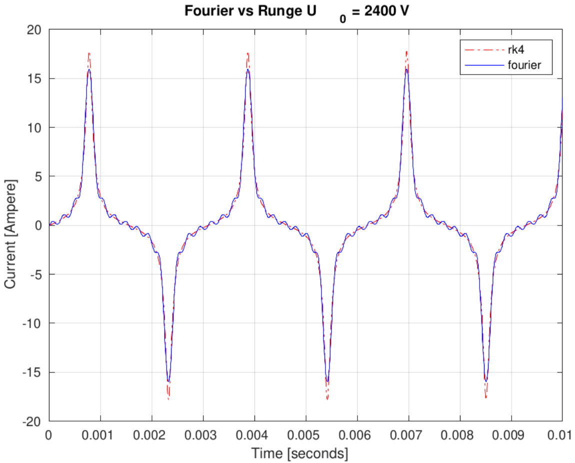


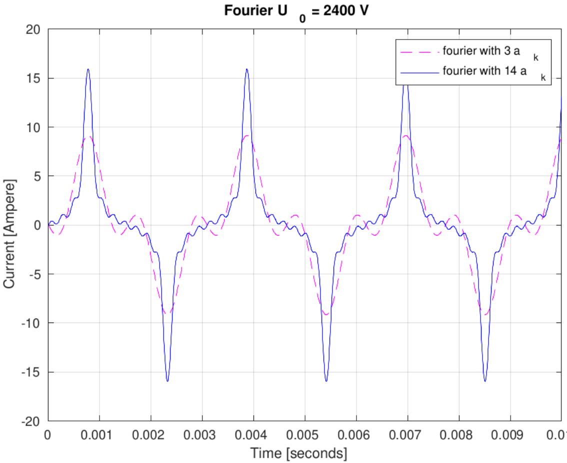
Fourier vs Runge U₀ = 1200 V
rk4
fourier
Current [Ampere]
Time [seconds]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

Fourier U₀ = 1200 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

# $U_0 = 2400V$


Fourier vs Runge U 0 = 2400 V
rk4
fourier
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier U 0 = 2400 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

# Voluntary part 1

$$I_{max} := 10$$

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$$I_\alpha = max\left\{ \quad I(U_0^*, L_0) : U_0 \in ]240, 2400[, L_0 \in \mathbb{R} \quad \right\}$$

# Voluntary part 1

$$I_{max} := 10$$

$$I_\alpha = max\left\{ \quad I(U_0^*, L_0) : U_0 \in ]240, 2400[, L_0 \in \mathbb{R} \quad \right\}$$

$$g(U_0^*, L_0) = 0 = I_{max} - I_\alpha$$

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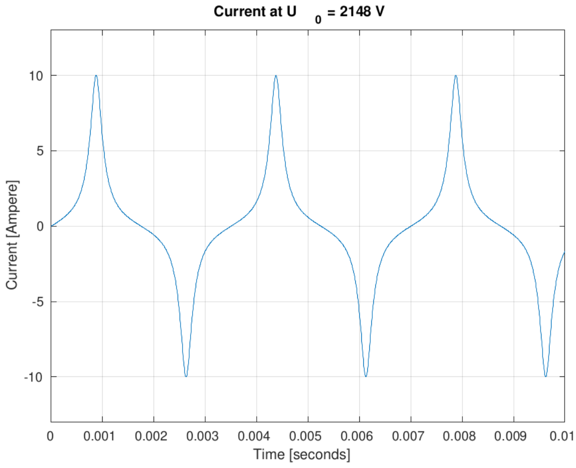
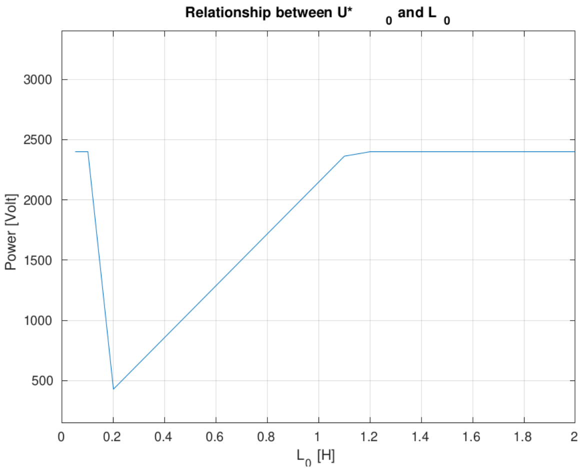
$$I_\alpha = max\left\{ \quad I(U_0^*, L_0) : U_0 \in ]240, 2400[, L_0 \in \mathbb{R} \quad \right\}$$

$$g(U_0^*, L_0) = 0 = I_{max} - I_\alpha$$

- Bisection method

# Voluntary part 1



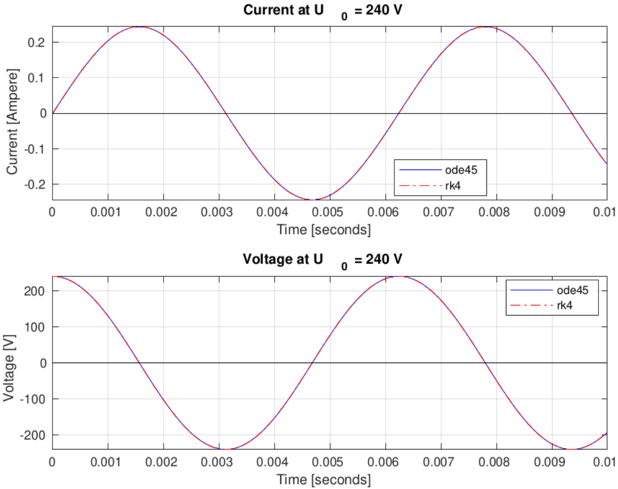
# Voluntary part 2

$$U = L\frac{dI}{dt} \qquad I = -C\frac{dU}{dt} \qquad L = \frac{L_0}{1 + I^2}$$

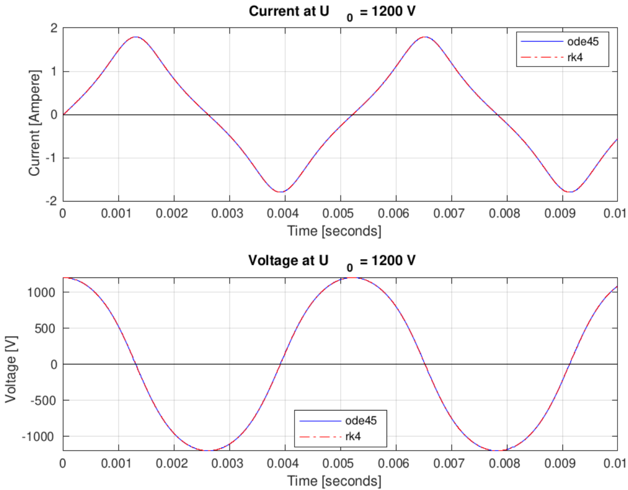
## Voluntary part 2

$$\tilde{y}' = \begin{bmatrix} \frac{dI}{dt} \\ \\ \frac{dU}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1+I^2}{L_0}U \\ \\ \frac{\left(\frac{dI}{dt}\right)}{-C} \end{bmatrix}$$

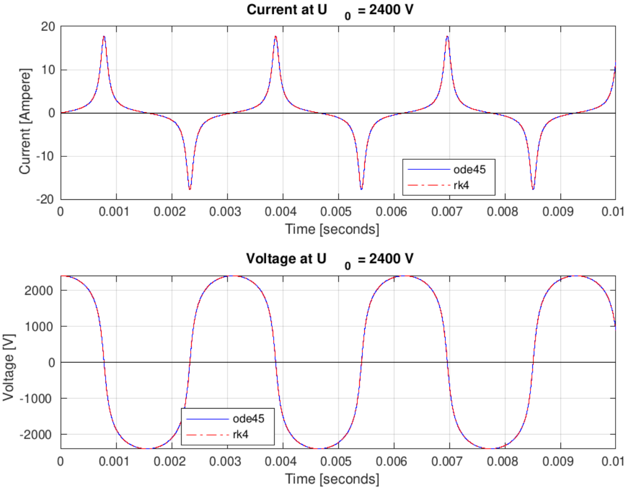
# Voluntary part 2



# Voluntary part 2



# Voluntary part 2



# Thanks

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