

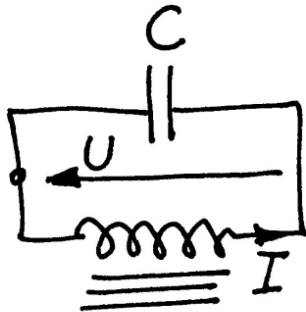


SF1518/19 Mästarprov 13: Strömkretsen

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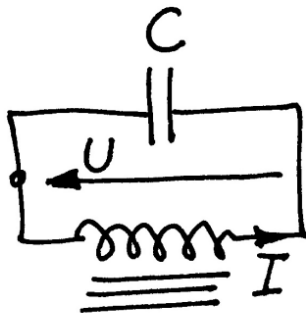


LC-circuit



LC-circuit

- What is an LC-circuit?







Electronic article surveillance





Project statement



Project statement

- ▶ What is the current in the circuit?



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- ▶ What is the current in the circuit?
- ▶ How is the current affected by the voltage?



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- ▶ Prove $E(t)$ is constant



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- ▶ Fourier transform



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- ▶ Prove $E(t)$ is constant
- ▶ How big is the error?
- ▶ Fourier transform
- ▶ Voluntary part

Mathematical formulation

$$L = \frac{L_0}{1 + I^2}$$

$$U = L \frac{dI}{dt}$$

$$I = -C \frac{dU}{dt} \quad t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

Mathematical formulation

$$\frac{d^2 I}{dt^2} = \frac{2I}{1 + I^2} \left(\frac{dI}{dt} \right)^2 - \frac{I (1 + I^2)}{L_0 C}$$

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{y_1(t)(1+y_1(t)^2)}{L_0C} \end{bmatrix}$$

Numerical Methods - Runge Kutta 4

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$K_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4) + O(h^5)$$



More math

$$E(t) = U(t)^2 - \log(1 + I(t)^2)$$



More math

$$\frac{dE}{dt} = \frac{d}{dt} (U(t)^2 - \log(1 + I(t)^2))$$

$$= \frac{d}{dt} \left(\left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \log (1 + I(t)^2) \right)$$

$$= \frac{d}{dt} \left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \frac{d}{dt} (\log (1 + I(t)^2))$$

The derivative

$$= 2 \left(\frac{\frac{dI}{dt} \left(1 + I(t)^2 \right) \left(\frac{d^2 I}{dt^2} \right) - 2I(t) \left(\frac{dI}{dt} \right)^2}{\left(1 + I(t)^2 \right)^3} - \frac{I(t) \left(\frac{dI}{dt} \right)}{\left(1 + I(t)^2 \right)} \right)$$

$$\frac{dE}{dt} = (0 - 0)$$

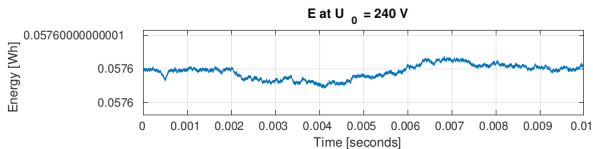
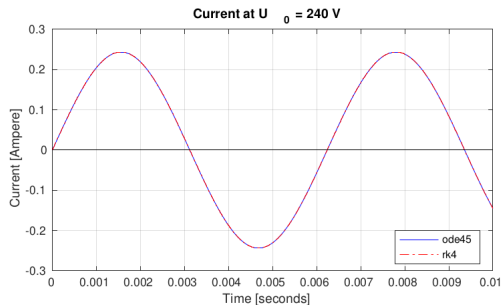
$$= \left(\cancel{2 \frac{\frac{dI}{dt} \left(1 + I(t)^2 \left(\frac{d^2 I}{dt^2} \right) - 2I(t) \left(\frac{dI}{dt} \right)^2}{(1 + I(t)^2)^3}} \xrightarrow{0}} - \cancel{2 \frac{I(t) \left(\frac{dI}{dt} \right)}{(1 + I(t)^2)} \xrightarrow{0}} \right)$$



so much work for nothing

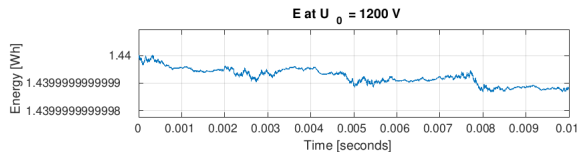
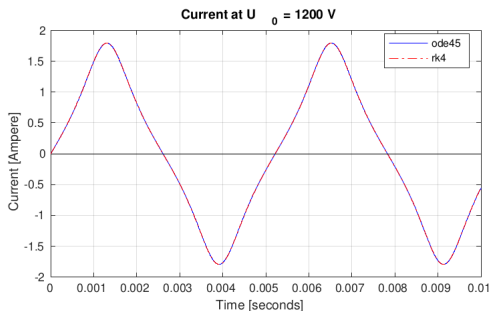
$$\frac{dE}{dt} = 0$$

$$U_0 = 240V$$



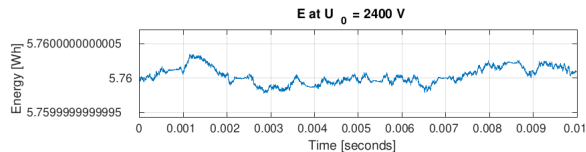
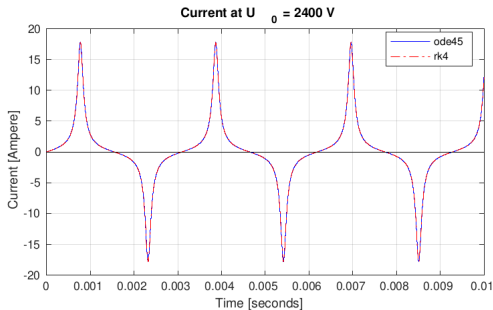
Iteration	Period [seconds]	approx error ($E_n - E_{2n}$)
1	6.280000000000000e-03	2.000000000000000e-05
2	6.260000000000000e-03	1.000000000000000e-05
3	6.250000000000000e-03	5.000000000000000e-06
4	6.245000000000000e-03	5.000000000000000e-06
5	6.240000000000000e-03	1.250000000000000e-06
6	6.238750000000000e-03	0
7	6.238750000000000e-03	3.125000000000000e-07
8	6.238437500000000e-03	3.125000000000000e-07
9	6.238125000000000e-03	0
10	6.238125000000000e-03	3.906200000000000e-08
11	6.238085937500000e-03	3.906300000000000e-08
12	6.238046875000000e-03	9.765599999999999e-09
13	6.238037109375000e-03	4.882800000000000e-09
14	6.238032226562000e-03	0
15	6.238032226562000e-03	2.441400000000000e-09
16	6.238029785156000e-03	6.103500000000000e-10
17	6.238029174805000e-03	NaN

$$U_0 = 1200V$$



Iteration	Period [seconds]	approx error ($E_n - E_{2n}$)
1	5.240000000000000e-03	0
2	5.240000000000000e-03	2.000000000000005e-05
3	5.230000000000000e-03	0
4	5.220000000000000e-03	2.499999999999898e-06
5	5.220000000000000e-03	2.499999999999898e-06
6	5.218750000000000e-03	0
7	5.218125000000000e-03	6.250000000001914e-07
8	5.217500000000000e-03	0
9	5.217343750000000e-03	7.812500000007813e-08
10	5.217265625000000e-03	7.81249999964445e-08
11	5.217226562500000e-03	1.953125000012795e-08
12	5.217207031250000e-03	0
13	5.217207031250000e-03	9.765625000063977e-09
14	5.217197265625000e-03	2.441406000107393e-09
15	5.217194824219000e-03	1.220703000053697e-09
16	5.217193603516000e-03	0
17	5.217193603516000e-03	NaN

$$U_0 = 2400V$$



Iteration	Period [seconds]	approx error ($E_n - E_{2n}$)
1	3.120000000000000e-03	0
2	3.120000000000000e-03	2.000000000000005e-05
3	3.100000000000000e-03	0
4	3.100000000000000e-03	2.499999999999898e-06
5	3.097500000000000e-03	2.499999999999898e-06
6	3.095000000000000e-03	0
7	3.095000000000000e-03	6.2500000000001914e-07
8	3.094375000000000e-03	0
9	3.094375000000000e-03	7.8125000000007813e-08
1	3.094296875000000e-03	7.81249999964445e-08
1	3.094218750000000e-03	1.953125000012795e-08
1	3.094199218750000e-03	0
1	3.094199218750000e-03	9.765625000063977e-09
1	3.094189453125000e-03	2.441406000107393e-09
1	3.094187011719000e-03	1.220703000053697e-09
1	3.094185791016000e-03	0
1	3.094185791016000e-03	NaN

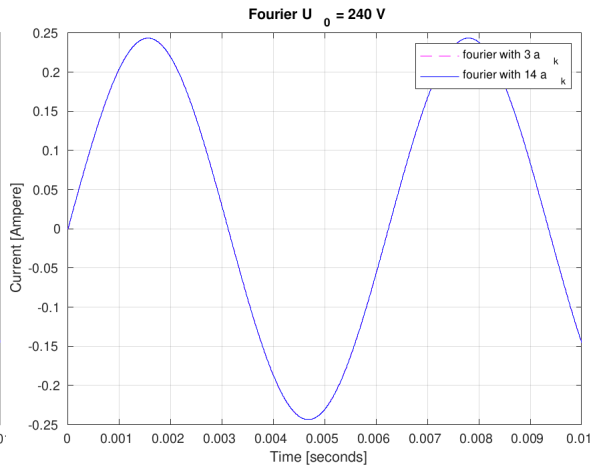
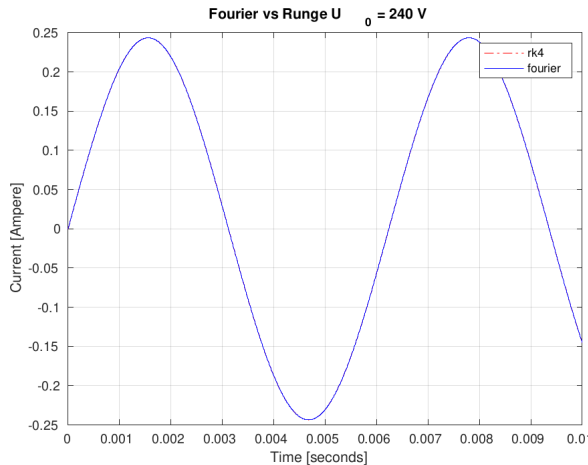


Fourier analysis

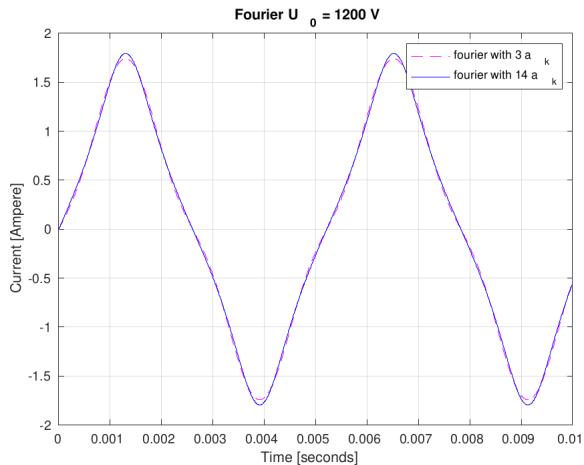
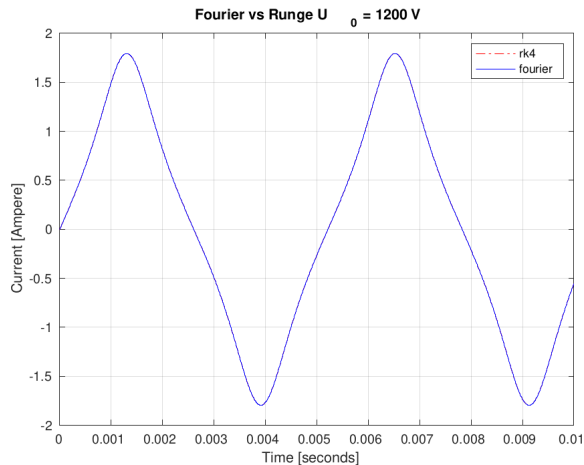
$$I(t) = a_1 \sin(\omega t) + a_2 \sin(2\omega t) + a_3 \sin(3\omega t) + \dots, \quad \omega = 2\pi/T$$

$$a_k = \frac{2}{T} \int_0^T I(t) \sin(k\omega t) dt, \quad k = 1, 2, 3, \dots$$

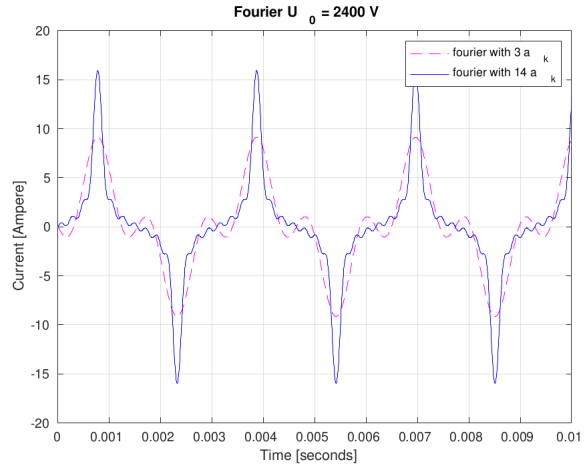
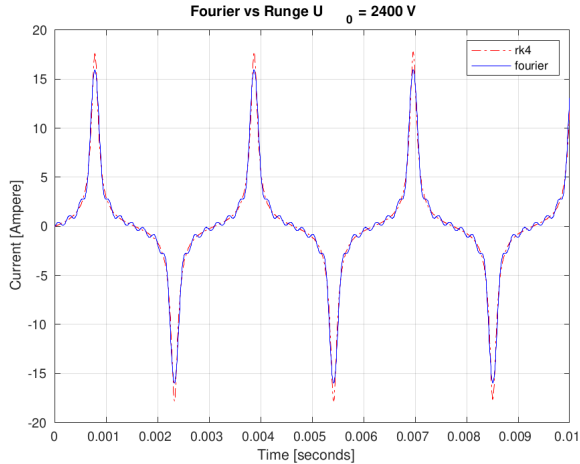
$$U_0 = 240V$$



$$U_0 = 1200V$$



$$U_0 = 2400V$$





Voluntary part 1

$$I_{max} := 10$$



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$$I_{\alpha} = \max \{ \quad I(U_0^*, L_0) : U_0 \in]240, 2400[, L_0 \in \mathbb{R} \quad \}$$

Voluntary part 1

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$$I_{\alpha} = \max \{ \quad I(U_0^*, L_0) : U_0 \in]240, 2400[, L_0 \in \mathbb{R} \quad \}$$

$$g(U_0^*, L_0) = 0 = I_{max} - I_{\alpha}$$

Voluntary part 1

$$I_{max} := 10$$

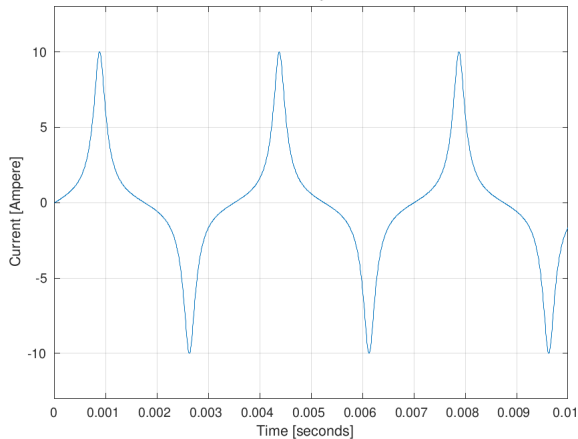
$$I_{\alpha} = \max \{ \quad I(U_0^*, L_0) : U_0 \in]240, 2400[, L_0 \in \mathbb{R} \quad \}$$

$$g(U_0^*, L_0) = 0 = I_{max} - I_{\alpha}$$

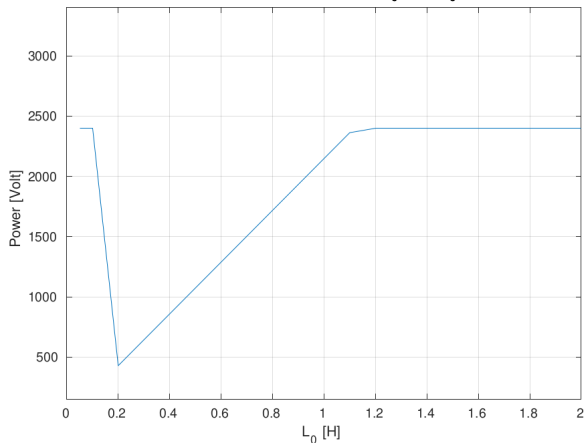
- Bisection method

Voluntary part 1

Current at $U_0 = 2148 \text{ V}$



Relationship between U_0^* and L_0



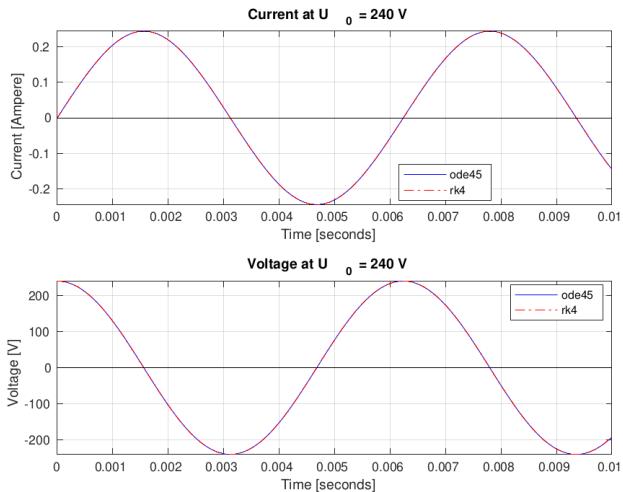


Voluntary part 2

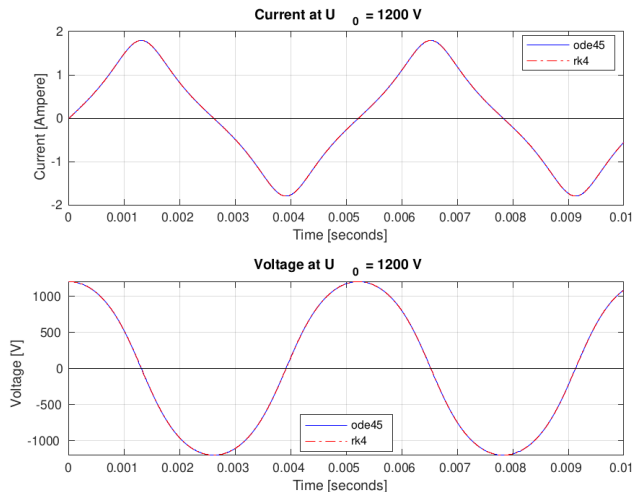
$$U = L \frac{dI}{dt} \quad I = -C \frac{dU}{dt} \quad L = \frac{L_0}{1 + I^2}$$

$$\tilde{y}' = \begin{bmatrix} \frac{dI}{dt} \\ \frac{dU}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1+I^2}{L_0} U \\ \left(\frac{dI}{dt} \right) \frac{1}{-C} \end{bmatrix}$$

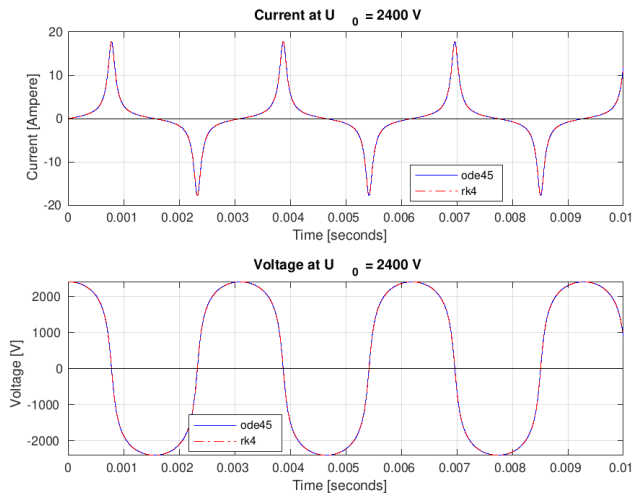
Voluntary part 2



Voluntary part 2



Voluntary part 2





Thanks

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