SF1518/19 Mästarprov 13: Strömkretsen

Fun version

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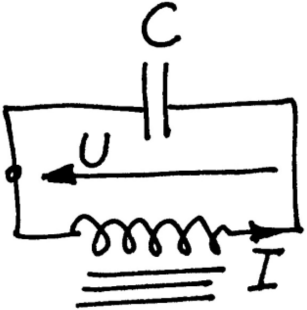
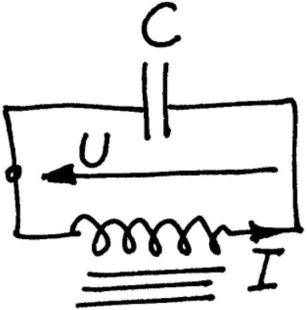
LC-circuit



LC-circuit

- What is an LC-circuit?



RFID

Electronic article surveillance

5 012345 678900

Project statement

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- What is the current in the circuit?

Project statement

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- How is the current affected by the voltage?

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- Prove $E(t)$ is constant

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- Fourier transform

Project statement

- What is the current in the circuit?
- How is the current affected by the voltage?
- Prove $E(t)$ is constant
- How big is the error?
- Fourier transform
- Voluntary part

Mathematical formulation

$$L = \frac{L_0}{1 + I^2} \qquad U = L\frac{dI}{dt}$$

$$I = -C\frac{dU}{dt} \qquad t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

Mathematical formulation

$$\frac{d^2 I}{dt^2} = \frac{2I}{1+I^2}\left(\frac{dI}{dt}\right)^2 - \frac{I\left(1+I^2\right)}{L_0 C}$$

Mathematical formulation

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{y_1(t)\left(1+y_1(t)^2\right)}{L_0 C} \end{bmatrix}$$

Numerical Methods - Runge Kutta 4

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$K_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$K_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) + O(h^5)$$

Mathematical formulation

$$E(t) = U(t)^2 - log(1 + I(t)^2)$$

Mathematical formulation

$$\frac{dE}{dt} = \frac{d}{dt}\left(U(t)^2 - log(1 + I(t)^2)\right)$$

Mathematical formulation

$$= \frac{d}{dt}\left(\left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2}\right)\right)^2 - log\left(1 + I(t)^2\right)\Bigg)$$

Mathematical formulation

$$= \frac{d}{dt}\left(\frac{L_0\frac{dI}{dt}}{1+I(t)^2}\right)^2 - \frac{d}{dt}\left(log\left(1+I(t)^2\right)\right)$$

Mathematical formulation

$$= \dots$$

Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}\left(1 + I(t)^2\left(\frac{d^2I}{dt^2}\right) - 2I(t)\left(\frac{dI}{dt}\right)^2\right)}{\left(1 + I(t)^2\right)^3} - \frac{I(t)\left(\frac{dI}{dt}\right)}{\left(1 + I(t)^2\right)}\right)$$

Mathematical formulation

$$= 2\left(\frac{\frac{dI}{dt}}{1 + I(t)^2}\right)\left(\frac{1 + I(t)^2\left(\frac{d^2I}{dt^2}\right) - 2I(t)\left(\frac{dI}{dt}\right)^2}{\left(1 + I(t)^2\right)^2} - \frac{I(t)}{1}\right)$$

Mathematical formulation

$$\frac{dE}{dt} = 0 \quad \forall \quad t \in \mathbb{R} \quad \iff \frac{d}{dt}I(0) = 0 \quad or \quad I(0) = 0 \quad \because$$

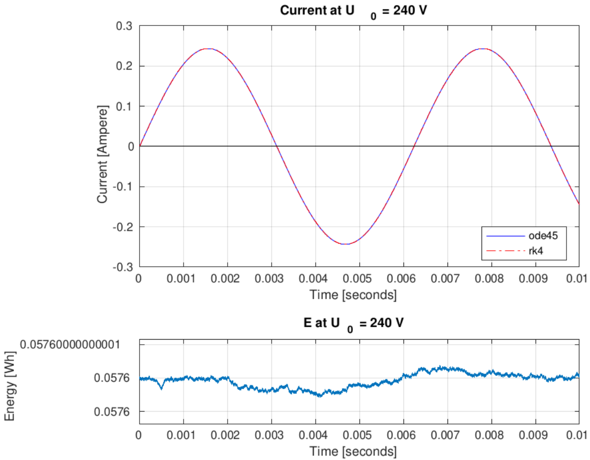
$$\frac{dE}{dt} = 0 \implies \frac{d^2 I}{dt^2} = \left(\frac{2I(t)\frac{dI}{dt}}{1 + I(t)^2} + \frac{4\left(\frac{dI}{dt}\right)^3 I(t)}{\left(1 + I(t)^2\right)^3} \right) \frac{\left(1 + I(t)^2\right)^2}{2\left(\frac{dI}{dt}\right)}$$

Which by definition is a second order autonomous ODE. Thus, If $I = c$ is a specific solution, then its phase line is indenpendent of the time at which initial conditions are applied. Hence

$$\frac{d^2 I}{dt^2} = 0, \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0$$

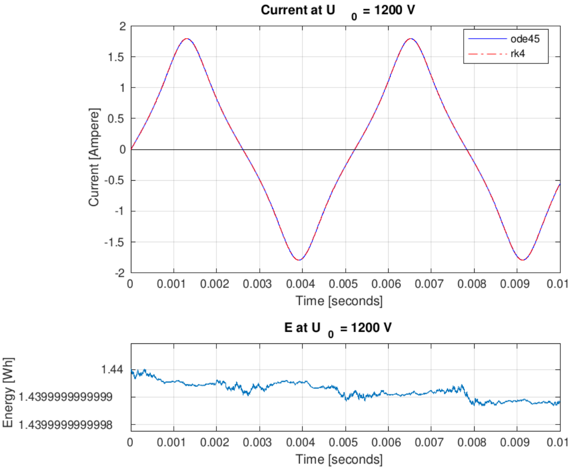
$\square$

$U_0 = 240V$



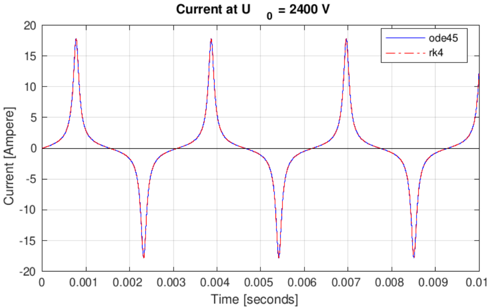
Iteration	Period [seconds]	Error
1	6.280000000000000e-03	2.000000000000000e-05
2	6.260000000000000e-03	1.000000000000000e-05
3	6.250000000000000e-03	5.000000000000000e-06
4	6.245000000000000e-03	5.000000000000000e-06
5	6.240000000000000e-03	1.250000000000000e-06
6	6.238750000000000e-03	0
7	6.238750000000000e-03	3.125000000000000e-07
8	6.238437500000000e-03	3.125000000000000e-07
9	6.238125000000000e-03	0
10	6.238125000000000e-03	3.906200000000000e-08
11	6.238085937500000e-03	3.906300000000000e-08
12	6.238046875000000e-03	9.765599999999999e-09
13	6.238037109375000e-03	4.882800000000000e-09
14	6.238032226562000e-03	0
15	6.238032226562000e-03	2.441400000000000e-09
16	6.238029785156000e-03	6.103500000000000e-10
17	6.238029174805000e-03	NaN

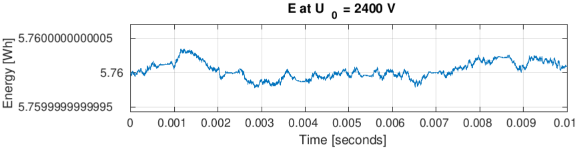
$U_0 = 1200V$



Iteration	Period [seconds]	Error
1	5.240000000000000e-03	0
2	5.240000000000000e-03	2.000000000000005e-05
3	5.230000000000000e-03	0
4	5.220000000000000e-03	2.499999999999898e-06
5	5.220000000000000e-03	2.499999999999898e-06
6	5.218750000000000e-03	0
7	5.218125000000000e-03	6.250000000001914e-07
8	5.217500000000000e-03	0
9	5.217343750000000e-03	7.812500000007813e-08
10	5.217265625000000e-03	7.812499999964445e-08
11	5.217226562500000e-03	1.953125000012795e-08
12	5.217207031250000e-03	0
13	5.217207031250000e-03	9.765625000063977e-09
14	5.217197265625000e-03	2.441406000107393e-09
15	5.217194824219000e-03	1.220703000053697e-09
16	5.217193603516000e-03	0
17	5.217193603516000e-03	NaN

$U_0 = 2400V$





Iteration	Period [seconds]	Error
1	3.120000000000000e-03	0
2	3.120000000000000e-03	2.000000000000005e-05
3	3.100000000000000e-03	0
4	3.100000000000000e-03	2.499999999999898e-06
5	3.097500000000000e-03	2.499999999999898e-06
6	3.095000000000000e-03	0
7	3.095000000000000e-03	6.250000000001914e-07
8	3.094375000000000e-03	0
9	3.094375000000000e-03	7.812500000007813e-08
1	3.094296875000000e-03	7.812499999964445e-08
1	3.094218750000000e-03	1.953125000012795e-08
1	3.094199218750000e-03	0
1	3.094199218750000e-03	9.765625000063977e-09
1	3.094189453125000e-03	2.441406000107393e-09
1	3.094187011719000e-03	1.220703000053697e-09
1	3.094185791016000e-03	0
1	3.094185791016000e-03	NaN

Fourier analysis

$$I(t) = a_1 sin(\omega t) + a_2 sin(2\omega t) + a_3 sin(3\omega t) + \cdots, \qquad \omega = 2\pi/T$$

$$a_k = \frac{2}{T}\int_0^T I(t) sin(k\omega t) dt, \quad k = 1, 2, 3....$$

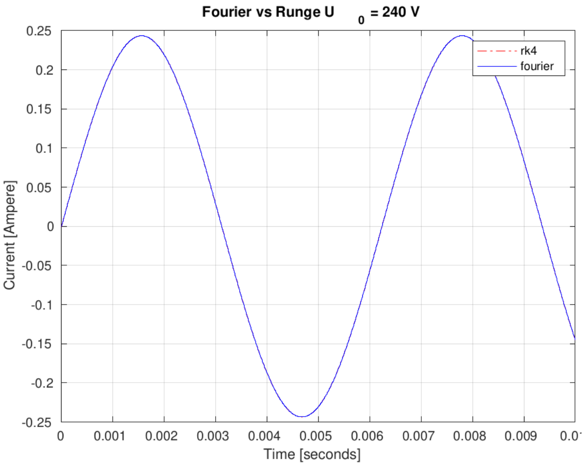
Fourier analysis

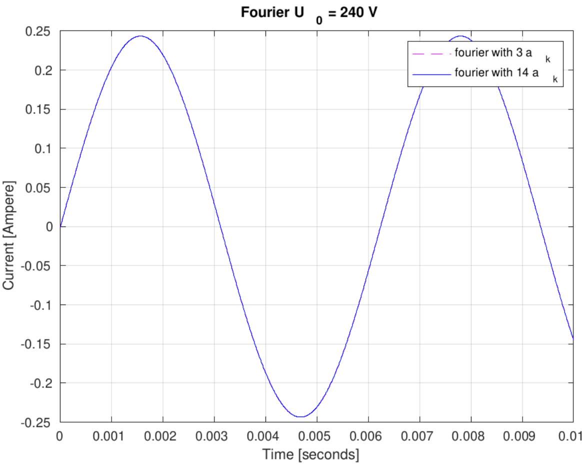
- Trapezoidal integration for periodic functions

$$T(h) = \frac{h}{2}\left(f_0 + f_m + 2\sum_{m-1}^{i=1} y_i \right)$$

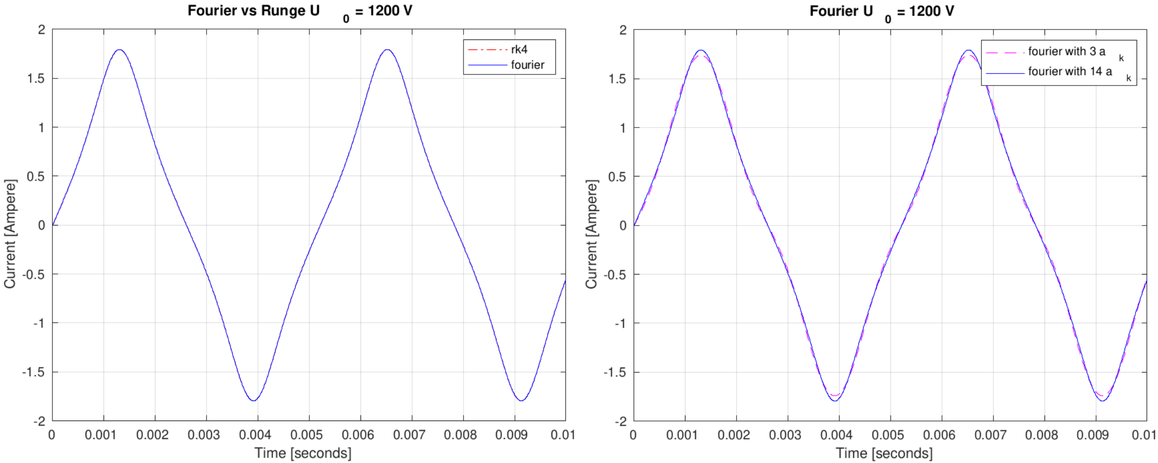
$$I = T(h) - \frac{f'(b) - f'(a)}{12}h^2 + \frac{f'''(b) - f'''(a)}{720}h^4 - \frac{f^{(5)}(b) - f^{(5)}(a)}{30240}h^6 + O(h^8)$$

$U_0 = 240V$


Fourier vs Runge U 0 = 240 V
rk4
fourier
Current [Ampere]
Time [seconds]
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009

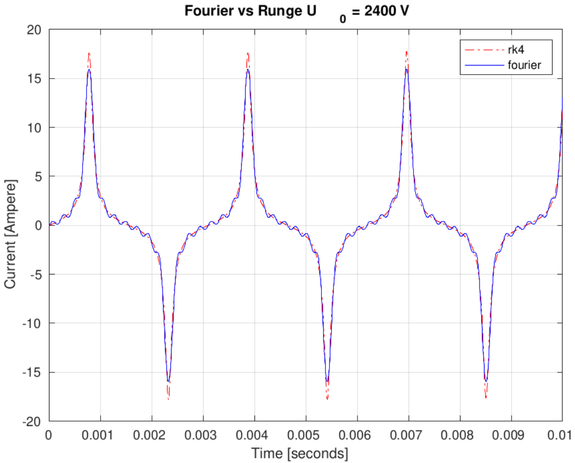

Fourier U 0 = 240 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009

$U_0 = 1200V$

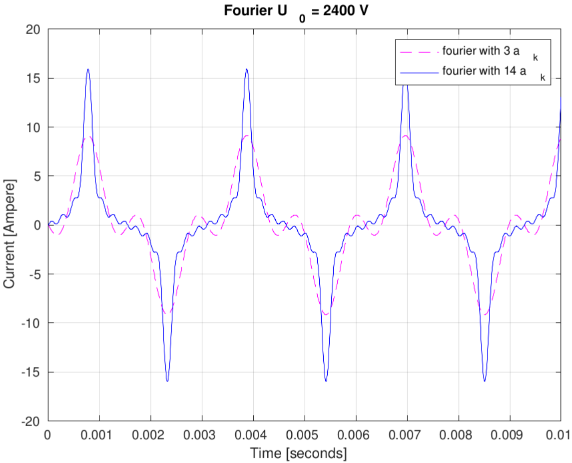


Fourier vs Runge U_0 = 1200 V
rk4
fourier
Current [Ampere]
Time [seconds]
Fourier U_0 = 1200 V
fourier with 3 a_k
fourier with 14 a_k
Current [Ampere]
Time [seconds]

$U_0 = 2400V$



Fourier vs Runge U 0 = 2400 V
rk4
fourier
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01



Fourier U 0 = 2400 V
fourier with 3 a k
fourier with 14 a k
Current [Ampere]
Time [seconds]
20
15
10
5
0
-5
-10
-15
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

Voluntary part 1

$$I_{max} := 10$$

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$$I_\alpha = max\left\{ \quad I(U_0^*, L_0) : U_0 \in]240, 2400[, L_0 \in \mathbb{R} \quad \right\}$$

Voluntary part 1

$$I_{max} := 10$$

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$$g(U_0^*, L_0) = 0 = I_{max} - I_\alpha$$

Voluntary part 1

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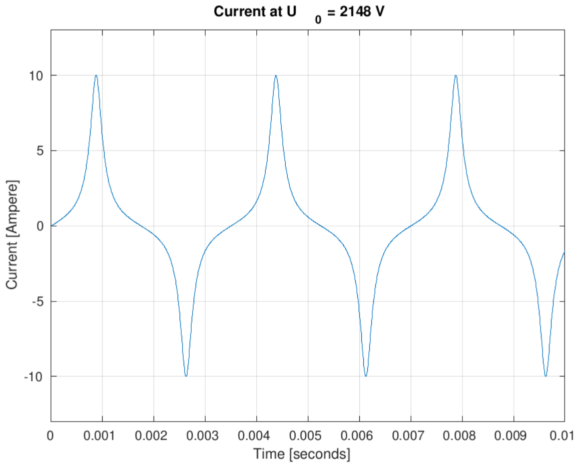
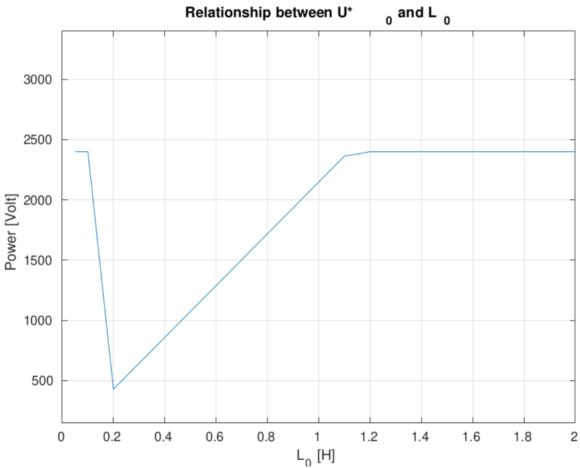
$$g(U_0^*, L_0) = 0 = I_{max} - I_\alpha$$

- Bisection method

Voluntary part 1





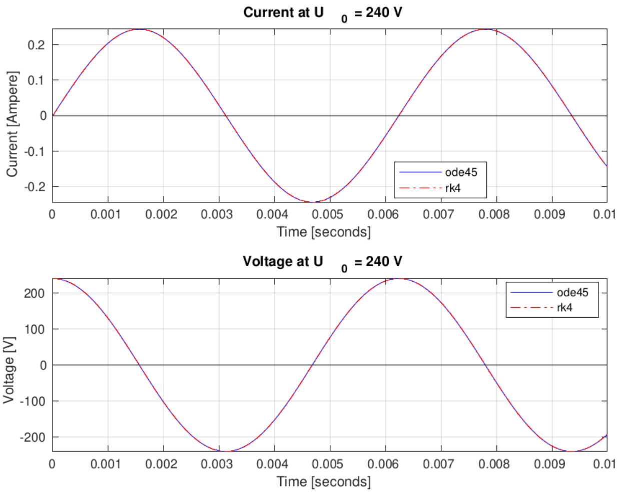
Voluntary part 2

$$U = L\frac{dI}{dt} \qquad I = -C\frac{dU}{dt} \qquad L = \frac{L_0}{1+I^2}$$

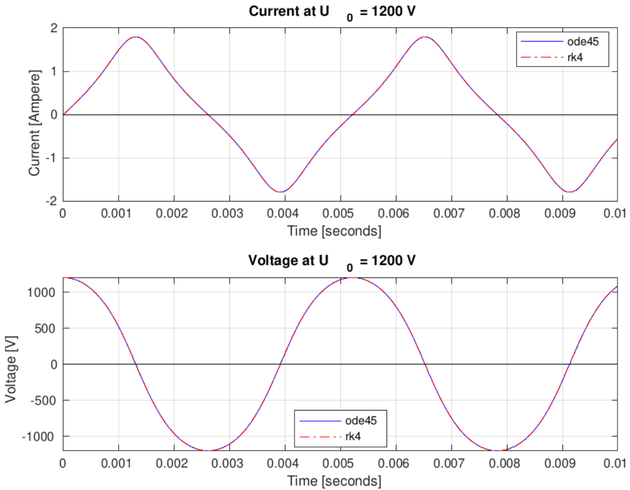
Voluntary part 2

$$\tilde{y}' = \begin{bmatrix} \frac{dI}{dt} \\ \\ \frac{dU}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1+I^2}{L_0}U \\ \\ \frac{\left(\frac{dI}{dt}\right)}{-C} \end{bmatrix}$$

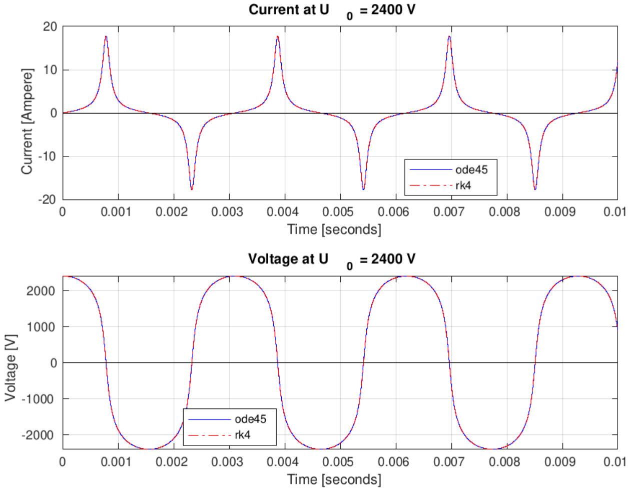
Voluntary part 2



Voluntary part 2



Voluntary part 2



Thanks

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