

$$L = \frac{L_0}{1 + I^2}$$

$$U = L \frac{dI}{dt}$$

$$I = -C \frac{dU}{dt}$$

$$\frac{d^2 I}{dt^2} = \frac{2I}{1 + I^2} \left(\frac{dI}{dt} \right)^2 - \frac{I(1 + I^2)}{L_0 C}$$

$$t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{1+y_1(t)^2}{L_0 C} \end{bmatrix}$$

$$\begin{aligned}
E(t) &= U(t)^2 - \log(1 + I(t)^2) \\
\frac{dE}{dt} &= \frac{d}{dt} (U(t)^2 - \log(1 + I(t)^2)) \\
&= \frac{d}{dt} \left(\left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \log(1 + I(t)^2) \right) \\
&= \frac{d}{dt} \left(\frac{L_0 \frac{dI}{dt}}{1 + I(t)^2} \right)^2 - \frac{d}{dt} (\log(1 + I(t)^2)) \\
&= \dots \\
&= 2 \left(\frac{\frac{dI}{dt} \left(1 + I(t)^2 \left(\frac{d^2 I}{dt^2} \right) - 2I(t) \left(\frac{dI}{dt} \right)^2 \right)}{(1 + I(t)^2)^3} - \frac{I(t) \left(\frac{dI}{dt} \right)}{(1 + I(t)^2)} \right) \\
&= 2 \left(\frac{\frac{dI}{dt}}{1 + I(t)^2} \right) \left(\frac{1 + I(t)^2 \left(\frac{d^2 I}{dt^2} \right) - 2I(t) \left(\frac{dI}{dt} \right)^2}{(1 + I(t)^2)^2} - \frac{I(t)}{1} \right) \\
\frac{dE}{dt} = 0 \quad \forall \quad t \in \mathbb{R} &\iff \frac{d}{dt} I(0) = 0 \quad \text{or} \quad I(0) = 0 \quad \therefore \\
\frac{dE}{dt} = 0 \implies \frac{d^2 I}{dt^2} &= \left(\frac{2I(t) \frac{dI}{dt}}{1 + I(t)^2} + \frac{4 \left(\frac{dI}{dt} \right)^3 I(t)}{(1 + I(t)^2)^3} \right) \frac{(1 + I(t)^2)^2}{2 \left(\frac{dI}{dt} \right)}
\end{aligned}$$

Which by definition is a second order autonomous ODE. Thus, If $I = c$ is a specific solution, then its phase line is independent of the time at which initial conditions are applied. Hence

$$\frac{d^2 I}{dt^2} = 0$$

Since

$$\frac{d^2 I}{dt^2} = I(c) \neq \frac{dI}{dt} \quad \text{and} \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0$$

□

Which is given by the fact that the capacitor and magnetic field had been fully charged at $t = 0$.

$$= 2 \left(\frac{\frac{dI}{dt} \left(1 + I(t)^2 \left(\frac{d^2 I}{dt^2} \right) - 2I(t) \left(\frac{dI}{dt} \right)^2 \right)}{(1 + I(t)^2)^3} + \frac{I(t) \left(\frac{dI}{dt} \right)}{(1 + I(t)^2)} \right)$$