$$L = \frac{L_0}{1 + I^2}$$

$$U = L\frac{dI}{dt}$$

$$I = -C\frac{dU}{dt}$$

$$\frac{d^2 I}{dt^2} = \frac{2I}{1 + I^2}\left(\frac{dI}{dt}\right)^2 - \frac{I(1 + I^2)}{L_0 C}$$

$$t = 0, \quad I = 0, \quad \frac{dI}{dt} = \frac{U_0}{L_0}$$

$$\tilde{y}'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} y_2(t) \\ \frac{2y_1(t)y_2^2(t)}{1+y_1(t)^2} - \frac{1+y_1(t)^2}{L_0 C} \end{bmatrix}$$

$$\frac{d^2 I}{dt^2} = \frac{2I}{1 + I^2}\left(\frac{dI}{dt}\right)^2 - \frac{I\left(1 + I^2\right)}{L_0 C}$$

$$
\begin{aligned}
E(t) &= U(t)^2 - log(1 + I(t)^2) \\
\frac{dE}{dt} &= \frac{d}{dt}\left(U(t)^2 - log(1 + I(t)^2)\right) \\
&= \frac{d}{dt}\left(\left(\frac{L_0\frac{dI}{dt}}{1+I(t)^2})\right)^2 - log\left(1+I(t)^2\right)\right) \\
&= \frac{d}{dt}\left(\frac{L_0\frac{dI}{dt}}{1+I(t)^2}\right)^2 - \frac{d}{dt}\left(log\left(1+I(t)^2\right)\right) \\
&= ... \\
&= 2\left(\frac{\frac{dI}{dt}\left(1+I(t)^2\left(\frac{d^2I}{dt^2}\right) - 2I(t)\left(\frac{dI}{dt}\right)^2\right)}{\left(1+I(t)^2\right)^3} - \frac{I(t)\left(\frac{dI}{dt}\right)}{\left(1+I(t)^2\right)}\right) \\
&= 2\left(\frac{\frac{dI}{dt}}{1+I(t)^2}\right)\left(\frac{1+I(t)^2\left(\frac{d^2I}{dt^2}\right) - 2I(t)\left(\frac{dI}{dt}\right)^2}{\left(1+I(t)^2\right)^2} - \frac{I(t)}{1}\right) \\
\frac{dE}{dt} &= 0 \quad \forall \quad t \in \mathbb{R} \quad \iff \frac{d}{dt}I(0) = 0 \quad or \quad I(0) = 0 \quad \because \\
\frac{dE}{dt} &= 0 \implies \frac{d^2I}{dt^2} = \left(\frac{2I(t)\frac{dI}{dt}}{1+I(t)^2} + \frac{4\left(\frac{dI}{dt}\right)^3 I(t)}{\left(1+I(t)^2\right)^3}\right)\frac{\left(1+I(t)^2\right)^2}{2\left(\frac{dI}{dt}\right)}
\end{aligned}
$$

Which by definition is a second order autonomous ODE. Thus, If $I = c$ is a specific solution, then its phase line is indenpendent of the time at which initial conditions are applied. Hence

$$
\frac{d^2I}{dt^2} = 0, \quad I_0 = 0 \quad \therefore \quad \frac{dE}{dt} = 0
$$

$\square$

Which is given by the fact that the capacitor and magnetic field had been fully charged at $t = 0$.

$$= 2 \left(\cancelto{0}{\frac{\frac{dI}{dt} \left(1 + I(t)^2 \left(\frac{d^2 I}{dt^2}\right) - 2I(t) \left(\frac{dI}{dt}\right)^2\right)}{\left(1 + I(t)^2\right)^3}} + \cancelto{0}{\frac{I(t) \left(\frac{dI}{dt}\right)}{(1 + I(t)^2)}} \right)$$

$$\tilde{y}' = \begin{bmatrix} \frac{dU}{dt} \\ \\ \frac{dI}{dt} \end{bmatrix} = \begin{bmatrix} \frac{L_0 \left(\frac{dI}{dt}\right)}{1+I^2} \\ \\ -C \frac{dU}{dt} \end{bmatrix}$$