Traffic Flow on highways

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December 24, 2022

1 The Description of Problem

Now let' s consider the traffic flow in the highways from Guangzhou to Shenzhen. As shown by the map, there are three main highways from Guangzhou to Shenzhen:

- Guangzhou-Shenzhen Highway (G4)
- Guangzhou-Shenzhen Yanjiang Highway (S3)
- Jing-Gang-Ao Highway (S296)(S296 is connected to S3 and G4 through the Nansha Bridge or the Humen Bridge)

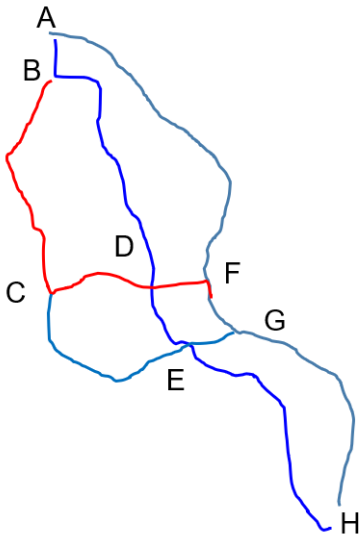

Figure 1: figure of lakes model.

2 Question1

2.1 The Description of Question 1

The traffic volume and average vehicle speed of the Guangzhou-Shenzhen Highway (one checkpoint in G4) were monitored, and the following data were obtained.

Based on the given data, please try to build a model to predict the average vehicle speed on the Guangzhou-Shenzhen Highway, and find out what is the maximum capability of the Guangzhou-Shenzhen Highway (G4).

2.2 Model

2.2.1 Assumptions

1. According to the laws in China, if the cars average speed is lower than 60km/h it means the highway is congestion.
2. The point of intersection of normal prediction curve and congestion prediction curve is the maximum capability.
3. By observation, the scatter points of the given data are similar to the quadratic curve, we use the quadratic function to regression and prediction the average vehicle speed.

2.2.2 Symbols

Symbols	Definition
V	Average vehicle speed
x	Volume of flow

Table 1: Table of Symbols and their definition

2.2.3 Algorithm to predict the average vehicle speed

Algorithm 1 Regression model

```
Read the data from the xls file.
Classify data according to speed.
if speed larger than 60km/h then
    Put the data into the set of normal flow
else
    Put the data into the set of congesion flow
end if
Make use of the tool of matlab and make the polynomial regression.Get the function.
Plot the picture and give the result.
```

2.3 Result

2.3.1 Regression result

For the normal flow we can get the result:

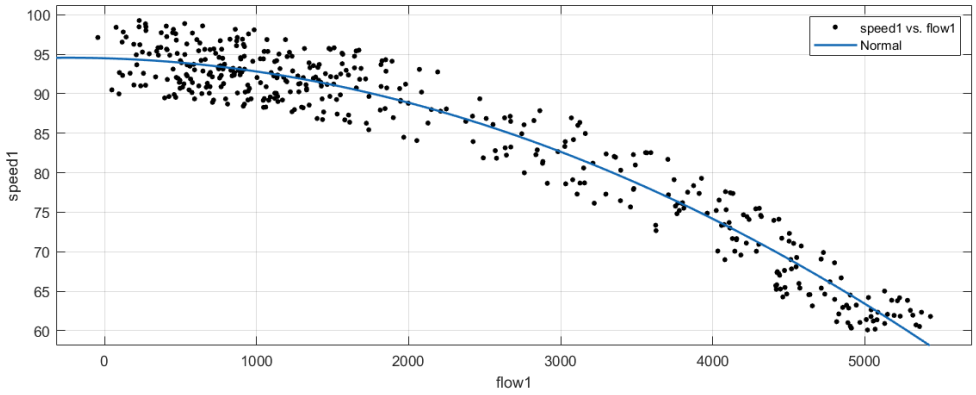

Figure 2: prediction of normal flow

Regression function of the normal flow :

$$V = 0.0000011134 * x^2 - 0.0005443 * x + 94.48 \tag{1}$$

For the congestion flow we can get the result: Congestion function of the normal flow :

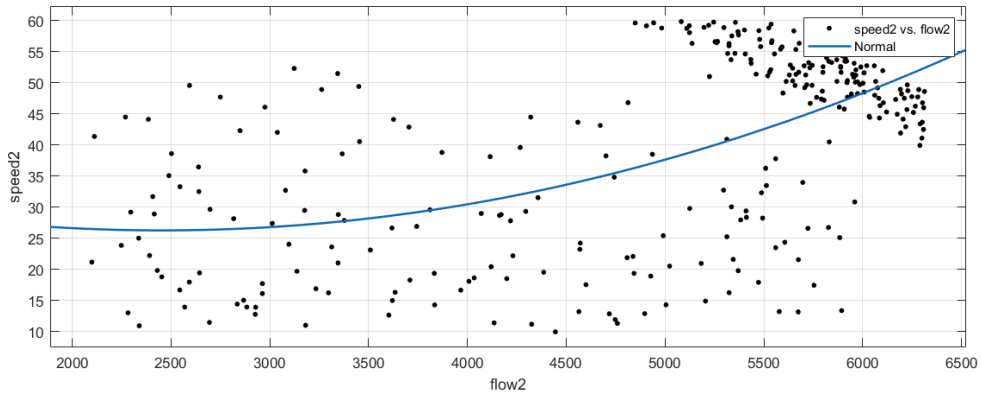

Figure 3: prediction of congestion flow

$$V = 0.000001751 * x^2 - 0.008593 * x + 36.8 \tag{2}$$

SSE of the functions:

Flow	SSE
Normal	3320
Congestion	4473

Table 2: Table of Symbols and their definition

2.3.2 Maximum Capability

We can get the maximum capability by find the intersection of the curves.

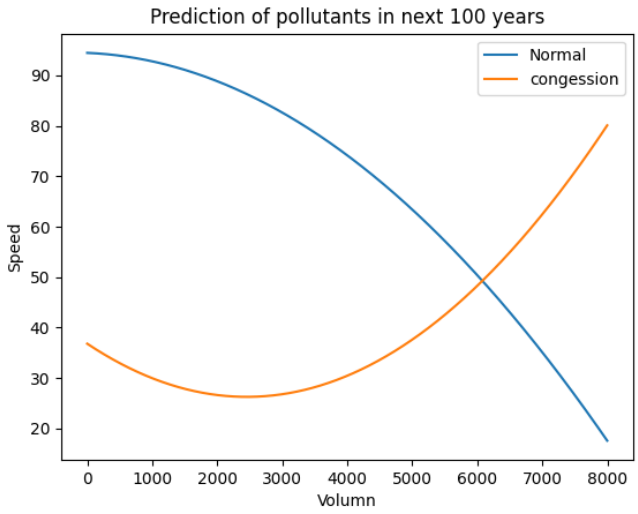

Figure 4: figure of two regression function

Finally its result is **6079**

2.4 effectiveness of model

Regression can be useful in the prediction according the given data. The SSE of two functions are also relatively small. For preciser, we also divide the data into 2 group according to different situations of the traffic flow. Therefore, the model is effectiveness.

3 Question 2

3.1 The Description of Question 2

As the number of vehicles arrived at A grows (number of vehicles/hour), how to assign the traffic flows on all these highways so that the average travel time can be minimized? What is the maximum capacity for the whole highway network from Guangzhou to Shenzhen?

Please build a model to answer the above questions.

3.2 Model

3.2.1 Assumptions

1. The condition and the capability of all the highways are the same.
2. The vehicle density and average speed are the same along the whole road section.(e.g., A-B, B-C, C-D, A-F, F-G, etc. are considered as a section)

3. 2/3 vehicles begin from A, and 1/3 vehicles begin from C. The destination of all vehicle is H.

4. The flow of vehicles arriving at point A and C is constant.

5. We only consider the traffic flows from Guangzhou to Shenzhen, i.e., the vehicles only enter the highway at A or C, and leave at H

6. Possible routes from A-H: A-F-G-H, A-B-D-E-H, A-B-C-H

7. Possible routes from C-H: C-D-H, C-E-H, C-F-H, C-G-H

8. We consider the traffic flows are in equilibrium that means how many vehicles enter the highway at A and C, the same number of vehicles leave at H at the same time.

3.2.2 Symbols

Symbols	Definition
V_y	Speed of traffic in different path $y \in from\ A\ to\ H$
w_y	weight of speed of traffic in different path $y \in from\ A\ to\ H$
L_y	length of speed of traffic in different path $y \in from\ A\ to\ H$
x	Volume of flow enter to A
a	Volume of flow in path AB
b	Volume of flow in path AF
c	Volume of flow in path BC
d	Volume of flow in path BD
e	Volume of flow in path CD
f	Volume of flow in path DF
g	Volume of flow in path FG
h	Volume of flow in path CE
i	Volume of flow in path DE
j	Volume of flow in path EH
k	Volume of flow in path GH
l	Volume of flow leave CD enter DE
m	Volume of flow from C enter CE
n	Volume of flow in path EG

Table 3: Table of Symbols and their definition

3.2.3 Linear optimization model

To minimize average travel time and find the maximum capacity for the whole highway network from Guangzhou to Shenzhen, we can use the library of python **gurobipy**.

First we should add constrains,about flows in different paths according to the assumptions,to the model.

$$x = a + b \tag{3}$$

$$a = c + d \tag{4}$$

$$x = 2 * e + 2 * m \tag{5}$$

$$f = e - l \tag{6}$$

$$g = f + b \tag{7}$$

$$h = m + c \tag{8}$$

$$i = l + d \tag{9}$$

$$j = i + m - n \tag{10}$$

$$k = n + g \tag{11}$$

In order to minimize average travel time, we should build a linear function about it, which means we can not simply make the function like $time = \frac{distance}{speed}$.Therefore, we build a function here to represent minimize average travel time.

$$max \sum w_y V_y \tag{12}$$

This function means we can maximize the sum of speed in different path with a corresponding weight, so the average travel time will be minimized. V_y is the speed of traffic in different paths and w_y is the weight of corresponding speed.

We can calculate V_y by the equation we can on the question1 :

$$V = 0.0000011134 * v^2 - 0.0005443 * v + 94.48 \tag{13}$$

v is the flow volume on the specific path.

We can calculate w_y by this:

$$w_y = \frac{L_y}{\sum_{z \in all\ paths} L_z} \tag{14}$$

L represent each path's length. Therefore, longer path has larger weight. Finally, we can maximize the $\sum w_y V_y$ to get the minimum average travel time.

In order to find out the maximum capacity for the whole highway network from Guangzhou to Shenzhen. We also build a linear function to represent it.

$$maximum\ capacity\ =\ max\{\ a + b + c + d + e + f + g + h + i + j + k + n\} \tag{15}$$

All the variables represent the flow on different paths.And they are limited in smaller than 6079 which is the maximum capacity of each path. Finally, use the constarins above, we can get the result.

3.2.4 Algorithm to minimize average travel time and find maximum capacity

Algorithm 2 Minimize average travel time

1: Initial L_y
2: Calculate the w_y
3: **for** $x = 1$ to 15000 **do**
4: Build the gurobipy objective model
5: Add constrains functions
6: Calculate the V_y
7: optimize the objective function with the help of gurobipy library
8: **end for**
9: Plot the result we get

Algorithm 3 Find maximum capacity

1: **for** $x = 1$ to 15000 **do**
2: Build the gurobipy objective model
3: Add constrains functions
4: optimize the objective function with the help of gurobipy library
5: **end for**
6: Plot the result we get

3.3 Result

3.3.1 Minimum average travel time

After calculation, we can draw a graph about the flow assignment on different path:

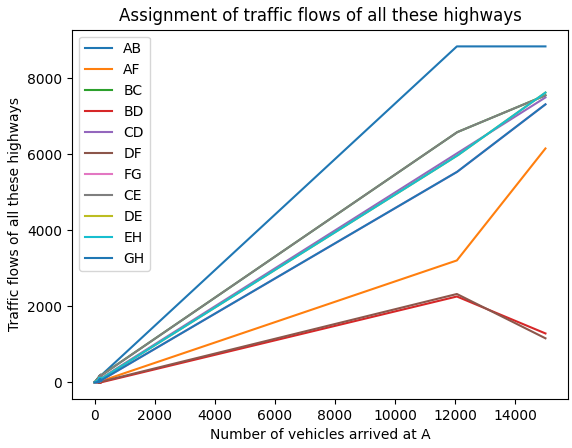

Figure 5: flow assignment on different path

We can see the figure, as the number of vehicles arrived at A increasing, all the paths' flow increase at first, but some of then will decrease since some vehicles are assigned to another way.

We can also observe the figure of corresponding speed.:

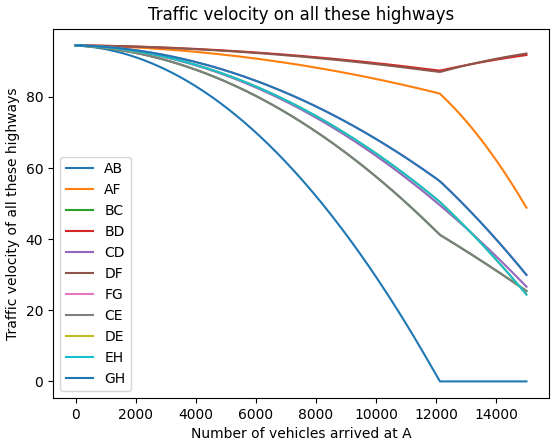

Figure 6: different path speed

3.3.2 Maximum capacity

After calculation, we can draw a graph about the flow assignment on different path:

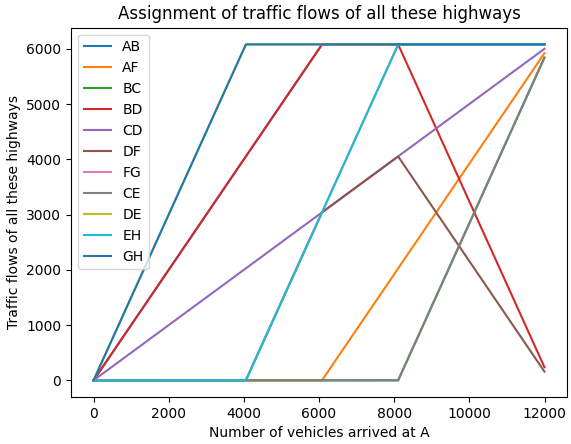

Figure 7: flow assignment on different path

Observing the result, we can find that as the number of vehicles arrived at A increasing more and more roads are used to contain the increasing number of vehicles. However, there are also some paths are abandoned since they can not link more paths to contain more vehicles than the others.

Then we can find the maximum capacity by adding them together:

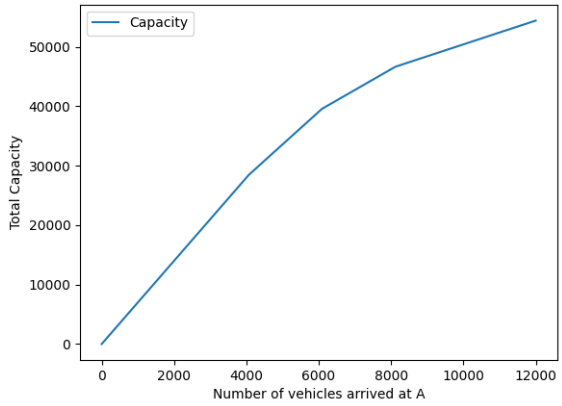

Figure 8: maximum capacity as the number of vehicles arrived at A increasing

3.4 effectiveness of model

This linear optimization model can be useful for us to solve the question like find the minimum and maximum, which only require us find a linear function and its constrains. Also the result the model is reliable since it can be explained by the daily logic.

4 Conclusion

To solve the question 1 we use the regression model by taking advantage of given data to find out the regression function and use the function to predict. By dividing the data into two group and getting two regression functions' intersection, we find the maximum capacity. To solve the question 2, we make use of the python library gurobipy and build corresponding objective linear function and constrains functions to solve the problem of minimum average travel time and maximum capacity