Control Principles

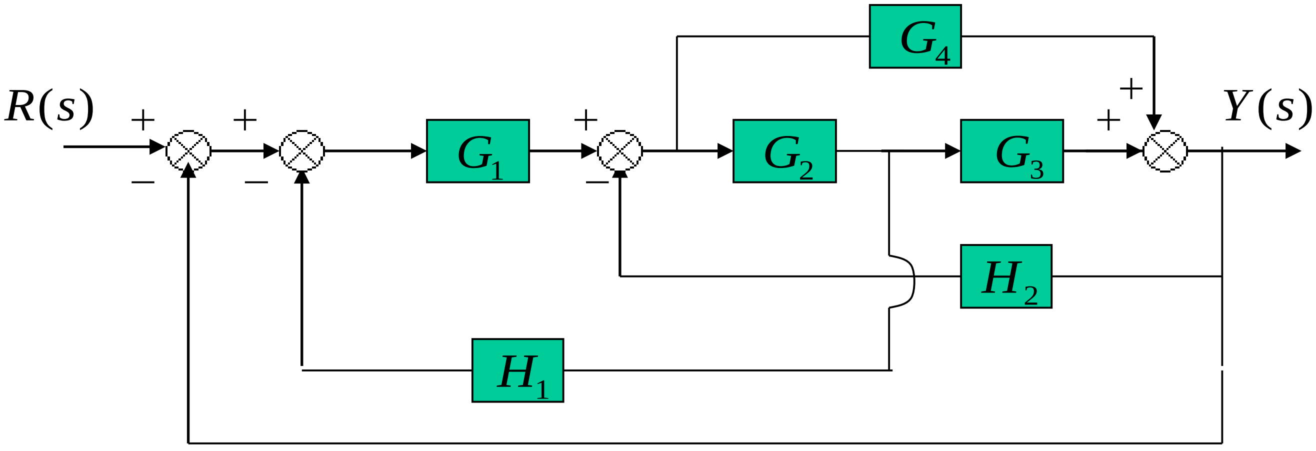
Block Diagram

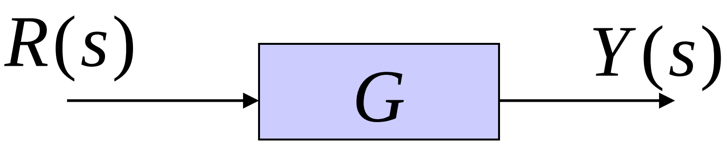
Block diagram

Transfer Function

Consists of Blocks

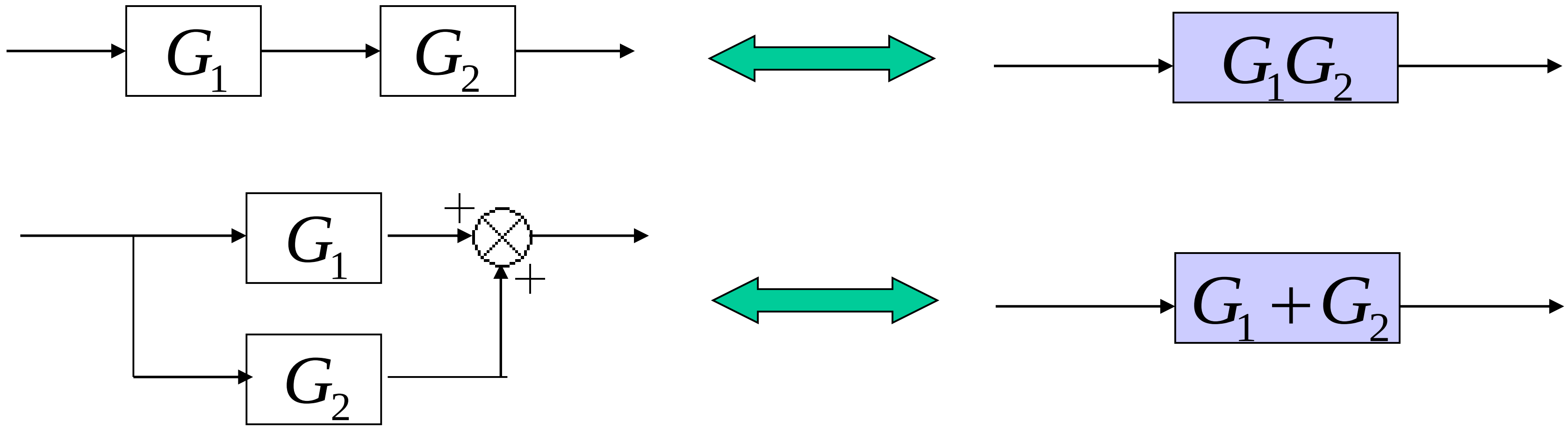
Can be reduced



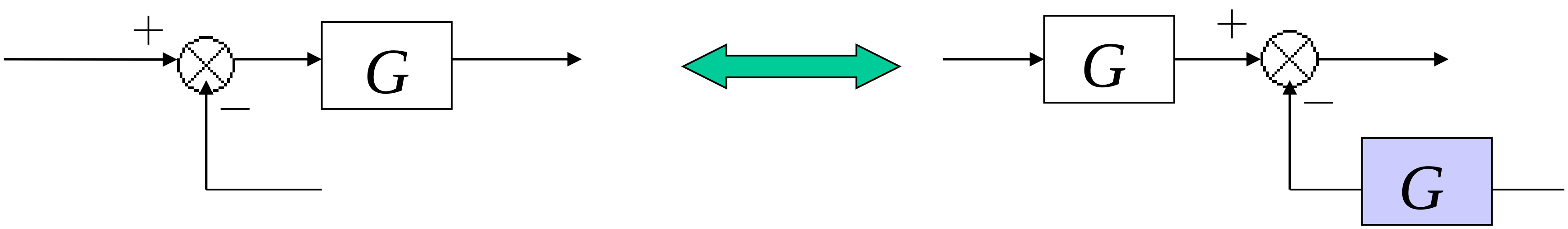


Reduction techniques

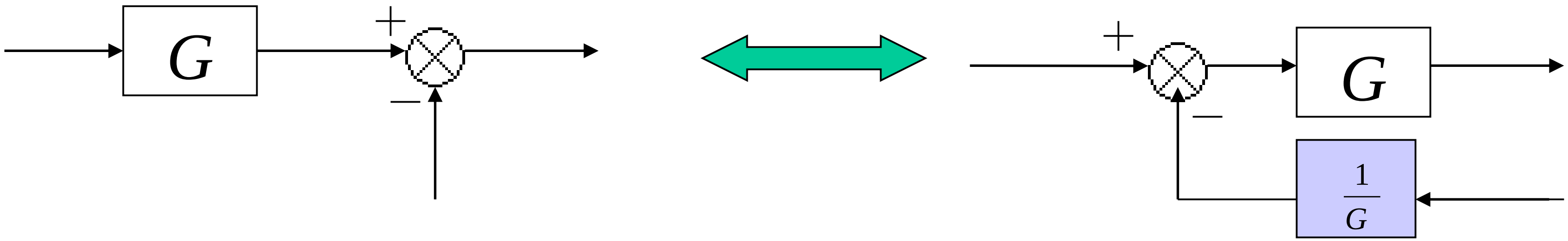
1. Combining blocks in cascade or in parallel



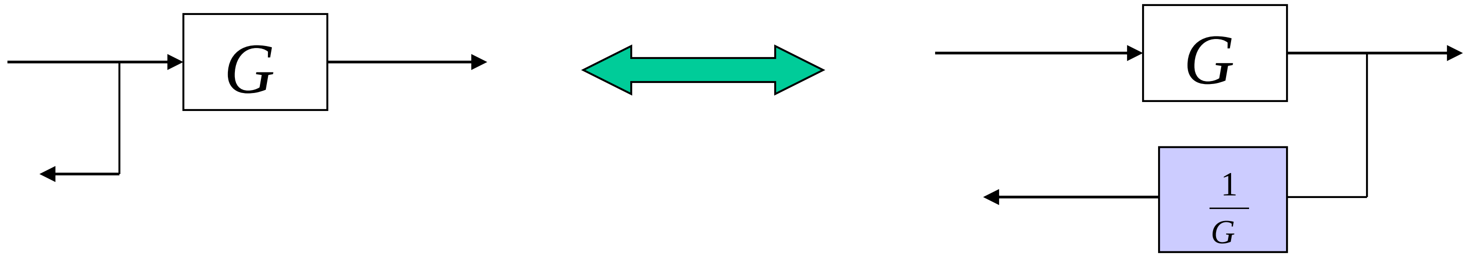
2. Moving a summing point behind a block



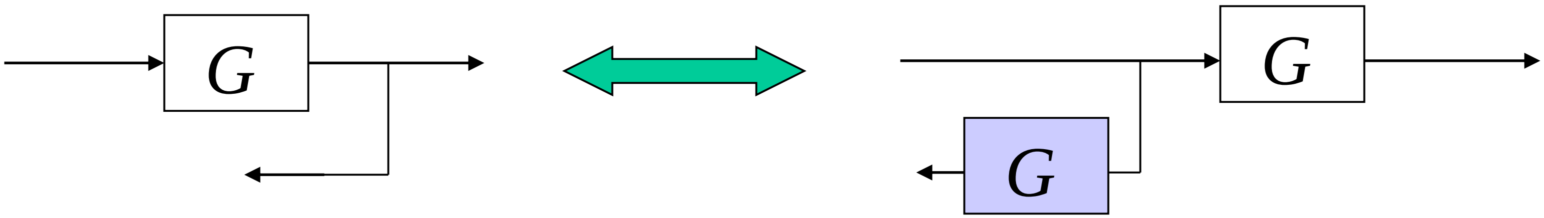
3. Moving a summing point ahead of a block



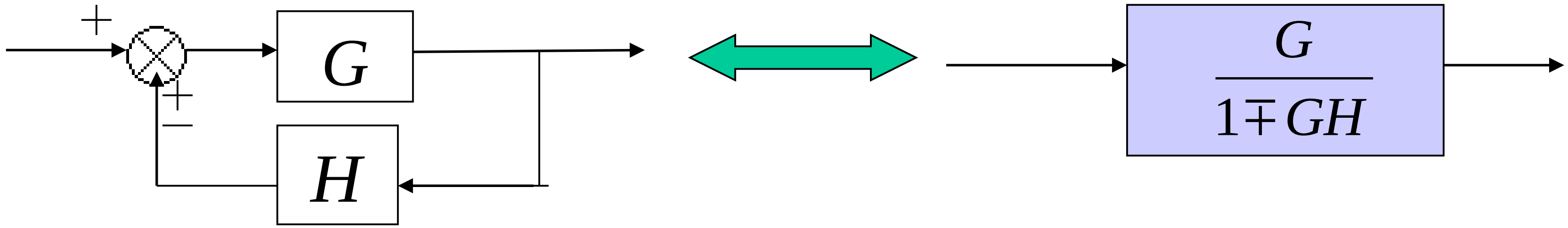
4. Moving a pickoff point behind a block

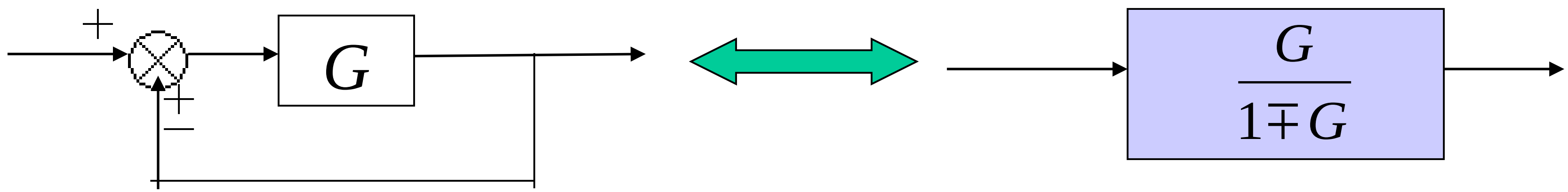


5. Moving a pickoff point ahead of a block



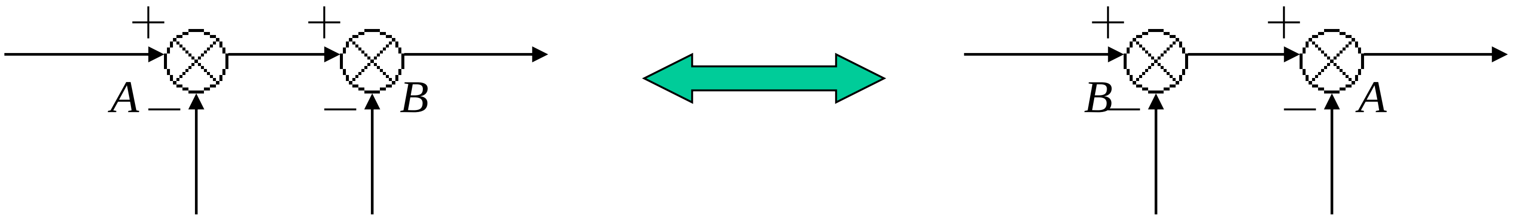
6. Eliminating a feedback loop





$$H = 1$$

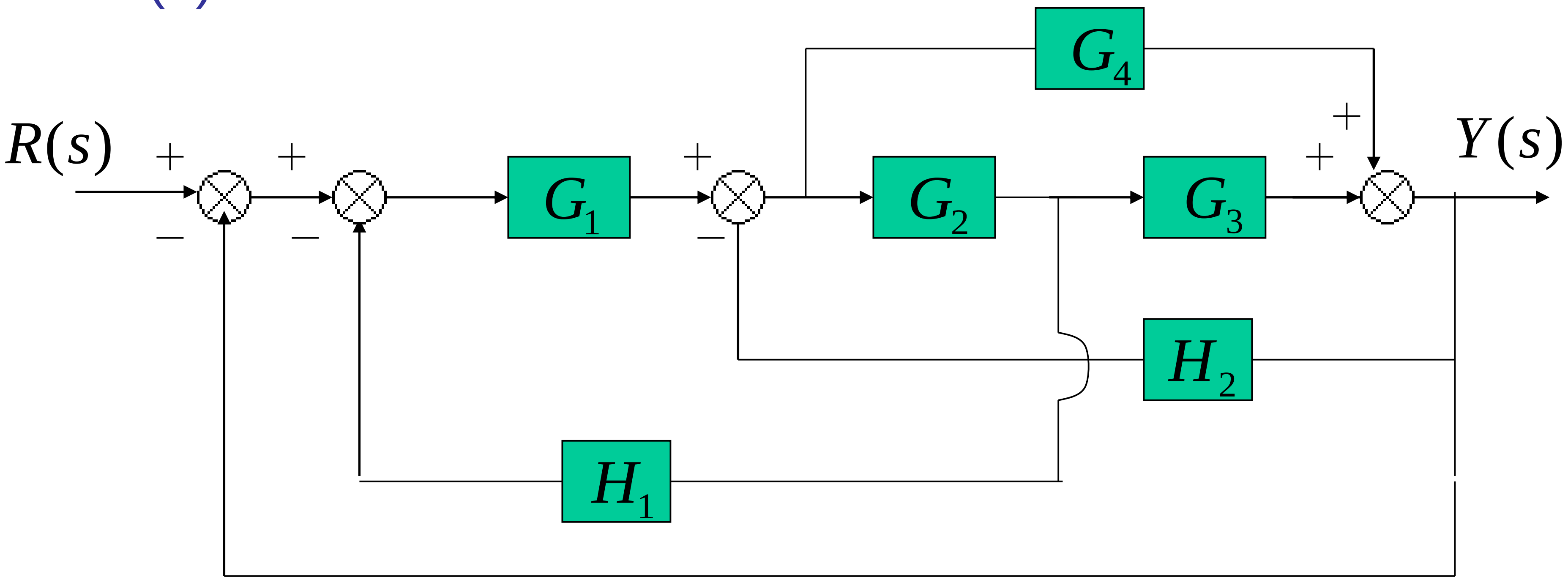
7. Swap with two neighboring summing points

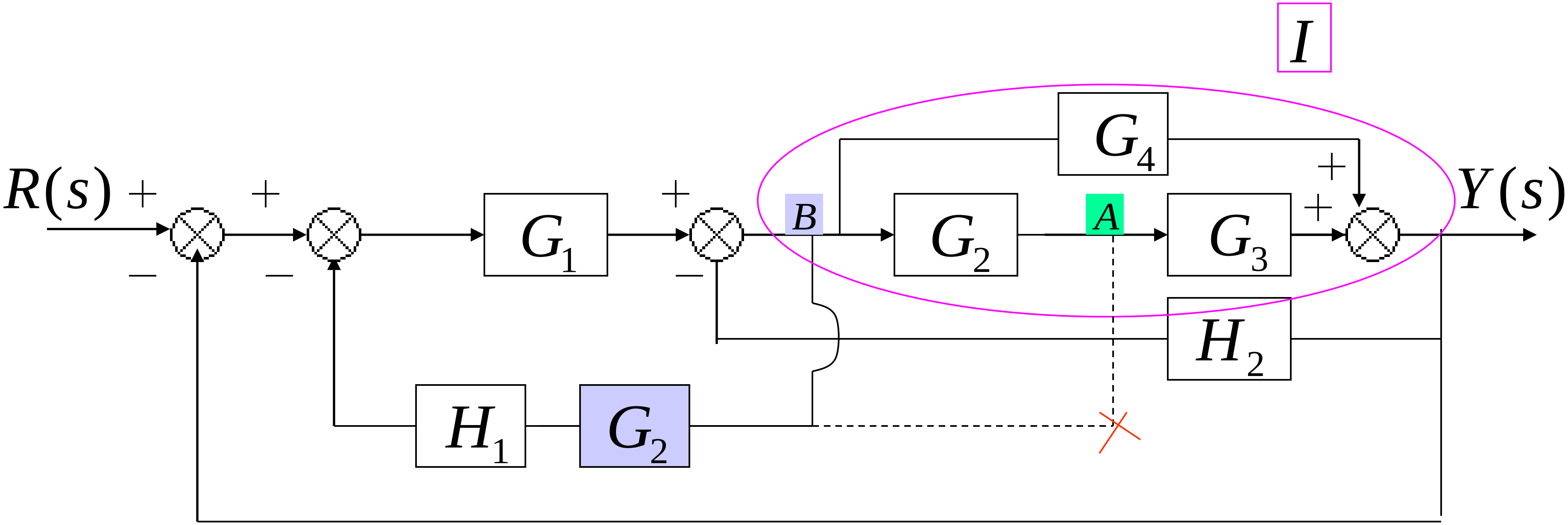


Example 1

Find the transfer function of the following block diagrams

(a)

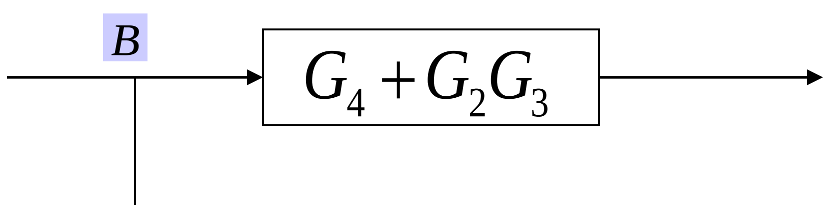


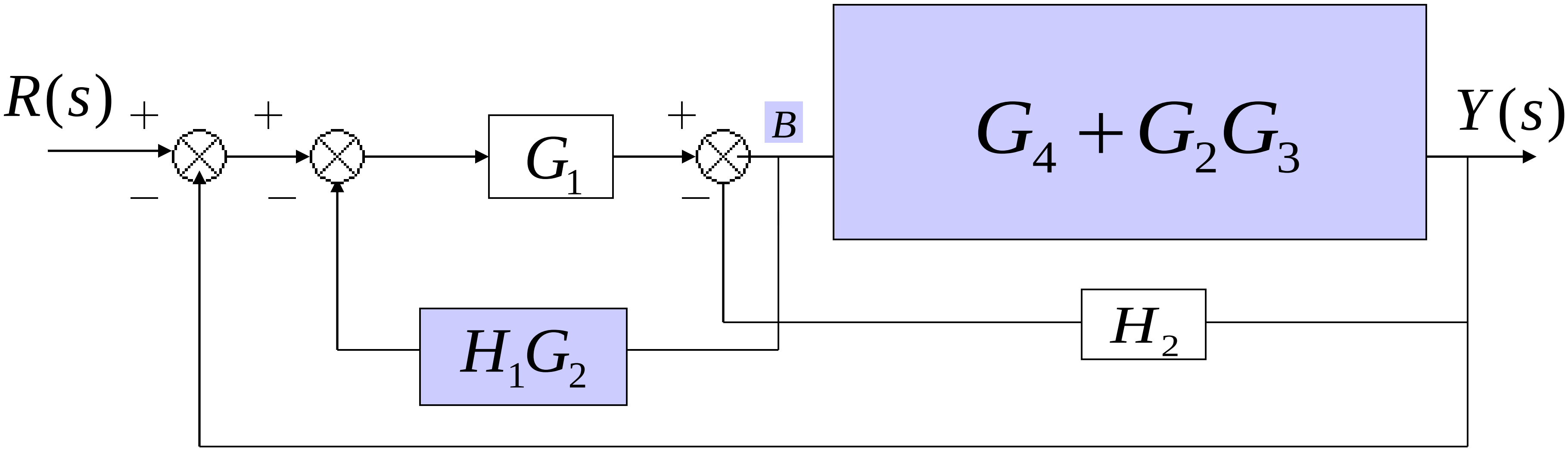


Solution:

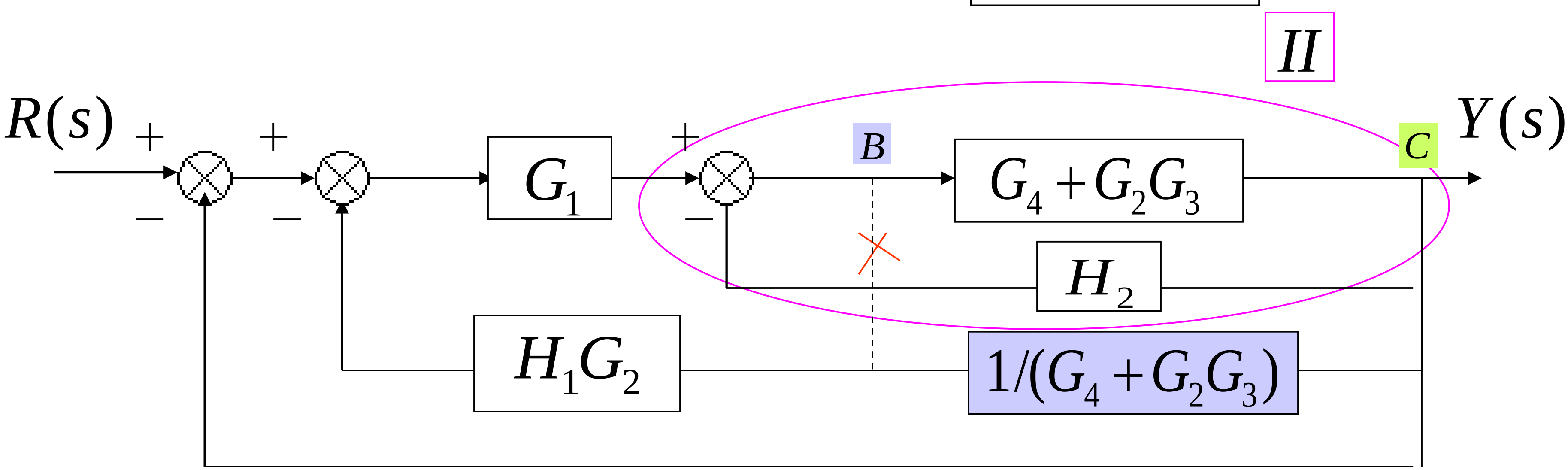
1. Moving pickoff point A ahead of block $\boxed{G_2}$

2. Eliminate loop I & simplify

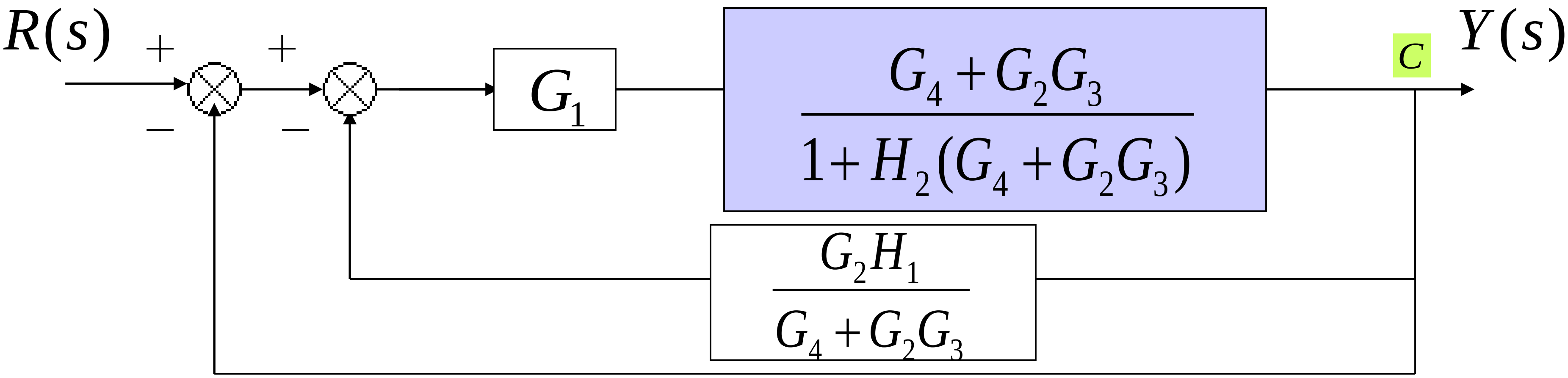




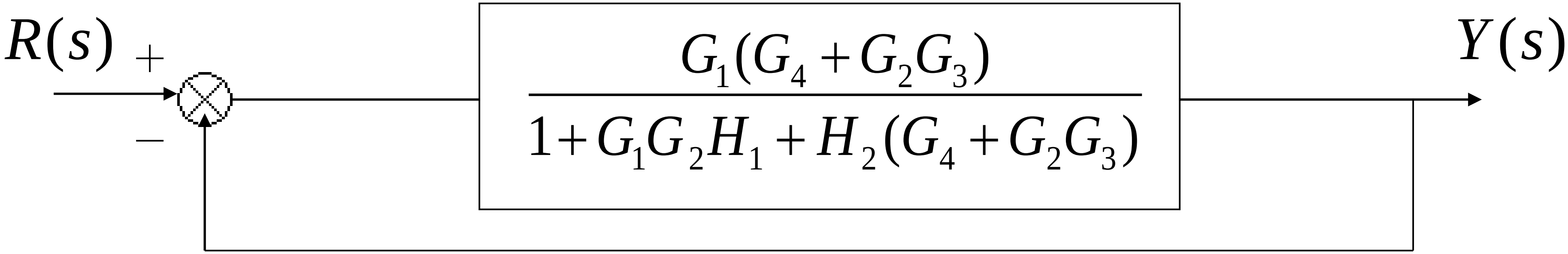
3. Moving pickoff point B behind block $\boxed{G_4+G_2G_3}$



4. Eliminate loop III

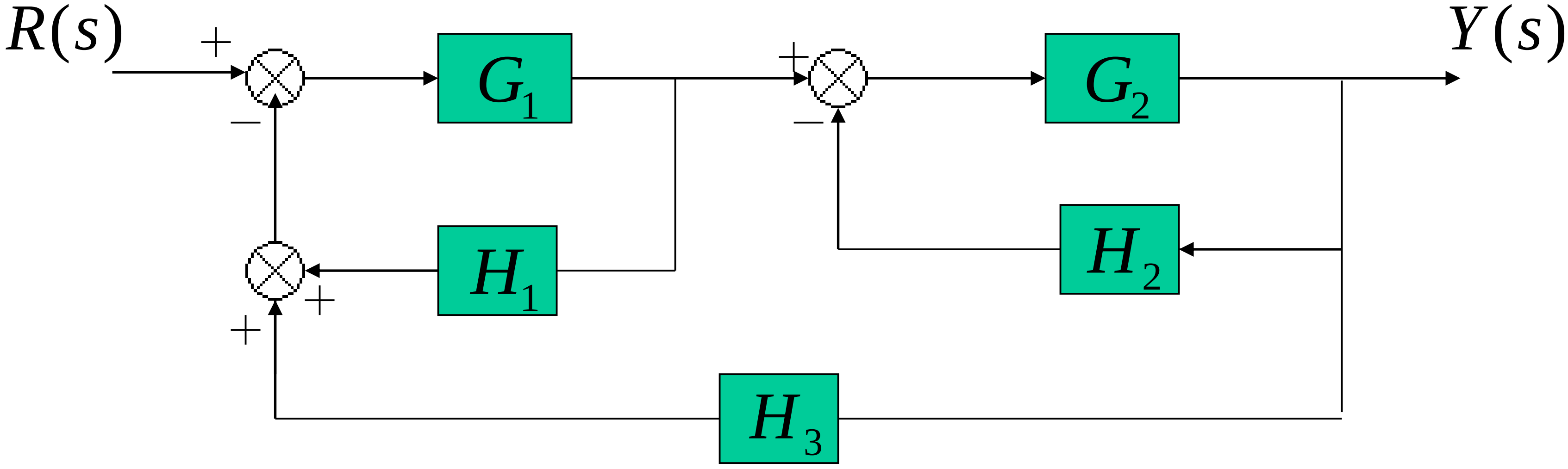


Using rule 6



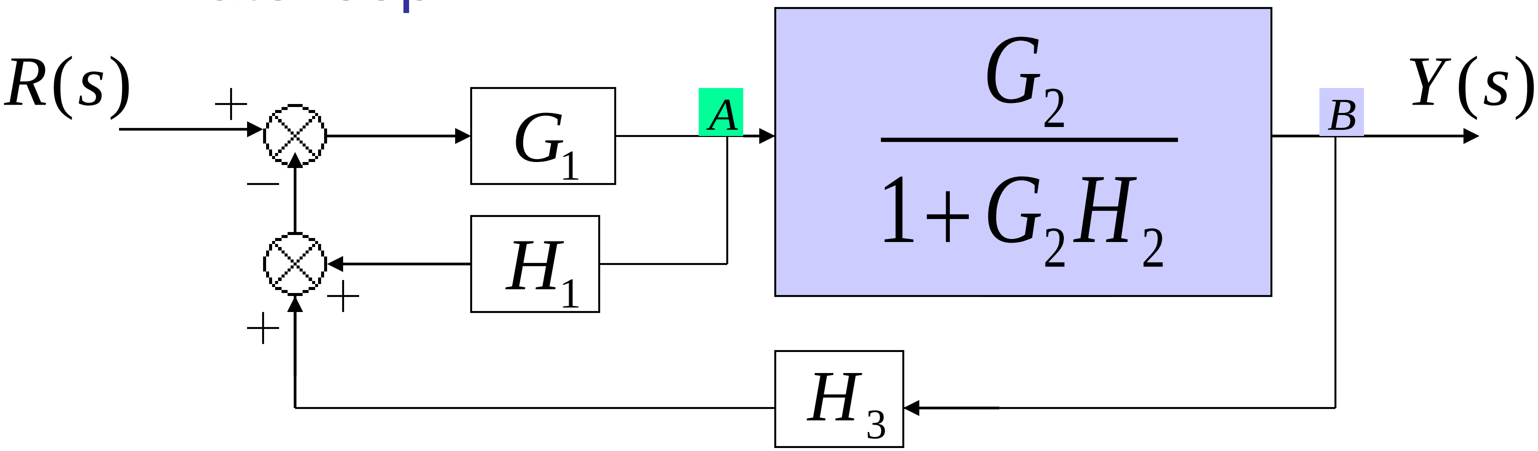
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1+H_2(G_4+G_2G_3)+G_1(G_4+G_2G_3)}$$

(b)

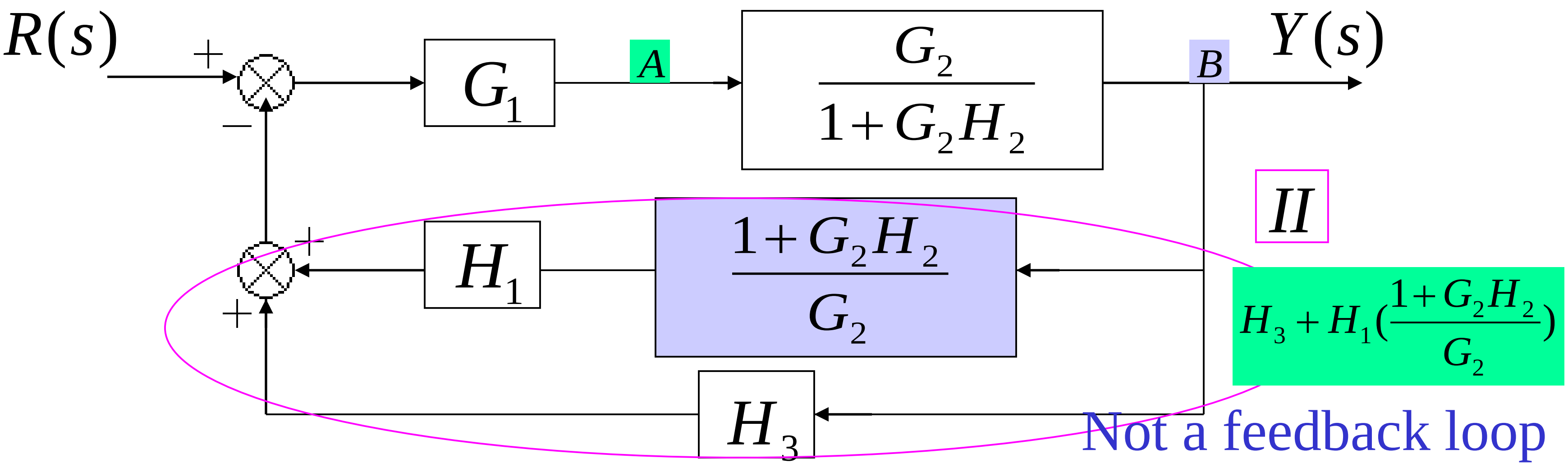


Solution:

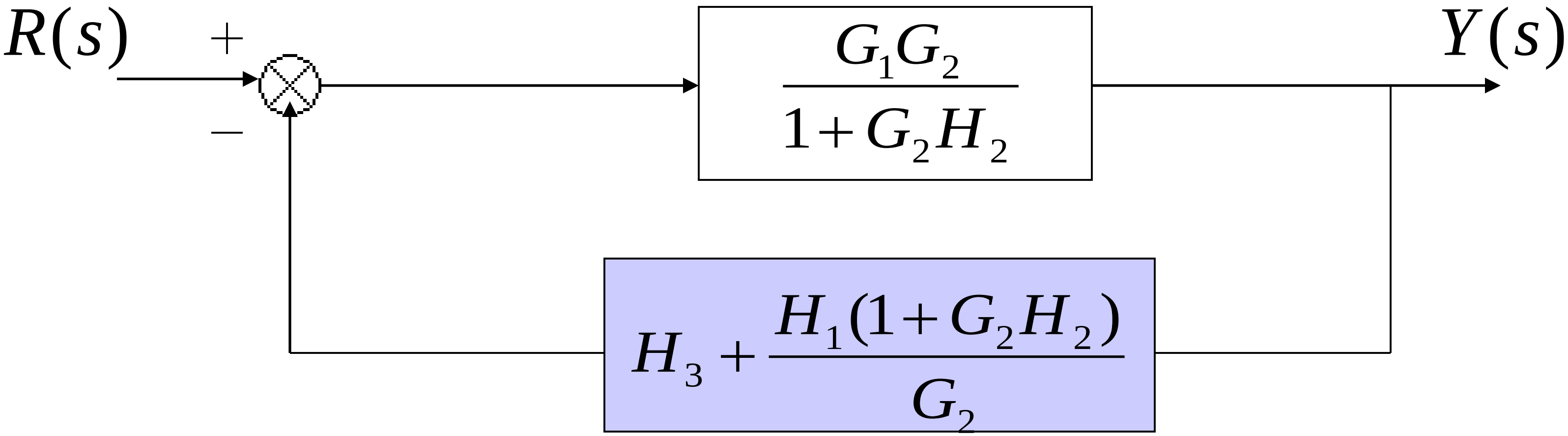
1. Eliminate loop I



2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



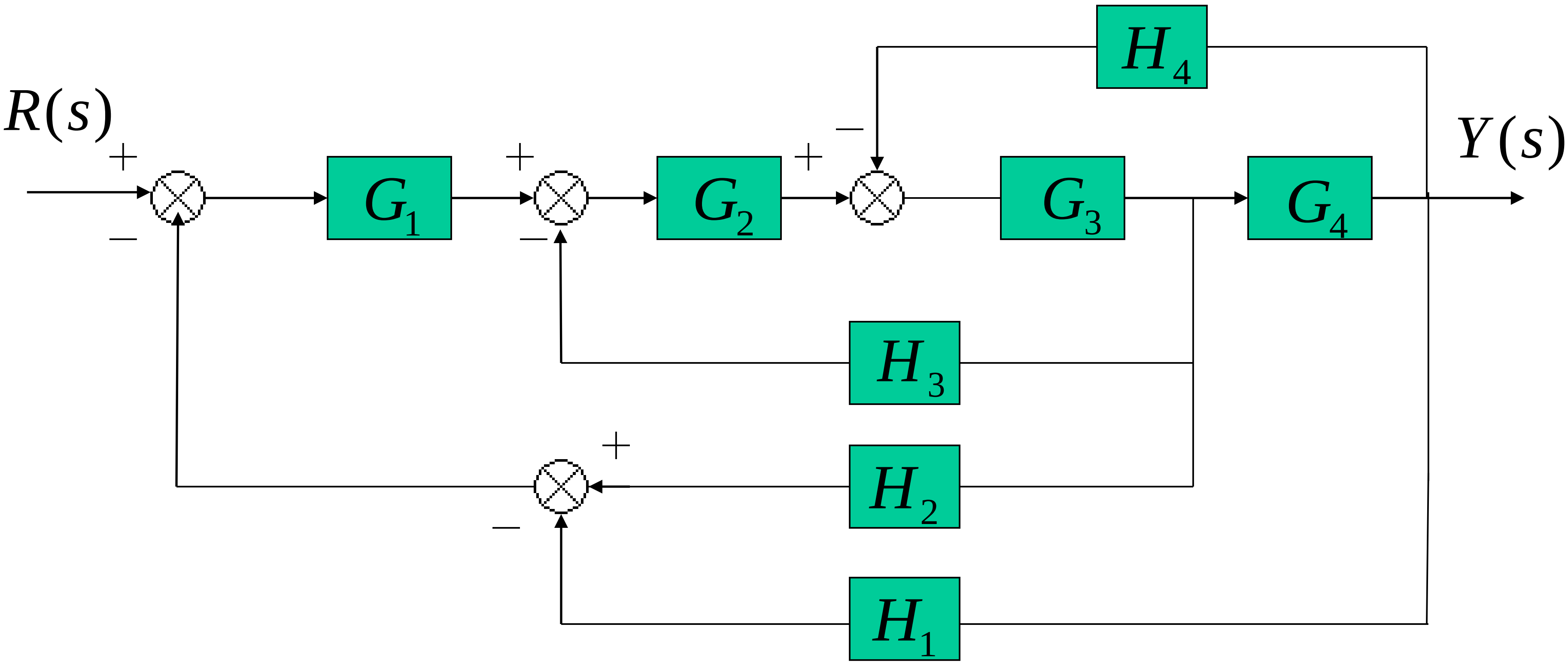
3. Eliminate loop II



Using rule 6

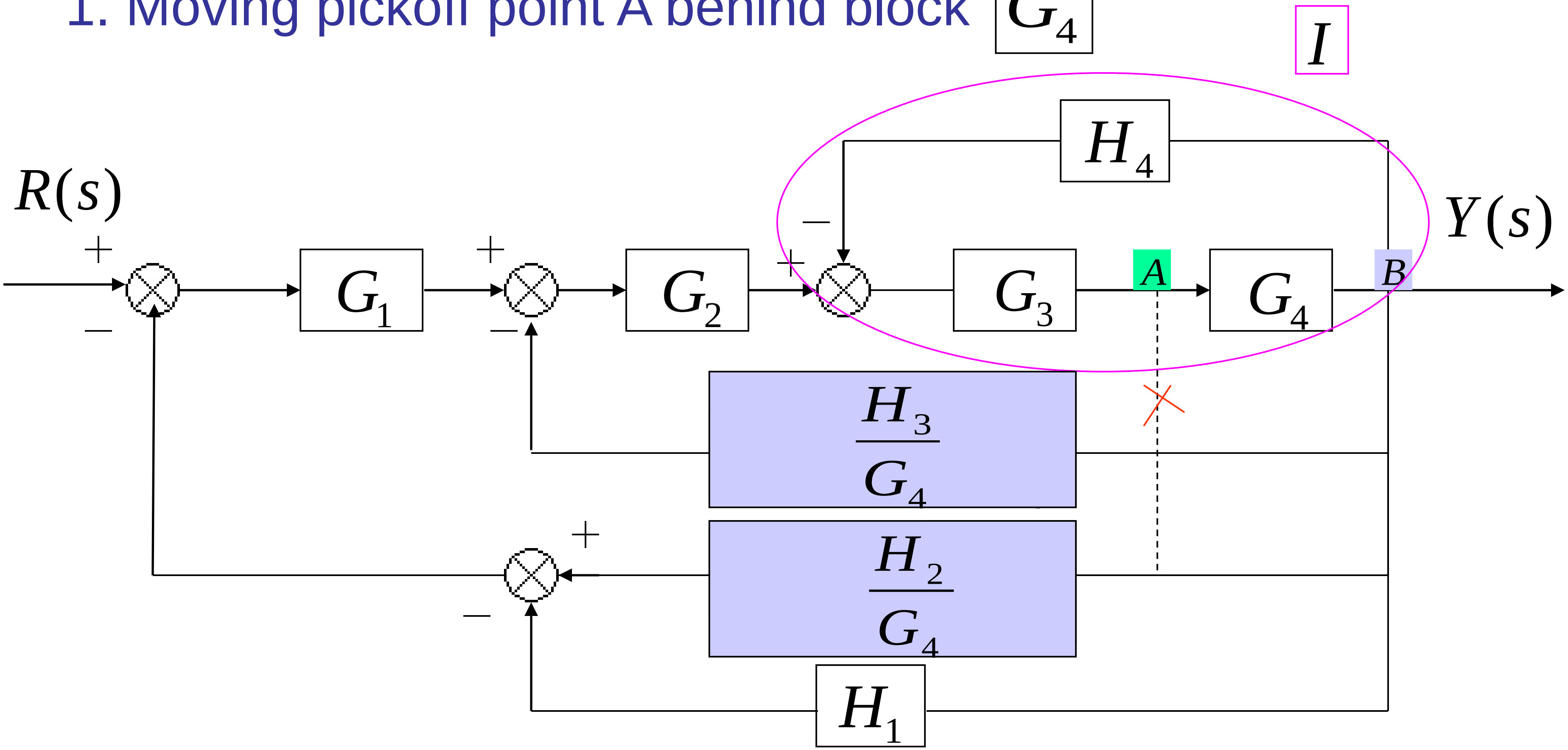
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2}{1+G_2H_2+G_1G_2H_3+G_1H_1+G_1G_2H_1H_2}$$

(c)

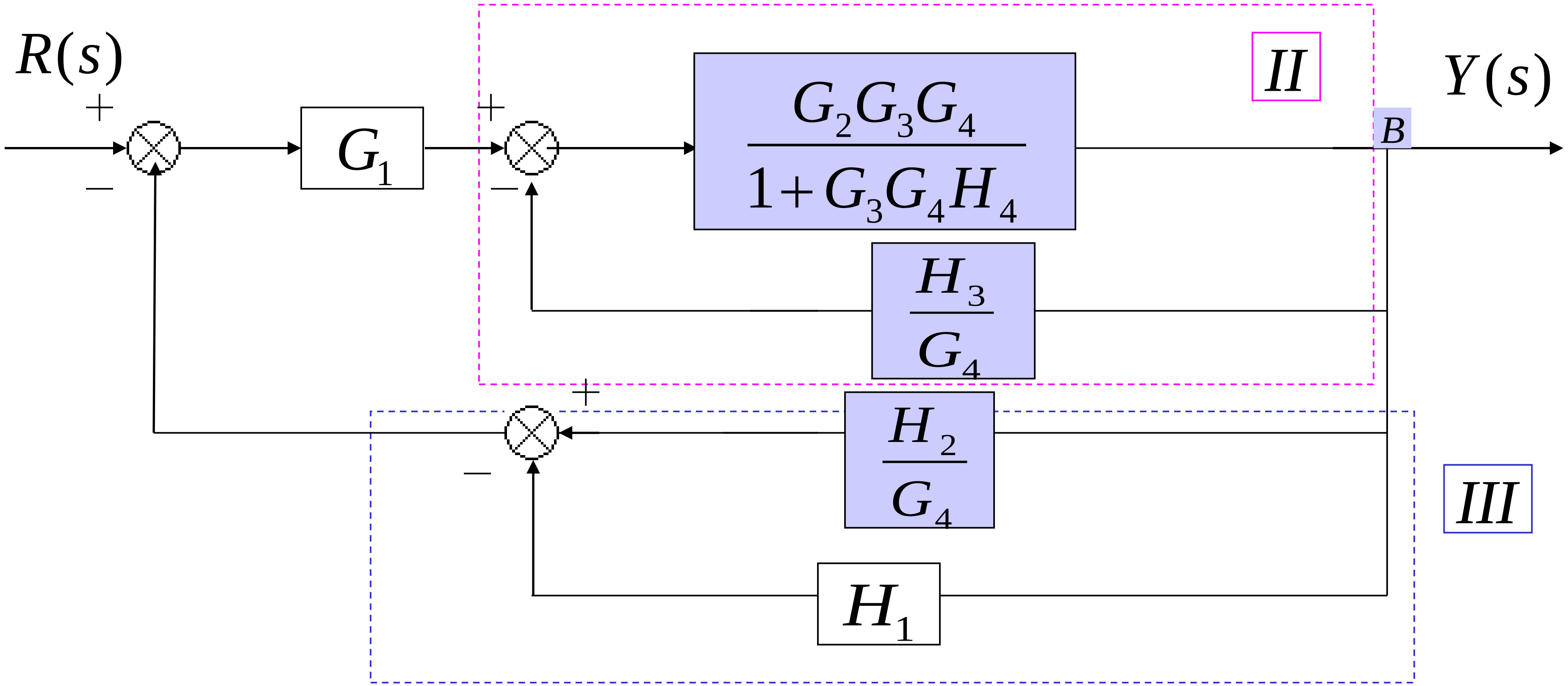


Solution:

1. Moving pickoff point A behind block $\boxed{G_4}$



2. Eliminate loop I and Simplify



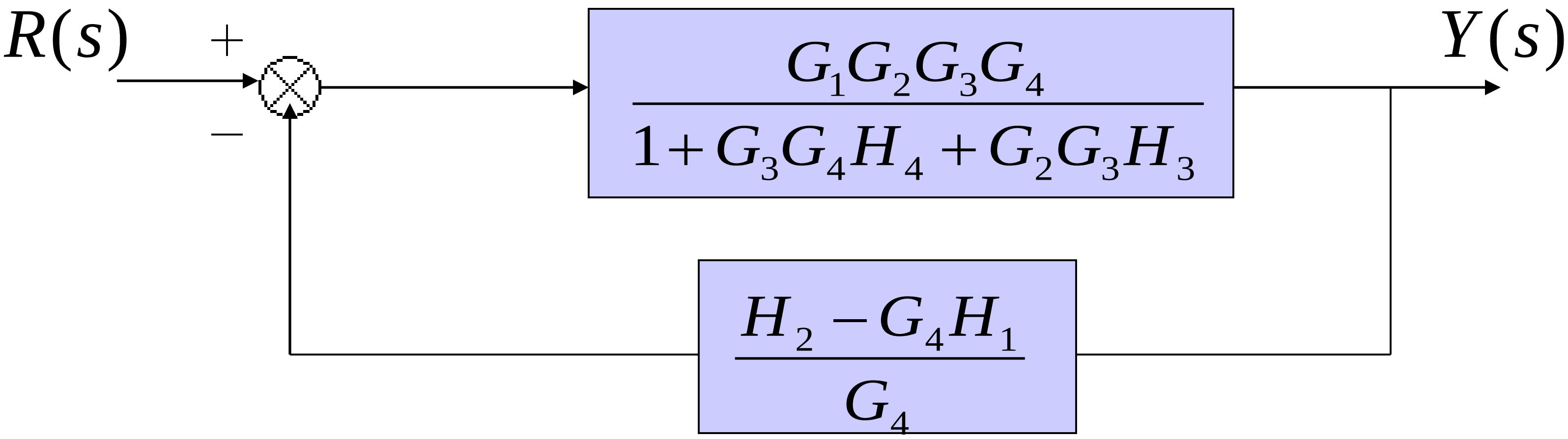
II feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III Not feedback

$$\frac{H_2-G_4H_1}{G_4}$$

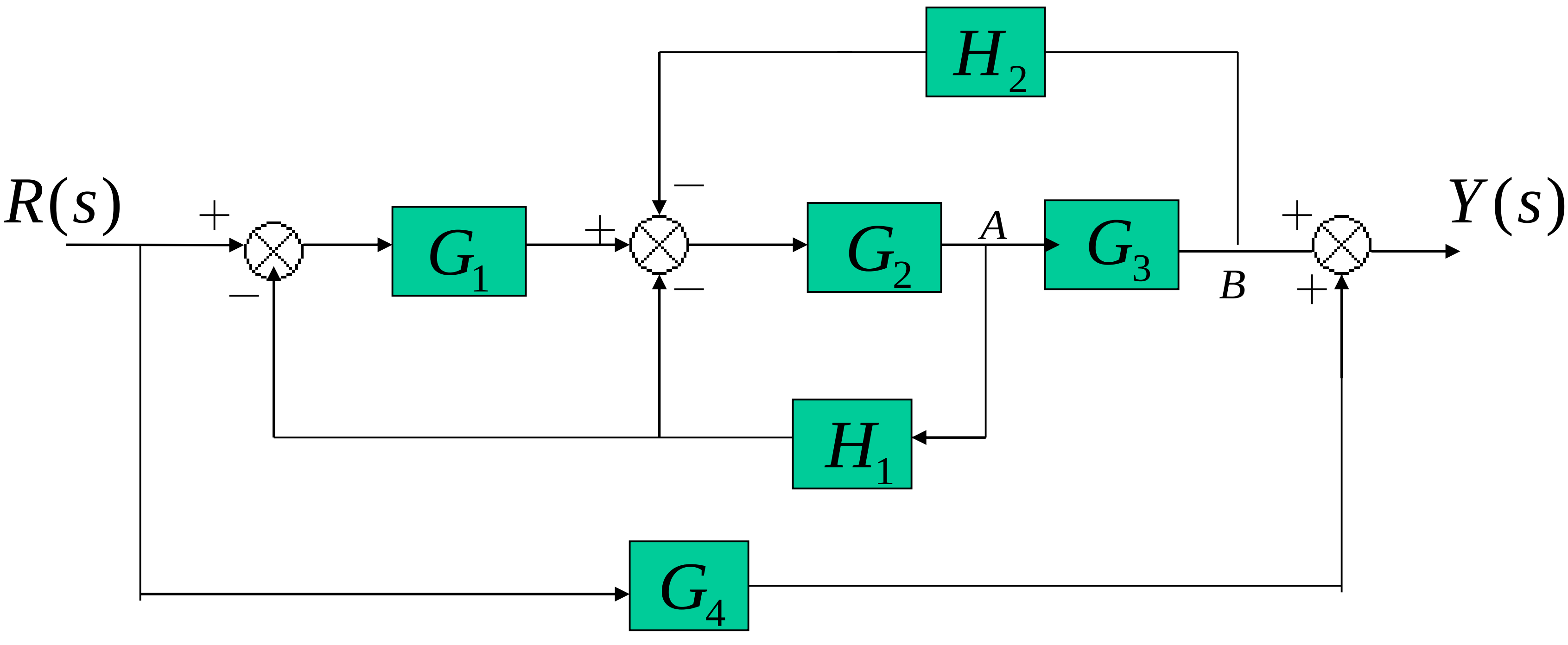
3. Eliminate loop II & IIII



Using rule 6

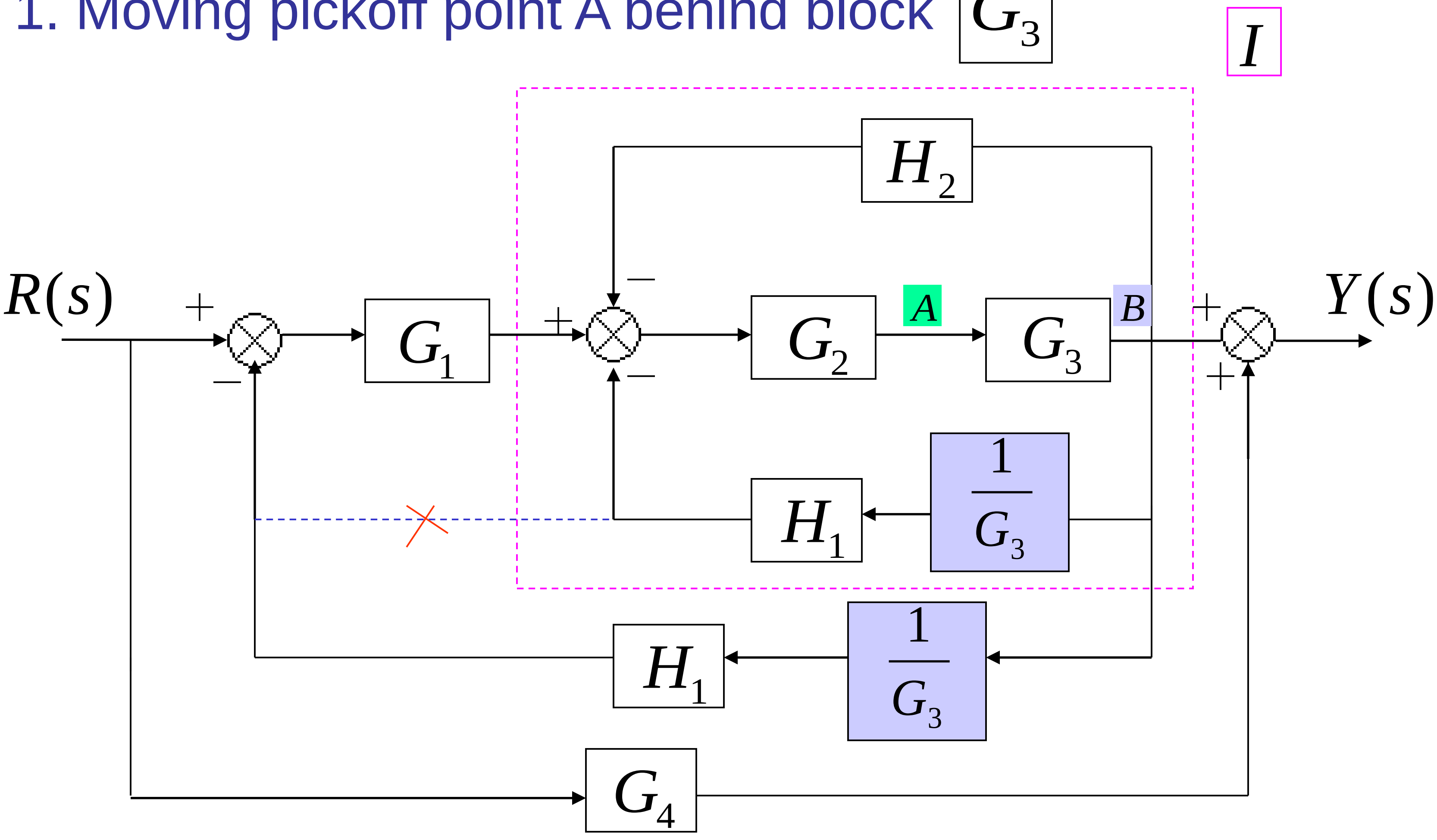
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

(d)

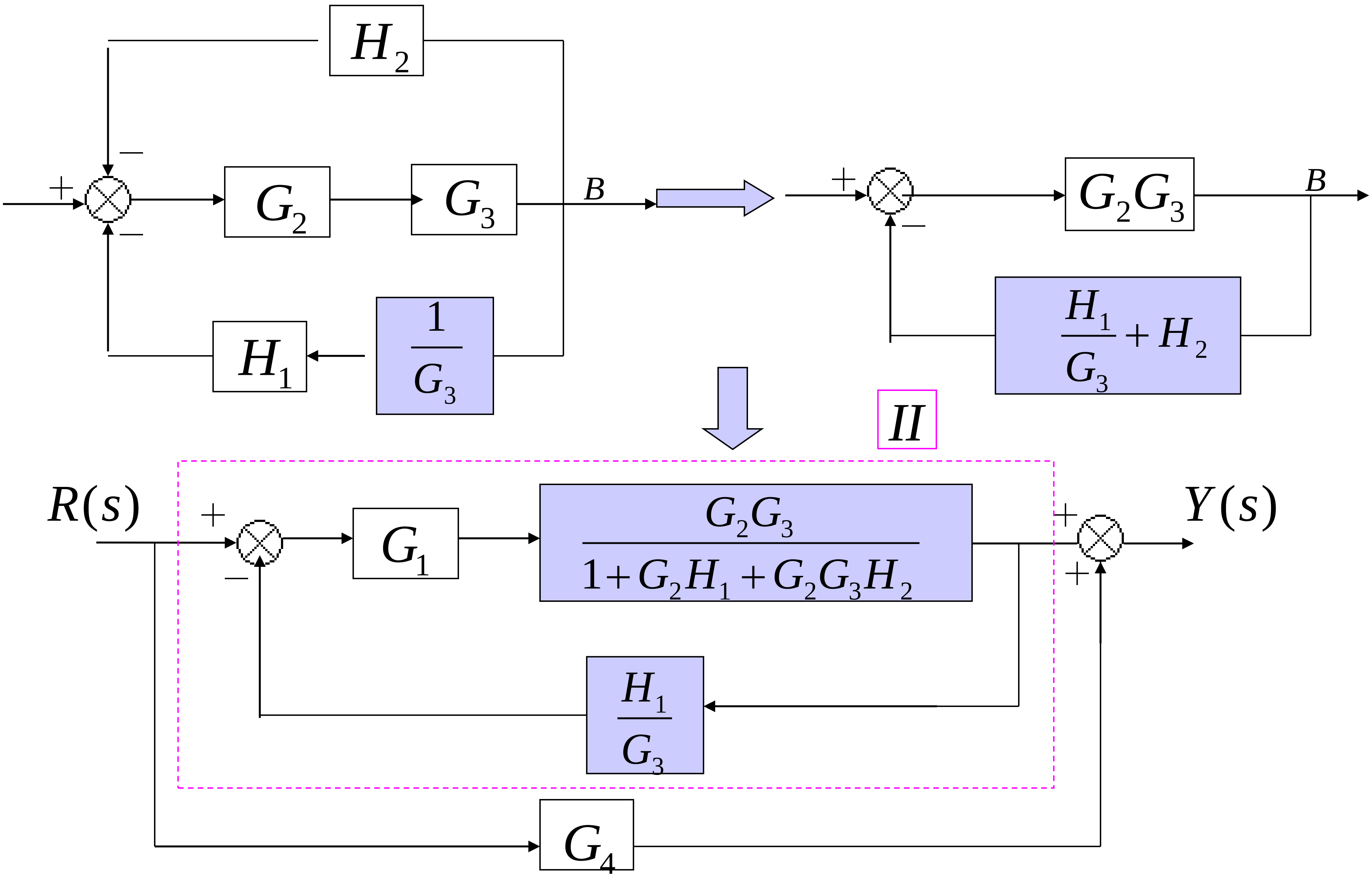


Solution:

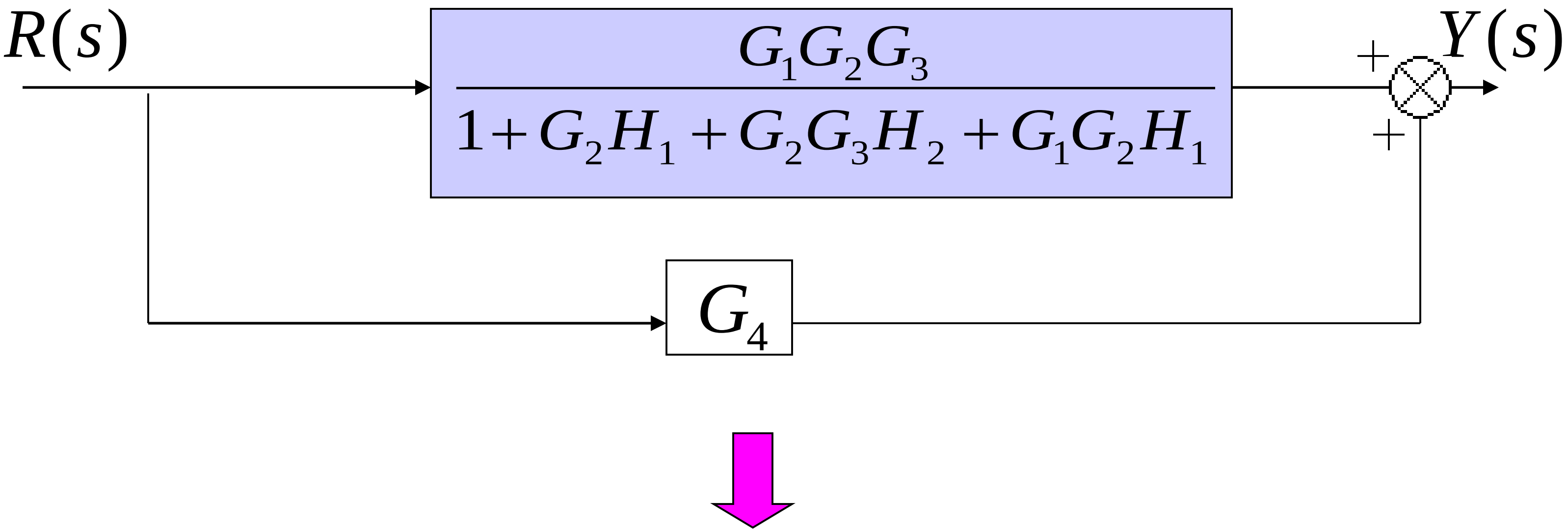
1. Moving pickoff point A behind block $\boxed{G_3}$



2. Eliminate loop I & Simplify



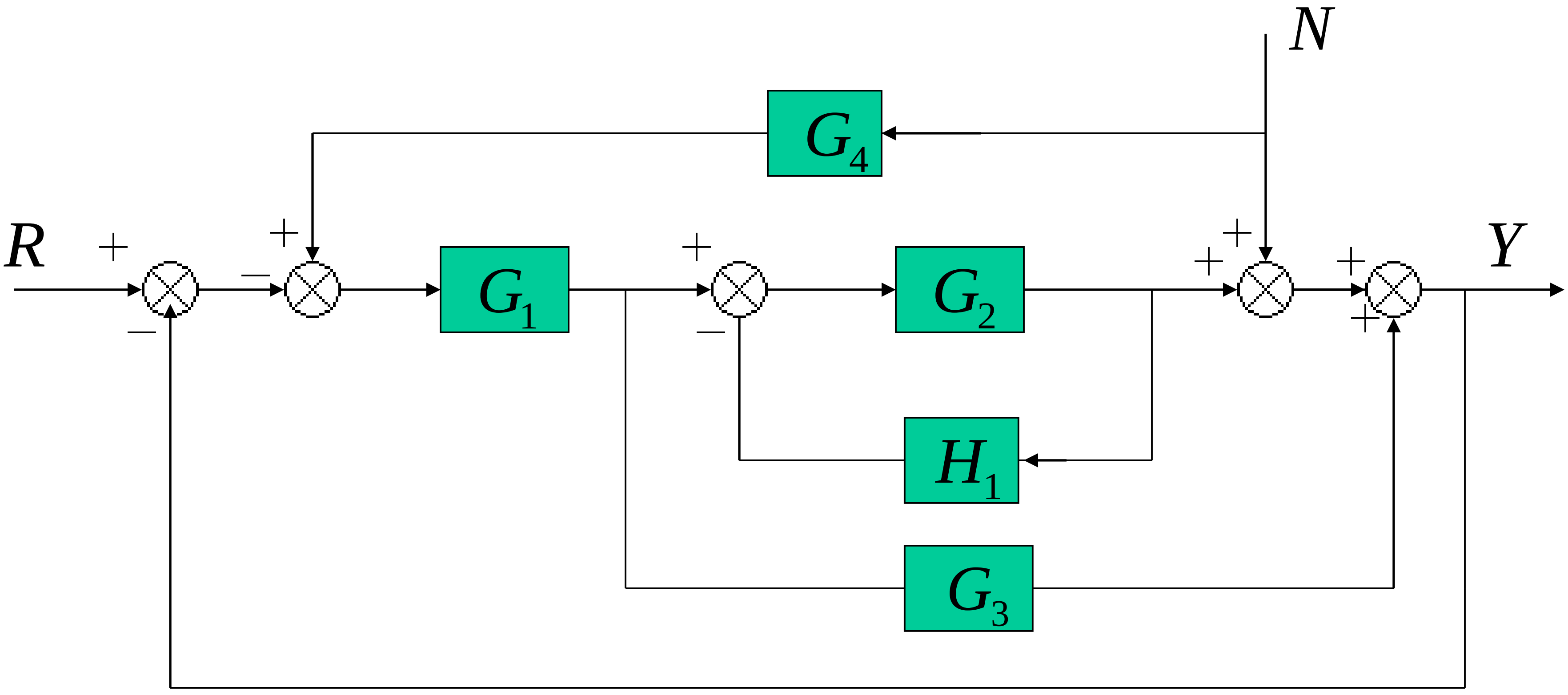
3. Eliminate loop II



$$T(s)=\frac{Y(s)}{R(s)}=G_4+\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2+G_1G_2H_1}$$

Example 2

Determine the effect of R and N on Y in the following diagram



In this linear system, the output Y contains two parts, one part is related to R and the other is caused by N:

$$Y = Y_1 + Y_2 = T_1 R + T_2 N$$

If we set N=0, then we can get Y1:

$$Y_1 = Y_{N=0} = T_1 R$$

The same, we set R=0 and Y2 is also obtained:

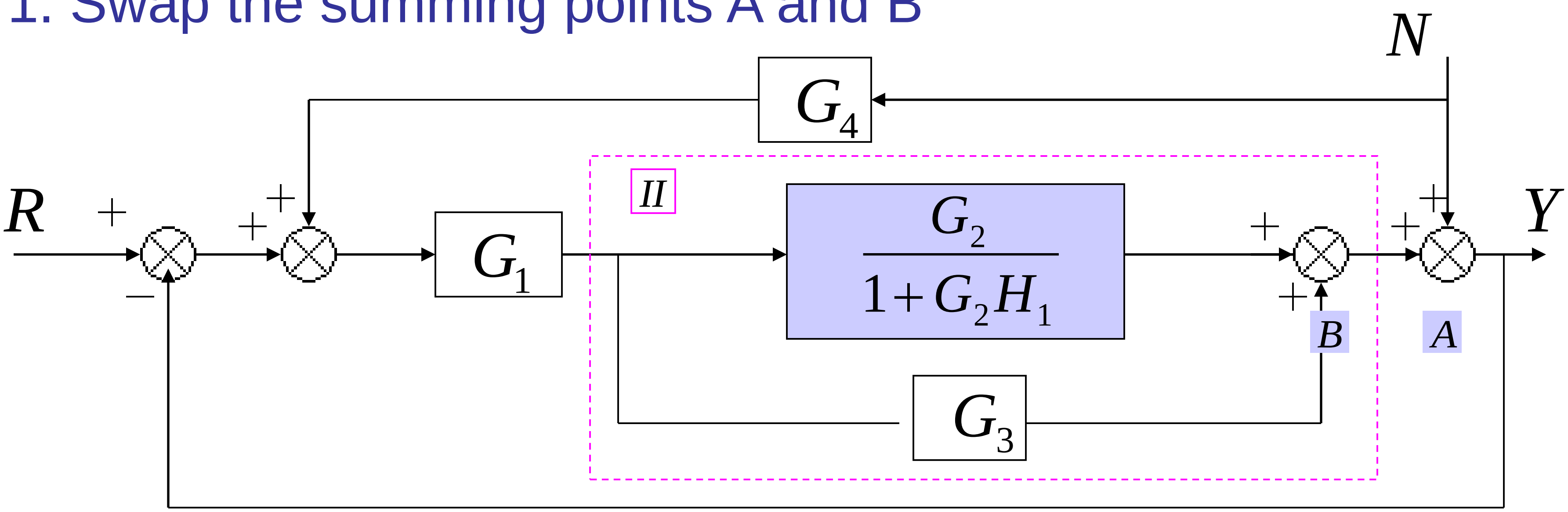
$$Y_2 = Y_{R=0} = T_2 N$$

Thus, the output Y is given as follows:

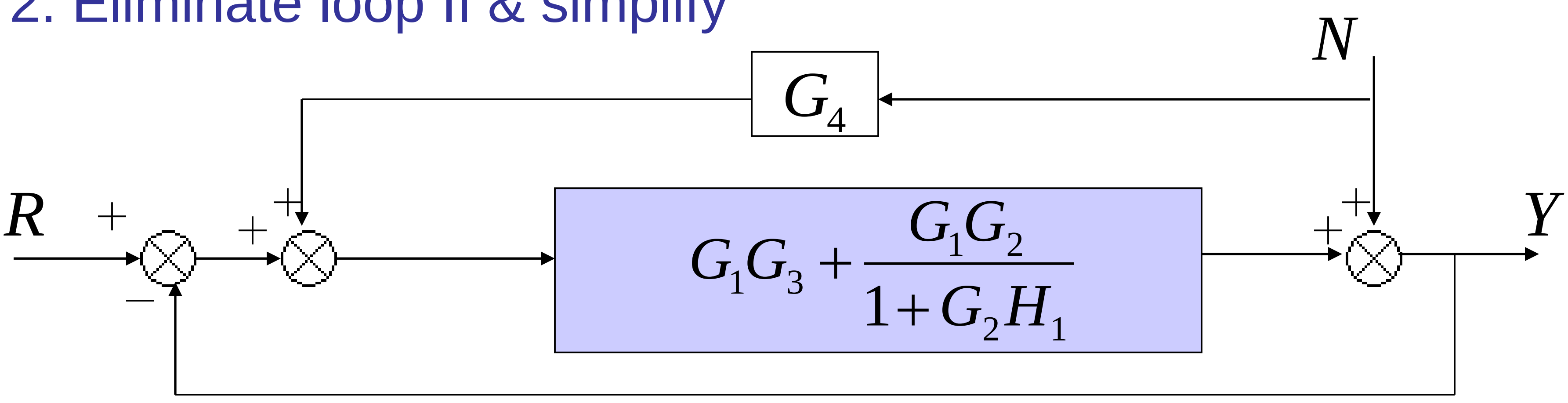
$$Y = Y_1 + Y_2 = Y_{N=0} + Y_{R=0}$$

Solution:

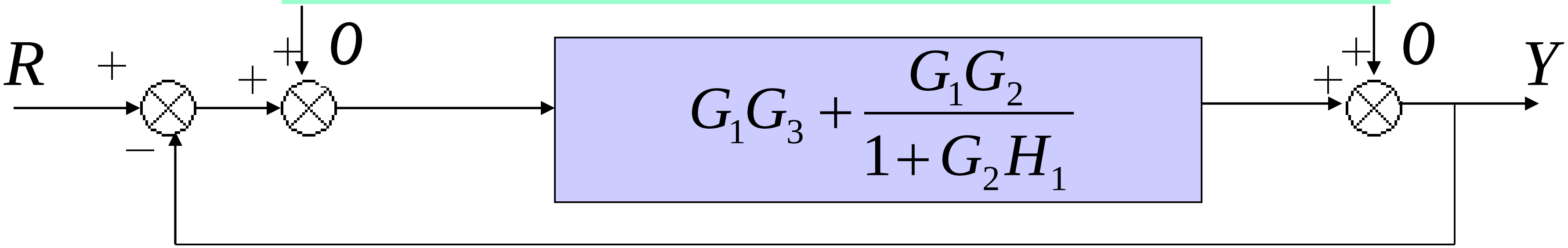
1. Swap the summing points A and B



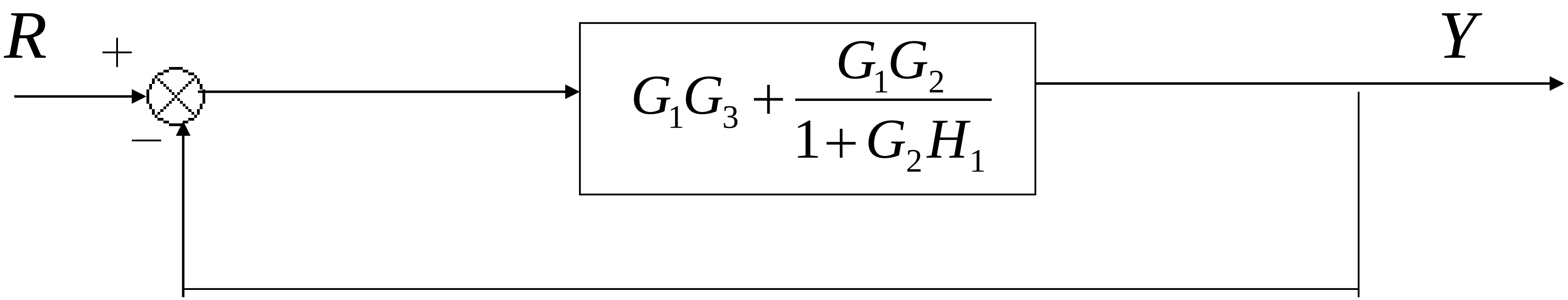
2. Eliminate loop II & simplify



Rewrite the diagram:



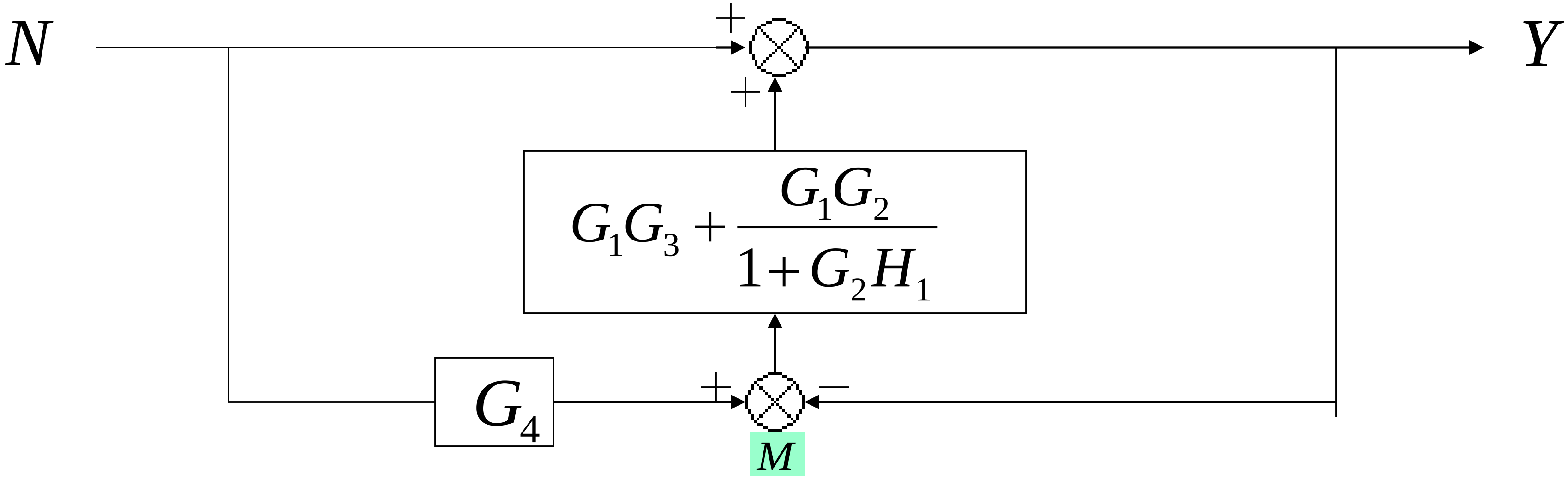
3. Let N=0



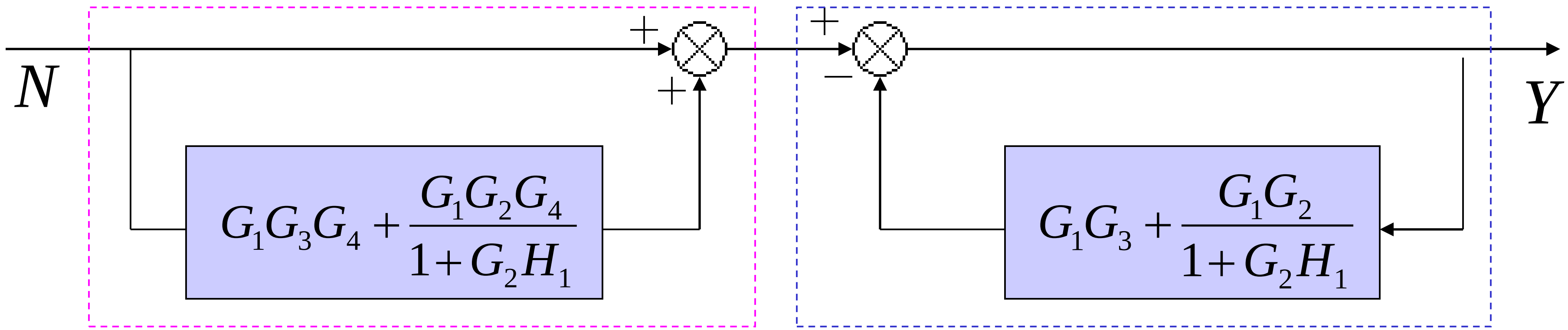
We can easily get Y_1

$$Y_1 = \frac{G_1G_2 + G_1G_3 + G_1G_2G_3H_1}{1 + G_2H_1 + G_1G_2 + G_1G_3 + G_1G_2G_3H_1} R$$

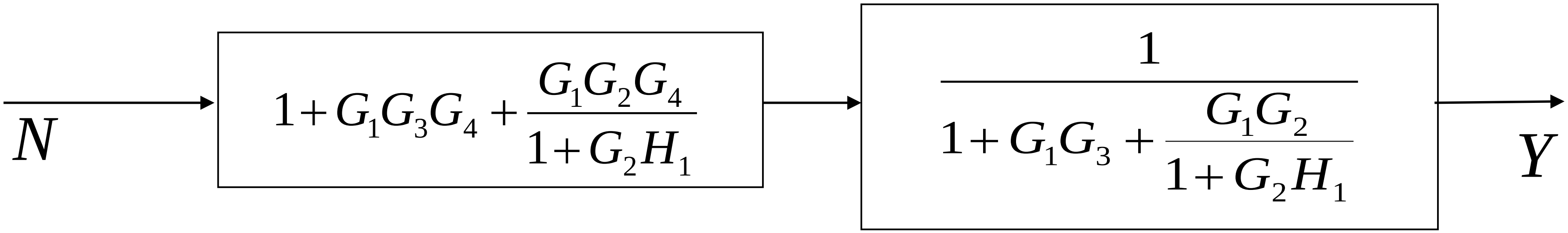
4. Let R=0, we can get:



5. Break down the summing point M:



6. Eliminate above loops:



$$Y_2=\frac{1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}N$$

7. According to the principle of superposition, Y_1 and Y_2 can be combined together, So:

$$Y=Y_1+Y_2$$
$$=\frac{1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}[(G_1G_2+G_1G_3+G_1G_2G_3H_1)R$$
$$+(1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1)N]$$

End