# 1 Overview

GMAT models measurements as either a primitive measurement type, a compound measurement type (sequence of primitives), or a custom measurement type that is specific to an individual system or is proprietary. Primitive measurements include range, range rate, Doppler and angles measurements. For each primitive type, models are provided for different sensor combinations. For example, an angle measurement between a telescope and a target is modelled differently than an angles measurement between a Radar and a target.

For compound and custom measurements, solving for the measurement value and its partial derivatives involves locating discrete events involved in the measurement process. This in turn requires knowledge of the states and STM over portions of the event duration. Hence, in general, real-world measurements are complex and involve the interaction of several system components to evaluate. In the following pages, we present a detailed example of a how GMAT evaluates a real world measurement.

Assume the measurement we wish to evaluate is the average range rate.

# 2 Mathematical Model

For compound and custom measurements, solving for the measurement value and its partial derivatives involves locating discrete events involved in the measurement process. This in turn requires knowledge of the states and STM over portions of the event duration. Hence, in general, real-world measurements are complex and involve the interaction of several system components to evaluate. In the following pages, we present a detailed example of a how GMAT evaluates a real world measurement.

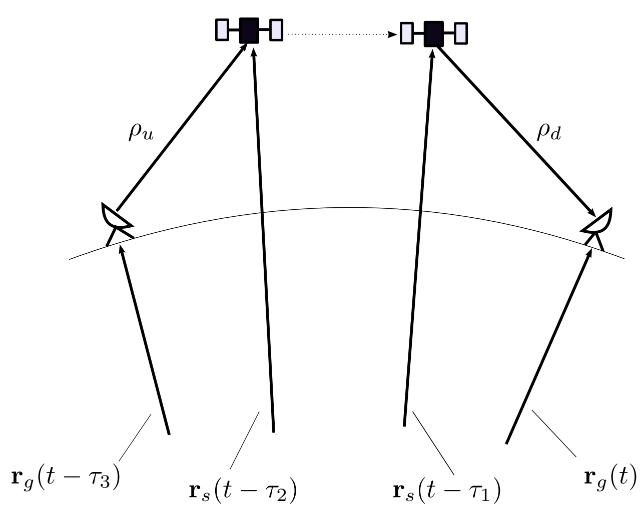


Figure 1: Illustration of Average Range Rate Measurement

- $\rho_d$ is the downlink range
- $\rho_u$ is the uplink range
- $t$ is the measurement anchor time
- $\Delta t_j$ is the time offset, from $t$ for the $j^{th}$ subevent in the measurement process.
- $\mathbf{r}_s$ is the spacecraft position vector
- $\mathbf{r}_g$ is the ground station position vector

- $\Delta t_a$ is the known averaging interval

## Suggested steps in measurement process:

1. Prop all participants to measurement anchor time $t$
2. Save state and STM
3. Locate epoch of event at $\tau_1$
4. Save state and STM
5. Locate epoch of event at $\tau_2$
6. Save state and STM
7. Locate epoch of event at $\tau_3$
8. Save state and STM
9. Calculate measurements and partials using the following equations.

$$\bar{\dot{\rho}} = \frac{\rho_d - \rho_u}{\tau_2 - \tau_1} \tag{1}$$

where

$$\rho_d = \|\mathbf{r}_g(t) - \mathbf{r}_s(t - \tau_1)\| \tag{2}$$

$$\rho_u = \|\mathbf{r}_s(t - \tau_2) - \mathbf{r}_g(t - \tau_3)\| \tag{3}$$

The partial derivatives of the measurement w/r/t $\mathbf{r}_s(t)$, $\mathbf{v}_s(t)$ and $\mathbf{r}_g(t)$ are

$$\frac{\partial \bar{\dot{\rho}}}{\partial \mathbf{r}_s(t)} = \frac{1}{\tau_2 - \tau_1}\left(\hat{\rho}_u^T \mathbf{A}_s(t - \tau_2, t) + \hat{\rho}_d^T \mathbf{A}_s(t - \tau_1, t)\right) \tag{4}$$

$$\frac{\partial \bar{\dot{\rho}}}{\partial \mathbf{v}_s(t)} = \frac{1}{\tau_2 - \tau_1}\left(\hat{\rho}_u^T \mathbf{B}_s(t - \tau_2, t) + \hat{\rho}_d^T \mathbf{B}_s(t - \tau_1, t)\right) \tag{5}$$

$$\frac{\partial \bar{\dot{\rho}}}{\partial \mathbf{r}_g(t)} = \frac{1}{\tau_2 - \tau_1}\left(\hat{\rho}_u^T \mathbf{B}_g(t - \tau_3, t) + \hat{\rho}_d^T\right) \tag{6}$$

The event function for the downlink and uplink legs are respectively:

$$\mathcal{E}_d = \|\mathbf{r}_g(t) - \mathbf{r}_s(t - \tau_1)\| - c\tau_1 = 0 \qquad \text{(Solve for } \tau 1\text{)} \tag{7}$$

$$\mathcal{E}_u = \|\mathbf{r}_g(t - \tau_3) - \mathbf{r}_s(t - \tau_2)\| - c(\tau_3 - \tau_2) = 0 \qquad \text{(Solve for } \tau_3\text{)} \tag{8}$$

where we know that $\tau_2 = \tau_1 - \Delta t_a$.