Wavelet-based Edge Detection

Project – 1

Group # - 28

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Literature Review:

Wavelet Transform:

1. Fourier transform will give the localization of the signal only in one domain that is in Frequency or Spatial domain and not in both.
2. This drawback of the Fourier transform can be solved by using the Wavelet transform of the signal which localizes the signal in both frequency and spatial domain.
3. The wavelet has 2 parts Mother wavelet and the Father wavelets. The father wavelets are constructed using the mother wavelets by changing the values of the scale and the shifting which are related to the frequency and the spatial domain.

Edge detection:

1. Edge detection is an important process in Image processing and computer vision. It can be identified as the boundary separating one region from the other.
2. It separates 2 region with maximum homogenous difference. Example the intensity gradient is high at the edge.

Image noise:

1. There are several noises in an image and some of the noise that are used in this project are the Gaussian and the Impulse noise.
2. These noise are applied to the signal/image using the IMNOISE function.

Hierarchical Data structure:

1. There are several types of data structures that can be used to represent the image after various process. The data structures used in this project are Matrix to represent the image. And the hierarchical data structure to represent the wavelet transform which provide the various levels of the Wavelet transform as a pyramid.

Threshold:

1. Most edge detectors will generate non zero values for almost all the image pixels in edge image.
2. Small edge values correspond to insignificant gray level changes not correlated to region boundaries usually caused by quantization noise, imaging system noise, and lighting irregularities.
3. Thresholding should be able to remove these small values from the edge map image.

Edge detection by scale Multiplication:

1. This method of edge detection is similar to edge detection using wavelet transform but it requires the image to be wavelet transformed using 2 wavelets with 2 different scales.
2. The wavelet transformed image has 2 components Horizontal and vertical.
3. These components are multiplied with corresponding Horizontal and vertical in the next scale.

Introduction:

The project involves detection of the edges in a new manner. It requires the image to be decomposed using a wavelet transform for wavelets with 2 different scales and multiplying the corresponding result to find the edge map of the image.

Unlike many multi-scale edge detectors, where the edge maps were formed at several scales and then synthesized together, our scheme determines edges as the local maxima in the product function after a thresholding. The scale multiplication enhances image structures and suppresses noise. An integrated edge map will be formed efficiently while avoiding the ill-posed edge synthesis process.

Hence this method of edge detection will prove to be very efficient for the image with or without noises as the noises are removed when the scale multiplication is performed.

Edge detection Process:

The process of the edge detection involved selection of suitable mother wavelet and scaling of it to produce the father wavelets. This will give us 2 wavelets with different scales. Then the image given is wavelet transformed using the wavelet with 2 scales. The transformed image from one scale is multiplied with the other transformed image. Then some post processing are done on the edge map to get the desired edge map.

Approach to the Project: (With reference to the paper)

Initially the main thing for the finding the edge map is the proper selection of the wavelets. There are many wavelets available in which can be used to find the wavelet transformation. But only few wavelet can be used for the purpose of this project execution. This is because few wavelets like Mexican hat, Gaussian etc. which are continuous and cannot be used in this project as the image is considered to be discrete. Hence we have to select a discrete wavelet. Even when selecting one of the discrete wavelet we have to consider the fact that the project involves scaling of the mother wavelet which is possible only if the wavelet has a proper scaling function. By proper research on the available wavelets and based on the conditions 2 wavelets are chose to find the edge map using scale multiplication. **The 2 wavelet transforms used are HAAR and DB2.**

In the reference paper the scale multiplication involves using of Gaussian filter differentiated in X and Y direction to find X and Y gradient.

The Gaussian filter is then scaled for a bigger scale and the X and Y gradients are found. This gives the edges in X and Y direction found using the Gaussian derivative filter.

By using the below formula the 2 wavelet functions for the wavelet is found. The θ is the Gaussian equation which is differentiated in X and Y direction.

$$\psi^1(x,y) = \frac{\partial \theta(x,y)}{\partial x}, \qquad \psi^2(x,y) = \frac{\partial \theta(x,y)}{\partial y}$$

The given image is filtered using the 2 wavelets calculated above. This gives the W^1_1 and W^2_1 for the scale J. Now the other wavelet is constructed using the same way but with a larger Gaussian filter scale. This will yield 2 wavelets which is applied to the image to get the wavelet transformed image W_2^1 and W_2^2. Thus the edge in the X and Y direction of the image is found using the wavelets.

Now after finding the edges in X and Y direction using the 2 scales of the wavelet (i.e. Gaussian filter here) we multiply the edge image in X direction with the corresponding edge image in X direction found using the higher scale. The same is done for Y direction edge image.

$$P_j^{f,1}(x,y) = W_j^1 f(x,y) \cdot W_{j+1}^1 f(x,y)$$

Now using the below formula the edge map of the image is found using the scale multiplication process.

$$M_j f(x,y) = \sqrt{P_j^{f,1}(x,y) + P_j^{f,2}(x,y)}$$

Now the obtained edge map M f(x, y) is thresholded and some post processing is performed. One of them is finding the edge point only at the point where edge has the local maximum in the direction of the gradient given by A f(x, y):

$$A_j f(x,y) = \arctan\left(\frac{\operatorname{sgn}(W_j^2 f(x,y)) \cdot \sqrt{P_j^{f,2}(x,y)}}{\operatorname{sgn}(W_j^1 f(x,y)) \cdot \sqrt{P_j^{f,1}(x,y)}}\right)$$

This is process is called the non-maximal suppression. By deleting the points in the edge image whose magnitude is less than the neighboring pixels which are in the direction of the pixel under consideration.

Then proper thresholding is done using the threshold value selected using the following formulae.

$$t_{sc}^i(j) = c \cdot \|\psi_j^i\| \cdot \|\psi_{j+1}^i\| \cdot \sigma^2 \cdot (\sigma_{j,+}^i)^2 \tag{25}$$

where c is a constant and

$$\|\psi_j^i\| = \sqrt{\iint (\psi_j^i(x,y))^2 \,\mathrm{d}x\,\mathrm{d}y} \tag{26}$$

$$\sigma_{j,+}^i = \frac{1}{2}\sqrt{\iint (\psi_j^i(x,y)/\|\psi_j^i\| + \psi_j^i(x,y)/\|\psi_j^i\|)^2 \,\mathrm{d}x\,\mathrm{d}y} \tag{27}$$

$$t_{sc}(j) = 0.8 \ast \sqrt{t_{sc}^1(j) + t_{sc}^2(j)} \tag{28}$$

The equation (28) gives the threshold value to be used to remove the non-significant grey levels in the edge image to get the proper edge map of the input image.

The above process was strictly followed in the project execution in which the wavelet selected is HAAR in one case and DB2 in the other. The wavelet that was constructed is properly scaled to get the scaled wavelets which is needed to perform the wavelet transform based edge detection by scale multiplication.

Software Technologies used for the implementation of the project:

1. Wavefun (): This function will give the wavelet and scaling function for the given wavelet at the given iteration or the scale. The scaling function obtained from this function is used to get the filter for wavelet decomposition.
2. Orthfilt(): This function gets a scaling function as an input and gives the corresponding decomposition and reconstruction Low pass filter and the high pass filter which is used to convolve with the given image to get the decomposed image.
3. Conv2(): This function will takes 2 filters and the given input image as the input and will convolve the image with the filter and produce the decomposed image.
4. Atan(): Gives the angle of the edge thus gives us the direction of the edge in the edge image.
5. Imresize(): This function is used to sub-sample the image to get the next level of decomposition.
6. Imnoise (): This function will induce both Gaussian and Impulse noise in to the image.

Some of the methodologies used are:

- Wavelet Decomposition/Transform.
- Wavelet scaling.
- Scale Multiplication.
- Non Maximal Suppression.
- Noising the image.
- Thresholding.

Step of process in implementing the project:

1. A Wavelet is selected to perform the wavelet decomposition. One by each of the team member and the corresponding decomposition was done separately.
2. The wavelet decomposition is done for 4 levels for the image and the output image is displayed.
3. Now the wavelet is scaled by using the Wavefun() function and the filter values are found using the orthfilt() function. The wavelet decomposition is again done on the input image for the scaled wavelet.
4. Now level 1 of the decomposition is taken from the 2 wavelet decomposition and the corresponding Verticals and Horizontals are multiplied. This is done as it will reduce the noise present in the image and will retain the edge in the image.
5. The M (x, y) is found which gives the edge map of the image. Thus the edge map of an image is found using the scale multiplication technique of the wavelet transform. Some post processing of the image is performed to remove spurious edges and finally thresholded.
6. The same process is repeated for all 3 input images (Peppers, Lena, and Carriage) and their Gaussian and impulse noise affected image.
7. The noise are introduced in to the image using IMNOISE function in MATLAB.

Outcome and Deviations:

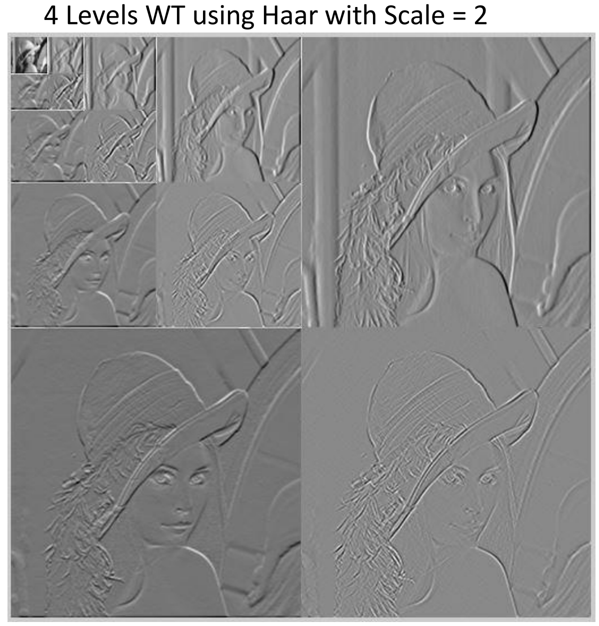
Outcome of the algorithm: (Shown for the first image using both the wavelet transforms chosen and with the corresponding noisy image.)

Lena Original Image

Lena with Gaussian Noise

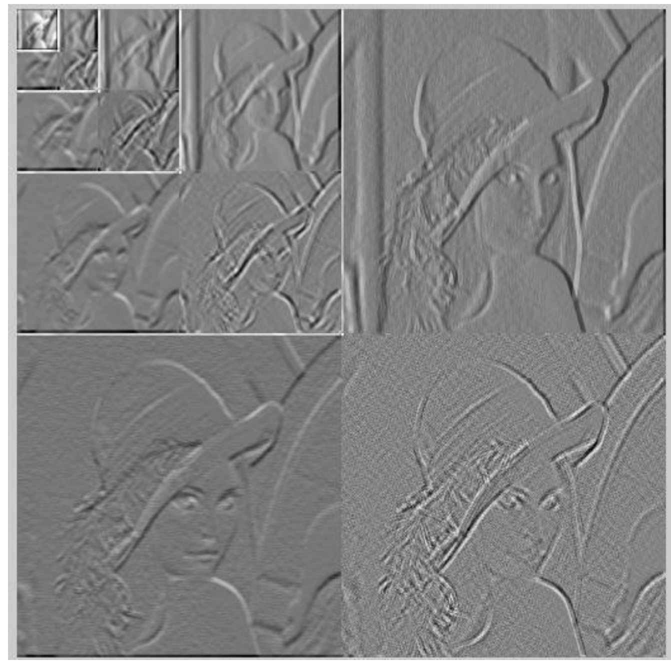
Lena with Impulse Noise

4 Levels WT using Haar with Scale = 2

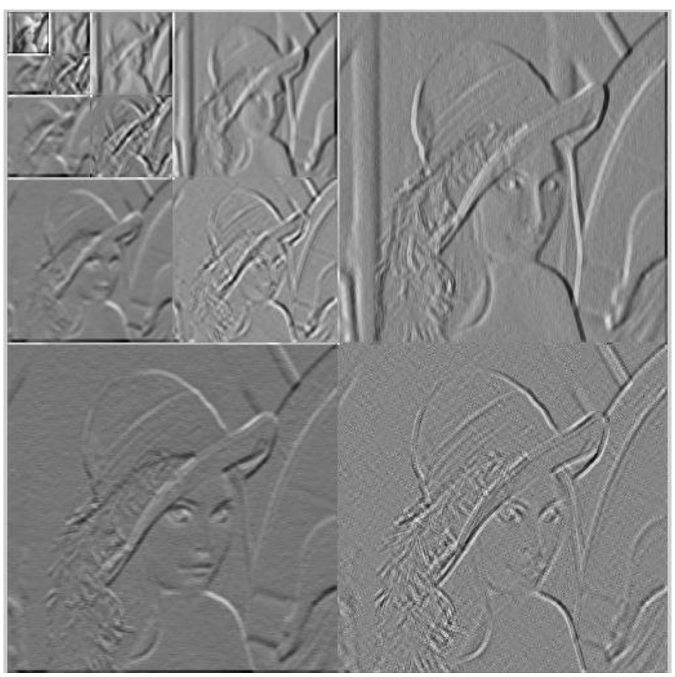


4 Levels WT using Haar with Scale = 3

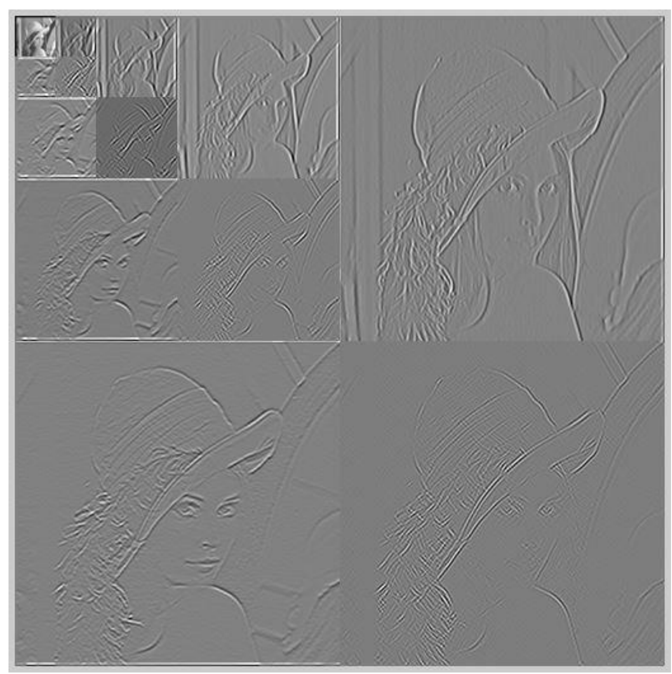
Lena with Gauss – Haar with Scale = 3



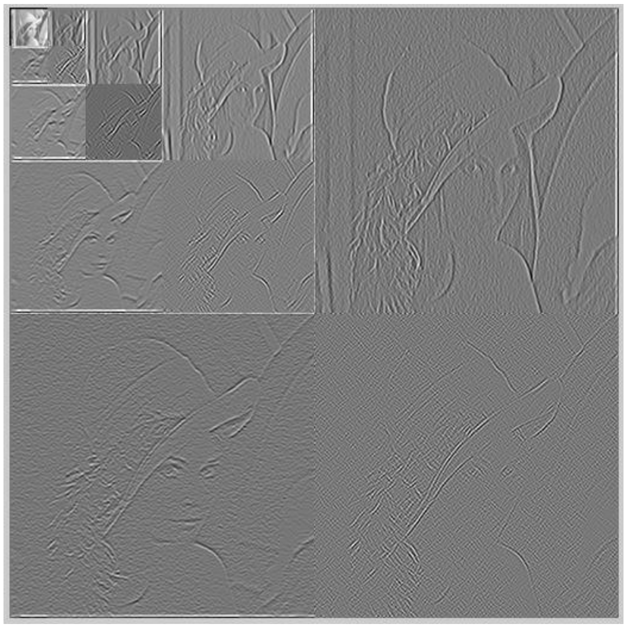
Lena with Impulse – Haar with Scale = 3



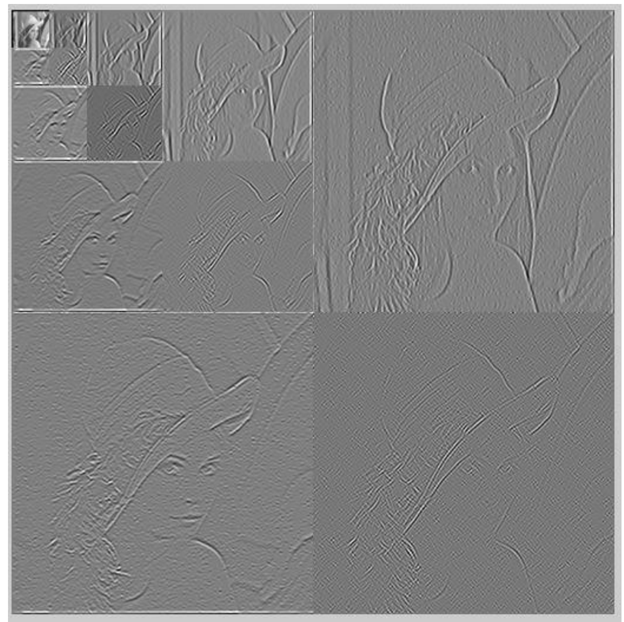
4 Levels WT using DB2 with Scale = 2



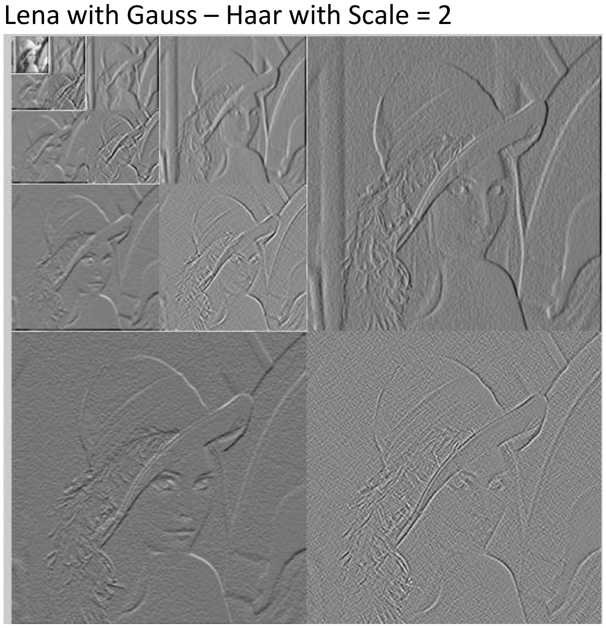
Lena with Gauss – DB2 with Scale = 2



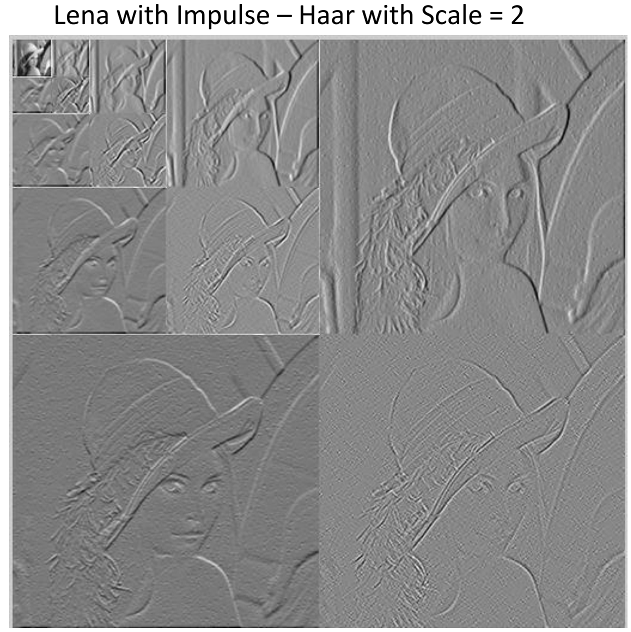
Lena with Impulse – DB2 with Scale = 2



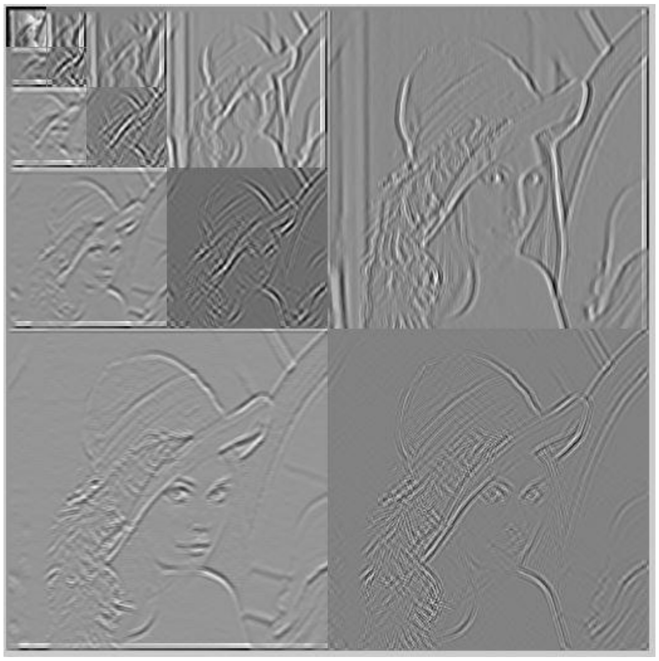
Lena with Gauss – Haar with Scale = 2



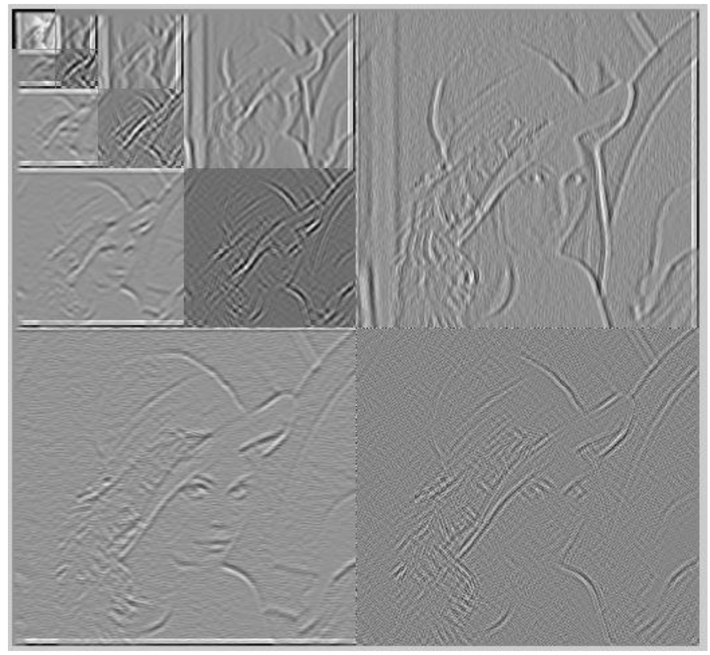
Lena with Impulse – Haar with Scale = 2



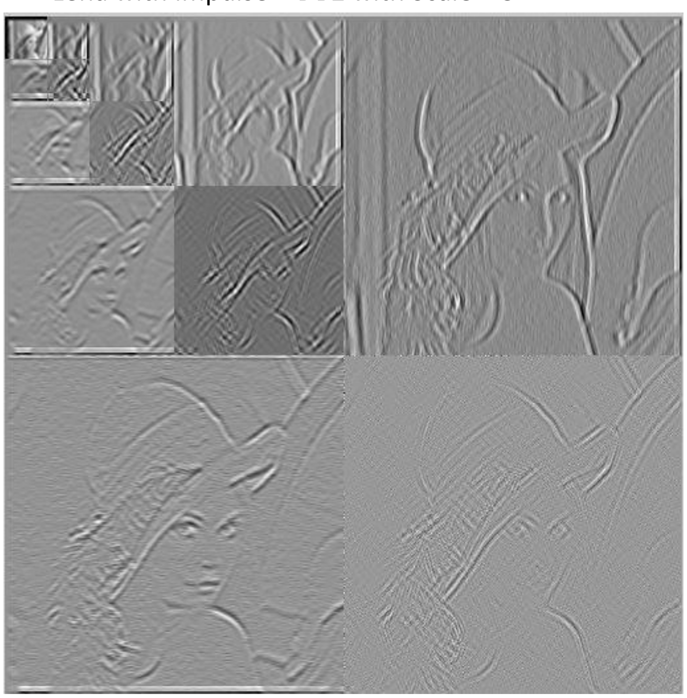
4 Levels WT using DB2 with Scale = 3



Lena with Gauss – DB2 with Scale = 3



Lena with Impulse – DB2 with Scale = 3



Edge Map of Lena – Haar

Edge Map Lena with Gaussian Noise – Haar

Edge Map Lena with Impulse Noise – Haar

Edge Map of Lena – DB2

Edge Map Lena with Gaussian Noise – DB2

Edge Map Lena with Impulse Noise – DB2

Deviation in the outcome:

1. The algorithm developed to find the edge map provides an efficient way to identify the edges in an image. But the output of the algorithm contains non-continuous edges as a result of the final thresholding. This can be resolved by performing an Edge relaxation technique to link the edges by analyzing the neighboring significance.
2. The edge of the image even in the presence of the Gaussian or the impulse noise was easily detected using the Scale multiplication technique Wavelet based edge detection. The amount of noise in the edge map is drastically reduced due to the scale multiplication performed which removes the noises in the image.
3. The impulse noise still persists in the edge map in small scale even when the scale multiplication was performed.

Lesson Learnt from the algorithm development:

1. When a large scale wavelet is used to decompose the image then the edges are very precisely identified.
2. The scale multiplication will results in the removal of the noises and will retain the image edges in the both edge image formed using the wavelet decomposition using 2 different scale wavelets.
3. Noised induced image will pose a difficulty in detecting the edge. Moreover it was easier to remove the Gaussian noise using the scale multiplication but the impulse noise still persisted in the final edge map.

Software and the program development:

Work done by Hariharan Kumar:

a. Developed the scale multiplication technique using the wavelet transform to find the edge map of the image. There was 3 image given and the image was introduced to noise of both Gaussian and Impulse noise.
b. A wavelet is chosen to perform the wavelet decomposition. HAAR wavelet is chosen to perform the decomposition.
c. The wavelet transform is performed for 4 levels. Then the HAAR wavelet is scaled and the wavelet decomposition is again done.
d. Then the X gradient of the Level 1 of first Scale decomposition is multiplied to the 2^{nd} scale.
e. Then using the formulae the scale multiplication is done and then the thresholding is performed.
f. Then the edge map of the image is displayed.
g. The 4 levels of the decomposition is done for the different images and their noised form. Then the 4 levels are displayed by proper concatenation.
h. The various functions used in this projects are highlighted above.
i. A wavelet transformation function was defined which will give the output of the wavelet transformation for 4 levels. This was done to reduce the redundancy in writing the same code.
j. A function for thresholding and edge map detection is also done for the same reason.
k. Another function was created to properly display the 4 level decomposition of the image using the wavelet.

Work done by Aswin Gokulachandran:

a. Wavelet transform for 4 levels using the other wavelet DB2.
b. The corresponding scale multiplication to find the edge map after the scaled version of the DB2 was used to find the decomposed levels. The same process used for HAAR was followed to find the edge map using different wavelet.
c. Gathering the screenshot for the output from the algorithm.

Adoption of the code from online:

Non Maximal suppression used in this project was referred from the online resource. This was the part of the algorithm and few idea and concepts was adopted to perform the non-maximal suppression to remove the spurious edges in the edge map. This non-maximal suppression used is similar to the one that is used in the canny edge detection.

Lesson Learnt from Program code development:

The lesson learnt by coding the program for implementing this projects are the different ways to find the edges and also the post processing techniques used to remove the unwanted noises in the edge image after the detection of the image. It also gave us the perfect lesson to master MATLAB programming and use the various function and features available in it.

Project Questions:

1. **Select from several possible wavelet transforms and determine which two to use with convincing reasons?**

The wavelet transforms used in this project are the HAAR and DB2 based wavelet transforms. As in the project description it was said that the edge map has to be found for the image along with noise. Hence a more robust and a rigid wavelet transform has to be selected. Also the wavelet has to be scaled so that the scale multiplication is made possible which implies that the wavelets to be selected must have a scaling function. Based on these condition the best suited wavelets are the HAAR and the DB2. The wavelet selected are orthogonal as it will be easy to scale an orthogonal wavelets than that are not. Hence an orthogonal wavelets are preferred in this project.

2. **Add Gaussian noise and impulse noise, respectively, to the test images for your implementation. You may select the level of noise with explanation in your report?**

The amount of the Gaussian noise are chosen such a way that they doesn't impact much on the edge detection. The Gaussian noise is efficiently removed from the image while performing the scale multiplication but the Impulse noise still persists in the edge map and hence a nominal value of 25% was selected to induce the Gaussian noise in the image and a value of 0.02 for the Impulse noise is chosen. The function Imnoise is used to induce noise in the image.

3. **Compare the results of edge detection from two different wavelet transforms and from two different noise types. Explain the difference among these results?**

   a. **Edge detection using different wavelets:**
      i. The edges are very efficiently detected when the HAAR wavelet is used as the HAAR wavelet is very robust to the noise. Also when the image contains noise there was no effect in finding the edge map but the image gets distorted when Impulse noise is induced in the image. Hence a very small amount of impulse noise was induced in to the image to find the edge map.
      ii. The edges found using the other wavelet transform i.e. DB2 was less effective than HAAR to clearly identify the edge. Scale multiplication only resulted in removal of only certain amount of the image. The final edge map still contains few noises comparatively to the HAAR wavelet.
      iii. The Gaussian noise was effectively removed by the HAAR wavelet and but DB2 wavelet was not as effective for image with Gaussian noise.
   b. **Edge detection for noised image:**
      i. The edge map found using this algorithm proves to be effective and finds all the edges in the image and removes any spurious edges. It satisfies the localization criteria such that the location of the actual edge in the image and the edge found in the image don't have much difference in the location.
      ii. But when a noise is induced in the image the edge map seems to distort a little when applying the wavelet transform. When the scale multiplication is done and followed by the thresholding the edge are easily detected without any deviation. But one of the consequence is that the edges are not continuous and some spurious edges appears in the edge map. This was the deviation in the

edge map of the image with noises. In case if the noise is impulse type then the spurious edges in the edge map is more than that of the Gaussian noise.

iii. It proves to be so difficult to obtain the perfect edge of the image when the image consists of noise as it distorts the image and will need some more techniques to differentiate the noise and image which in turn will help us in detecting the image edges with ease.

Summary:

In this project we performed the edge detection using the discrete wavelet transform using scale multiplication technique. A scale product function is defined as the multiplication of the 2 adjacent scales of wavelet coefficient to amplify the edge structures and remove the noise from the image. Then the edge is determined as the local maxima of the product. A properly determined threshold is used to remove the insignificant grey scales in the edge image that arises due to the noise in the image. Thus the Edge map of the given input images are found using the scale multiplication technique based wavelet transformation.

Lesson Learnt from the class:

1. How the image processing will improve the images and will help the vision process for computers.
2. How the computer vision works.
3. Image processing levels.
4. Image Understanding.
5. Various problems associated in image acquisition.

Reference List:

a. Edge detection by scale multiplication in wavelet domain Lei Zhang *, Paul Bao Ziou, D., Tabbone, S., 1993. A multi-scale edge detector. Pattern Recogn. 26, 1305–1314.

b. Aydn, T. et al., 1996. Multidirectional and multiscale edge detection via M-band wavelet transform. IEEE Trans.Image Processing 5, 1370–1377.

c. Canny, J., 1986. A computational approach to edge detection. IEEE Trans. PAMI PAMI-8, 679–698. Daubechies, I., 1992. Ten Lectures on Wavelets. SIAM,Philadelphia, PA.

d. Lu, Y., Jain, R.C., 1989. Behavior of edges in scale space. IEEE Trans. PAMI 11, 337–356.

e. Lu, Y., Jain, R.C., 1992. Reasoning about edges in scale space. IEEE Trans. PAMI 14, 450–468.

f. Mallat, S., Hwang, W.L., 1992. Singularity detection and processing with wavelets. IEEE Trans. Information Theory 32, 617–643.

g. Mallat, S., Zhong, S., 1992. Characterization of signals from multiscale edges. IEEE Trans. PAMI 14, 710–732.

h. Marr, D., Hildreth, E., 1980. Theory of edge detection. Proc. Royal Soc. London 207, 187–217.

i. Park, D.J., Nam, K.N., Park, R.H., 1995. Multiresolution edge detection techniques. Pattern Recogn. 28, 211–229.

j. Petrou, M., Kittler, J., 1991. Optimal edge detectors for ramp edges. IEEE Trans. PAMI 13, 483–491.
k. Pratt, W.K., 1991. Digital Image Processing, second ed. John Wiley & Sons, New York.

l. Sadler, B.M., Swami, A., 1999. Analysis of multiscale products for step detection and estimation. IEEE Trans. Information Theory 45, 1043–1051.

m. Sarkar, S., Boyer, K.L., 1991. On optimal infinite impulse response edge detection filters. IEEE Trans. PAMI 13, 1154–1171.

n. Shen, J., Castan, S., 1992. An optimal linear operator for step edge detection. CGVIP: Graphical Models Image Process. 54, 112–133.
o. http://www.mathworks.com/help/wavelet/
p. http://www.mathworks.com/help/wavelet/ref/wavefun.html
q. http://www.cs.ucf.edu/~mikel/Research/Edge_Detection.htm