# Hilbert Range Query Results (Order 10, RTT 5 ms and 15 ms)

**Experimental setup**

Topology: 30 nodes arranged in three clusters. Each cluster has 10 nodes connected to one switch. The three switches connect via a router.

Clusters:

- Cluster 1 = nodes connected to switch_a (S1-S10)
- Cluster 2 = nodes connected to switch_b (S11-S20)
- Cluster 3 = nodes connected to switch_c (S21-S30)

Query nodes:

- S1, S5, S10 (Cluster 1)
- S11, S15, S20 (Cluster 2)
- S21, S25, S30 (Cluster 3)

RTT thresholds tested: 5 ms and 15 ms. Hilbert order: 10. The topology is as follows. The topology diagram shows configured link RTTs, while all tables below use the Vivaldi-predicted RTT matrix for evaluation. The query nodes are marked in “Blue” color.

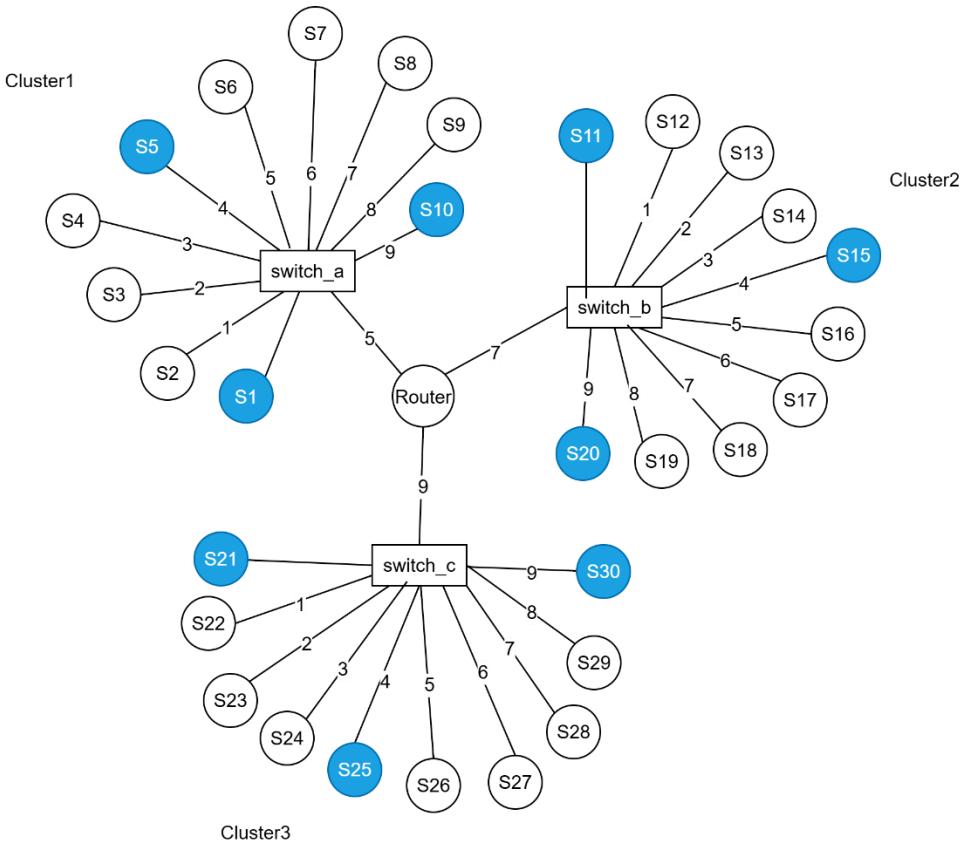


*Figure 1:Topology*

**Ground truth and evaluation definitions**

For a query node $q$ and RTT threshold $T$:

- Ground truth (GT): All nodes *n* such that predicted_RTT($q$, *n*) ≤ $T$
- Returned set: All nodes returned by the range query for $q$ and $T$
- True Positive (TP): The node was returned by the query and its RTT ≤ $T$
- False Positive (FP): The node was returned by the query but its RTT > $T$
- False Negative (FN): The node was not returned by the query even though its RTT ≤ $T$

**Result Matrix Layout and TP/FP/FN**

The following matrix tables summarize the outcome of multiple range queries using the Vivaldi-predicted RTT matrix as the reference.

- Rows (left side) represent individual query executions. Each row is identified by the query node ($q$) and the RTT threshold ($T$) colored in "Blue". Eg: S1, 5 ms or S1, 15 ms.
- Columns (top) list all nodes S1–S30, grouped by Cluster 1 (S1–S10), Cluster 2 (S11–S20) and Cluster 3 (S21–S30).
- Cell values show the predicted RTT between the query node (row) and the corresponding node (column).
- Cell colors indicate the classification of that node for the given query and threshold.
    - TP: Green
    - FP: Red
    - FN: Orange

**Evaluation scenarios**

Three different scenarios were used during this experiment. All scenarios use the same topology, the same set of query nodes, the same RTT thresholds, and the same evaluation procedure.

They differ only in how the coordinate information is used during indexing and query execution. For each scenario, the results are presented using cluster-wise result matrices that show the classification outcomes (TP, FP, FN) based on the Vivaldi-predicted RTTs.

**Scenario 1: 5D Vivaldi vector**

In this scenario, only the 5-dimensional Vivaldi coordinate vector is used for indexing and querying.

Each node is represented solely by its 5D vector, and the Hilbert range query operates entirely in this 5D space. The RTT threshold is mapped to a spherical region in the 5D coordinate space, which is approximated by a set of small axis-aligned 5D sub-regions (cells). Only those sub-regions that intersect the sphere are considered during query execution. No additional information, such as height or adjustment, is used to restrict or refine the query.

**Expected behavior**

Because the query relies only on the 5D vector geometry, nodes whose coordinates appear close in the vector space may still have a predicted RTT larger than the threshold which can lead to false positives.

**Results**

The following matrix tables show the results for Scenario 1, presented separately for Cluster 1, Cluster 2, and Cluster 3.

## Scenario 1 Results: Cluster 1

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster1 | **S1, 5ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S1, 15 ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S5, 5ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S5, 15 ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S10,5 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |
| | **S10,15 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |

**Scenario 1 Results: Cluster 2**

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster2 | **S11,5 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S11,15 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S15,5 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S15,15 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S20,5 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |
| | **S20,15 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |

## Scenario 1 Results: Cluster 3

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster3 | **S21,5 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S21,15 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S25,5 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S25,15 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S30,5 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |
| | **S30,15 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |

**Scenario 2: 6D Coordinates (5D Vector + Height and Adjustment)**

In this scenario, the node representation is extended from 5D to 6D by adding a single correction scalar derived from height and adjustment. The Hilbert index and the range query both operate in this 6D space. The RTT threshold is mapped to a region in 6D, and cells intersecting this region are queried accordingly.

**Expected behavior**

Adding height and adjustment as extra dimensions (6D or 7D) does not eliminate false positives because these terms are not geometric coordinates. In Serf, height and adjustment are applied as additive corrections after the Euclidean vector distance is computed. Embedding them into the coordinate space forces these additive terms to behave like spatial dimensions, which changes the distance geometry and can only approximate their effect. As a result, false positives may be reduced but cannot be eliminated entirely.

**Results**

The following matrix tables show the results for Scenario 2, presented separately for Cluster 1, Cluster 2, and Cluster 3.

## Scenario 2 Results: Cluster 1

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster1 | **S1, 5ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S1, 15 ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S5, 5ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S5, 15 ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S10,5 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |
| | **S10,15 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |

**Scenario 2 Results: Cluster 2**

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster2 | **S11,5 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S11,15 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S15,5 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S15,15 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S20,5 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |
| | **S20,15 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |

## Scenario 2 Results: Cluster 3

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster3 | **S21,5 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S21,15 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S25,5 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S25,15 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S30,5 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |
| | **S30,15 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |

**Scenario 3: 5D Coordinates with Pruning Using Height and Adjustment**

In this scenario, the index remains 5-dimensional using only the Vivaldi vector for spatial ordering. However, height and adjustment are used as additional constraints during query execution and not as index dimensions.

The range query is executed by recursively exploring sub-ranges of the 5D coordinate space, where each sub-range is represented by a lower-bound and upper-bound vector (LB/UB) and can be issued as a Lawder range query. Before issuing a range query for a given LB/UB sub-range, the algorithm performs an early feasibility check to determine whether the RTT threshold can possibly be satisfied within that sub-range. This check extends the standard 5D geometric filtering by additionally considering the smallest height and adjustment values observed among nodes mapped into that LB/UB sub-range.

If this lower bound already exceeds the RTT threshold, the LB/UB sub-range is pruned and no Lawder range query is executed for it since no node within that sub-range can satisfy the constraint.

**Expected behavior**

By pruning sub-ranges that cannot possibly satisfy the RTT constraint, this approach reduces false positives before candidate nodes are returned. For the tested scenarios, no false positives were observed, while recall was preserved. This separates the roles of:

- The 5D vector for ordering and locality
- Height and adjustment for latency feasibility checks.

**Results**

The following matrix tables show the results for Scenario 3, presented separately for Cluster 1, Cluster 2, and Cluster 3

**Scenario 3 Results: Cluster 1**

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster1 | **S1, 5ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S1, 15 ms** | 0.0 | 1.784 | 2.83 | 3.773 | 4.531 | 5.478 | 7.354 | 9.041 | 10.476 | 12.044 | 12.757 | 13.437 | 14.483 | 15.557 | 16.534 | 17.355 | 18.244 | 20.088 | 20.553 | 21.839 | 14.499 | 15.607 | 16.331 | 17.155 | 18.281 | 19.458 | 20.547 | 21.473 | 22.289 | 23.235 |
| | **S5, 5ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S5, 15 ms** | 4.531 | 6.771 | 7.544 | 8.732 | 0.0 | 10.244 | 11.943 | 13.877 | 15.173 | 16.532 | 17.476 | 18.161 | 19.203 | 20.283 | 21.244 | 22.053 | 22.965 | 24.877 | 25.262 | 26.441 | 19.31 | 20.42 | 21.141 | 21.964 | 23.085 | 24.229 | 25.349 | 26.281 | 27.157 | 27.997 |
| | **S10,5 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |
| | **S10,15 ms** | 12.044 | 14.19 | 14.868 | 16.118 | 16.532 | 17.029 | 14.474 | 14.443 | 15.054 | 0.0 | 22.865 | 23.622 | 24.607 | 25.662 | 26.52 | 26.448 | 28.365 | 29.946 | 28.512 | 29.497 | 24.307 | 25.569 | 26.312 | 27.055 | 28.196 | 28.032 | 30.264 | 30.286 | 32.197 | 32.154 |

**Scenario 3 Results: Cluster 2**

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster2 | **S11,5 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S11,15 ms** | 12.7577 | 14.6633 | 15.49 | 16.51 | 17.476 | 18.242 | 19.005 | 20.151 | 21.095 | 22.865 | 0.0 | 2.502 | 3.358 | 4.388 | 5.18 | 6.929 | 8.117 | 11.126 | 11.365 | 12.791 | 17.109 | 18.366 | 19.119 | 19.77 | 20.951 | 22.133 | 23.329 | 24.08 | 24.997 | 25.9 |
| | **S15,5 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S15,15 ms** | 16.534 | 18.445 | 19.259 | 20.293 | 21.244 | 22.017 | 22.716 | 23.959 | 24.831 | 26.52 | 5.18 | 6.323 | 6.869 | 8.258 | 0.0 | 10.332 | 11.563 | 14.7 | 14.888 | 15.98 | 20.962 | 22.224 | 22.975 | 23.624 | 24.797 | 25.959 | 27.179 | 27.946 | 28.891 | 29.675 |
| | **S20,5 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |
| | **S20,15 ms** | 21.839 | 23.807 | 24.387 | 25.658 | 26.441 | 27.269 | 26.574 | 29.369 | 29.037 | 29.497 | 12.791 | 13.789 | 13.798 | 15.771 | 15.98 | 14.215 | 14.294 | 15.695 | 16.691 | 0.0 | 25.864 | 27.305 | 28.06 | 28.584 | 29.614 | 30.393 | 32.136 | 33.151 | 34.15 | 32.477 |

**Scenario 3 Results: Cluster 3**

| | | Cluster1 | | | | | | | | | | Cluster2 | | | | | | | | | Cluster3 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $q, T$ | | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** | **S19** | **S20** | **S21** | **S22** | **S23** | **S24** | **S25** | **S26** | **S27** | **S28** | **S29** | **S30** |
| Cluster3 | **S21,5 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S21,15 ms** | 14.499 | 16.487 | 17.322 | 18.363 | 19.31 | 20.063 | 20.648 | 21.656 | 22.675 | 24.307 | 17.109 | 18.044 | 18.941 | 20.019 | 20.962 | 21.658 | 22.518 | 24.151 | 24.777 | 25.864 | 0.0 | 2.134 | 3.506 | 3.195 | 4.752 | 6.08 | 6.81 | 8.897 | 8.805 | 10.216 |
| | **S25,5 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S25,15 ms** | 18.281 | 20.281 | 21.088 | 22.172 | 23.085 | 23.777 | 24.442 | 25.689 | 25.954 | 28.196 | 20.951 | 21.888 | 22.793 | 23.877 | 24.797 | 25.682 | 26.117 | 28.268 | 28.578 | 29.614 | 4.752 | 6.393 | 7.69 | 8.025 | 0.0 | 10.769 | 11.163 | 12.256 | 13.306 | 13.721 |
| | **S30,5 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |
| | **S30,15 ms** | 23.235 | 25.252 | 25.892 | 27.139 | 27.997 | 28.71 | 28.271 | 30.483 | 30.205 | 32.154 | 25.9 | 26.837 | 27.606 | 28.903 | 29.675 | 30.408 | 30.767 | 32.573 | 34.123 | 32.477 | 10.216 | 12.682 | 13.975 | 13.846 | 13.721 | 13.657 | 17.22 | 16.034 | 15.647 | 0.0 |

**Overall Result Comparison**

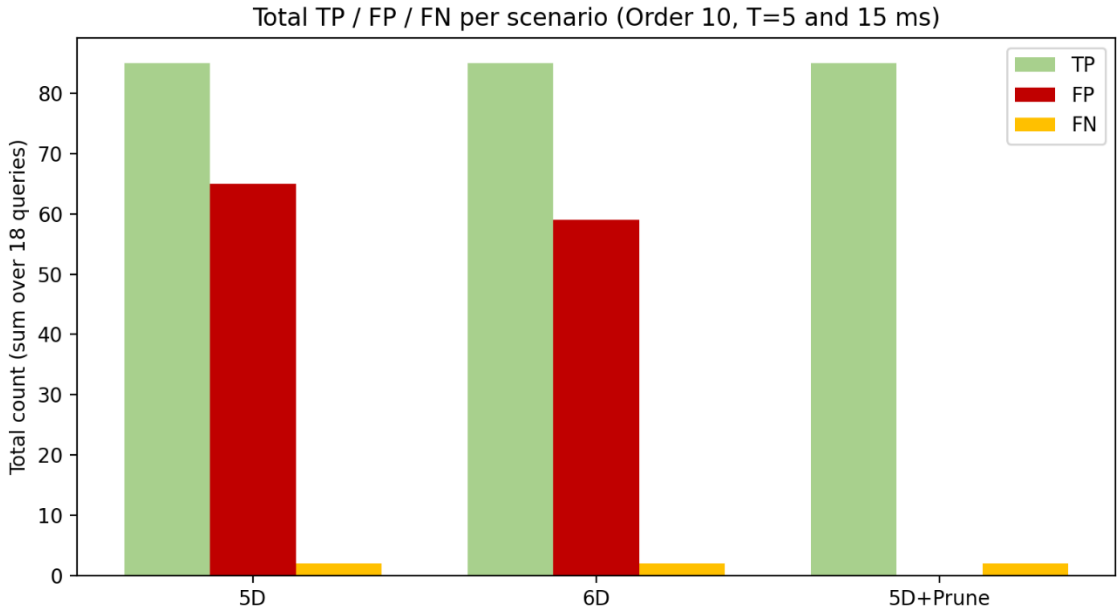


*Figure 2: Total TP,FP,FN per scenario*

Across all 18 queries, the total number of true positives remains constant for all three scenarios (TP = 85), indicating that none of the approaches reduces recall. In contrast, the number of false positives decreases from 65 in the 5D case to 59 in the 6D case and is fully eliminated in the 5D + pruning scenario (FP = 0). The number of false negatives remains low and identical across all scenarios (FN = 2).

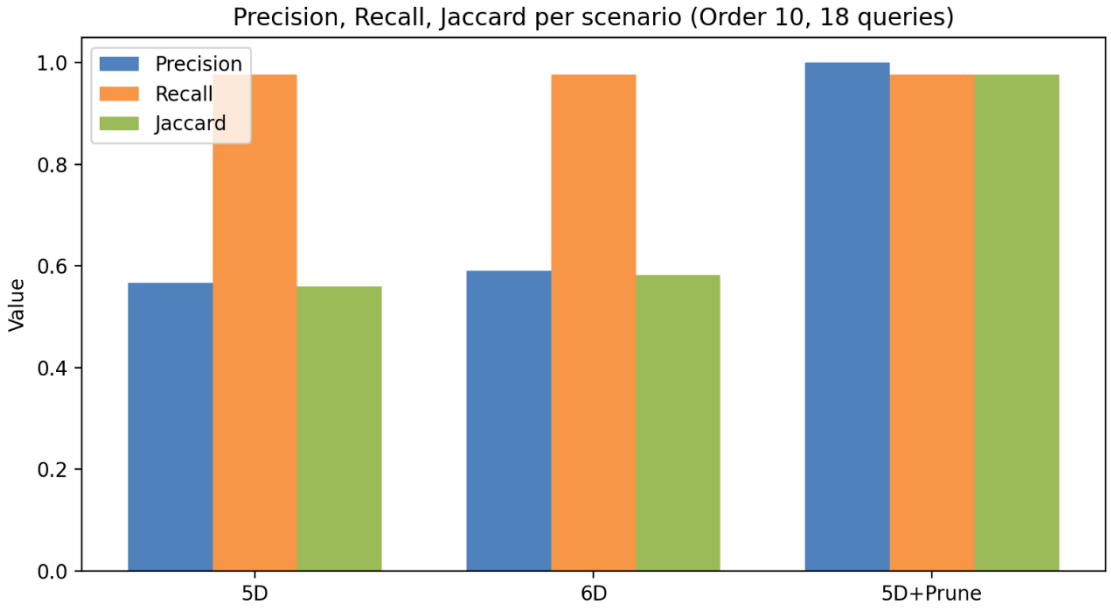


*Figure 3:Precision, Recall, Jaccard per scenario*

According to Figure 3 Precision increases from 0.567 in the 5D scenario to 0.590 in the 6D scenario and reaches 1.000 when pruning is applied. Recall remains unchanged at 0.977 for all three scenarios,

confirming that the pruning strategy does not reduce the number of correctly retrieved nodes. The Jaccard index follows the same trend, increasing from 0.559 (5D) to 0.582 (6D) and to 0.977 in the 5D + pruning case.

**Analysis of False Positive and False Negative Causes**

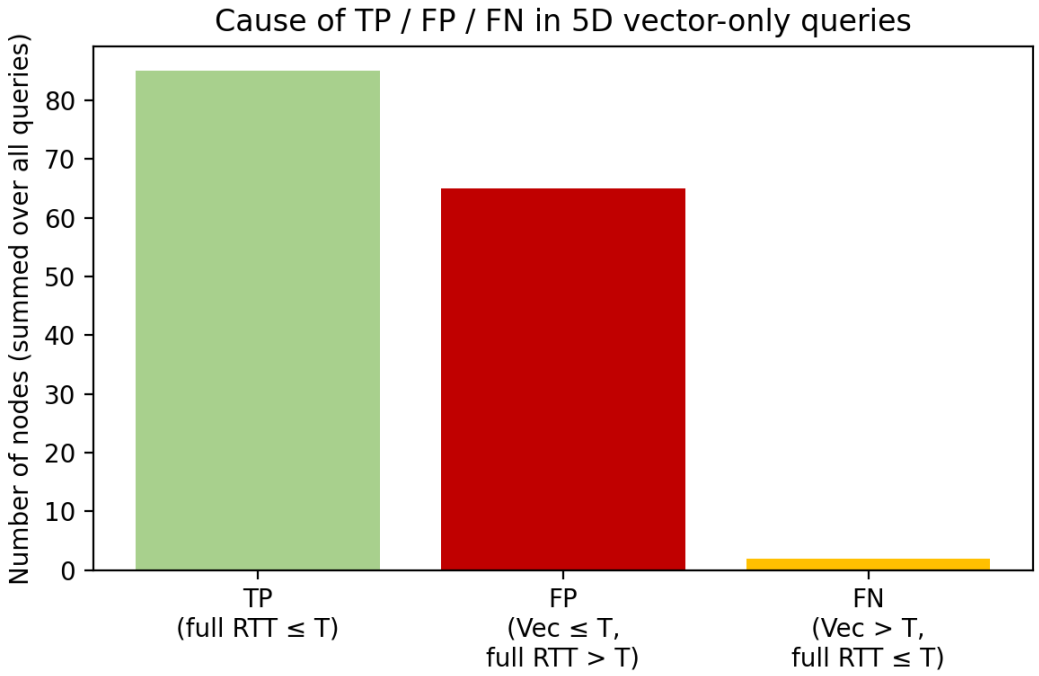


*Figure 4: TP, FP, FN cause*

Figure 4 explains why nodes are classified as true positives (TP), false positives (FP), or false negatives (FN) when range queries are executed using only the 5D vector component. True positives occur when both the vector distance and the full predicted RTT (including height and adjustment) are within the threshold. False positives arise when the vector distance alone is within the threshold, but the full predicted RTT exceeds it after applying height and adjustment. False negatives occur rarely and only near the query boundary, where the vector distance slightly exceeds the threshold while the full predicted RTT remains within it. All nodes that are not returned by the query and not shown in the figure are true negatives. For these nodes, both the vector distance and the full predicted RTT are already greater than the threshold, so they are correctly excluded.

**Conclusion**

This study shows that Hilbert-based range queries over 5D Vivaldi coordinates provide efficient spatial access but inherently return false positives due to region-based approximation. By applying a conservative RTT feasibility check using height and adjustment during query execution, regions that cannot possibly satisfy the RTT constraint are excluded before querying the index. This reduces false positives without affecting recall.