Electronic Companion – Distribution and Transmission Coordinated Dispatch Under Joint Electricity and Carbon Day-Ahead Markets

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NOMENCLATURE

In this section, additional variables related to the lower-level dual problem are defined.

$\overline{\alpha}_{g,t,s}$ Dual variable associated with the upper bound for the power injection of transmission-connected generators (eq. 28/29).

$\underline{\alpha}_{g,t,s}$ Dual variable associated with the lower bound for the power injection of transmission-connected generators (eq. 28/29).

$\overline{\rho}_{g,b,t,s}$ Dual variable associated with the upper bound for the accepted bids of transmission-connected generators (eq. 31).

$\underline{\rho}_{g,b,t,s}$ Dual variable associated with the lower bound for the accepted bids of transmission-connected generators (eq. 31).

$\overline{\omega}_{l,t,s}$ Dual variable associated with the upper bound for the power flow across transmission lines (eq. 32).

$\underline{\omega}_{l,t,s}$ Dual variable associated with the lower bound for the power flow across transmission lines (eq. 32).

$\overline{\kappa}_{i,t,s}$ Dual variable associated with the upper bound for the substations' capacities (eq. 33).

$\underline{\kappa}_{i,t,s}$ Dual variable associated with the lower bound for the substations' capacities (eq. 33).

$\lambda_{i,t,s}$ Dual variable associated with the power balance at transmission nodes (eq. 25/26).

$\mu_{l,t,s}$ Dual variable associated with the transmission system power flow calculation (eq. 27).

$\psi_{t,s}$ Dual variable associated with the transmission system's carbon balance (eq. 35).

$\phi_{g,t,s}$ Dual variable associated with the transmission-connected generators' carbon emissions (eq. 34).

$\varphi_{g,t,s}$ Dual variable associated with the sum of the transmission-connected generators' accepted bids (eq. 30).

$\xi_{t,s}$ Dual variable associated with the transmission system's carbon allowance limit (eq. 36).

I. REFORMULATION AS A MPEC

As a general rule, adopting the strong duality theorem to formulate the mathematical programming with equilibrium constraints (MPEC) model leads to lower computational burden than using Karush–Kuhn–Tucker (KKT) conditions, as shown in [EC.1]. However, for this specific problem, the linearization of the bilinear terms in the leader's objective function has an even heavier computational effort than the KKT conditions if the primal-dual model is employed to describe the MPEC model, as observed in [EC.2],[EC.3]. Thus, the MPEC model is formulated using KKT conditions as follows.

$$\begin{aligned}\min\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}p_{s,t}\Bigg[&\sum_{g\in\mathcal{G}_D}\sum_{b\in\mathcal{B}_g}\left(C^D_{g,b}\rho^D_{g,b,t,s}\right)\\ &+\psi_{t,s}\varrho^{SE}_{t,s}+\sum_{i\in\mathcal{N}_\infty}\lambda_{i,t,s}P^{SE}_{i,t,s}\Bigg]\end{aligned} \tag{EC.1}$$

subject to:

$$\text{Constraints (2)–(23): Upper-level constraints} \tag{EC.2}$$

$$\text{Constraints (25)–(36): ISO's primal constraints} \tag{EC.3}$$

$$-\sum_{i\in\mathcal{N}_T\cap\mathcal{G}_T}(\lambda_{i,t,s})-\phi_{g,t,s}\gamma^T_g+\overline{\alpha}_{g,t,s}-\underline{\alpha}_{g,t,s}+\varphi_{g,t,s}=0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.4}$$

$$C^T_{g,b}-\varphi_{g,t,s}+\overline{\rho}_{g,b,t,s}-\underline{\rho}_{g,b,t,s}=0 \quad \forall g\in\mathcal{G}_T,b\in\mathcal{B}_g,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.5}$$

$$-\sum_{l\in\mathcal{L}_T}\left(\frac{A_{i,l}\mu_{l,t,s}}{X_l}\right)=0 \quad \forall i\in\mathcal{N}_T\cup\mathcal{N}_\infty,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.6}$$

$$\mu_{l,t,s}+\overline{\omega}_{l,t,s}-\underline{\omega}_{l,t,s}-\sum_{i\in\mathcal{N}_T\cup\mathcal{N}_\infty}A_{i,l}\lambda_{i,t,s}=0 \quad \forall l\in\mathcal{L}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.7}$$

$$-\pi^E_{i,t,s}+\lambda_{i,t,s}+\overline{\kappa}_{i,t,s}-\underline{\kappa}_{i,t,s}=0 \quad \forall i\in\mathcal{N}_\infty,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.8}$$

$$\phi_{g,t,s}+\overline{\xi}_{t,s}=0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.9}$$

$$\psi_{t,s}+\overline{\xi}_{t,s}=0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.10}$$

$$\pi^C_{t,s}-\psi_{t,s}=0 \quad \forall t\in\mathcal{T},s\in\mathcal{S} \tag{EC.11}$$

$$\overline{\alpha}_{g,t,s}(-P^T_{g,t,s}+\overline{P}^T_g)=0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.12}$$

$$\underline{\alpha}_{g,t,s}(P^T_{g,t,s})=0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.13}$$

$$\overline{\omega}_{l,t,s}(-PF_{l,t,s}+\overline{PF}_l)=0 \quad \forall l\in\mathcal{L}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.14}$$

$$\underline{\omega}_{l,t,s}(PF_{l,t,s}+\overline{PF}_l)=0 \quad \forall l\in\mathcal{L}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.15}$$

$$\overline{\rho}_{g,b,t,s}(-\rho^T_{g,b,t,s}+\overline{P}^T_{g,b})=0 \ \forall g\in\mathcal{G}_T,b\in\mathcal{B}_g,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.16}$$

$$\underline{\rho}_{g,b,t,s}(\rho^T_{g,b,t,s})=0 \quad \forall g\in\mathcal{G}_T,b\in\mathcal{B}_g,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.17}$$

$$\overline{\kappa}_{i,t,s}(-P^{SE}_{i,t,s}+\overline{P}^{SE}_i)=0 \quad \forall i\in\mathcal{N}_\infty,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.18}$$

$$\underline{\kappa}_{i,t,s}(P^{SE}_{i,t,s}+\overline{P}^{SE}_i)=0 \quad \forall i\in\mathcal{N}_\infty,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.19}$$

$$\xi_{t,s}\left(\overline{\Gamma}_t^T-\sum_{g\in\mathcal{G}_T}\left(\nu^T_{g,t,s}+\varrho^T_{g,t,s}\right)\right)=0 \quad \forall t\in\mathcal{T},s\in\mathcal{S} \tag{EC.20}$$

$$\overline{\alpha}_{g,t,s}\geq 0 \qquad \underline{\alpha}_{g,t,s}\geq 0 \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.21}$$

$$\overline{\rho}_{g,b,t,s}\geq 0 \qquad \underline{\rho}_{g,b,t,s}\geq 0 \quad \forall g\in\mathcal{G}_T,b\in\mathcal{B}_g,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.22}$$

$$\overline{\kappa}_{i,t,s}\geq 0 \qquad \underline{\kappa}_{i,t,s}\geq 0 \quad \forall i\in\mathcal{N}_\infty,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.23}$$

$$\overline{\omega}_{l,t,s}\geq 0 \qquad \underline{\omega}_{l,t,s}\geq 0 \quad \forall l\in\mathcal{L}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.24}$$

$$\xi_{t,s}\geq 0 \qquad \forall t\in\mathcal{T},s\in\mathcal{S} \tag{EC.25}$$

Equations (EC.4)–(EC.11) are the KKT stationary conditions, while equations (EC.12)–(EC.20) represents the complementary slackness conditions. Finally (EC.21)–(EC.25) are the dual feasibility constraints.

II. Mixed-Integer Problem Formulation

The nonlinear equations described in (EC.12)–(EC.20) can be linearized using the Big-M method. An example is presented for (EC.12), which is rewritten as (EC.26)–(EC.27). The same procedure is applied to the other complementary slackness equations.

$$\overline{\alpha}_{g,t,s}\leq M z^{\overline{\alpha}}_{g,t,s} \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.26}$$

$$(-P^T_{g,t,s}+\overline{P}^T_g)\leq M(1-z^{\overline{\alpha}}_{g,t,s}) \quad \forall g\in\mathcal{G}_T,t\in\mathcal{T},s\in\mathcal{S} \tag{EC.27}$$

being $z^{\overline{\theta}}$ a binary variable that indicates that the constraint $P^T_{g,t,s}\leq \overline{P}^T_g$ is active, i.e., $P^T_{g,t,s}=\overline{P}^T_g$. M is a sufficiently large constant.

As for the linearization of the bilinear terms in (EC.1), namely ($\psi_{t,s}\varrho^{SE}_{t,s}$) and ($\lambda_{i,t,s}P^{SE}_{i,t,s}$), the following steps were taken.

From the strong duality theorem, we know that the primal and dual objective functions of an optimization problem assume the same value at the optimal solution. Thus, applying the strong duality theorem for the ISO model, one can write the following equation.

$$\begin{aligned}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}p_{s,t}\left[\sum_{g\in\mathcal{G}_T}\sum_{b\in\mathcal{B}_g}(C^T_{g,b}\rho^T_{g,b,t,s})-\pi^C_{t,s}\varrho^{SE}_{t,s}-\sum_{i\in\mathcal{N}_\infty}\pi^E_{i,t,s}P^{SE}_{i,t,s}\right]=\\ \sum_{i\in\mathcal{N}_T}\lambda_{i,t,s}L^T_{i,t,s}-\sum_{l\in\mathcal{L}_T}PF_l\left(\overline{\omega}_{l,t,s}+\underline{\omega}_{l,t,s}\right)-\sum_{g\in\mathcal{G}_T,b\in\mathcal{B}_g}\overline{\rho}_{g,b,t,s}\overline{P}^T_{g,b}\\ -\xi_{t,s}\Gamma^T_t-\sum_{i\in\mathcal{N}_\infty}\overline{P}^{SE}_i\left(\overline{\kappa}_{i,t,s}+\underline{\kappa}_{i,t,s}\right)\end{aligned} \tag{EC.28}$$

Henceforth, the right-hand side of (EC.28) will be referred to as **B**. Observe that **B** is a linear function. Thus, $\pi^E_{t,s}P^{SE}_{i,t,s}+\pi^C_{t,s}\varrho^{SE}_{t,s}$ can be written as:

$$\begin{aligned}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}p_{s,t}\left(\pi^C_{t,s}\varrho^{SE}_{t,s}+\sum_{i\in\mathcal{N}_\infty}\pi^E_{i,t,s}P^{SE}_{i,t,s}\right)=-\mathbf{B}\\ +\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}p_{s,t}\sum_{g\in\mathcal{G}_T}\sum_{b\in\mathcal{B}_g}\left(C^T_{g,b}\rho^T_{g,b,t,s}\right)\end{aligned} \tag{EC.29}$$

III. Test System Data

The proposed model was validated for modified versions of IEEE's 14 and 34-bus systems, employed as the transmission and distribution networks, respectively. The adopted power systems are

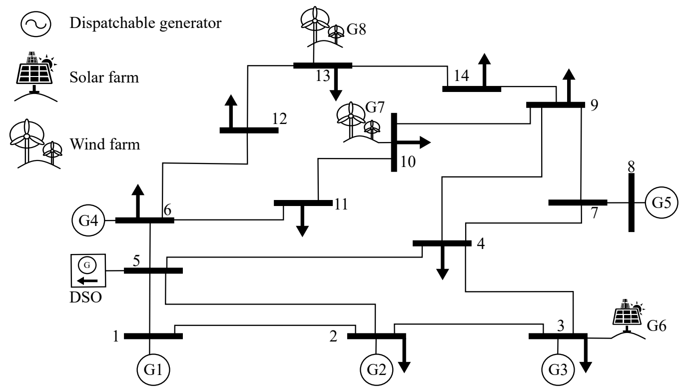

Fig. EC.1. Transmission System Illustration

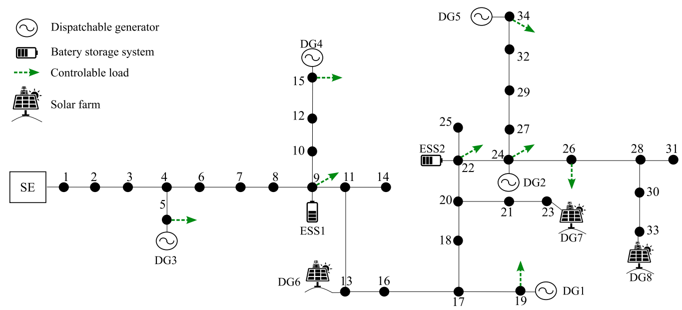

Fig. EC.2. Distribution System Illustration

illustrated in figs. EC.1 and EC.2. The parameters of each generator, energy storage system (ESS) and controllable load are presented in Tables EC.I, EC.II and EC.III, respectively.

An illustration of each conventional generator's production cost function, whose parameters are listed in Table EC.I, is shown in figs. EC.3 and EC.4. These figures can be used to easily determine the cheapest and most expensive power sources. Nonetheless, given the considerable difference in the capacities of generators connected to the transmission and distribution systems, it is difficult to directly compare the two figures. In this sense, fig. EC.5 depicts the optimal dispatch of each generator for a given energy price. An example considering a \$2.55/MWh clearing price is highlighted in the figure. Observe that the lines that represent the dispatch of DGs 1, 2, and 4 are below the highlighted clearing price. This means that for this clearing price, these DGs should dispatch their full capacity. The lines that represent the dispatch of DGs 3 and 5 intersect the clearing price line. In this sense, these generators should dispatch, respectively, 5.8067 MWh and 6.0667 MWh. Thus, the optimal combined injection of DGs for a \$2.55/MWh clearing price is 31.8734 MWh.

Data regarding the networks' physical parameters, nodal load shapes and stochastic parameters are available in [EC.4].

TABLE EC.I
GENERATORS PARAMETERS

Generator	Cost Parameters a	Cost Parameters b	Power Limits $\overline{P}$ (MW)	Generator	Cost Parameters a	Cost Parameters b	Power Limits $\overline{S}$ (MVA)
G1	0.017300	1.834500	50	DG1	0.040100	1.850000	7
G2	0.016600	1.079930	75	DG2	0.052020	1.750000	5.5
G3	0.023700	2.038054	55	DG3	0.030200	2.200000	6.5
G4	0.019700	2.158896	50	DG4	0.063800	1.000000	7.5
G5	0.017700	1.981700	40	DG5	0.045600	2.000000	10
G6	-	-	5	DG6	-	-	1.5
G7	-	-	15	DG7	-	-	1.0
G8	-	-	10	DG8	-	-	0.5

TABLE EC.II
ENERGY STORAGE SYSTEMS PARAMETERS

ESS	Charging Limit (MW)	Discharging Limit (MW)	$\underline{SOC}$ (MWh)	$\overline{SOC}$ (MWh)	$SOC^{(0)}$ (MWh)
9	1	1	1	5	5
22	1	1	1	5	5

TABLE EC.III
CONTROLLABLE LOADS PARAMETERS

Load	Maximum Hourly Load Increase (%)	Maximum Hourly Load Decrease (%)
L5, L15, L19	20	20
L9, L24, L34	15	15
L22, L26	10	10

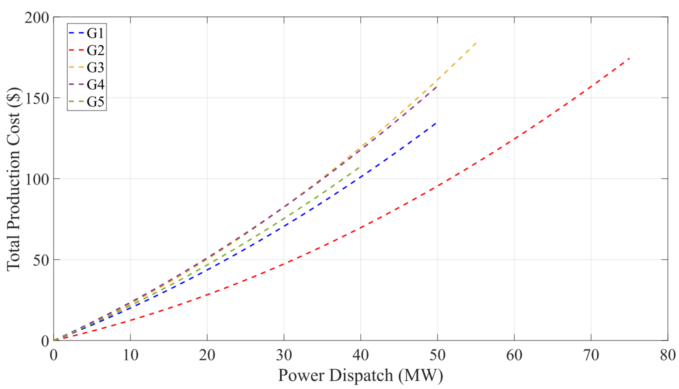

Fig. EC.3. Production costs of transmission-connected generators.

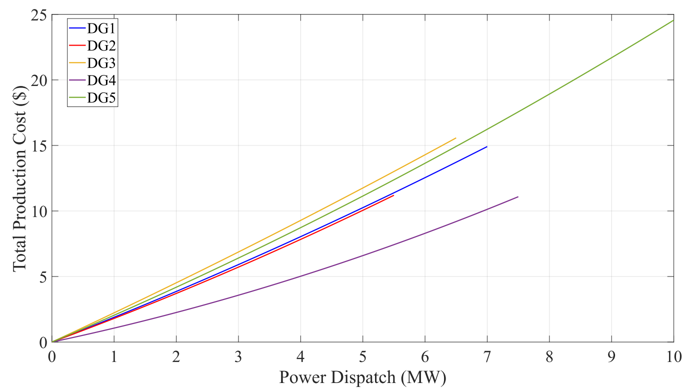

Fig. EC.4. Production costs of distribution-connected generators.

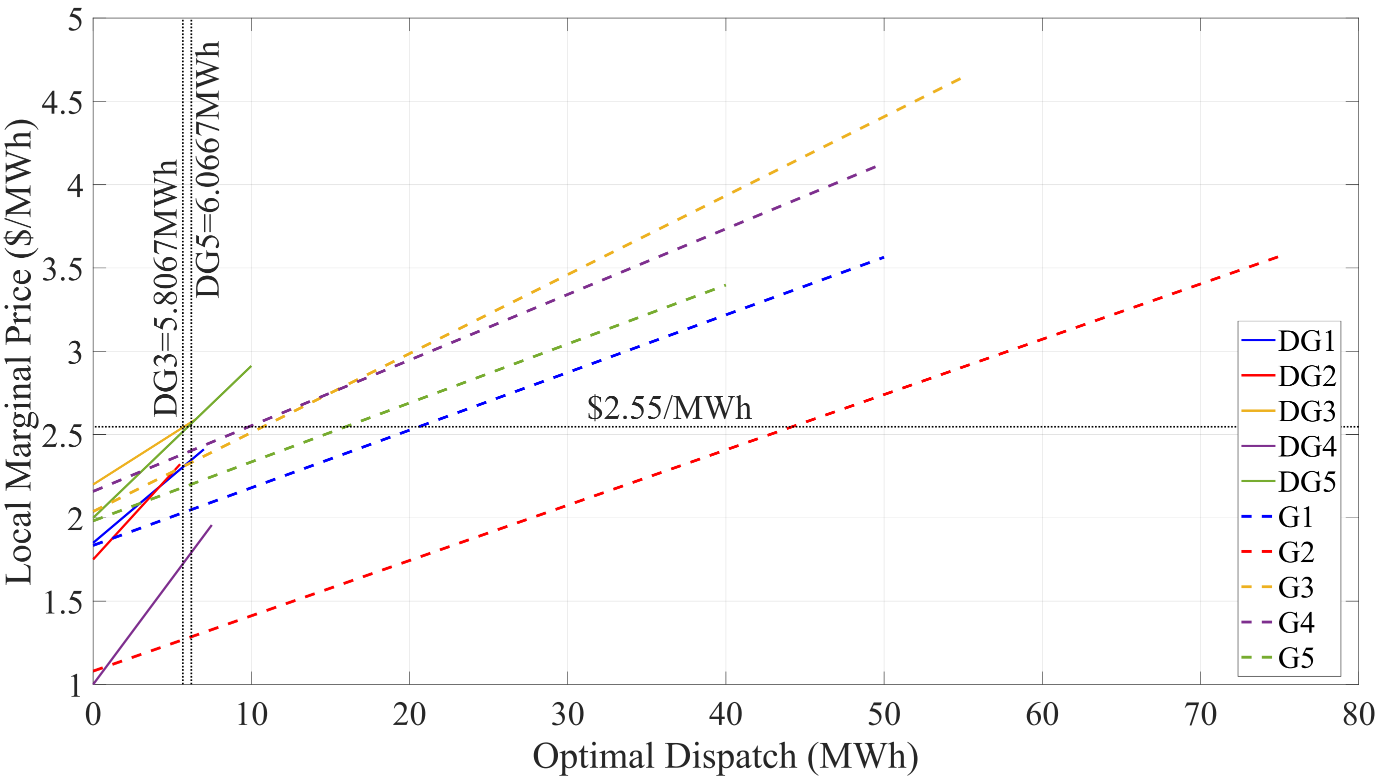

Fig. EC.5. Energy price *v.s.* optimal dispatch.

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