

# Bilevel approach for optimal location and contract pricing of distributed generation in radial distribution systems using mixed-integer linear programming

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**Abstract:** In this study, a novel approach for the optimal location and contract pricing of distributed generation (DG) is presented. Such an approach is designed for a market environment in which the distribution company (DisCo) can buy energy either from the wholesale energy market or from the DG units within its network. The location and contract pricing of DG is determined by the interaction between the DisCo and the owner of the distributed generators. The DisCo intends to minimise the payments incurred in meeting the expected demand, whereas the owner of the DG intends to maximise the profits obtained from the energy sold to the DisCo. This two-agent relationship is modelled in a bilevel scheme. The upper-level optimisation is for determining the allocation and contract prices of the DG units, whereas the lower-level optimisation is for modelling the reaction of the DisCo. The bilevel programming problem is turned into an equivalent single-level mixed-integer linear optimisation problem using duality properties, which is then solved using commercially available software. Results show the robustness and efficiency of the proposed model compared with other existing models. As regards to contract pricing, the proposed approach allowed to find better solutions than those reported in previous works.

## 1 Introduction

### 1.1 Distributed generation

In recent years a multitude of events have created a new environment for the electric power infrastructure. The presence of small-scale generation near load spots is becoming common. This type of generation is known as distributed generation (DG). DG can be broadly defined as the electric power produced by (typically) small-scale generators located within the distribution network or on the customer side of the meter [1, 2]. DG units can be powered by both conventional energy resources (gas turbines, fuel cells and microturbines) and renewable ones (biofuels, wind turbines and photovoltaic generation). The factors that have led to an upsurge in interest in the development and utilisation of DG include:

- deregulation of the electric utility industry;
- advances in small-scale generation technologies;
- public opposition to building new transmission lines based on environmental grounds;
- awareness of the potential benefits of DG, such as reduction in power losses and deferral of investments in network expansion; and

- rapid growth of electric power demand.

Owing to the factors listed above, many countries have encouraged DG through their energy policies [3]. The assessment of DG's impact has been the focus of a number of studies [4–7]. The main advantages of implementing DG include power loss reduction, improvement of voltage profile, reduction of network congestion and deferral of investment in network expansion. These benefits depend on the location and sizing of the DG units, as well as the parameters of the network. It is well known that DG also has the potential to cause technical problems such as overvoltages or overloads. In this regard, several studies have been conducted in order to determine the optimal sizing and location of DG in electrical systems. Such studies include analytical approaches [8, 9], metaheuristics [10–12] and non-linear programming techniques [13, 14]. Although most of these methodologies are addressed from the standpoint of the Distribution Company (DisCo) and are aimed at maximising the potential benefits of DG, in the model proposed in this paper we consider not only the point of view of the DisCo, but also that of the DG owner. We envisage a market structure in which DisCos are free to purchase energy from either the wholesale energy market, DG units within their networks or both.

## 1.2 Market structure

With the unbundling of the operation of electric power systems, retailers emerged to fill the gap between the wholesale energy market and small consumers. In some markets it is now commonplace to find retailers playing a dual role, acting as the DisCo as well. The main role of a DisCo is to supply the energy demanded by its consumers while remaining within network constraints. To meet the expected demand, the DisCo purchases energy from the wholesale energy market. Most of the energy purchased by the DisCo is negotiated through long-term bilateral contracts at a price based on the wholesale energy market price. Although it is well known that most DG technologies cannot compete with centrally dispatched generation, it is also true that DG provides technical benefits to the distribution network. In this regard, buying energy from a DG source can be an attractive option for a DisCo. This market structure is illustrated in Fig. 1. Apart from the wholesale energy market, DisCos can also buy energy from DG units owned by independent producers. The main advantage of such a market structure is that it allows DisCos to purchase energy near the loads. Locating energy supplies closer to customers might have desirable effects on the distribution system, such as reduction of power losses and improvement of voltage profile.

The aforementioned market structure is based on the hypothesis that DG owners are interested in engaging in business with DG units. Furthermore, as DisCos must determine the amount of energy to be purchased, the proposed methodology is restricted to dispatchable DG technologies.

## 1.3 Decision-making problem of the DisCo

Bearing in mind the market structure described above, at first glance the easiest solution would be for the DisCo to buy energy from the cheapest source. However, the decision is not that simple, as there are some issues related to physical limits of the distribution network that must be taken into account such as minimum and maximum voltage limits for every node of the system; power flow limits through lines and transformers; and power losses. Consequently, the

DisCo must consider not only the price offers of the DG units but also the impact of the power injected by these units. For instance, if the power injected by a DG unit contributes to the enforcement of a voltage constraint and/or has a positive impact on reducing power losses, then buying energy from this DG unit might be a good option, even if costs slightly more than the wholesale market price. If a DG unit negatively impacts the distribution network, buying energy from this unit might not be a good option, even if it costs less than the wholesale market price. Deciding how much energy to buy from one source or another is not a trivial task; however, an optimal power flow (OPF) will help the DisCo to decide how much energy to purchase from both the wholesale energy market and DG units. To account for load variation over time, the DisCo might have to run several OPFs, leading to what is known as an OPF-based dispatch.

## 1.4 Decision-making problem of the DG owner

In the last section, the DisCo's decision-making problem regarding the minimisation of the payments incurred in meeting expected demand was outlined. However, there is another decision-making problem that must be considered: that of the DG owner. In this case, we have modelled two decision variables for the DG owner, contract price (CP) and location. Consequently, the DG owner must decide the CP and location that will render the maximum profit; however, the reaction of the DisCo must be taken into consideration. That is, the DG owner must be aware of the fact that, for a given location and CP of his units, the DisCo will solve an OPF-based dispatch in order to determine the amount of energy to be bought from them. If the DG owner offers his energy at a very low price, he might sell a great amount of energy, but this will not guarantee maximum profits. Conversely, if he decides to raise the price in order to obtain higher profits, the DisCo might decide not to buy energy from the DG units, and instead supply the entire demand of its network from the wholesale energy market. Regarding location, there are some strategic nodes at which the DG owner might make higher profits; generally those locations are at the end of heavily loaded feeders, far from a substation.

## 1.5 Bilevel modelling framework

The decision-making problems of the DisCo and the DG owner can be combined into a bilevel programming problem (BPP). A BPP is a decision-making problem involving two optimisation levels. In this case, the DG owner is positioned in the upper optimisation level, choosing the allocation and CP of DG units to maximise the profits obtained from the energy sold to the DisCo. The DisCo is positioned on the lower optimisation level and calculates the energy purchased from the DG units and from the wholesale energy market. The allocation and CP make up a parameter set of the lower-level problem, to which the DisCo reacts by buying more or less energy to minimise the total payments incurred in satisfying the expected demand. This relationship is depicted in Fig. 2.

Owing to their hierarchical structure, BPPs are intrinsically non-convex. This applies even for BPPs with linear upper and lower-level optimisation problems [15]. A common approach to dealing with BPPs is to transform the lower-level optimisation problem into a set of constraints so that the original BPP becomes a single-level optimisation problem.

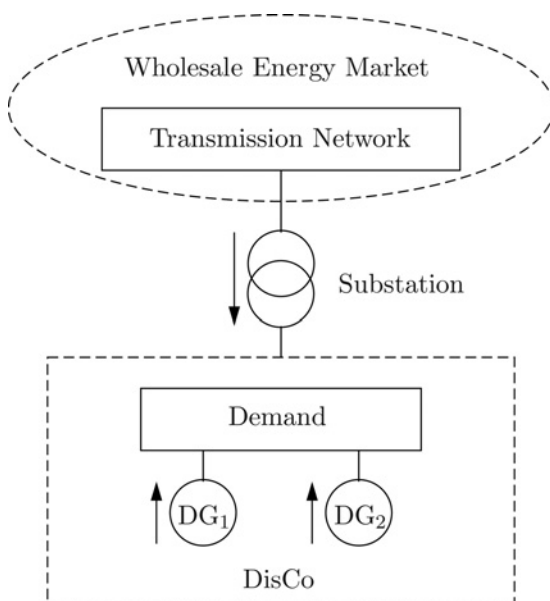


Fig. 1 Market structure

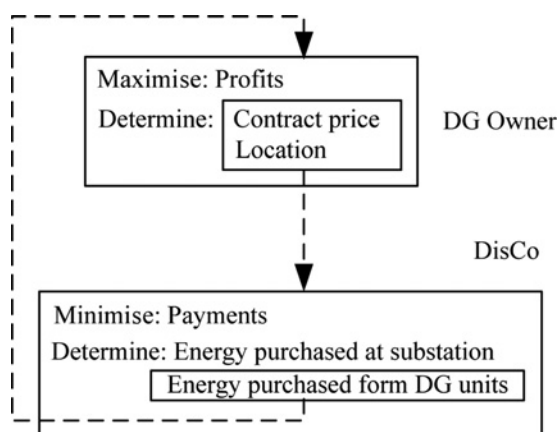


Fig. 2 Bilevel programming diagram

Such a transformation can be achieved either by applying Karush–Kuhn–Tucker (KKT) optimality conditions or by using duality theory. From a rigorous mathematical standpoint, both methodologies are equivalent; however, the latter is more suitable when using commercially available branch-and-cut solvers. This is because the equivalent number of constraints and binary variables involved in using duality theory is considerably lower than the one needed when using KKT optimality conditions [16].

In [17], the authors propose a bilevel programming model for the optimal contract pricing of DG. Such a methodology is based on a non-linear approximation of the power flow equations (the mathematical expressions used to model the distribution of power flows among the elements of a network). In this case, the lower-level problem is substituted by its KKT optimality conditions. The main drawback of such an approach is that, being the lower-level model of a non-linear programming problem, the KKT conditions constitute necessary but not sufficient conditions for optimality. As a consequence, the quality and type of solutions are sensitive to the initial values assigned to the state variables in the solver.

## 1.6 Literature review

There are several aspects to be considered when conducting studies concerning the operation and planning of DG. In this regard, the location and sizing of new DG units has been the focus of several studies. In [8], analytical methods are presented for the optimal location of DG in both radial and meshed networks. The objective function considered in [8] is the minimisation of power losses. In [9], an analytical expression to calculate the optimal size of DG is presented. In that study, a loss sensitivity factor is used to find the corresponding location that minimises power losses. Several metaheuristic techniques have also been used to determine the optimal location of DG units. In [10], an evolutionary algorithm is presented for the optimal sizing and location of DGs: this algorithm is based on a multi-objective approach that allows the planner to decide the best trade-off between the cost of power losses, cost of energy not supplied, cost of network upgrading and cost of energy required by the served customers. In [18], a genetic algorithm (GA) is used for the optimal location of DG for profit maximisation, loss reduction and voltage improvement. The approach proposed in [18] is based on a pricing mechanism usually found in transmission systems. In [19], a GA is implemented to maximise the potential benefits of DG. The approach

presented in [19] can be used in a single or multi-objective fashion. Other applications of GAs for the optimal location of DG are presented in [20, 21]. In [11], a Tabu search approach is developed for the optimal location of DG units from a viewpoint of power loss minimisation. In [22], the authors propose a particle swarm optimisation (PSO) technique to find the optimal location and sizing of DG units using a multi-objective index. Technical issues considered in [22] include power losses, voltage profile, line loading and the effect of DG on voltage collapse. PSO has also been proposed as a suitable technique for the location of DG units in [12, 23, 24]. In [25], an artificial bee-colony algorithm is proposed to determine the optimal size, location and power factor of DG units in order to minimise power losses. An extensive review of artificial intelligence techniques applied to the location and sizing of DG can be consulted in [26]. Several optimisation techniques based on mathematical programming have also been applied to the optimal location of DG [13, 27, 28]. In [13, 14], the optimal location and sizing of DG is calculated using conventional OPF techniques and locational marginal pricing. In [27], the optimal location of DG is calculated using power flow and linear programming. In [28], the authors propose a mixed-integer linear programming model for the optimal location of DG considering different scenarios.

The methodologies presented above do not explicitly consider the interaction between the DG owner and the DisCo. This interaction is indicated by the fact that these agents do not have the same objective functions, and can be modelled using multi-objective optimisation or bilevel programming. In [29], the interaction between the DisCo and DG owner is considered in terms of a number of objectives, providing win–win strategies for both parties. Bilevel programming has also been applied to markets [30, 31] and the interdiction problem [32, 33] to model the interaction between two agents that act sequentially and have two different objective functions. In [17, 34], the authors used a bilevel programming model to model the interaction between the DisCo and the DG owner.

## 1.7 Contributions

In this paper, we propose a more comprehensive model for the allocation and contract pricing of DG units through a BPP. In our model, the lower-level programming problem is substituted by a set of constraints using duality properties and linearisation schemes. The BPP is then recast as a mixed-integer linear programming problem solvable via commercially available branch-and-cut solvers.

The main contributions of this research are 3-fold:

1. We provide a robust framework for a DG owner based on bilevel programming, which allows for the selection of optimal allocation and CP.
2. Duality theory and linearisation schemes are used to turn the BPP into a mixed-integer linear programming problem.
3. We improve on the methodologies and models previously reported in the literature.

## 2 Mathematical model formulation

In this section, we present the proposed bilevel programming model and how it is recast as a mixed-integer linear programming model.



## 2.1 Bilevel programming model

The bilevel formulation for the optimal allocation and contract pricing of dispatchable DG in electrical distribution systems (EDS) is given by (1)–(4). The dual variables associated with each set of constraints appear next to the corresponding equation. To provide a linear formulation, the square of voltage and current flow magnitude have been linearised, as explained in Appendix.

$$\text{Max}_{C_p, w} \sum_{j \in J} \sum_{t \in T} \Delta t (C_{p_j} - c_j) P_{j,t}^{\text{dg}} \quad (1)$$

subject to

$$\sum_{j \in J} w_j \leq \bar{w} \quad (2a)$$

$$w_j \in \{0, 1\} \quad \forall j \in J \quad (2b)$$

$$\text{Min}_{\substack{P^{\text{se}}, P^{\text{dg}}, V, V^{\text{sqr}}, \\ \Delta V, \Delta V^{\text{sqr}}, P^{\text{fm}}, \\ P^{\text{to}}, I, I^{\text{sqr}}, \Delta I, I^+, I^-}} \sum_{k \in K} \sum_{t \in T} \Delta t \rho_{k,t} P_{k,t}^{\text{se}} + \sum_{j \in J} \sum_{t \in T} \Delta t C_{p_j} P_{j,t}^{\text{dg}} \quad (3)$$

subject to

$$P_{i,t}^{\text{se}} + P_{i,t}^{\text{dg}} - \sum_{ij \in L} P_{ij,t}^{\text{fm}} - \sum_{ki \in L} P_{ki,t}^{\text{to}} = P_{i,t}^{\text{d}}, \quad (4a)$$

$$\forall i \in I, \forall t \in T: \pi_{i,t}$$

$$P_{ij,t}^{\text{fm}} + P_{ij,t}^{\text{to}} = R_{ij} I_{ij,t}^{\text{sqr}}, \quad \forall ij \in L, \forall t \in T: \lambda_{ij,t} \quad (4b)$$

$$P_{ij,t}^{\text{fm}} - P_{ij,t}^{\text{to}} = \frac{R_{ij}}{Z_{ij}^2} (V_{i,t}^{\text{sqr}} - V_{j,t}^{\text{sqr}}), \quad (4c)$$

$$\forall ij \in L, \forall t \in T: \alpha_{ij,t}$$

$$I_{ij,t} = \frac{V_{i,t} - V_{j,t}}{Z_{ij}}, \quad \forall ij \in L, \forall t \in T: \varphi_{ij,t} \quad (4d)$$

$$I_{ij,t} - \bar{I}_{ij} \leq 0, \quad \forall ij \in L, \forall t \in T: \bar{\phi}_{ij,t} \quad (4e)$$

$$-I_{ij,t} - \bar{I}_{ij} \leq 0, \quad \forall ij \in L, \forall t \in T: \underline{\phi}_{ij,t} \quad (4f)$$

$$V_{i,t} - \bar{V} \leq 0, \quad \forall i \in I, \forall t \in T: \bar{w}_{i,t} \quad (4g)$$

$$-V_{i,t} + \underline{V} \leq 0, \quad \forall i \in I, \forall t \in T: \underline{w}_{i,t} \quad (4h)$$

$$P_{j,t}^{\text{dg}} - w_j \bar{P}_j^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T: \bar{\beta}_{j,t} \quad (4i)$$

$$-P_{j,t}^{\text{dg}} + w_j \underline{P}_j^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T: \underline{\beta}_{j,t} \quad (4j)$$

$$P_{k,t}^{\text{se}} - \bar{P}_k^{\text{se}} \leq 0, \quad \forall k \in K, \forall t \in T: \bar{\delta}_{k,t} \quad (4k)$$

$$-P_{k,t}^{\text{se}} + \underline{P}_k^{\text{se}} \leq 0, \quad \forall k \in K, \forall t \in T: \underline{\delta}_{k,t} \quad (4l)$$

$$\left. \begin{array}{ll} (18.a) & : \rho_{i,t} \\ (18.b) & : \varepsilon_{i,t} \\ (18.c) & : \sigma_{i,t} \\ (18.d) & : \bar{v}_{i,t,p} \\ (18.e) & : \underline{v}_{i,t,p} \end{array} \right\} \quad \left. \begin{array}{ll} (19.a) & : \eta_{ij,t} \\ (19.b) & : \theta_{ij,t} \\ (19.c) & : \tau_{ij,t} \\ (19.d) & : \bar{\kappa}_{ij,t,p} \\ (19.e) & : \underline{\kappa}_{ij,t,p} \\ (19.f) & : \bar{l}_{ij,t} \\ (19.g) & : \underline{l}_{ij,t} \end{array} \right\} \quad (4m)$$

The upper-level optimisation problem is used to model the profit maximisation of the DG owner given by (1), where  $J$

and  $T$  are the sets of DG units and time intervals, respectively.  $\Delta t$  is the length of time interval  $t$  in hours.  $C_{p_j}$  and  $c_j$  are the CP and production cost of DG unit  $j$  in €/MWh, respectively.  $P_{j,t}^{\text{dg}}$  is the active power supplied by DG at node  $j$  in period  $t$ .

Equation (2) is used to model the upper-level constraints, where  $w_j$  is the binary variable for the allocation of the DG unit  $j$  and  $\bar{w}$  is the maximum number of DG units that can be added to the system. The maximum number of DG units that can be installed in the system is modelled by (2a). Equation (2b) is used to model the binary nature of the allocation of DG units. A DG unit is allocated if the corresponding value is equal to 1 and is not allocated if it is equal to 0.

The binary investment variables  $w_j$  are decision variables, and a feasible operation solution for the EDS depends on their values. The remaining variables are the operating state of a feasible solution. For a feasible investment proposal, defined through a specified value of  $w_j$ , several feasible operation states are possible.

The lower-level optimisation problem is given in (3)–(4), where  $K$  and  $L$  are the sets of DG units and lines, respectively.  $\rho_{k,t}$  is the wholesale energy price at substation  $k$  in period  $t$  in €/MWh.  $P_{k,t}^{\text{se}}$  is the active power supplied by a substation at node  $k$  in period  $t$ .  $P_{i,t}^{\text{d}}$  is the active power demand at node  $i$  in period  $t$ .  $P_{ij,t}^{\text{fm}}$  is the active power flow that leaves node  $i$  towards node  $j$  in period  $t$ .  $P_{ij,t}^{\text{to}}$  is the active power flow that leaves node  $j$  towards node  $i$  in period  $t$ .  $I$  is the set of network nodes.  $R_{ij}$  and  $Z_{ij}$  are the resistance and impedance of circuit  $ij$ .  $V_{i,t}$  is the voltage magnitude at node  $i$  in period  $t$ .  $I_{ij,t}$  is the current flow magnitude of circuit  $ij$  in period  $t$ .  $V_{i,t}^{\text{sqr}}$  and  $I_{ij,t}^{\text{sqr}}$  are the square of  $V_{i,t}$  and  $I_{ij,t}$ , respectively.  $\bar{V}$  and  $\underline{V}$  are the maximum and minimum voltage magnitude, respectively.  $\bar{I}_{ij}$  is the maximum current flow magnitude of circuit  $ij$ .  $\bar{P}_j^{\text{dg}}$  and  $\underline{P}_j^{\text{dg}}$  are the maximum and minimum active power limits of DG unit  $j$ , respectively.  $\bar{P}_k^{\text{se}}$  and  $\underline{P}_k^{\text{se}}$  are the maximum and minimum active power limits of substation  $k$ .

Equation (3) corresponds to the minimisation of the energy payments by the DisCo. This objective function is composed of two terms. The first term corresponds to the payments of the energy purchased from the wholesale energy market and delivered through the substations. The second term corresponds to the energy purchased from DG units. The CPs at which DG owners are willing to sell their energy are not decision variables, but parameters of the inner optimisation problem.

Equation (4) represents the lower-level constraints. Equations (4a)–(4d) are used to model the steady-state operation of the EDS. Constraints (4b) and (4c) are used to model the active power flows in the systems (see Appendix). The limit of current flow magnitude of circuit  $ij$  is modelled by (4e) and (4f). Equations (4g) and (4h) are used to model the maximum and minimum operating limits of the voltage magnitude of the nodes. The limit of power generated by DG units is modelled by (4i) and (4j). If  $w_j = 0$ , then  $P_{j,t}^{\text{dg}} = 0$ ; otherwise  $\underline{P}_j^{\text{dg}} \leq P_{j,t}^{\text{dg}} \leq \bar{P}_j^{\text{dg}}$ . Equations (4k) and (4l) are used to model the maximum and minimum operating limits of the power supplied through the substations. Equation (4m) represents the linearisation of the quadratic terms  $V_{i,t}^{\text{sqr}}$  and  $I_{ij,t}^{\text{sqr}}$  (see Appendix).

$\pi_{i,t}$  is the dual variable associated with the constraint of the power balance equation in node  $i$  in period  $t$ .  $\lambda_{ij,t}$  is the dual variable associated with the constraint of the active power losses of circuit  $ij$  in period  $t$ .  $\alpha_{ij,t}$  is the dual variable associated with the constraint of the difference of active power flows of circuit  $ij$  in period  $t$ .  $\phi_{ij,t}$  is the dual variable associated with the constraint of the current flow magnitude of circuit  $ij$  in period  $t$ .  $\bar{\phi}_{ij,t}$  and  $\underline{\phi}_{ij,t}$  are the dual variables associated with the constraints of the maximum and minimum current flow limits of circuit  $ij$  in period  $t$ , respectively.  $\bar{w}_{i,t}$  and  $\underline{w}_{i,t}$  are the dual variables associated with the constraints of the maximum and minimum voltages in node  $i$  in period  $t$ .  $\bar{\beta}_{j,t}$  and  $\underline{\beta}_{j,t}$  are the dual variables associated with the constraints of the maximum and minimum active power generated by DG unit  $j$  in period  $t$ .  $\bar{\delta}_{k,t}$  and  $\underline{\delta}_{k,t}$  are dual variables associated with the constraints of the maximum and minimum active power generated by substation  $k$  in period  $t$ .

Finally,  $\rho_{i,t}$ ,  $\varepsilon_{i,t}$ ,  $\sigma_{i,t}$ ,  $\bar{v}_{i,t,p}$ ,  $\underline{v}_{i,t,p}$ ,  $\eta_{ij,t}$ ,  $\theta_{ij,t}$ ,  $\tau_{ij,t}$ ,  $\bar{\kappa}_{ij,t,p}$ ,  $\underline{\kappa}_{ij,t,p}$ ,  $\bar{l}_{ij,t}$  and  $\underline{l}_{ij,t}$  are the dual variables of the constraints associated with the square of voltage and current flow magnitude presented in Appendix.

## 2.2 Transforming the bilevel programming model into a single-level optimisation problem

For a given set of decision variables ( $Cp_j$  and  $w_j$ , from the upper-level problem), the problem given by (3)–(4) is a linear programming problem. Therefore it can be transformed into a set of constraints which correspond to the primal constraints, the constraints of the dual problem and the strong duality condition [35]. The bilevel problems (1)–(4) can be transformed into a single-level optimisation problem, substituting the lower-level problem by the

aforementioned set of constraints and incorporating them into the upper-level problem.

$$\begin{aligned} & \sum_{i \in I} \sum_{t \in T} (P_{i,t}^d \pi_{i,t} - \underline{V}^2 \rho_{i,t} + \bar{V} \bar{w}_{i,t} + \underline{V} (\sigma_{i,t} - \underline{w}_{i,t})) \\ & + \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} (\bar{\Delta}^V \bar{v}_{i,t,p}) + \sum_{ij \in L} \sum_{t \in T} \sum_{p \in P} (\bar{\Delta}_{ij}^L \bar{\kappa}_{ij,t,p}) \\ & + \sum_{ij \in L} \sum_{t \in T} (\bar{l}_{ij} (\bar{\phi}_{ij,t} + \underline{\phi}_{ij,t})) \\ & + \sum_{j \in J} \sum_{t \in T} (\bar{P}_j^{\text{dg}} w_j \bar{\beta}_{j,t} - P_j^{\text{dg}} w_j \underline{\beta}_{j,t}) \\ & + \sum_{k \in K} \sum_{t \in T} (\bar{P}_k^{\text{sc}} \bar{\delta}_{k,t} - P_k^{\text{sc}} \underline{\delta}_{k,t}) \end{aligned} \quad (5)$$

**2.2.1 Dual problem corresponding to the lower-level problems:** The dual problem associated with the lower-level problems (3) and (4) is

$$\begin{aligned} & \text{Max} \quad (\text{see (5)}) \\ & \pi, \lambda, \alpha, \phi, \theta, \tau, \rho, \\ & \varepsilon, \sigma, \bar{\kappa}, \underline{\kappa}, \bar{l}, \underline{l}, \\ & \bar{\phi}, \underline{\phi}, \bar{w}, \underline{w}, \bar{\beta}, \underline{\beta}, \bar{\delta}, \underline{\delta} \end{aligned} \quad (6)$$

subject to (see (7))

**2.2.2 Non-linear programming formulation:** The strong duality condition states that a primal feasible solution and a dual feasible solution are optimal solutions of the primal and dual problems, respectively, if and only if the values of the objective functions of both problems are equal (see (9) and (13)).

$$\left. \begin{aligned} & \pi_{k,t} + \bar{\delta}_{k,t} - \underline{\delta}_{k,t} = \Delta t \rho_{k,t}, & \forall k \in K, \forall t \in T \\ & \pi_{j,t} + \bar{\beta}_{j,t} - \underline{\beta}_{j,t} = \Delta t Cp_j, & \forall j \in J, \forall t \in T \\ & - \sum_{ij \in L} \frac{\varphi_{ij,t}}{Z_{ij}} + \sum_{ki \in L} \frac{\varphi_{ki,t}}{Z_{ki}} - 2V \rho_{i,t} + \sigma_{i,t} + \bar{w}_{i,t} - \underline{w}_{i,t} = 0, & \forall i \in I, \forall t \in T \\ & - \sum_{ij \in L} \frac{R_{ij}}{Z_{ij}^2} \alpha_{ij,t} + \sum_{ki \in L} \frac{R_{ki}}{Z_{ki}^2} \alpha_{ki,t} + \rho_{i,t} = 0, & \forall i \in I, \forall t \in T \\ & - \rho_{i,t} + \varepsilon_{i,t} = 0, & \forall i \in I, \forall t \in T \\ & - m_p^V \varepsilon_{i,t} - \sigma_{i,t} + \bar{v}_{i,t,p} - \underline{v}_{i,t,p} = 0, & \forall i \in I, \forall t \in T, \forall p = 1 \dots P \\ & \varphi_{ij,t} + \theta_{ij,t} + \bar{\phi}_{ij,t} - \underline{\phi}_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & - R_{ij} \lambda_{ij,t} + \eta_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & - m_{ij,p}^I \eta_{ij,t} - \tau_{ij,t} + \bar{\kappa}_{ij,t,p} - \underline{\kappa}_{ij,t,p} = 0, & \forall ij \in L, \forall t \in T, \forall p = 1 \dots P \\ & - \theta_{ij,t} + \tau_{ij,t} - \bar{l}_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & \theta_{ij,t} + \tau_{ij,t} - \underline{l}_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & - \pi_{i,t} + \lambda_{ij,t} + \alpha_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & - \pi_{j,t} + \lambda_{ij,t} - \alpha_{ij,t} = 0, & \forall ij \in L, \forall t \in T \\ & \bar{\kappa}_{ij,t,p}, \underline{\kappa}_{ij,t,p} \leq 0, & \forall ij \in L, \forall t \in T, \forall p = 1 \dots P \\ & \bar{v}_{i,t,p}, \underline{v}_{i,t,p} \leq 0, & \forall i \in I, \forall t \in T, \forall p = 1 \dots P \\ & \bar{\phi}_{ij,t}, \underline{\phi}_{ij,t}, \bar{l}_{ij,t}, \underline{l}_{ij,t} \leq 0, & \forall ij \in L, \forall t \in T \\ & \bar{w}_{i,t}, \underline{w}_{i,t} \leq 0, & \forall i \in I, \forall t \in T \\ & \bar{\beta}_{j,t}, \underline{\beta}_{j,t} \leq 0, & \forall j \in J, \forall t \in T \\ & \bar{\delta}_{k,t}, \underline{\delta}_{k,t} \leq 0, & \forall k \in K, \forall t \in T \end{aligned} \right\} \quad (7)$$

The equivalent single-level problem is

$$\text{Max} \sum_{j \in J} \sum_{t \in T} \Delta t C_{p_j} P_{j,t}^{\text{dg}} - \sum_{j \in J} \sum_{t \in T} \Delta t c_j P_{j,t}^{\text{dg}} \quad (8)$$

subject to

constraint (2):Upper-level constraints;

constraint (4):Lower-level primal constraints;

constraint (7):Lower-level dual constraints;

$$\sum_{k \in K} \sum_{t \in T} \Delta t \rho_{k,t} P_{k,t}^{\text{se}} + \sum_{j \in J} \sum_{t \in T} \Delta t C_{p_j} P_{j,t}^{\text{dg}} = (5):\text{Non-linear strong duality condition} \quad (9)$$

The above formulation corresponds to a non-linear programming problem because of the products of decision variables: (i)  $C_{p_j}$  and  $P_{j,t}^{\text{dg}}$ ; (ii)  $w_j$  and  $\bar{P}_{j,t}$ ; and (iii)  $w_j$  and  $\underline{P}_{j,t}$ . In order to use a conventional mixed integer linear programming (MILP) solver, it is preferable to obtain a linear equivalent for (8) and (9).

### 2.3 Mixed-integer linear programming formulation

The CP of a DG unit can be discretised into a set of  $Q$  steps, for example,  $[C_1^{\text{pd}}, C_2^{\text{pd}}, C_3^{\text{pd}}, \dots, C_{Q+1}^{\text{pd}}]$ , where  $C_q^{\text{pd}}$  is the  $q$ th value of the DG units CP discretisation and  $Q$  is the number of discretisations associated with the DG units CP. Thus, the product  $C_{p_j} P_{j,t}^{\text{dg}}$  is linearised by the use of the binary variables  $x_{j,q}$  to select the  $q$ th value of CP of DG unit  $j$  and the auxiliary variables  $CP_{j,t,q}^{\text{dg}} \forall q = 1 \dots Q$ , as shown in (9). Also, the binary expansion approach can be used, as shown in [36, 37].

$$\min_q (C_q^{\text{pd}}) \underline{P}_{j,t}^{\text{dg}} x_{j,q} \leq CP_{j,t,q}^{\text{dg}} \leq \max_q (C_q^{\text{pd}}) \bar{P}_{j,t}^{\text{dg}} x_{j,q}, \quad \forall j \in J, \forall t \in T, \forall q = 1 \dots Q \quad (10a)$$

$$\min_q (C_q^{\text{pd}}) \underline{P}_{j,t}^{\text{dg}} (1 - x_{j,q}) \leq C_q^{\text{pd}} P_{j,t}^{\text{dg}} - CP_{j,t,q}^{\text{dg}} \leq \max_q (C_q^{\text{pd}}) \bar{P}_{j,t}^{\text{dg}} (1 - x_{j,q}), \quad \forall j \in J, \forall t \in T, \forall q = 1 \dots Q \quad (10b)$$

$$\sum_{q=1}^Q x_{j,q} = 1, \quad \forall j \in J \quad (10c)$$

$$x_{j,q} \text{ binary} \quad \forall j \in J, \forall q = 1 \dots Q \quad (10d)$$

Constraints (10a) and (10b) define the values of  $CP_{j,t,q}^{\text{dg}}$ ,  $\forall j \in J, \forall t \in T, \forall q = 1 \dots Q$ . If  $x_{j,q} = 0$ , then  $CP_{j,t,q}^{\text{dg}} = 0$  and  $\min_q (C_q^{\text{pd}}) \underline{P}_{j,t}^{\text{dg}} \leq C_q^{\text{pd}} P_{j,t}^{\text{dg}} \leq \max_q (C_q^{\text{pd}}) \bar{P}_{j,t}^{\text{dg}}$ , otherwise  $CP_{j,t,q}^{\text{dg}} x = C_q^{\text{pd}} P_{j,t}^{\text{dg}}$  and  $\min_q (C_q^{\text{pd}}) \underline{P}_{j,t}^{\text{dg}} \leq CP_{j,t,q}^{\text{dg}} \leq \max_q (C_q^{\text{pd}}) \bar{P}_{j,t}^{\text{dg}}$ , where  $\min_q (C_q^{\text{pd}}) \underline{P}_{j,t}^{\text{dg}}$  and  $\max_q (C_q^{\text{pd}}) \bar{P}_{j,t}^{\text{dg}}$  provide a sufficient degree of freedom to  $CP_{j,t,q}^{\text{dg}}$ . Equation (10c) assures that it is possible to choose only one CP for the DG unit  $j$ .

The products  $\bar{P}_{j,t}^{\text{dg}} w_j \bar{\beta}_{j,t}$  and  $\underline{P}_{j,t}^{\text{dg}} w_j \underline{\beta}_{j,t}$  can be linearised using the auxiliary variables  $PB_{j,t}^{\text{dg}}$  and  $Pb_{j,t}^{\text{dg}}$ , respectively,

and the disjunctive formulation as shown in (11).

$$-Mw_j \leq PB_{j,t}^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T \quad (11a)$$

$$-M(1 - w_j) \leq \bar{P}_{j,t}^{\text{dg}} \bar{\beta}_{j,t} - PB_{j,t}^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T \quad (11b)$$

$$-Mw_j \leq Pb_{j,t}^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T \quad (11c)$$

$$-M(1 - w_j) \leq \underline{P}_{j,t}^{\text{dg}} \underline{\beta}_{j,t} - Pb_{j,t}^{\text{dg}} \leq 0, \quad \forall j \in J, \forall t \in T \quad (11d)$$

where  $M$  is a factor that provides a sufficient degree of freedom to  $PB_{j,t}^{\text{dg}}$  and  $Pb_{j,t}^{\text{dg}}$ . The constraints listed in (11) define the values of  $PB_{j,t}^{\text{dg}}$  and  $Pb_{j,t}^{\text{dg}}$ ,  $\forall j \in J, \forall t \in T$ . If  $w_j = 0$ , then  $PB_{j,t}^{\text{dg}} = Pb_{j,t}^{\text{dg}} = 0$ ,  $-M \leq \bar{P}_{j,t}^{\text{dg}} \bar{\beta}_{j,t} \leq 0$  and  $-M \leq \underline{P}_{j,t}^{\text{dg}} \underline{\beta}_{j,t} \leq 0$ ; otherwise  $PB_{j,t}^{\text{dg}} = \bar{P}_{j,t}^{\text{dg}} \bar{\beta}_{j,t}$ ,  $Pb_{j,t}^{\text{dg}} = \underline{P}_{j,t}^{\text{dg}} \underline{\beta}_{j,t}$ ,  $-M \leq PB_{j,t}^{\text{dg}} \leq 0$  and  $-M \leq Pb_{j,t}^{\text{dg}} \leq 0$ .

Finally, the problem equivalent to (8) and (9) is

$$\text{Max}_{C_{p,w}} \sum_{j \in J} \sum_{t \in T} \sum_{q=1}^Q \Delta t CP_{j,t,q}^{\text{dg}} - \sum_{j \in J} \sum_{t \in T} \Delta t c_j P_{j,t}^{\text{dg}} \quad (12)$$

subject to

constraint (2):Upper-level constraints;

constraint (4):Lower-level primal constraints;

constraint (7):Lower-level dual constraints;

constraints (10)–(11):Linearisations;

$$\begin{aligned} & \sum_{k \in K} \sum_{t \in T} \Delta t \rho_{k,t} P_{k,t}^{\text{se}} + \sum_{j \in J} \sum_{t \in T} \sum_{q=1}^Q \Delta t CP_{j,t,q}^{\text{dg}} \\ &= \sum_{i \in I} \sum_{t \in T} (P_{i,t}^{\text{d}} \pi_{i,t} - V^2 \varrho_{i,t} + \bar{V} \bar{w}_{i,t} + V(\sigma_{i,t} - \underline{w}_{i,t})) \\ &+ \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} (\bar{\Delta}^V \bar{v}_{i,t,p}) + \sum_{ij \in L} \sum_{t \in T} \sum_{p \in P} (\bar{\Delta}_{ij}^T \bar{K}_{ij,t,p}) \\ &+ \sum_{ij \in L} \sum_{t \in T} (\bar{I}_{ij} (\bar{\phi}_{ij,t} + \underline{\phi}_{ij,t})) + \sum_{j \in J} \sum_{t \in T} (PB_{j,t}^{\text{dg}} - Pb_{j,t}^{\text{dg}}) \\ &+ \sum_{k \in K} \sum_{t \in T} (\bar{P}_k^{\text{se}} \bar{\delta}_{k,t} - \underline{P}_k^{\text{se}} \underline{\delta}_{k,t}): \text{Strongduality condition} \end{aligned} \quad (13)$$

The above formulation corresponds to a mixed-integer linear programming problem. Constraints (12) and (13) replace constraints (8) and (9), respectively. As it will be illustrated in Section 3, this kind of optimisation problems can be solved with the help of standard optimisation software.

### 3 Tests and results

The proposed methodology was tested with a modified single-phase version of the IEEE 34-bus test system (see Fig. 3). The IEEE 34-bus test system is a real distribution system located in Arizona; it is based on a medium-voltage industrial 24.9 kV distribution grid [38]. We will consider three different scenarios (named A, B and C) for high, medium and low demand, respectively. Annual load

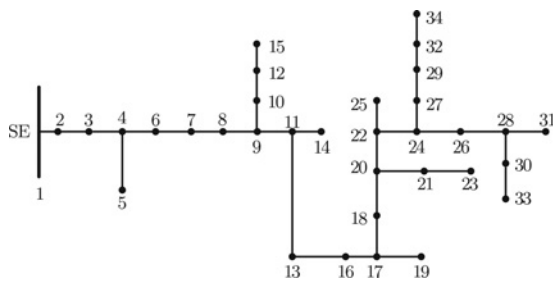


Fig. 3 IEEE 34-bus distribution system

duration curves for the different scenarios are shown in Fig. 4. Fig. 5 shows the wholesale energy market prices corresponding to the scenarios illustrated in Fig. 4. These prices are based on data obtained from the Spanish market operator OMEL [39] for the year 2008. Figs. 4 and 5 are related, as higher prices on the wholesale market are expected to occur during peak hours; conversely, lower prices are expected during off-peak hours. The data used in this system can be consulted in [17]. The CP of DG unit  $j$  is discretised using 30 steps with a lower value of 65 €/MWh and a higher value of 95 €/MWh. The number of blocks of the piecewise linearisation is equal to 20. The model (12) and (13) have been implemented in AMPL [40] and solved with CPLEX [41] (called with default options).

For this system, two different tests were performed: 'Test 1' – optimal contract pricing calculation, considering the locations of the DG units as known; and 'Test 2' – optimal location and contract pricing calculation of the DG units.

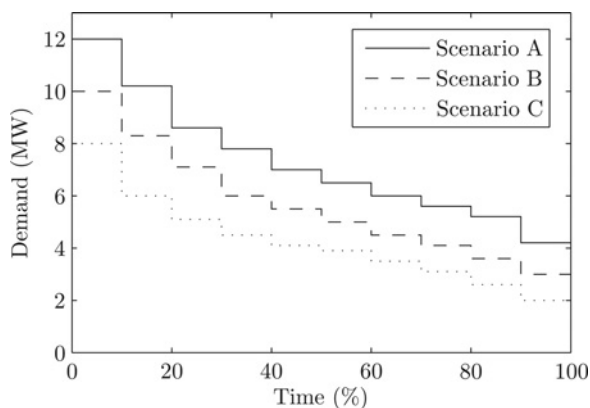


Fig. 4 Annual load duration curves for different scenarios

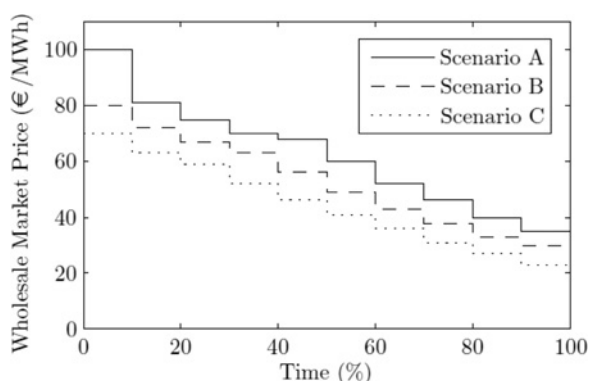


Fig. 5 Wholesale energy prices for different scenarios

### 3.1 Test 1 – optimal contract pricing of DG units in distribution systems

The aim of this test is to reproduce the results shown in [17]. In this case, suppose that there are two DG units located in buses 17 and 24 (denoted as DG1 and DG2, respectively) with a capacity of 1.5 MW and a production cost of 60 €/MWh for both DG units (this production cost corresponds to short run marginal costs). In Table 1 are shown the DG owner profits obtained by the proposed methodology and the total profits reported in [17] for the different scenarios. As it is expected the greatest profits are obtained in Scenario A. That is because in this scenario the highest wholesale market prices and forecasted load are considered; moreover, the smallest profits are obtained in Scenario C, where wholesale market prices are expected to be the lowest. For all scenarios, the total profits obtained by the proposed methodology are higher than those reported in [17] (enabling profits to reach up to nearly 41% in Scenario C), illustrating the robustness of the proposed model.

In Table 2 are shown the optimal CPs of DG1 and DG2 for the different scenarios obtained by the proposed methodology and those reported in [17]. These CPs are fixed for 1 year (the period of time under consideration). In Scenario A, the optimal CP of DG1 is higher than that of DG2; however, for Scenarios B and C, the optimal CPs of DG2 are higher than that of DG1. For all scenarios, the optimal CPs obtained by the proposed methodology are higher than those reported in [17].

In Table 3 are shown the capacity factors of the DG units for the different scenarios obtained by the proposed methodology and those reported in [17]. The capacity factor is defined as the ratio of the actual power generated by a DG unit over a period of time, and the output if it had operated at full capacity for all of that period of time, as shown in [17]. It can be observed that the greatest amount

Table 1 Profits for different scenarios (€)

DG unit	Scenario A	Scenario B	Scenario C
DG1	86 724.00	55 188.00	21 024.00
DG2	105 779.19	49 932.00	21 024.00
total	192 503.19	105 120.00	42 048.00
total [17]	183 152.68	91 087.33	29 895.19
difference	+5.11%	+15.41%	+40.65%

Table 2 Contract prices for different scenarios (€/MWh)

DG unit	Scenario A		Scenario B		Scenario C	
	Proposed	[17]	Proposed	[17]	Proposed	[17]
DG1	82.0	75.2	74.0	71.0	68.0	65.6
DG2	78.0	76.1	79.0	71.6	76.0	66.0

Table 3 Capacity factors of DG units for different scenarios (%)

DG unit	Scenario A		Scenario B		Scenario C	
	Proposed	[17]	Proposed	[17]	Proposed	[17]
DG1	30.0	41.3	30.0	30.0	20.0	19.1
DG2	44.7	47.7	20.0	31.2	10.0	20.2



of energy is sold in Scenario A, in which DG1 and DG2 present capacity factors of 30.0 and 44.7%, which correspond to a generation of 3942.00 and 5876.62 MWh, respectively. In Scenario C, the capacity factors of DG1 and DG2 drop to 20.0 and 10.0%, respectively. This means that the DG units are used mainly during peak hours. That is because in Scenario C we consider the lowest wholesale energy market prices as well as demand; thus, from the viewpoint of the DisCo, purchasing energy from the DG units is not as attractive as it is in Scenario A. For all scenarios, the capacity factors obtained by the proposed methodology are lower than those reported in [17], which means that the proposed methodology uses the DG units less than the model proposed in [17].

In Table 4 are shown the DisCo payments for different scenarios with and without DG obtained by the proposed methodology, and the savings reported in [17]. The greatest savings are obtained in Scenario A, where wholesale market prices are expected to be the highest; the DisCo savings obtained by the proposed methodology are lower than those reported in [17].

### 3.2 Test 2 – optimal location and contract pricing of DG units in distribution systems

To allow for a comparison with the results shown above without losing generality, all DG units are considered to have a capacity of 1.5 MW and a production cost of 60 €/MWh. All load buses are possible candidates for installation of a DG unit. The maximum number of DG units that can be installed in the system is equal to 2.

In Table 5 is illustrated the optimal locations and CPs obtained with the proposed methodology for different scenarios. The DG units are located at the last buses, far from the substation, for all scenarios. Also, the highest CPs are obtained in Scenario A, which corresponds to the scenario with the highest demand and wholesale energy market prices.

In Table 6 is illustrated the optimal profits obtained by the DG owner through the proposed methodology for the different scenarios, considering the locations and CPs shown in Table 5. As in 'Test 1' case the greatest profits were found in Scenario A. That is because, in this scenario, we consider the higher wholesale market prices and a forecasted load; additionally, the smallest profits are

**Table 4** DisCo payments for different scenarios (€)

	Scenario A	Scenario B	Scenario C
without DG	4 728 713.68	3 125 692.32	1 978 203.33
with DG	4 541 491.51	3 058 390.71	1 956 589.46
savings	187 222.17	67 301.61	21 613.87
savings [17]	228 960.98	101 234.00	43 156.13
difference	-18.23%	-33.52%	-49.92%

**Table 5** Optimal locations and contract prices for different scenarios (€/MWh)

DG unit	Scenario A		Scenario B		Scenario C	
	Bus	CP	Bus	CP	Bus	CP
DG1	21	82.0	20	75.0	21	76.0
DG2	33	79.0	24	79.0	25	69.0

**Table 6** Profits for different scenarios considering the optimal locations and contract prices (€)

DG unit	Scenario A	Scenario B	Scenario C
DG1	86 724.00	57 812.89	20 960.42
DG2	106 795.16	49 444.22	21 662.47
total	193 519.16	107 257.12	42 622.89

**Table 7** DisCo payments for different scenarios considering the optimal locations and contract prices (€)

	Scenario A	Scenario B	Scenario C
without DG	4 728 713.68	3 125 692.32	1 978 203.33
with DG	4 542 028.97	3 060 101.13	1 957 842.57
savings	186 684.71	65 591.19	20 360.76

obtained in Scenario C, where wholesale market prices are expected to be the lowest. For all scenarios, the total profits obtained in this 'Test 2' case are higher than those presented in the 'Test 1' case.

In Table 7 is illustrated the DisCo payments for different scenarios with and without DG obtained by the proposed methodology, considering the locations and CPs shown in Table 5. As in the 'Test 1' case, the greatest savings are obtained in Scenario A, where wholesale market prices are expected to have the highest values.

## 4 Conclusions

A robust bilevel programming framework for the optimal location and contract pricing of DG in distribution system is presented. Simultaneously and in a single optimisation problem, the proposed model considers the minimisation of energy payments expected by the DisCo and the maximisation of profits expected by the DG owner. From a regulatory point of view, the proposed model can be used to provide efficient incentives to both the DisCo and the DG owner. The proposed bilevel programming model takes into account the interests of both agents, expressed by two different objective functions, and provides a solution in which both agents benefit. It exploits the willingness of DisCos to pay above the wholesale market price in order to avoid power losses, and properly rewards DG resources.

The main contribution of this paper lies in the combination of the aforementioned optimisation problems into a single problem, providing a solution in the best interests of both DG owners and DisCos. Duality properties and linearisation schemes were used to turn the BPP into an equivalent mixed-integer linear single-level optimisation problem. The use of a mixed-integer linear programming model guarantees convergence to optimality using conventional MILP solvers.

In the tests carried out on a test-distribution system, it was found that the total profits and optimal CPs obtained by the proposed methodology are higher than those reported in specialised literature, and that the DisCo savings and capacity factors obtained by the proposed methodology are lower than those reported in specialised literature, illustrating the robustness and efficiency of the proposed model. Further work will consider the inclusion of other types of DG units, as well as a more complex market modelling.



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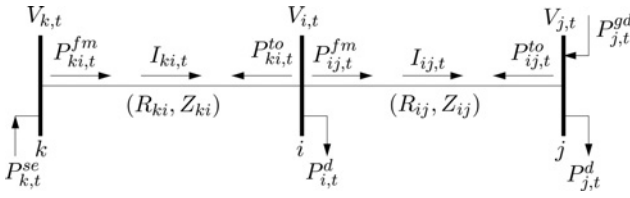
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## 7 Appendix

### 7.1 Appendix 1: Power flow approximations

The goal of a power flow study is to calculate the voltage angle and magnitude information of all buses in a power system by means of numerical analysis. In distribution systems, the angle is often neglected and the power flow equations are given in terms of voltages and parameters of the network (resistance  $R_{ij}$  and reactance  $Z_{ij}$ ). In the proposed model we have considered an approximation of the power flow equations in distribution systems similar to



**Fig. 6** Three-bus branch of a distribution system

the one presented in [17]. In this case, only active power, current flow and voltage magnitudes, are considered as decision variables. A three-bus branch of a distribution system is depicted in Fig. 6. This figure is used to represent a typical section (or branch) of a distribution system and we can use it to obtain the conventional equation of load balance, given in (14).

$$P_{i,t}^{se} + P_{i,t}^{dg} - \sum_{ij \in L} P_{ij,t}^{fm} - \sum_{ki \in L} P_{ki,t}^{to} = P_{i,t}^d, \quad \forall i \in I, \forall t \in T \quad (14)$$

where  $P_{i,t}^{se}$  is the active power supplied by a substation at node  $i$  in period  $t$ .  $P_{i,t}^{dg}$  is the active power supplied by DG at node  $i$  in period  $t$ .  $P_{i,t}^d$  is the active power demand at node  $i$  in period  $t$ .  $P_{ij,t}^{fm}$  is the active power flow that leaves node  $i$  towards node  $j$  in period  $t$ .  $P_{ij,t}^{to}$  is the active power flow that leaves node  $j$  towards node  $i$  in period  $t$ .  $I$  is the set of network nodes. The active power flows and the current flow magnitude of circuit  $ij$  in period  $t$  are shown in (15) and (16), respectively.

$$P_{ij,t}^{fm} = \frac{R_{ij}}{Z_{ij}^2} V_{i,t} (V_{i,t} - V_{j,t}), \quad \forall ij \in L, \forall t \in T \quad (15a)$$

$$P_{ij,t}^{to} = \frac{R_{ij}}{Z_{ij}^2} V_{j,t} (V_{j,t} - V_{i,t}), \quad \forall ij \in L, \forall t \in T \quad (15b)$$

$$I_{ij,t} = \frac{V_{i,t} - V_{j,t}}{Z_{ij}}, \quad \forall ij \in L, \forall t \in T \quad (16)$$

where  $R_{ij}$  and  $Z_{ij}$  are the resistance and impedance of circuit  $ij$ .  $V_{i,t}$  is the voltage magnitude at node  $i$  in period  $t$ .  $I_{ij,t}$  is the current flow magnitude of circuit  $ij$  in period  $t$ . By adding and subtracting (15a) and (15b) and considering (16), the following expressions are obtained

$$P_{ij,t}^{fm} + P_{ij,t}^{to} = R_{ij} I_{ij,t}^2, \quad \forall ij \in L, \forall t \in T \quad (17a)$$

$$P_{ij,t}^{fm} - P_{ij,t}^{to} = \frac{R_{ij}}{Z_{ij}^2} (V_{i,t}^2 - V_{j,t}^2), \quad \forall ij \in L, \forall t \in T \quad (17b)$$

where (17a) is the active power losses in circuit  $ij$ . Constraints (17a) and (17b) are used to replace constraints (15a) and (15b), respectively. Equations (14), (16) and (17) are used to model the steady-state operation of the EDS. Equations (14) and (16) are linear, whereas (17) contains square terms. These square terms can be linearised using a piecewise linearisation approach as shown below.

## 7.2 Appendix 2: Linearisation

The quadratic terms  $V_{i,t}^2$  and  $I_{ij,t}^2$  appear in (17). The objective of this section is to find linear expressions for both terms. This

linearisation is an approximate method; however, a more accurate result may be obtained by increasing the number of blocks of the piecewise linearisation.

**7.2.1 Appendix 2.1: Square of the voltage magnitude:** First, assume that the voltage magnitude  $V_{i,t}$  has a minimum voltage magnitude value of  $\underline{V}$  and a maximum voltage magnitude value of  $\bar{V}$ . Variable  $V_{i,t}^{sq}$  is the square voltage magnitude, as shown in (18).

$$V_{i,t}^{sq} = -\underline{V}^2 + 2\underline{V}V_{i,t} + \Delta V_{i,t}^2, \quad \forall i \in I, \forall t \in T \quad (18)$$

where  $\Delta V_{i,t} = V_{i,t} - \underline{V}$  has a minimum value of 0 and a maximum value of  $\bar{V} - \underline{V}$ . From (18), the quadratic term  $\Delta V_{i,t}^2$  is linearised as described in [42] and shown in Fig. 7. Thus, the square of voltage magnitude  $V_{i,t}^{sq}$  is defined in (19).

$$V_{i,t}^{sq} = -\underline{V}^2 + 2\underline{V}V_{i,t} + \Delta V_{i,t}^{sq}, \quad \forall i \in I, \forall t \in T \quad (19a)$$

$$\Delta V_{i,t}^{sq} = \sum_{p=1}^P m_p^V \Delta_{i,t,p}^V, \quad \forall i \in I, \forall t \in T \quad (19b)$$

$$V_{i,t} = \underline{V} + \sum_{p=1}^P \Delta_{i,t,p}^V, \quad \forall i \in I, \forall t \in T \quad (19c)$$

$$\Delta_{i,t,p}^V - \bar{\Delta}^V \leq 0, \quad \forall i \in I, \forall t \in T, p = 1 \dots P \quad (19d)$$

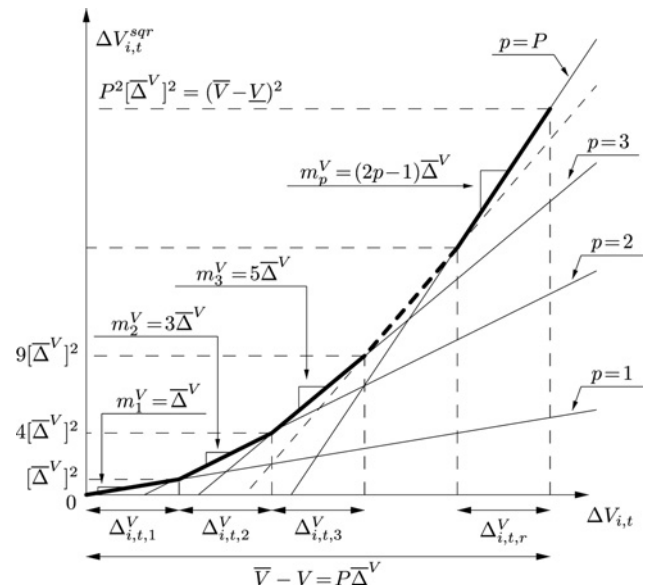
$$-\Delta_{i,t,p}^V \leq 0, \quad \forall i \in I, \forall t \in T, p = 1 \dots P \quad (19e)$$

where

$$m_p^V = (2p-1)\bar{\Delta}^V, \quad p = 1 \dots P$$

$$\bar{\Delta}^V = \frac{\bar{V} - \underline{V}}{P}$$

where  $\Delta V_{i,t}^{sq}$  is the square of  $\Delta V_{i,t}$ .  $m_p^V$  is the slope of the  $p$ th block of voltage magnitude deviation.  $\Delta_{i,t,p}^V$  is the value of the  $p$ th block of voltage magnitude deviation at node  $i$  in period  $t$ .  $\bar{\Delta}^V$  is the upper bound of the voltage magnitude deviation



**Fig. 7** Modelling the piecewise linear  $\Delta V_{i,t}^{sq}$  function

blocks.  $P$  is the number of blocks of the piecewise linearisation. Equation (19) is a set of linear expressions, and  $m_p^V$  and  $\bar{\Delta}^V$  are constant parameters. Constraints (19a) are the linear approximations of the square voltage magnitude at node  $i$  in period  $t$ . Constraints (19b) are the linear approximations of the square of  $\Delta V_{i,t}$ . Constraints (19c) state that the voltage magnitude at node  $i$  in period  $t$  is equal to the sum of the values in each block of the discretisation plus  $\underline{V}$ . Constraints (19d) and (19e) set the upper and lower limits of the contribution of each block of the difference between the voltage magnitude at node  $i$  in period  $t$  and  $\underline{V}$ .

**7.2.2 Appendix 2.2: Square of the current flow magnitude:** Analogously, assume that the current flow magnitude  $I_{ij,t}$  has a maximum current limit of  $\bar{I}_{ij}$ . Variable  $I_{ij,t}^{\text{sqr}}$  is the square current flow magnitude.  $I_{ij,t}^{\text{sqr}}$  is linearised in the same way as  $V_{i,t}^{\text{sqr}}$  (see Section 7.2.1), as shown in (20).

$$I_{ij,t}^{\text{sqr}} = \sum_{p=1}^P m_{ij,p}^I \Delta_{ij,t,p}^I, \quad \forall ij \in L, \forall t \in T \quad (20a)$$

$$I_{ij,t}^+ - I_{ij,t}^- = I_{ij,t}, \quad \forall ij \in L, \forall t \in T \quad (20b)$$

$$I_{ij,t}^+ + I_{ij,t}^- = \sum_{p=1}^P \Delta_{ij,t,p}^I, \quad \forall ij \in L, \forall t \in T \quad (20c)$$

$$\Delta_{ij,t,p}^I - \bar{\Delta}_{ij}^I \leq 0, \quad \forall ij \in L, \forall t \in T, p = 1 \dots P \quad (20d)$$

$$-\Delta_{ij,t,p}^I \leq 0, \quad \forall ij \in L, \forall t \in T, p = 1 \dots P \quad (20e)$$

$$-I_{ij,t}^+ \leq 0, \quad \forall ij \in L, \forall t \in T \quad (20f)$$

$$-I_{ij,t}^- \leq 0, \quad \forall ij \in L, \forall t \in T \quad (20g)$$

where

$$m_{ij,p}^I = (2p - 1) \bar{\Delta}_{ij}^I, \quad \forall ij \in L, p = 1 \dots P$$

$$\bar{\Delta}_{ij}^I = \frac{\bar{I}_{ij}}{P}, \quad \forall ij \in L$$

where  $m_{ij,p}^I$  is the slope of the  $p$ th block of the current flow magnitude of circuit  $ij$ .  $\Delta_{ij,t,p}^I$  is the value of the  $p$ th block of the current flow magnitude of circuit  $ij$  in period  $t$ .  $\bar{\Delta}_{ij}^I$  is the upper bound of the current flow magnitude blocks of branch  $ij$ . As in (19), (20) is a set of linear expressions, and  $m_{ij,p}^I$  and  $\bar{\Delta}_{ij}^I$  are constant parameters. Constraint (20a) are the linear approximations of the square current flow magnitude on circuit  $ij$  in period  $t$ .  $I_{ij,t}^+$  and  $I_{ij,t}^-$  are non-negative auxiliary variables utilised to obtain  $|I_{ij,t}|$ , as is shown in (20b). Constraint (20c) states that  $|I_{ij,t}|$  is equal to the sum of the values in each block of the discretisation. Constraint (20d) sets the upper and lower bounds of the contribution of each block of  $|I_{ij,t}|$ .