12.9.2

$$xy'' + y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n-1)\, a_n\, x^{n-2}$$

$$0 = x \cdot \sum_{n=0}^{\infty} n(n-1) a_n\, x^{n-2} + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} n(n-1)\, a_n\, x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=-1}^{\infty} (n+1) \cdot n\, a_{n+1}\, x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} \left[(n+1)n\, a_{n+1} + a_n \right] x^n$$

$\Rightarrow \quad (n+1)n\, a_{n+1} + a_n = 0 \qquad$ for alle n

$\Rightarrow \quad a_{n+1} = -\dfrac{a_n}{(n+1)n} \qquad$ for $n = 1, 2, 3, \ldots$

$n=1: \quad a_2 = -\dfrac{a_1}{(2)(1)}$

$n=2: \quad a_3 = -\dfrac{a_2}{(3)(2)} = \dfrac{a_1}{(3)(2)(2)(1)}$

$n=3: \quad a_4 = -\dfrac{a_3}{(4)(3)} = -\dfrac{a_1}{(4)(3)(3)(2)(2)(1)}$

$a_n = (-1)^n \dfrac{a_1}{(n+1)!\; n!} \qquad$ for $n = 1, 2, 3, 4 \ldots$

$$y = a_0 + a_1 \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{(n+1)!\; n!}$$

$$y = C + D\,e^{-x} \cdot \sum_{n=1}^{\infty} \frac{x^n}{(n+1)!}$$

Oppgave 1

$$\sum_{n=1}^{\infty} \frac{(x+1)^{3n}}{n^2 \cdot 8^n}$$

Forholdstest $\quad \lim_{n \to \infty} \left| \frac{\frac{(x+1)^{3n+3}}{(n+1)^2\, 8^{n+1}}}{\frac{(x+1)^{3n}}{n^2\, 8^n}} \right| = \lim_{n \to \infty} \left| \frac{(x+1)^3 \quad n^2}{(n+1)^2 \quad 8} \right| = \left| \frac{(x+1)^3}{8} \right| < 1$

$$\frac{(x+1)^3}{8} < 1 \quad \Rightarrow \quad x < 1$$

$$\left| \frac{(x+1)^3}{8} \right| < 1 \quad \Rightarrow \quad -3 < x < 1$$

Rekka konvergerer for $\underline{\underline{x < 1}}$

Rekka konvergerer absolutt for $\underline{\underline{-3 < x < 1}}$

Oppgave 2

$$f(x) = \frac{1}{1 + x^2}$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{-2x}{(x^2+1)^2}$$

$$f'(1) = \frac{-2}{4} = -\frac{1}{2}$$

$$f''(x) = \frac{u'v - uv'}{v^2} = \frac{-2(x^2+1)^2 + 2x \cdot 4x(x^2+1)}{(x^2+1)^4} = \frac{8x^2 - 2(x^2+1)}{(x^2+1)^3}$$

$$f''(1) = \frac{8-4}{8} = \frac{4}{8} = \frac{1}{2}$$

$$T_2 f_{x=1} = \frac{1}{2} - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2$$

Oppgave 3

$$y'' + 2y' + 5y = 0 \quad , \quad y(0) = 0 \qquad y'(0) = 1$$

Løser den karakteristiske likningen

$$r^2 + 2r + 5 = 0$$

$$r = -1 \pm 2i$$

$$y = e^{-x}\left(C\cos(2x) + D\sin(2x)\right)$$

$$y(0) = 0 \Rightarrow C\cos(0) + D\sin(0) = 0$$
$$C = 0$$

$\Rightarrow y = D \cdot e^{-x} \sin(2x)$

$y'(x) = D \cdot \left(-e^{-x} \sin 2x + 2e^{-x} \cos 2x\right)$

$y'(0) = 1 \quad \Rightarrow \quad D \cdot (0 + 2) = 1$

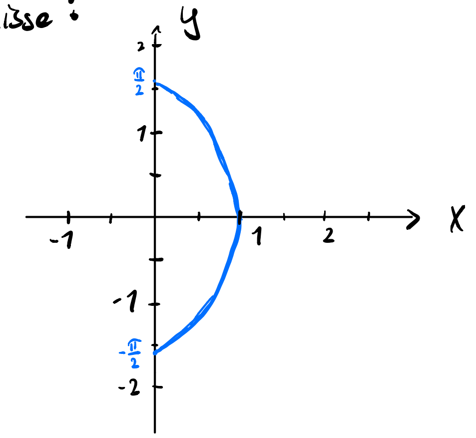
$D = \frac{1}{2}$

$\Rightarrow \underline{\underline{y = \frac{1}{2} e^{-x} \sin(2x)}}$

<u>Oppgave 4</u>

$\vec{r}(t) = \left(1 - t^2,\ t\sqrt{1-t^2} + \arcsin t\right) \qquad -1 \le t \le 1$

Skisse:



b)

$$r(t) = \int_{-1}^{1} \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

$$= \int_{-1}^{1} \sqrt{4t^2 + \left(\sqrt{1-t^2} - \frac{t}{\sqrt{1-t^2}} + \frac{1}{\sqrt{1-t^2}}\right)^2}\, dt$$

$$= \int_{-1}^{1} \sqrt{4t^2 + 4 - 4t^2}\, dt$$

$$= \int_{-1}^{1} 2\,dt = \left[2t\right]_{-1}^{1} = 2 - (-2) = \underline{\underline{4}}$$

Oppgave 5

$$f(x) = x \sin x$$

$$f'(x) = \sin x + x \cos x$$

Vi må vise at $f'(x)$ bare har ett nullpunkt for $x \in (0, \pi)$

Har prøvd ganske lenge nå men får det ikke til :(

Newtons metode rundt $x = 2$

$$f'(x) = \sin x + x \cos x$$

$$x_0 = 2$$

$$f''(x) = \cos x + \cos x - x \sin x$$

$$= 2\cos x - x \sin x$$

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = 2 - \frac{\sin(2) + 2\cos(2)}{2\cos(2) - 2\sin(2)} = 2.029$$

Vår tilnærming til x-verdien av maksimumspunktet

$$f(2.029) = 2.029 \cdot \sin(2.029) = 1.820$$

Altså er tilnærmingen vår av maksimumspunktet : $\underline{(2.029,\ 1.820)}$