Øving 4 - Matte 4D

Håvard [illegible]

1)

a) $(f,g) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\overline{g(x)}\,dx$

$(1, \sin(nx)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1\cdot\sin(nx)\,dx$

$= \frac{1}{2\pi}\left[-\frac{1}{n}\cos nx\right]_{-\pi}^{\pi}$

$= \frac{1}{2\pi}\left(-\frac{1}{\pi}\cos\pi n - \left(-\frac{1}{n}\cos\pi n\right)\right)$

$= 0$

$(\sin nx, \cos mx) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(nx)\cos(mx)\,dx$

$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2}\left(\sin(n+m)x + \sin(m-n)x\right)dx$

$= \frac{1}{4\pi}\int_{-\pi}^{\pi}\sin(n+m)x\,dx + \frac{1}{4\pi}\int_{-\pi}^{\pi}\sin(m-n)x\,dx$

$= \frac{1}{4\pi}\left[-\frac{1}{n+m}\cos(n+m)x\right]_{-\pi}^{\pi} + \frac{1}{4\pi}\left[\frac{1}{n-m}\cos(n-m)x\right]_{-\pi}^{\pi}$

$= 0$

b) $(\sin mx, \sin nx) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sin mx\,\sin nx\,dx$

$\sin mx\cdot\sin nx = \frac{1}{2}\left[\cos(m-n) - \cos(m+n)\right]$

$\Rightarrow \frac{1}{4\pi}\cdot\int_{-\pi}^{\pi}\cos(m-n)x\,dx - \frac{1}{4\pi}\int_{-\pi}^{\pi}\cos(m+n)x\,dx$

$= \frac{1}{4\pi}\left[\frac{1}{m-n}\sin(m-n)x\right]_{-\pi}^{\pi} - \frac{1}{4\pi}\left[\frac{1}{m+n}\sin(m+n)x\right]_{-\pi}^{\pi}$

$= 0$

For cos:

$$\frac{1}{4\pi}\int_{-\pi}^{\pi}\cos(m-n)x\,dx + \frac{1}{4\pi}\int_{-\pi}^{\pi}\cos(m+n)x\,dx$$

$$= \frac{1}{4\pi}\left[\frac{1}{m-n}\cos(m-n)x\right]_{-\pi}^{\pi} + \left[\frac{1}{m+n}\cos(m+n)x\right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi}\left(\frac{1}{m-n}\cos(m-n)\pi - \frac{1}{m-n}\cos(m-n)\pi + \frac{1}{m+n}\cos(m+n)\pi - \frac{1}{m+n}\cos(m+n)\pi\right)$$

$$= 0$$

När $n=m$ har vi:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin^2 nx\,dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2}(1-\cos(2nx))\,dx$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi}1\,dx - \frac{1}{4\pi}\int_{-\pi}^{\pi}\cos(2nx)\,dx$$

$$= \frac{1}{4\pi}\left[x\right]_{-\pi}^{\pi} - \frac{1}{4\pi}\left[\frac{1}{2n}\sin(2nx)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi}(\pi+\pi) - \frac{1}{4\pi}\left(\frac{1}{2n}\sin(2n\pi) - \frac{1}{2n}\sin(-2n\pi)\right)$$

$$= \frac{1}{4\pi}(\pi+\pi) - \frac{1}{4\pi}(0)$$

$$= \frac{1}{2}$$

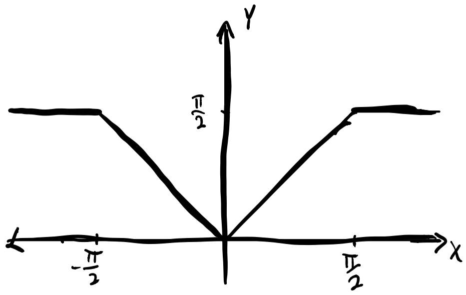
c) Assume that:

$$f = a_0 + \sum^{\infty}(a_n\cos nx + b_n\sin nx)$$

2)

$$f(x) = \begin{cases} x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$$

$$f_e(x) = \begin{cases} \pi/2 & -\pi \le x < -\frac{\pi}{2} \\ -x & -\pi/2 \le x < 0 \\ x & 0 \le x < \frac{\pi}{2} \\ \pi/2 & \frac{\pi}{2} \le x \le \pi \end{cases}$$



$$f_o(x) = \begin{cases} -\pi/2 & -\pi \le x < -\frac{\pi}{2} \\ x & -\pi/2 \le x < \pi/2 \\ \pi/2 & \pi/2 \le x \le \pi \end{cases}$$