Oppgave 25 s, 52 $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Vis at for alle hele tall $n \geq 2$

$$\sum_{k=2}^{n} \binom{k}{2} = \binom{n+1}{3}$$

For $n = 2$:

$$\sum_{k=2}^{2} \binom{k}{2} = \binom{2}{2} = \frac{2!}{2!(0)!} = \frac{2!}{2!} = \frac{6}{6} = \frac{3!}{3!} = \binom{2+1}{3} = \binom{n+1}{3} \text{ OK}$$

Anta

$$\sum_{k=2}^{a} \binom{k}{2} = \binom{a+1}{3} = \frac{(a+1)!}{3!(a-2)!}$$

$$\sum_{k=2}^{a+1} \binom{k}{2} = \sum_{k=2}^{a} \binom{k}{2} + \binom{a+1}{2} = \binom{a+1}{3} + \binom{a+1}{2} = \frac{(a+1)!}{3!(a-2)!} + \frac{(a+1)!}{2!(a-1)!}$$

$$= \frac{(a+1)!}{3!(a-2)!} + \frac{(a+1)!}{2!(a-2)!\cdot(a-1)} = \frac{(a-1)(a+1)!}{3!(a-2)!\cdot(a-1)} + \frac{3(a+1)!}{3!(a-2)!\cdot(a-1)}$$

$$= \frac{(a-1)(a+1)! + 3(a+1)!}{3!(a-1)!} = \frac{(a+1)!(a-1+3)}{3!(a-1)!} = \frac{(a+1)!(a+2)}{3!(a-1)!}$$

$$= \frac{(a+2)!}{3!(a-1)!} = \frac{((a+1)+1)!}{3!((a+1)-3)!} = \binom{(a+1)+1}{3} = \binom{n+1}{3}$$

q.e.d

Oppgave 1 s. 90

c) $(-3,4) \cup [-2,1) = (-3,4)$

f) $(2,4) \setminus (1,3) = [3,4)$

Oppgave 2 s. 90

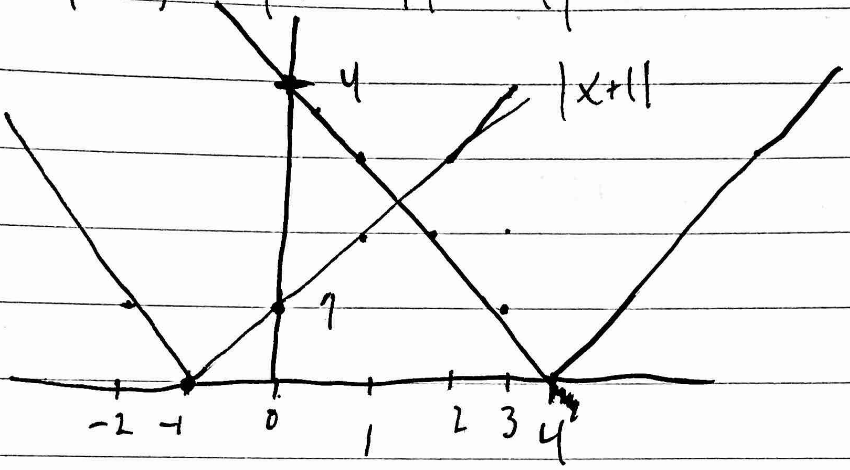
a) $\emptyset \cup (1,5) = (1,5)$

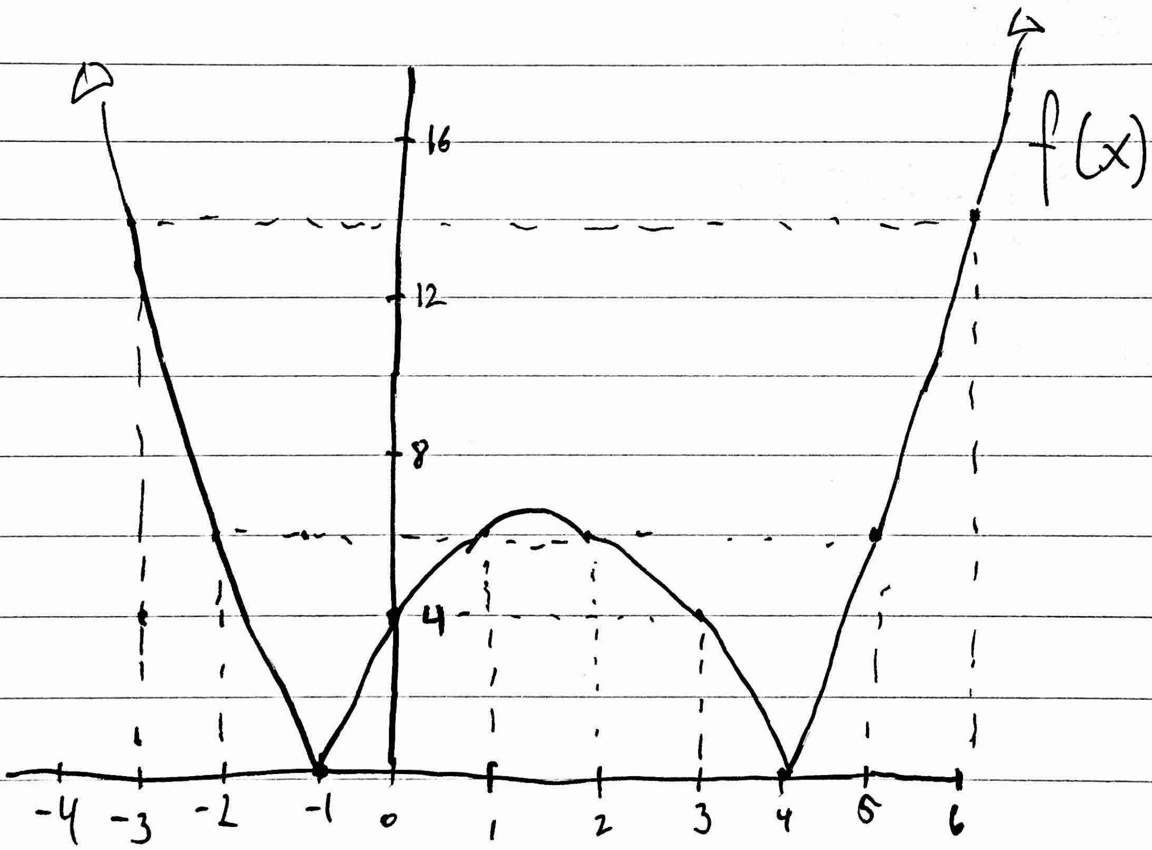
b) $\emptyset \cap (1,5) = \emptyset$

c) $\emptyset \setminus (1,5) = \emptyset$

Oppgave 4d s. 91

$f(x) = |x+1||x-4|$





Oppgave 5a s. 91

$$|x-2| < |x+3|$$

$$\rightarrow (x-2)^2 < (x+3)^2$$

$$x^2 - 4x + 4 < x^2 + 6x + 9$$

$$x^2 - x^2 + 4 - 9 < 6x + 4x$$

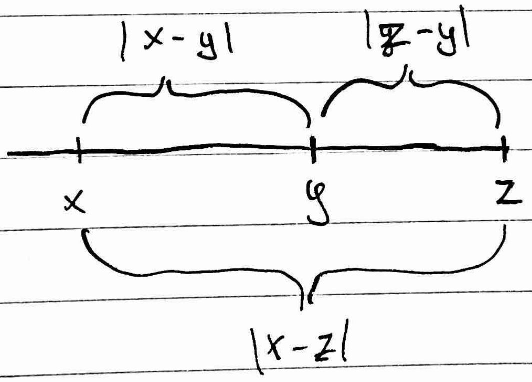
$$-5 < 10x$$

$$x > -\frac{5}{10}$$

$$x > -\frac{1}{2}$$

Oppgave 9

Vis: $|x-y| \leq |x-z| + |z-y|$



Trekantulikheten

$$|a| + |b| \geq |a+b| \rightarrow |x-z| + |z-y| \geq |x-z+z-y|$$

$$|x-z| + |z-y| \geq |x-y|$$

q.e.d

Oppgave 2d s. 98

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}}{2}$$

$$\frac{(\sqrt{5}+1)(\sqrt{5}-1)}{(\sqrt{5}-1)^2} - \frac{\sqrt{5}}{2}$$

$$\frac{\sqrt{5}^2 - 1^2}{\sqrt{5}^2 - 2\sqrt{5} + 1^2} - \frac{\sqrt{5}}{2}$$

$$\frac{5-1}{5-2\sqrt{5}+1} - \frac{\sqrt{5}}{2}$$

$$\frac{4}{2(3-\sqrt{5})} - \frac{\sqrt{5}(3-\sqrt{5})}{2(3-\sqrt{5})}$$

$$\frac{4-3\sqrt{5}+\sqrt{5}^2}{2(3-\sqrt{5})} =$$

$$\frac{9-3\sqrt{5}}{2(3-\sqrt{5})} = \frac{3\cancel{(3-\sqrt{5})}}{2\cancel{(3-\sqrt{5})}} = \underline{\underline{\frac{3}{2}}}$$

Oppgave 14a s.99

$a > 1$

$$a^n = (a+1-1)^n = (1+(a-1))^n$$

Ifølge Bernoullis ulikhet

$$a^n \geq 1 + n\cdot(a-1)$$

$$a^n \geq 1 + na - n$$

$$1 + na - n > b$$

$$1 + n(a-1) > b$$

$$n > \frac{b-1}{a-1}$$

Siden $a > 1$, vil nevneren på høyre side av ulikhetstegnet alltid være positivt, altså vil

$\frac{b-1}{a-1}$ alltid være et reelt tall

og ifølge arkimedes prinsipp kan vi derfor velge at n skal være et større tall.

Oppgave 2 s. 105

a) Ja, oppad begrenset til 1

c) Nei, ingen oppad begrensing.