T1.2.43

$\vec{r}(t) = (\cos t,\ t \sin t)$

$\vec{v}(t) = (-\sin t,\ \sin t + t\cos t)$

$$v(t) = \sqrt{(-\sin t)^2 + (\sin t + t\cos t)^2}$$

$$= \sqrt{\sin^2 t + \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t}$$

$$= \sqrt{2\sin^2 t + 2t\sin t\cos t + t^2\cos^2 t}$$

$\vec{a}(t) = (-\cos t,\ 2\cos t - t\sin t)$

$$a(t) = \sqrt{(-\cos t)^2 + (2\cos t - t\sin t)^2}$$

$$= \sqrt{t^2\sin^2 t - 4t\cos t\sin t + 5\cos^2 t}$$

T1.2.46

$\vec{r}(t) = (t,\ \ln(\cos t))$ for $t \in [0, \pi/4]$

a)

$$\vec{v}(t) = \left(1, \frac{1}{\cos t} \cdot -\sin t\right)$$

$$= \left(1, -\frac{\sin t}{\cos t}\right) = (1, -\tan t)$$

$$v(t) = \sqrt{1^2 + (-\tan t)^2}$$

$$= \sqrt{1 + \tan^2 t}$$

b)

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 t}\, dt = 0.88$$

11.1.1

$f(x) = e^{x^2} \qquad a = 0$

$f'(x) = 2x e^{x^2} \qquad f^{(2)}(x) = 4x^2 e^{x^2} + 2e^{x^2}$

$f^{(3)}(x) = 12x e^{x^2} + 8x^3 e^{x^2} \qquad f^{(4)}(x) = 48x^2 e^{x^2} + 16x^4 e^{x^2} + 12 e^{x^2}$

$$T_4(x;0) = \frac{f^{(4)}(0)}{4!}(x-0)^4 + \dots + \frac{f^{(0)}(0)}{0!}(x-0)^0$$

$$= \frac{12}{24}x^4 + 0x^3 + \frac{2}{2}x^2 + 0x + 1$$

$$= \underline{\underline{0.5x^4 + x^2 + 1}}$$

11.1.2

$f(x) = \sqrt{x} \qquad a = 1$

$f^{(1)}(x) = \frac{1}{2\sqrt{x}} \qquad f^{(2)}(x) = -\frac{1}{4x\sqrt{x}}$

$f^{(3)}(x) = \frac{3\sqrt{x}}{8x^3}$

$$T_3(x;1) = \underline{\underline{\frac{1}{16}(x-1)^3 - \frac{1}{8}(x-1)^2 + \frac{1}{2}(x-1) + 1}}$$

11.1.5

$f(x) = \sinh x \qquad a = 0$

$f'(x) = \cosh x \qquad f^{(2)}(x) = \sinh x$

$f^{(3)}(x) = \cosh x \qquad f^{(4)}(x) = \sinh x$

$f^{(5)}(x) = \cosh x$

$$T_5(x;0) = \underline{\underline{0 + 1x + \frac{1}{6}x^3 + \frac{1}{120}x^5}}$$

11.1.10

$f(x) = x^4 - 3x^2 + 2x - 7 \qquad a = 1$

$f'(x) = 4x^3 - 6x + 2$

$f^{(2)}(x) = 12x^2 - 6$

$f^{(3)}(x) = 24x$

$$T_3(x;1) = -7 + 0x + \frac{6}{2}(x-1)^2 + \frac{24}{6}(x-1)^3$$

$$= \underline{\underline{-7 + 3(x-1)^2 + 4(x-1)^3}}$$

11.2.2

$f(x) = \sin x \qquad a = 0$

$f^{(1)} = \cos x \qquad f^{(2)} = -\sin x \qquad f^{(3)} = -\cos x \qquad f^{(4)} = \sin x \qquad f^{(5)} = \cos x$

$$T_4 f(x;0) = 0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4$$

$$= x - \frac{1}{6}x^3$$

Lagranges restleddsformel

$$\Rightarrow R_4 f(b) = \frac{f^{(5)}(c)}{5!}(b-0)^5$$

$$= \frac{1}{120}\cos(c) \cdot b^5$$

$$\underline{\underline{|R_4 f(b)| \leq \frac{|b|^5}{120}}} \quad \text{fordi } 0 \leq |\cos(c)| \leq 1$$

11.2.6

$$T_2(e^x;0) = 1 + x + \frac{1}{2}x^2$$

$$\Rightarrow \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \approx \lim_{x \to 0} \frac{1 + x + \frac{1}{2}x^2 - 1 - x}{x^2} = \frac{\frac{1}{2}x^2}{x^2} = \underline{\underline{\frac{1}{2}}}$$

11.2.10

a)

$f(x) = \sin x \qquad a = 0$

$f^{(1)} = \cos x$
$f^{(2)} = -\sin x$
$f^{(3)} = -\cos x$
$f^{(4)} = \sin x$
$f^{(5)} = \cos x$
$f^{(6)} = -\sin x$

$$T_6(x;a) = 0 + x + 0x^2 - \frac{1}{6}x^3 + 0x^4 + \frac{1}{120}x^5 + 0x^6$$

$$= \underline{\underline{x - \frac{1}{6}x^3 + \frac{1}{120}x^5}}$$

$$f(x) = T_n(f(x)) + R_n(f(x))$$

b)

$$\int_0^1 \sin(x^2)\,dx \qquad 0{,}00002$$

$$T_n \sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$$

$$\int_0^1 \sin(x^2)\,dx = \int_0^1 x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} + \dots \, dx$$

$$\sin(x^2) = T_n \sin(x^2) + R_n \sin(x^2)$$

$$\int_0^1 \sin(x^2)\,dx = \int_0^1 T_n \sin(x^2)\,dx + \int_0^1 R_n \sin(x^2)\,dx$$

$$|R_n \sin(x^2)| = \left|\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}\right| \leq \frac{1}{(n+1)!}x^{n+1}$$

$$\Rightarrow \int_0^1 R_n \sin(x^2)\,dx \leq \frac{1}{(n+1)!}\int_0^1 x^{n+1}\,dx = \frac{1}{(n+1)!}\left[\frac{1}{(n+2)}x^{n+2}\right]_0^1$$

$$= \frac{1}{(n+2)!}$$

Som er mindre enn eller lik 0,00002 ved $n = 7$

$$\Rightarrow \int_0^1 \sin(x^2)\,dx \approx \int_0^1 T_7 \sin(x^2)\,dx = \int_0^1 \left(x^2 - \frac{x^6}{3!}\right)dx = \underline{\underline{\frac{13}{42}}}$$

11.2.15

a)

$g(x) = \sqrt[3]{1+x} = (1+x)^{\frac{1}{3}}$ $\quad g^{(1)}(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}$ $\quad g^{(2)}(x) = -\frac{2}{9}(1+x)^{-\frac{5}{3}}$ $\quad g^{(3)} = \frac{10}{27}(1+x)^{-\frac{8}{3}}$

$g(0) = \sqrt[3]{1} = 1$ $\quad g^{(1)}(0) = \frac{1}{3}(1)^{-\frac{2}{3}} = \frac{1}{3}$ $\quad g^{(2)}(0) = -\frac{2}{9}$

$$T_2(g(x);0) = 1 + \frac{1}{3}x - \frac{2}{18}x^2$$

$$= \underline{\underline{1 + \frac{1}{3}x - \frac{1}{9}x^2}}$$

b)

$$R_2\, g(x) = \frac{g^{(3)}(c)}{3!}(x)^3$$

$$= \frac{10}{162}(1+c)^{-\frac{8}{3}}(x)^3$$

$$= \frac{5}{81}x^3 \cdot \underbrace{(1+c)^{-\frac{8}{3}}}_{\leq 1 \text{ for } c \geq 0}$$

$$\Rightarrow |R_{g_2}(x)| \leq \frac{5}{81}x^3 \quad \text{for } x \geq 0$$

■

c)

$$\sqrt[3]{1003} = \underline{\underline{10.0099900}}$$