$u = 5x \quad u' = 5$

1) $(\cos(5x))' = (\cos(u))' \cdot u'$

Bruker kjerneregelen.

$= -\sin(5x) \cdot 5 = \underline{\underline{-5\sin(5x)}}$

$(\sin(5x))' = (\sin(v))' \cdot v'$ der $v = 5x \quad v' = 5$

$= \cos(5x) \cdot 5$

$= \underline{\underline{5\cos(5x)}}$

2)

$u' = \sin 3x \qquad u = -\frac{1}{3}\cos 3x$

$v = x \qquad v' = 1$

$$\int_{-\pi}^{\pi} x \sin 3x \, dx = \int_{-\pi}^{\pi} u'v \, dx$$

$$= uv - \int uv' \, dx$$

$$= -\frac{x}{3}\cos(3x) - \int -\frac{1}{3}\cos(3x) \, dx$$

$$= \left[-\frac{x}{3}\cos(3x) - -\frac{1}{9}\sin(3x)\right]_{-\pi}^{\pi}$$

$$= \left[\frac{1}{9}\sin(3x) - \frac{x}{3}\cos(3x)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{9}\sin(3\pi) - \frac{\pi}{3}\cos(3\pi) - \frac{1}{9}\sin(-3\pi) - \frac{\pi}{3}\cos(-3\pi)$$

$$= \underline{\underline{\frac{2\pi}{3}}}$$

3)

a

$$f(t) = 5 + 2t^2 - 7t^3$$

$$F(s) = \mathcal{L}(f(t)) = \mathcal{L}(5) + \mathcal{L}(2t^2) - \mathcal{L}(7t^3)$$

$$= \frac{5}{s} + 2\frac{2!}{s^3} - 7\frac{3!}{s^4}$$

$$= \underline{\underline{\frac{1}{s}\left(5 + \frac{4}{s^2} - \frac{42}{s^3}\right)}}$$

b)

$$f(t) = te^{-3t}$$

$$F(s) = \mathcal{L}(f(t)) = e^{-3t} \cdot t$$

$$F(s+3) = e^{-3t} \cdot \mathcal{L}(t)$$ Bruker ~~første~~ skifteteorem.

$$F(s) = \underline{\underline{\frac{1}{(s+3)^2}}}$$

c)

$$f(t) = e^{t} \sin(3t)$$

$$F(s) = \mathcal{L}(f(t))$$

$$F(s-1) = e^{t} \cdot \mathcal{L}(\sin 3t)$$

$$F(s) = \underline{\underline{\frac{3}{(s-1)^2 + 3^2}}}$$

d)

$$f(t) = \begin{cases} 1 & \text{if } 0 < t < \pi \\ 0 & \text{if } t \geq \pi \end{cases}$$

$$\underline{\underline{\mathcal{L}(f(t)) = \begin{cases} \frac{1}{s} & \text{if } 0 < \frac{1}{s^2} < \pi \\ 0 & \text{if } \frac{1}{s^2} \geq \pi \end{cases}}}$$

e)

$$f(t) = \begin{cases} 0 & \text{if } 0 < t < \pi \\ \sin t & \text{if } t \geq \pi \end{cases}$$

$$\mathcal{L}(f(t)) = \begin{cases} 0 & \text{if } 0 < \frac{1}{s^2} < \pi \\ \frac{1}{s^2+1} & \text{if } \frac{1}{s^2} \geq \pi \end{cases}$$

4

a) $$F(s) = \frac{2}{s^3} - \frac{4}{s^5}$$

$$f(t) = \mathcal{L}^{-1}\left(\frac{2}{s^3} - \frac{4}{s^5}\right) = \mathcal{L}^{-1}\left(\frac{2}{s^3}\right) - \mathcal{L}^{-1}\left(\frac{4}{s^5}\right)$$

$$= t^2 - \frac{1}{6} \cdot t^4$$

b) $$F(s) = \frac{11-s}{s^2-2s-3} = \frac{11-s}{(s-3)(s+1)} = \frac{A}{s-3} + \frac{B}{s+1}$$

$$\Rightarrow 11 - s = A(s+1) + B(s-3)$$

$s = 3 \Rightarrow 8 = 4A$
$A = 2$

$s = -1 \Rightarrow 12 = -4B$
$B = -3$

$$\Rightarrow F(s) = \frac{2}{s-3} - \frac{3}{s+1}$$

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left(\frac{2}{s-3}\right) - \mathcal{L}^{-1}\left(\frac{3}{s+1}\right)$$

$$= 2e^{3t} - 3e^{-t}$$

c)

$$F(s) = \frac{2}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{B+Cs}{s^2+1}$$

$$\Rightarrow \quad 2 = A(s^2+1) + (B+Cs)(s-1)$$

$s=1 \Rightarrow 2 = 2A$
$A = 1$

$A = 1 \Rightarrow \quad 2 = s^2 + 1 + B(s-1) + Cs(s-1)$
$2 = s^2 + 1 + Bs - B + Cs^2 - Cs$

$0 = s^2(1+C) + s(B-C) - B - 1$

$s=0 \Rightarrow \quad 0 = -B - 1 \quad \Rightarrow B = -1$

$B = -1 \Rightarrow \quad 0 = s^2(1+c) + s(-1-c)$

$\Rightarrow 0 = 1 + c \quad \Rightarrow c = -1$

$$\Rightarrow F(s) = \frac{1}{s-1} + \frac{-s-1}{s^2+1}$$

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= e^t - \cos(t) - \sin(t)$$

5)

a) $y'' + 5y' + 6y = 0 \qquad y(0) = -2 \qquad y'(0) = 1$

$$\mathcal{L}(y'') + 5\,\mathcal{L}(y') + 6\,\mathcal{L}(y) = \mathcal{L}(0)$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + 5\left(s\,\mathcal{L}(y) - y(0)\right) + 6\,\mathcal{L}(y) = 0$$

$$s^2\mathcal{L}(y) + 2s - 1 + 5s\,\mathcal{L}(y) + 10 + 6\,\mathcal{L}(y) = 0$$

$$s^2\mathcal{L}(y) + 5s\,\mathcal{L}(y) + 6\mathcal{L}(y) + 2s + 9 = 0$$

$$\mathcal{L}(y) \cdot (s^2 + 5s + 6) = -2s - 9$$

$$\mathcal{L}(y) = \frac{-2s-9}{s^2+5s+6} = \frac{-2s-9}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$-2s - 9 = A(s+3) + B(s+2)$$

$s = -3 \Rightarrow 6 - 9 = -3B + 2$
$B = 3$

$s = -2 \Rightarrow -5 = -2A + 3A$
$A = -5$

$$\Rightarrow \mathcal{L}(y) = \frac{-5}{s+2} + \frac{3}{s+3}$$

$$\Rightarrow y = \mathcal{L}^{-1}\left(\frac{-5}{s+2}\right) + \mathcal{L}^{-1}\left(\frac{3}{s+3}\right)$$

$$y = -5 \cdot e^{-2t} + 3e^{-3t}$$

(e) $y'' + 3y' + 2y = e^{-t}$ $\quad y(0) = 0 \quad y'(0) = 0$

$\mathcal{L}(y'') + 3\mathcal{L}(y') + 2\mathcal{L}(y) = \mathcal{L}(e^{-t})$

$s^2\mathcal{L}(y) - sy(0) - y'(0) + 3(s\mathcal{L}(y) - y(0)) + 2\mathcal{L}(y) = \frac{1}{s+1}$

$s^2\mathcal{L}(y) + 3(s\mathcal{L}(y)) + 2\mathcal{L}(y) = \frac{1}{s+1}$

$s^2\mathcal{L}(y) + 3s\mathcal{L}(y) + 2\mathcal{L}(y) = \frac{1}{s+1}$

$\mathcal{L}(y)(s^2 + 3s + 2) = \frac{1}{s+1}$

$\mathcal{L}(y) = \frac{\frac{1}{s+1}}{s^2+3s+2} = \frac{s^2+3s+2}{s+1} = \frac{(s+1)(s+2)}{s+1} = s+2$

$\Rightarrow y = \mathcal{L}^{-1} s+2 = \underline{\underline{e^{-2t}}}$