

study	reported effect	variance	weights	weighted effects
1	$d_1$	$\text{var}(d_1)$	$w_1 = (1/\text{var}(d_1)) / \sum 1/\text{var}(d_i)$	$w_1 d_1$
2	$d_2$	$\text{var}(d_2)$	$w_2 = (1/\text{var}(d_2)) / \sum 1/\text{var}(d_i)$	$w_2 d_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n$	$d_n$	$\text{var}(d_n)$	$w_n = (1/\text{var}(d_n)) / \sum 1/\text{var}(d_i)$	$w_n d_n$

column sum = 1

column sum =  $d_+$  (summary)

$$\text{var}(d_+) = (\sum 1/\text{var}(d_i))^{-1}$$

$$95\% \text{ CI: } d_+ \pm 1.96 \sqrt{\text{var}(d_+)}$$

Test for heterogeneity:

$$Q = \sum d_i^2 / \text{var}(d_i) - \left( \sum d_i / \text{var}(d_i) \right)^2 / \sum (1/\text{var}(d_i))$$

$$Q \sim \chi^2_{df=n-1}$$

$$Q > \chi^2_{\alpha} ? \text{ yes, no}$$

random effects

$$\text{var}(d_i) = \text{var}_w(d_i) + \tau^2$$

$\text{var}_w(d_i) \rightarrow$  see fixed effects

$\tau^2 \rightarrow$  estimate using DerSimonian-Laird, MLE, etc.

$$DL: \tau^2 = (Q - (n-1)) / \left[ \sum \frac{1}{\text{var}_w(d_i)} - \left( \sum (1/\text{var}_w(d_i))^{-1} \right) / \sum \frac{1}{\text{var}_w(d_i)} \right]$$

fixed effects

Use within-study variance  $\text{var}_w(d_i)$ , whatever study  $i$  reports, or estimate it based on type of statistic

$$d_i = p_1 - p_2 \Rightarrow \text{var}_w d_i = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$

$$d_i = \log \frac{p_1}{p_2} \Rightarrow \text{var}_w d_i = \frac{1-p_1}{n_1 p_1} + \frac{1-p_2}{n_2 p_2}$$

$$d_i = M_1 - \mu_1 \Rightarrow \text{var}_w d_i = s_{d_1}^2 + s_{d_2}^2$$

Same type, eg, SMD, log odds, risk ratio, etc. May need to transform (see Olkin handout on estimating one statistic when study reports another)