(20) $e_{n-1}(0) = p_{00}\, e_n(0)\delta_0 + p_{01}\, e_n(1)\delta_1$, $e_{n-1}(1) = p_{10}\, e_n(0)\delta_0 + p_{11}\, e_n(1)\delta_1$, $e_{n-1} = \begin{pmatrix}\delta_0 p_{00} & \delta_1 p_{01}\\ \delta_0 p_{10} & \delta_1 p_{11}\end{pmatrix} e_n$ (41)

$= \begin{pmatrix}\delta_0 p & \delta_1(1-p)\\ \delta_0(1-p) & \delta_1 p\end{pmatrix} e_n$, $\mathbb{E}\,\delta_{L_t} = \frac12(\delta_0+\delta_1)$, $\mathbb{E}\,\delta_{L_1}\delta_{L_2} = \mathbb{E}\,\delta_{L_1}(p\,\delta_{L_1} + \bar p\,\delta_{\bar L_1}) = p\,\mathbb{E}\,\delta_{L_1}^2 + \bar p\,\delta_0\delta_1$

$= \frac{p}{2}(\delta_0^2+\delta_1^2) + \bar p\,\delta_0\delta_1$, $\mathbb{E}\,\delta_{L_1}\delta_{L_2}\delta_{L_3} = \frac{\delta_0}{2}\mathbb{E}(\delta_{L_2}\delta_{L_3}\,|\,L_1=0) + \frac{\delta_1}{2}\mathbb{E}(\delta_{L_2}\delta_{L_3}\,|\,L_1=1)$, $e_2 = P\begin{pmatrix} e_3(0)\\ e_3(1)\end{pmatrix}$

$= P\begin{pmatrix} p\delta_0+\bar p\delta_1\\ p\delta_1+\bar p\delta_0\end{pmatrix}$ $= p^2\delta_0 + 2p\bar p\,\delta_1 + \bar p^2\delta_0 = 2p(1-p)\delta_1 + (2p^2-2p+1)\delta_0$, $2p\bar p\,\delta_0 + \bar p^2\delta_1 + p^2\delta_1 = 2p(1-p)\delta_0 + (2p^2-2p+1)\delta_1$, $\mathbb{E}\prod^3 \delta_{L_t} = \cancel{\frac12(e_2(0)+e_2(1)) = p(1-p)\delta_1}$ $\frac12(\delta_0 e_2(0) + \delta_1 e_2(1))$

$= (p^2 - p + \frac12)(\delta_0^2+\delta_1^2) + 2p(1-p)\delta_0\delta_1$, $= \frac{\delta_0}{2}\left(p\delta_0\,\mathbb{E}(\delta_{L_3}|L_2=0) + \bar p\,\delta_1\,\mathbb{E}(\delta_{L_3}|L_2=1)\right) + \frac{\delta_1}{2}\left(p\delta_1\,\mathbb{E}(\delta_{L_3}|L_2=1)\right.$

$\left.+ \bar p\,\delta_0\,\mathbb{E}(\delta_{L_3}|L_2=0)\right) = \frac{\delta_0}{2}\left(p^2\delta_0^2 + p\bar p\,\delta_0\delta_1 + \bar p^2\delta_0\delta_1 + p\bar p\,\delta_1^2\right) + \frac{\delta_1}{2}\left(p\bar p\,\delta_0\delta_1 + p^2\delta_1^2 + p\bar p\,\delta_0^2 + \bar p^2\delta_0\delta_1\right)$

$\bar p(1+p) = (1-p^2)$

$= \frac12\left(\delta_0^3 p^2 + \delta_0^2\delta_1(2p\bar p + \bar p^2) + \delta_0\delta_1^2(2p\bar p + \bar p^2) + \delta_1^3 p^2\right)$, $e_2 = P\begin{pmatrix}p\delta_0+\bar p\delta_1\\ p\delta_1+\bar p\delta_0\end{pmatrix} = \begin{pmatrix}\delta_0^2p^2 + p\bar p\,\delta_0\delta_1 + p\bar p\,\delta_1^2 + \bar p^2\delta_0\delta_1\\ p\bar p\,\delta_0^2 + \bar p^2\delta_0\delta_1 + \delta_1^2p^2 + \delta_0\delta_1 p\bar p\end{pmatrix}$

$\cancel{\frac12(e_2(0)+e_2(1))}$ $\frac12(\delta_0 e_2(0) + \delta_1 e_2(1)) = \frac12\left(\delta_0^3p^2 + p\bar p\,\delta_0^2\delta_1 + p\bar p\,\delta_0\delta_1^2 + \bar p^2\delta_0^2\delta_1 + p\bar p\,\delta_1\delta_0^2 + \bar p^2\delta_0\delta_1^2 + \delta_1^3p^2 + \delta_0\delta_1^2 p\bar p\right)$

$= \frac12\left(\delta_0^3p^2 + \delta_0^2\delta_1(2p\bar p + \bar p^2) + \delta_0\delta_1^2(2p\bar p + \bar p^2) + \delta_1^3 p^2\right)$, (36) $\mathbb{P}(L_{t+1} = \ell_{t+1}\,|\,L_t=\ell_t) = \sum_a \mathbb{P}(L_{t+1}=\ell_{t+1}\,|\,A_{t+1}=a)\cdot$

$\mathbb{P}(A_{t+1}=a\,|\,L_t=\ell_t) = \sum_a p^{a=\ell_{t+1}}(1-p)^{a\neq\ell_{t+1}}\, p^{a=\ell_t}(1-p)^{a\neq\ell_t} = (1-p)^2\sum_a\left(\frac{p}{1-p}\right)^{a=\ell_{t+1}+a=\ell_t} = (1-p)^2\left\{\left(\frac{p}{1-p}\right)^2 + 1\right\}^{\ell_t=\ell_{t+1}}$

$T = 2p$, $D = 2p-1$

$\left\{2\frac{p}{1-p}\right\}^{\ell_t\neq\ell_{t+1}}$, $\begin{pmatrix}p & 1-p\\ 1-p & p\end{pmatrix}\begin{pmatrix}1/\sqrt2\\ 1/\sqrt2\end{pmatrix}$ $\lambda_2 = p + \sqrt{p^2-2p+1} = p\pm(p-1)$, $\lambda_1 = 2p-1$, $\lambda_2 = 1$,

$\begin{pmatrix}p-1\\ 1-p\end{pmatrix}, \begin{pmatrix}1-p\\ 1-p\end{pmatrix}$, $P = \begin{pmatrix}\delta_0 p & \delta_1(1-p)\\ \delta_0(1-p) & \delta_1 p\end{pmatrix}$, $T = p(\delta_0+\delta_1)$, $D = p^2\delta_0\delta_1 - \delta_0\delta_1(1-p)^2 = \delta_0\delta_1(2p-1)$,

$\lambda = \frac12 p(\delta_0+\delta_1) \pm\sqrt{\frac{p^2}{4}(\delta_0+\delta_1)^2 - \delta_0\delta_1(2p-1)} = \cancel{\frac12 p(\delta_0+\delta_1) \pm\sqrt{\left(\frac{p^2}{4}(\delta_0^2+\delta_1^2) + \frac{p^2}{2}\delta_0\delta_1 - \ldots\right)}} = \frac{p}{2}(\delta_0+\delta_1)$

$2\pm\sqrt2$

$= \frac p2(\delta_0+\delta_1) \pm\sqrt{\left(\frac{p^2}{4}(\delta_0^2+\delta_1^2) + \frac{p^2}{2}\delta_0\delta_1 - 2p\,\delta_0\delta_1 + \delta_0\delta_1\right)}$, $\frac{p^2}{2} - 2p + 1 = \frac12(p^2-4p+2) = \frac12(p-2+\sqrt2)(p-2-\sqrt2)$,

$\sqrt{\frac{p^2}{4}(\delta_0-\delta_1)^2 + (p^2-2p+1)\delta_0\delta_1} = \sqrt{\frac{p^2}{4}(\delta_0-\delta_1)^2 + \delta_0\delta_1(p-1)^2}$, $\mathbb{E}\prod\delta_{L_t} = (\frac12\delta_0, \frac12\delta_1)\, S P^{-1}\begin{pmatrix}\lambda_1^{T-2} & \\ & \lambda_2^{T-2}\end{pmatrix} P\begin{pmatrix}p\delta_0+\bar p\delta_1\\ p\delta_1+\bar p\delta_0\end{pmatrix}$

$(\delta_0=\delta_1)$ $\lambda = p\delta \pm\sqrt{(p^2\delta^2 - \delta^2(2p-1))} = p\delta\pm\delta(1-p) = \delta,\ (2p-1)\delta$, $v = \begin{pmatrix}\delta-\delta p\\ \delta-\delta p\end{pmatrix}, \begin{pmatrix}(p-1)\delta\\ \delta-\delta p\end{pmatrix}$,

$A = \delta\begin{pmatrix}1-p & p-1\\ 1-p & 1-p\end{pmatrix} = \delta(1-p)\begin{pmatrix}1&-1\\1&1\end{pmatrix}$, $A^{-1} = \frac{1}{2\delta(1-p)}\begin{pmatrix}1&1\\-1&1\end{pmatrix}$, $(\delta\ \delta)\,\delta^{T-2}\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}1 & \\ & (2p-1)^{T-2}\end{pmatrix}\begin{pmatrix}1&-1\\1&1\end{pmatrix} = (\delta\ \delta)\,\delta^{T-2}\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}1&-1\\ (2p-1)^{T-2} & (2p-1)^{T-2}\end{pmatrix}$

$= \delta^{T-2}(0\ \ 2\delta)(\quad)$

$e_{t_0}(\ell_{t_0-1}) = \mathbb{E}\left(\prod_{t=t_0}^{T}\frac{\delta_{A_{t-1}}}{\delta_{L_t}}\,\middle|\, A_{t_0-1}=a\right) = p\,\mathbb{E}\left(\prod_{t=t_0+1}^T\frac{\delta_{A_{t-1}}}{\delta_{L_t}}\,\middle|\,L_{t_0}=a\right) + (1-p)\frac{\delta_a}{\delta_{\bar a}}\,\mathbb{E}\left(\prod_{t_0+1}^T\cdots\,\middle|\,L_{t_0}=\bar a\right)$

$e_t(\ell_{t-1}) = \mathbb{E}\left(\prod_{j=t}^T\frac{\delta_{A_{j-1}}}{\delta_{L_j}}\,\middle|\,L_{t-1}=\ell_{t-1}\right) = p^2\,\mathbb{E}\left(\prod_{j=t+1}^T\cdots\,\middle|\,L_t=\ell_{t-1}\right) + p(1-p)\frac{\delta_{\ell_{t-1}}}{\delta_{\bar\ell_{t-1}}}\,\mathbb{E}(\cdots\,|\,L_t=\bar\ell_{t-1})$

$+(1-p)p\,\frac{\delta_{\bar\ell_{t-1}}}{\delta_{\bar\ell_{t-1}}}\,\mathbb{E}(\cdots\,|\,L_t=\bar\ell_{t-1}) + (1-p)^2\frac{\delta_{\bar\ell_{t-1}}}{\delta_{\ell_{t-1}}}\,\mathbb{E}(\cdots\,|\,L_t=\ell_{t-1}) = \left(p^2+(1-p)^2\frac{\delta_{\bar\ell_{t-1}}}{\delta_{\ell_{t-1}}}\right)e_{t+1}(\ell_{t-1}) + p(1-p)\left(\frac{\delta_{\ell_{t-1}}}{\delta_{\bar\ell_{t-1}}}+1\right)$

$e_{t+1}(\bar\ell_{t-1})$, $\begin{pmatrix}e_t(0)\\ e_t(1)\end{pmatrix} = \begin{pmatrix}p^2+(1-p)^2\,\delta_1/\delta_0 & p(1-p)\left(\frac{\delta_0}{\delta_1}+1\right)\\ p(1-p)\left(\frac{\delta_1}{\delta_0}+1\right) & p^2+(1-p)^2\frac{\delta_0}{\delta_1}\end{pmatrix}\begin{pmatrix}e_{t+1}(0)\\ e_{t+1}(1)\end{pmatrix}$, $\mathbb{E}\prod\frac{\delta_{A_{t-1}}}{\delta_{L_t}} = \frac12\left(\delta_0^{-1}e_2(0) + \delta_1^{-1}e_2(1)\right)$

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(21) $(T\geq 1)$, $e_1(0)=e_1(1)=1$

$$e_T(\ell_{t-1}) = \mathbb{E}\left(\frac{\delta_{A_{t-1}}}{\delta_{L_t}} \,\Big|\, L_{T-1}=\ell_{t-1}\right) = \mathbb{E}\left(\delta_{A_{t-1}} \,|\, L_{t-1}=\ell_{t-1}\right) = p\,\delta_{\ell_{t-1}} + (1-p)\,\delta_{\bar\ell_{t-1}}, \quad (T=1)\ \ \frac{L_1}{A_1},\ \ \mathbb{E}\frac{\delta_{A_0}}{\delta_{L_1}} = \frac{1}{2}\left(\frac{1}{\delta_0}+\frac{1}{\delta_1}\right),$$

$$(T=2)\quad \begin{matrix} L_1 & & L_2 \\ \downarrow & \nearrow & \downarrow \\ A_1 & & A_2 \end{matrix} \qquad \mathbb{E}\frac{1}{\delta_{L_1}}\frac{\delta_{A_1}}{\delta_{L_2}} = \frac{1}{2}\delta_1^{-1}\,\mathbb{E}\left(\frac{\delta_{A_1}}{\delta_{L_2}} \,\Big|\, \delta_{L_1}=1\right) + \frac{1}{2}\delta_0^{-1}\,\mathbb{E}\left(\frac{\delta_{A_1}}{\delta_{L_2}} \,\Big|\, \delta_{L_1}=0\right) = \frac{1}{2}\delta_1^{-1} e_1(1) + \frac{1}{2}\delta_0^{-1} e_1(0)$$

$$e_{T+1}(1) = e_{T+1}(0) = 1, \quad \left(\frac{1}{2\delta_0}, \frac{1}{2\delta_1}\right) P \begin{pmatrix} e_2(0) \\ e_2(1)\end{pmatrix}, \quad \frac{1}{2\delta_1}\left(p^2 + p(1-p)\frac{\delta_1}{\delta_0} + (1-p)p \cdots\right)$$

$$T = 2p^2 + (1-p)^2\left(\frac{\delta_1}{\delta_0}+\frac{\delta_0}{\delta_1}\right), \quad \Delta = p^4 + p^2(1-p)^2\left(\frac{\delta_0}{\delta_1}+\frac{\delta_1}{\delta_0}\right) + (1-p)^4 - p^2(1-p)^2\left(2+\frac{\delta_0}{\delta_1}+\frac{\delta_1}{\delta_0}\right)$$

$$= p^4 + (1-p)^4 - 2p^2(1-p)^2 = (p^2-(1-p)^2)^2, \quad \lambda_\pm = p^2 + \frac{(1-p)^2}{2}\left(\frac{\delta_1}{\delta_0}+\frac{\delta_0}{\delta_1}\right) \pm \sqrt{\frac{T^2}{4}-\Delta}, \quad \frac{T^2}{4}-\Delta = 4p$$

$$f(A_{t+1}|A_t) = \sum_\ell f(A_{t+1}|L_{t+1}=\ell)\,\mathbb{P}(L_{t+1}=\ell|A_t) = \sum_\ell p_1^{1_{\ell=A_{t+1}}}(1-p_1)^{1_{\ell\neq A_{t+1}}}\,p_2^{1_{\ell=A_t}}(1-p_2)^{1_{\ell\neq A_t}}$$

$$= (1-p_1)(1-p_2)\sum_\ell \left(\frac{p_1}{1-p_1}\right)^{1_{\ell=A_{t+1}}}\left(\frac{p_2}{1-p_2}\right)^{1_{\ell=A_t}} = (1-p_1)(1-p_2)\left(\left(\frac{p_1}{1-p_1}\right)\left(\frac{p_2}{1-p_2}\right)\right)^{1_{A_t=A_{t+1}}}\left(\frac{p_1}{1-p_1}+\frac{p_2}{1-p_2}\right)^{1_{A_t\neq A_{t+1}}}$$

$$= \left(p_1(1-p_2)+p_2(1-p_1)\right)\left(\frac{p_1p_2}{(1-p_1)(1-p_2)}\cdot\frac{(1-p_1)(1-p_2)}{p_1+p_2-2p_1p_2}\right)^{1_{A_t=A_{t+1}}} = (p_1+p_2-2p_1p_2)\left(\underbrace{\frac{1-p_1-p_2+2p_1p_2}{p_1+p_2-2p_1p_2}}_{(p_1+p_2-2p_1p_2)^{-1}-1}\right)^{1_{A_t=A_{t+1}}},$$

$$\mathbb{E}\frac{d^{1_{A_{t-1}=A_t}}}{c^{1_{A_t=L_t}}}, \quad \frac{1}{c}\mathbb{P}(A_t=L_t)\,\mathbb{E}\left(d^{1_{A_{t-1}=A_t}} \,|\, A_t=L_t\right) = \frac{p_1}{c}\left(d\,\mathbb{P}(A_{t-1}=A_t) + \mathbb{P}(A_{t-1}\neq A_t)\right) = \frac{p_1}{c}\left(1+(d-1)\,\mathbb{P}(A_{t-1}=A_t)\right)$$

$$\begin{matrix} & L_t \\ & \nearrow\ \downarrow \\ A_{t-1} & A_t \end{matrix} \qquad + \mathbb{P}(A_t\neq L_t)\,\mathbb{E}\left(d^{1_{A_{t-1}=A_t}} \,|\, A_t\neq L_t\right) = (1-p_1)\left(1+(d-1)\,\mathbb{P}(A_t=A_{t-1})\right), \quad \mathbb{E}\frac{d^{1_{A_{t-1}=A_t}}}{c^{1_{A_t=L_t}}} = \left(1-p_1+\frac{p_1}{c}\right)\left(1+(d-1)\,\mathbb{P}(A_{t-1}=A_t)\right)$$

$$\cancel{f(A_{t+1}|A_t) = (p_1p_2+(1-p_1)(1-p_2))^{1_{A_t=A_t}}}\cdot \quad \mathbb{E}\frac{d^{1_{A_{t-1}=A_t}}}{c^{1_{A_t=L_t}}} = p_1p_2\frac{d}{c} + p_2(1-p_1) + (1-p_2)p_1\frac{1}{c}$$

$$+ (1-p_1)(1-p_2)\,d, \quad d := \left(\frac{1}{p_1+p_2-2p_1p_2}-1\right)^{2}, \quad c := \left(\frac{p_1}{1-p_1}\right)^2, \quad \mathbb{E}\left(\prod\frac{f(A_t|A_{t-1})}{f(A_t|L_t)}\right)^2 =$$

$$\left(\mathbb{E}\left(\frac{f(A_t|A_{t-1})}{f(A_t|L_t)}\right)^2\right)^T = \left(\frac{\overbrace{p_1+p_2-2p_1p_2}^{\text{“}\alpha\text{”}}}{1-p_1}\right)^{2T}\left(\mathbb{E}\frac{c^{1_{A_t=A_{t-1}}}}{d^{1_{A_t=L_t}}}\right)^T = \left(\frac{p_1+p_2-2p_1p_2}{1-p_1}\right)^{2T}\left(p_1p_2\frac{d}{c} + p_2(1-p_1) + (1-p_2)p_1\frac{1}{c} + (1-p_1)(1-p_2)d\right),$$

$$\alpha = p_1+p_2-2p_1p_2 = p_1(1-p_2)+p_2(1-p_1)$$

$$\frac{p_1}{c}(1-p_2+p_2d) + (1-p_1)(p_2+(1-p_2)d) = \frac{p_1}{c}\left\{1+p_2\left(\frac{1}{\alpha^2}-\frac{2}{\alpha}\right)\right\} + (1-p_1)\left\{p_2+(1-p_2)\left(\frac{1}{\alpha^2}-\frac{2}{\alpha}+1\right)\right\}$$

$$= \frac{p_1}{c}\left\{1+\frac{p_2}{\alpha}\left(\frac{1}{p_1+p_2-2p_1p_2}-2\right)\right\} + (1-p_1)\left\{p_2+(1-p_2)\frac{(1-\alpha)^2}{\alpha^2}\right\} = \left(\frac{p_1p_2}{c}+(1-p_1)(1-p_2)\right)\left(\frac{1}{\alpha^2}-\frac{2}{\alpha}\right) + \frac{p_1}{c} + (1-p_1)$$

$$= \left(\frac{p_2}{p_1}(1-p_1)^2 + (1-p_1)(1-p_2)\right)\frac{1}{\alpha}\left(\frac{1}{\alpha}-2\right) + p_1\left(\frac{1}{c}-1\right) + 1 = \frac{1-p_1}{\alpha}\left(\frac{1}{\alpha}-2\right)\left(\frac{p_2}{p_1}-p_2+1-p_2\right) + \frac{(1-p_1)^2}{p_1} + 1-p_1$$

$$= \frac{1-p_1}{\alpha}\left(\frac{1}{\alpha}-2\right)\left(\frac{p_2}{p_1}+1-2p_2\right) + (1-p_1)\frac{1}{p_1} = \frac{1-p_1}{p_1}\left(\frac{1}{\alpha}-2\right) + \frac{1-p_1}{p_1} = \frac{1-p_1}{p_1}\left(\frac{1}{\alpha}-1\right), \quad \left(\frac{\alpha}{1-p_1}\right)^{2T}\left(\left(\frac{1-p_1}{p_1}\right)\left(\frac{1}{\alpha}-1\right)\right)^T$$

$$= \left(\frac{1}{p_1(1-p_1)}(\alpha-\alpha^2)\right)^T, \quad 1-\alpha = 1-p_1-p_2+2p_1p_2 = (1-p_1)(1-p_2)+p_1p_2, \quad \frac{p_1(1-p_2)+p_2(1-p_1)}{p_1(1-p_1)}\left((1-p_1)(1-p_2)\right.$$

$$\left.+p_1p_2\right) = \alpha\left(\frac{1-p_2}{p_1}+\frac{p_2}{1-p_1}\right) = (1-p_2)^2 + \frac{p_2}{p_1}(1-p_1)(1-p_2) + \frac{p_1p_2(1-p_2)}{1-p_1} + p_2^2 = 1+3p_2^2-2p_2+\frac{p_2}{p_1} - p_2 - \frac{p_2^2}{p_1} + \frac{p_1p_2(1-p_2)}{1-p_1}$$

$$2p_2^2-p_2+(1-p_2)^2$$

$$= (1-p_2)\left(1-p_2+\frac{p_2}{p_1}(1-p_1)+\frac{p_1p_2}{1-p_1}\right) + p_2^2 = (1-p_2)\left(1-2p_2+p_2\left(\frac{1}{p_1}+\frac{p_1}{1-p_1}\right)\right) + p_2^2 = (3p_2^2-3p_2+1) + \frac{p_2(1-p_2)}{p_1(1-p_1)}\,(p_1^2-p_1+1)$$

$(p_1=p_2)$ $\qquad (1-p_1)^2+p_1$

$$p^2-4p+2 = 2(p^2+p^2-2p+1) = 2(p^2+(1-p)^2), \quad (1-p_2)(1-2p_2)+p_2^2 = (1-p_2)^2+p_2(2p_2-1)$$

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(12) $\rho := \frac{p_2}{p_1}$, $1-\rho p_1 = 1-p_2$, $\frac{1-\rho p_1}{1-p_1} \cdot \rho$

$\sigma_2 > \sigma_1$

$(2\pi\sigma_1^2)^{-1/2}\exp\left(-\frac{x^2}{2\sigma_1^2}\right) = (2\pi\sigma_2^2)^{-1/2}\exp\left(-\frac{x^2}{2\sigma_2^2}\right)$, $\sqrt{\frac{2\pi\sigma_2^2}{2\pi\sigma_1^2}} = \exp\left(\frac{x^2}{2}\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_2^2}\right)\right)$, $x^2 = 2\left(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_2^2}\right)^{-1}$

$\log\left(\sqrt{\frac{2\pi\sigma_2^2}{2\pi\sigma_1^2}}\right)$, $= 2\frac{\sigma_1^2\sigma_2^2}{\sigma_2^2-\sigma_1^2}\log\left(\frac{\sigma_2}{\sigma_1}\right)$, $\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) - \frac{1}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{x^2}{2\sigma_2^2}\right) + \frac{1}{\sqrt{2\pi}\sigma_3}\exp\left(-\frac{x^2}{2\sigma_3^2}\right)\right.$

$\propto -x\exp\left(-\frac{x^2}{2}\right) + \frac{1}{\sigma_2}\frac{x}{\sigma_2^2}\exp\left(-\frac{x^2}{2\sigma_2^2}\right) - \frac{x}{\sigma_3^3}\exp\left(-\frac{x^2}{2\sigma_3^2}\right) = -x\exp\left(-\frac{x^2}{2}\right)\Big\{1-\sigma_2^{-3}\exp\left(\frac{x^2}{2}\left(1-\frac{1}{\sigma_2^2}\right)\right)+\sigma_3^{-3}\cdot$

$\exp\left(\frac{x^2}{2}\left(1-\frac{1}{\sigma_3^2}\right)\right)\Big\} := 0$, $1 = \sigma_2^{-3}\exp\left(\frac{x^2}{2}\left(1-\frac{1}{\sigma_2^2}\right)\right)\left\{1-\left(\frac{\sigma_2}{\sigma_3}\right)^3\exp\left(\frac{x^2}{2}\left(\frac{1}{\sigma_2^2}-\frac{1}{\sigma_3^2}\right)\right)\right\}$, $\sup_c \inf\{$

$\varphi(x)-\varphi(x/\sigma_2)/\sigma_2+\varphi(x/c)/c : 2\frac{\sigma_2^2}{\sigma_2^2-1}\log\sigma_2 < x < \frac{\sigma_2^2c^2}{c^2-\sigma_2^2}\log\left(\frac{c}{\sigma_2}\right)\} > 0)$, $\sigma_3^{-3}\exp\left(\frac{x^2}{2}\left(1-\frac{1}{\sigma_3^2}\right)\right) > \sigma_2^{-3}\cdot$

$\exp\left(\frac{x^2}{2}\left(1-\frac{1}{\sigma_2^2}\right)\right)$, $\left(\frac{\sigma_2}{\sigma_3}\right)^3 > \exp\left(\frac{x^2}{2}\left(\frac{1}{\sigma_3^2}-\frac{1}{\sigma_2^2}\right)\right)$, $x^2 < 6\left(\frac{\sigma_3^2\sigma_2^2}{\sigma_2^2-\sigma_3^2}\right)\log\left(\frac{\sigma_2}{\sigma_3}\right)$

$Y = \sum_t \left(L_t - \mathbb{E}(L_t|\bar L_{t-1},\bar A_{t-1})\right) + \sum\left(U_t - \mathbb{E}(U_t|\bar U_{t-1},\bar A_{t-1})\right) + \mu_B(\bar A) + \varepsilon$,

---

$\mathbb{E}\frac{g(Y,\bar A)}{\bar W} = \sum_{\bar a}\mathbb{E}\{\bar A=\bar a\}\frac{g(Y_a,\bar a)}{\bar W} = \sum_{\bar a}\mathbb{E}\Big\{\frac{g(Y_{\bar a},\bar a)}{\cancel{f_{z_T}(z_T|\overline{LU}_T,\bar A_{T-1})\Delta_T}}\mathbb{P}(\bar A=\bar a|$

$Y_{\bar a},\overline{LUZ}_T)\Big\} = \sum_{\bar a}\mathbb{E}\Big\{\frac{g(Y_{\bar a},\bar a)}{\bar W_T}\mathbb{P}(\bar A_T=a_T|\bar A_{T-1}=\bar a_{T-1},\overline{LUZ}_T)\mathbb{P}(\bar A_{T-1}=\bar a_{T-1}|\overline{LUZ}_T,Y_{\bar a})\Big\}$

$= \sum_{\bar a}\mathbb{E}\{\{\bar A=\bar a\}\bar W^{-1}\mathbb{E}(g(Y_a,\bar a)|AZLU)\} = \sum_{\bar a}\mathbb{E}\,\bar W^{-1}\mathbb{E}(g|\ldots)\mathbb{P}(\bar A = \bar a$, $\cancel{\int \bar W^{-1}\mathbb{E}(g|\bar A_{T-1},\overline{ZLU})f_{\bar A_{T-1},\overline{ZLU}}}$

$\mathbb{E}(\bar W^{-1}\mathbb{E}(g|\overline{AZLU})) =$

$\int\bar W^{-1}\mathbb{E}g(\overline{AZLU})f_{\overline{AZLU}}\mu_{\overline{AZLU}}(\overline{aZPU}) = \int\bar W(\bar a_{T-1},\overline{ZPU})^{-1}\mathbb{E}g(\overline{aZPU})f_{A_T|\bar A_{T-1},\overline{ZLU}}f_{\bar A_{T-1},\overline{ZLU}}\mu(\ldots) = \int\bar W(\bar a_{T-1},\overline{ZPU})^{-1}\mathbb{E}g(\overline{aPU})$

$= \int\bar W(\bar a_{T-2},\overline{ZPU}_{T-1})^{-1}\mathbb{E}g(\overline{aPU})f_{Z_T|\overline{ZA}_{T-1},\overline{LU}}^{-1}f_{\overline{AZ}_{T-1},\overline{LU}_T}(\ldots)^{1-z_T}\Delta_T^{-1}f_{A_T|\bar A_{T-1},\overline{ZLU}}\mu(\ldots) = \int\bar W_{T-1}^{-1}\mathbb{E}g(\bar a_{T-1},\overline{PU})$

$\left(\sum_{z_T=0,1}f_{A_T|\bar A_{T-1},\overline{ZLU}}(z_T,\ldots)\Delta_T^{-1}\right)\frac{f_{\overline{AZ}_{T-1},\overline{LU}}}{\cancel{f_{A_T|\bar A_{T-1},\overline{ZLU}}}}\mu(\ldots/z_T) = \int\bar W_{T-1}^{-1}\mathbb{E}g(\bar a_{T-1},\overline{PU})f_{\overline{LU}|\overline{AZLU}_{T-1}}f_{\overline{AZLU}_{T-1}}\mu($

$\ldots/z_T) = \int\bar W_{T-1}^{-1}\mathbb{E}g(\overline{aPU}_{T-1})f_{\overline{AZLU}_{T-1}}\mu(\ldots/PZU_T) = \cancel{\int\bar W_{T-1}^{-1}\mathbb{E}g(}\int_A\mathbb{E}(\bar W_{T-1}^{-1}g(Y_{a_T},(\bar A_{T-1},a_T))\mu(a_T)$,

$\int\bar W^{-1}$

$\varphi(x)\left\{1-c^{-1}\exp\left(\frac{x^2}{2}(1-c^{-2})\right)+d^{-1}\exp\left(\frac{x^2}{2}(1-d^{-2})\right)\right\} = \varphi(x)\Big\{1+d^{-1}\exp\left(\frac{x^2}{2}(1-d^{-2})\right)\Big($1

$\varphi(x)c^{-1}\exp\left(\frac{x^2}{2}(1-c^{-2})\right)\left\{c\exp\left(\frac{x^2}{2}(c^{-2}-1)\right)+\frac{c}{d}\exp\left(\frac{x^2}{2}(c^{-2}-d^{-2})\right)-1\right\}$, $c-1>-\frac{c}{d}$, $d<\frac{c}{c-1}$

$-\frac{d}{c}\exp\left(\frac{x^2}{2}\left(\frac{1}{d^2}-\frac{1}{c^2}\right)\right)\Big)\Big\}$, $\cancel{\exp\left(\frac{x^2}{2}\left(\frac{1}{d^2}-\frac{1}{c^2}\right)\right)} < \frac{c}{d}$ $1-\frac{d}{c}\exp\left(\frac{x^2}{2}\left(\frac{1}{d^2}-\frac{1}{c^2}\right)\right)$

$> -d\exp\left(\frac{x^2}{2}(d^{-2}-1)\right)$, $1 > \frac{d}{c}\exp\left(\frac{x^2}{2}(d^{-2}-c^{-2})\right)\left(1-c\exp\left(\frac{x^2}{2}(c^{-2}-1)\right)\right)$, $\frac{c}{d}\exp\left(\frac{x^2}{2}(c^{-2}-d^{-2})\right)+$

$c\exp\left(\frac{x^2}{2}(c^{-2}-1)\right)>1$, $c<1$, $c\frac{d+1}{d}>1$, $1>c>\frac{d}{d+1}$, $\cancel{c^2\left(\frac{d+1}{d}\right)^2}$ $d>c>\frac{d}{d+1}$, $1>d>0$, $c<d<\frac{c}{1-c}$, $c:=c$, $u:=d$

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$h(Y - W^{-1}\beta^T D) = 0$, $h W^{-1} Y = h W^{-1} D \beta$, $\hat\beta = (h^T W^{-1} D)^{-1} h^T W^{-1} Y$, $Y = D^T\beta + L + U + \varepsilon$, $\hat\beta \approx \beta$

$\hat\beta - \beta = (h^T W^{-1} D)^{-1} h^T W^{-1} (L + U + \varepsilon)$, $\mathbb{E}(\hat\beta - \beta) = \mathbb{E}\left((h^T W^{-1} D)^{-1} h^T W^{-1} (L+U)\right)$, $\mathbb{E}\, h^T \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar A_{T-1})}{\prod_t f(A_t \mid \bar L_t, \bar A_{t-1})}$

$= \mathbb{E}\left(\mathbb{E}(\cdots \mid \bar L_{T}, \bar A_{T-1})\right) = \mathbb{E}\left(\sum_{a_T} h(\bar A_{T-1}, a_T)^T \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar A_{T-1})}{\prod_t^{T-1} f(A_t \mid \bar L_t, \bar A_{t-1})}\right) = \mathbb{E}(\cdots \mid \bar L_{T-1}, \bar A_{T-1}) = 0.$

$\mathbb{E}\, h^T \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar A_{T-1})}{\bar W_{T-1}\, \Delta_T (-1)^{1-Z_T} f_{Z_T}} = \sum_{\bar a} \mathbb{E}\left(h^T(\bar a)\{\bar A = \bar a\} \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar A_{T-1})}{\bar W_{T-1}\, \Delta_T (-1)^{1-Z_T} f_{Z_T}}\right) = \sum_{\bar a} \mathbb{E}(\cdots \mid \bar A_{T-1}, \overline{LUZ}_T)$ $\quad (\bar U_{t+1} \perp\!\!\!\perp Z_t \mid \bar A_t)$, $\bar U_{T-1}$

$= \sum_{\bar a} \mathbb{E}\left(h^T(\bar a)\{\bar A_{T-1} = \bar a_{T-1}\} \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar a_{T-1})}{\bar W_{T-1}}\right) = \sum_{\bar a} \mathbb{E}\left(h^T(\bar a)\{\bar A_{T-1} = \bar a_{T-1}\} \dfrac{L_T - \mathbb{E}(L_T \mid \bar L_{T-1}, \bar a_{T-1}, \bar Z_{T-1})}{\bar W_{T-1}}\right)$

$(\bar U_{t+1}, \bar L_{t+1}, \bar Z_t) \perp\!\!\!\perp Z_t \mid \bar A_t)$, $= \sum_{\bar a} \mathbb{E}\left(h^T(\bar a)\{\bar A_{T-1} = \bar a_{T-1}\}\, \mathbb{E}(- \mid \overline{UL}_{T-1}, \bar a_{T-1}, \bar Z_{T-1})\right) = 0$

$$\frac{\frac{1}{2}\left(\binom{T}{2} + T\right)\left(T\gamma^{1-T} + \sigma^2\gamma^{-T}\right)}{\{2^{T-2}\, T(T+1)\}^2} = \frac{(T+1)(\gamma^{1-T} + \frac{\sigma^2}{T}\gamma^{-T})}{\{2^{T-1}(T+1)\}^2} = \frac{1}{T+1}\frac{1}{(4\gamma)^{T-1}}\left(1 + \frac{\sigma^2}{T\gamma}\right)$$

$p_2(1-p_2) = (\tfrac12 - u_2)(\tfrac12 + u_2) = \tfrac14 - u_2^2 = \tfrac14 - (\tfrac12 - p_2)^2$, $(1-p_1)^2 + p_1 = p_1^2 - p + 1$, $p_1^2 - p_1 + \tfrac14 = (p_1 - \tfrac12)^2$, $\left(1 - 3p_2(1-p_2)\right) + \dfrac{p_2(1-p_2)}{p_1(1-p_1)}\left(1 - p_1(1-p_1)\right)$, $-4p_1^2 + 4p_1 = 4p_1(1-p_1)$

$= 1 - p_2(1-p_2)\left(4 - \dfrac{1}{p_1(1-p_1)}\right) = 1 + 4\dfrac{p_2(1-p_2)}{p_1(1-p_1)}(p_1 - \tfrac12)^2 = 1 + 4p_2(1-p_2)\left(\dfrac{p_1(1-p_1)}{(p_1 - \frac12)^2}\right)^{-1} = 1 + 4p_2(1-p_2)\left(\dfrac{1}{4(p_1 - \frac12)^2} - 1\right)^{-1}$

$= 1 + 4p_2(1-p_2)\left(\dfrac{1}{4p_1(1-p_1)} - 1\right)$

$A_{t-1} \xrightarrow{p_2} L_t \xrightarrow{p_1} A_t$

$\mathbb{E}\prod_{t=1}^{T}\dfrac{\gamma_{A_{t-1}}}{\delta_{L_t}}$, $\varphi_t(\ell_{t-1}) := \mathbb{E}\left(\prod_{j=t}^{T}\dfrac{\gamma_{A_{j-1}}}{\delta_{L_j}} \,\Big|\, L_{t-1} = \ell_{t-1}\right) = p_1 p_2 \dfrac{\gamma_{\ell_{t-1}}}{\delta_{\ell_{t-1}}}\, \mathbb{E}\left(\prod_{j=t+1}^{T} \cdots \,\Big|\, L_{t} = \ell_{t-1}\right)$ $\quad \varphi_{t+1}(\ell_{t-1})$

$+ p_1(1-p_2)\dfrac{\gamma_{\ell_{t-1}}}{\delta_{\overline{\ell_{t-1}}}}\varphi_{t+1}(\overline{\ell_{t-1}}) + (1-p_1)p_2 \dfrac{\gamma_{\overline{\ell_{t-1}}}}{\delta_{\overline{\ell_{t-1}}}}\varphi_{t+1}(\overline{\ell_{t-1}}) + (1-p_1)(1-p_2)\dfrac{\gamma_{\overline{\ell_{t-1}}}}{\delta_{\ell_{t-1}}}\varphi_{t+1}(\ell_{t-1})$

$= \left(p_1 p_2 \dfrac{\gamma_{\ell_{t-1}}}{\delta_{\ell_{t-1}}} + (1-p_1)(1-p_2)\dfrac{\gamma_{\overline{\ell_{t-1}}}}{\delta_{\ell_{t-1}}}\right)\varphi_{t+1}(\ell_{t-1}) + \left(p_1(1-p_2)\dfrac{\gamma_{\ell_{t-1}}}{\delta_{\overline{\ell_{t-1}}}} + (1-p_1)p_2 \dfrac{\gamma_{\overline{\ell_{t-1}}}}{\delta_{\overline{\ell_{t-1}}}}\right)\varphi_{t+1}(\overline{\ell_{t-1}})$

$$P = \begin{pmatrix} p_1 p_2 \frac{\gamma_0}{\delta_0} + (1-p_1)(1-p_2)\frac{\gamma_1}{\delta_0} & p_1(1-p_2)\frac{\gamma_0}{\delta_1} + (1-p_1)p_2\frac{\gamma_1}{\delta_1} \\ p_1(1-p_2)\frac{\gamma_1}{\delta_0} + (1-p_1)p_2\frac{\gamma_0}{\delta_0} & p_1 p_2\frac{\gamma_1}{\delta_1} + (1-p_1)(1-p_2)\frac{\gamma_0}{\delta_1} \end{pmatrix}, \quad (p_1 = p_2),\ P = \begin{pmatrix} p^2\frac{\gamma_0}{\delta_0} + (1-p)^2\frac{\gamma_1}{\delta_0} & \frac{p(1-p)}{\delta_1}(\gamma_0 + \gamma_1) \\ \frac{p(1-p)}{\delta_0}(\gamma_0 + \gamma_1) & p^2\frac{\gamma_1}{\delta_1} + (1-p)^2\frac{\gamma_0}{\delta_1} \end{pmatrix}$$

$\dfrac{\gamma_0}{\delta_0} = p_2 + (1-p_2)\dfrac{\delta_1}{\delta_0}$, $\dfrac{\gamma_1}{\delta_0} = p_2\dfrac{\delta_1}{\delta_0} + (1-p_2)$, $\dfrac{\gamma_1}{\delta_1} = p_2 + (1-p_2)\dfrac{\delta_0}{\delta_1}$, $p^2\dfrac{\gamma_0}{\delta_0} + (1-p)^2\dfrac{\gamma_1}{\delta_1} = p^3 + p^2(1-p)\dfrac{\delta_1}{\delta_0} + p(1-p)^2 + (1-p)^3\dfrac{\delta_0}{\delta_1}$,

$\Delta = (p^4 + (1-p)^4)\dfrac{\gamma_0\gamma_1}{\delta_0\delta_1} + p^2(1-p)^2\left(\dfrac{\gamma_0^2}{\delta_0\delta_1} + \dfrac{\gamma_1^2}{\delta_0\delta_1}\right) - p^2(1-p)^2(\gamma_0 + \gamma_1)^2\dfrac{1}{\delta_0\delta_1} = (p^2 - (1-p)^2)^2\dfrac{\gamma_0\gamma_1}{\delta_0\delta_1} + \cancel{p^2(1-p)^2\left(\frac{\gamma_0^2}{\delta_0}\left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right) + \frac{\gamma_1^2}{\delta_1}\left(\frac{1}{\delta_1} - \frac{1}{\delta_0}\right)\right)}$

$\cancel{= (p^2 - (1-p)^2)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1} + p^2(1-p)^2\left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right)\left(\frac{\gamma_0^2}{\delta_0} - \frac{\gamma_1^2}{\delta_1}\right)}$, $\cancel{AT = (p^2 + (1-p)^2)\left(\frac{\gamma_0}{\delta_0} + \frac{\gamma_1}{\delta_1}\right)}$

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$\varphi_T = (p^2\dfrac{\delta_0}{\delta_0} +$

$\mathbb{E}(\eta \mid \bar{a}_{T-1}, \bar{L}_{T-1}) = \mathbb{E}\left(\mathbb{E}(Y \mid \bar{A}, \bar{L}) - \mu(\bar{A}) \mid \bar{a}_T, \bar{L}_{T-1}\right) = \int \mathbb{E}(Y_{\bar{a}} \mid \bar{a}, \bar{L})\, f(L_T \mid \bar{a}_T, \bar{L}_{T-1}) - \mu(\bar{a})$

$\mathbb{E}(Y \mid A, L) = \mu(A) + L, \quad \mathbb{E}\, L = 0$ $\qquad$ $\mathbb{E}(Y_{00} \mid 0, 0, \bar{L}_2) = \mu(\bar{A}_2) + L_1 + L_2, \quad \mathbb{E}(L_1) = 0, \quad \mathbb{E}(L_2 \mid a_1 = 0, L_1) = 0$

$\mathbb{E}(Y_1 \mid 1, L) = \mu(1) + L$

$\mathbb{E}(Y_1 \mid 0, L) = ?$

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$$\mathbb{E}\prod_{t=1}^{T}\frac{\gamma_{A_{t-1}}}{\delta_{L_t}} \overset{?}{=} \frac{1}{2}\left(\varphi_2(0)+\varphi_2(1)\right), \quad \mathbb{E}\prod_{t=1}^{2}\frac{\gamma_{A_{t-1}}}{\delta_{L_t}} = \mathbb{E}\left(\gamma_{A_0}^{\delta_{L_1}}\right)\mathbb{E}\left(\frac{1}{\delta_{L_1}}\frac{\gamma_{A_1}}{\delta_{L_2}}\right) = \mathbb{E}\left(\gamma_{A_0}^{\delta_{L_1}}\right)\left(\frac{1}{2\delta_0}\mathbb{E}\left(\frac{\gamma_{A_1}}{\delta_{L_2}}\middle| L_1=0\right)\right.$$

$$\left.+\frac{1}{2\delta_1}\mathbb{E}\left(\frac{\gamma_{A_1}}{\delta_{L_2}}\middle| L_1=1\right)\right) = \mathbb{E}\left(\gamma_{A_0}^{\delta_{L_1}}\right)\frac{1}{2}\sum_{l_1}\frac{1}{\delta_{l_1}}\mathbb{E}\left(\frac{\gamma_{A_1}}{\delta_{L_2}}\middle| L_1=l_1\right) = \mathbb{E}\left(\gamma_{A_0}^{\delta_{L_1}}\right)\frac{1}{2}\sum_{l_1}\varphi_2(l_1)$$

$$T = p^2\frac{\gamma_0\delta_1+\gamma_1\delta_0}{\delta_0\delta_1} + (1-p)^2\frac{\gamma_1\delta_1+\gamma_0\delta_0}{\delta_0\delta_1} = (1-2p)\frac{\gamma_0\delta_0+\gamma_1\delta_1}{\delta_0\delta_1} + p^2\frac{(\gamma_0-\gamma_1)(\delta_1-\delta_0)}{\delta_0\delta_1} = p^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)+(1-p)^2\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right),$$

$$T^2/4-\Delta = \frac{p^4}{4}\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)^2+\frac{(1-p)^4}{4}\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right)^2+\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right)-\left(p^2-(1-p)^2\right)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}$$

$$=\left[\frac{1}{2}\left(p^4+(1-p)^4\right)-\left(p^2-(1-p)^2\right)^2\right]\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{p^4}{4}\left(\frac{\gamma_0^2}{\delta_0^2}+\frac{\gamma_1^2}{\delta_1^2}\right)+\frac{(1-p)^4}{4}\left(\frac{\gamma_0^2}{\delta_1^2}+\frac{\gamma_1^2}{\delta_0^2}\right)+\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right)$$

$$=\left[-\frac{1}{2}p^4-\frac{1}{2}(1-p)^4+2p^2(1-p)^2\right]\cdots = p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}-\frac{1}{2}\left(p^2-(1-p)^2\right)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\cdots = 2p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{p^4}{4}\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)^2+\frac{(1-p)^4}{4}.$$

$$\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)^2+\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right) = \frac{1}{4}\left(p^2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)\pm(1-p)^2\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)\right)^2 \mp\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)$$

$$+2p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right) = \frac{1}{4}\left(\cdots\pm\cdots\right)^2+2p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{1}{2}p^2(1-p)^2\left(\frac{\gamma_0^2}{\delta_0\delta_1}+\frac{\gamma_0\gamma_1}{\delta_0^2}+\frac{\gamma_0\gamma_1}{\delta_1^2}\right.$$

$$\left.+\frac{\gamma_1^2}{\delta_0\delta_1}\mp\frac{\gamma_0^2}{\delta_0\delta_1}\pm\frac{\gamma_0\gamma_1}{\delta_0^2}\pm\frac{\gamma_0\gamma_1}{\delta_1^2}\mp\frac{\gamma_1^2}{\delta_0\delta_1}\right) \quad (-):\ = \frac{1}{4}\left(\cdots-\cdots\right)^2+2p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+p^2(1-p)^2\left(\gamma_0^2+\gamma_1^2\right)\frac{1}{\delta_0\delta_1}$$

$$=\frac{1}{4}\left(p^2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)-(1-p)^2\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)\right)^2+\frac{p^2(1-p)^2}{\delta_0\delta_1}(\gamma_0+\gamma_1)^2,\quad p^2(1-p)^2\left(\frac{(\gamma_0+\gamma_1)^2}{\delta_0\delta_1}-\frac{1}{2}\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)\right)$$

$$=p^2(1-p)^2\left|\frac{1}{2}\frac{\gamma_0^2}{\delta_0\delta_1}+\frac{1}{2}\frac{\gamma_1^2}{\delta_0\delta_1}+\frac{2\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{1}{2}\frac{\gamma_0\gamma_1}{\delta_0^2}+\frac{1}{2}\frac{\gamma_0\gamma_1}{\delta_1^2}\right| = p^2(1-p)^2\left(\frac{2\gamma_0\gamma_1}{\delta_0\delta_1}+\frac{1}{2}\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)\left(\frac{\gamma_0}{\delta_1}+\frac{\gamma_1}{\delta_0}\right)\right)$$

$$(+):\ = \frac{1}{4}\left(\cdots+\cdots\right)^2+2p^2(1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}+p^2(1-p)^2\gamma_0\gamma_1\left(\frac{1}{\delta_0^2}+\frac{1}{\delta_1^2}\right) = \frac{1}{4}\left(p^2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right)+(1-p)^2\left(\frac{\gamma_0}{\delta_1}-\frac{\gamma_1}{\delta_0}\right)\right)^2$$

$$+p^2(1-p)^2\gamma_0\gamma_1\left(\frac{1}{\delta_0}+\frac{1}{\delta_1}\right)^2 \quad (40) \quad \gamma_a = p\delta_a+(1-p)\delta_{1-a},\quad \frac{\gamma_0^2}{\delta_0^2}+\frac{\gamma_1^2}{\delta_1^2} = 2p^2+2p(1-p)\left(\frac{\delta_1}{\delta_0}+\frac{\delta_0}{\delta_1}\right)+(1-p)^2\left(\frac{\delta_1^2}{\delta_0^2}+\frac{\delta_0^2}{\delta_1^2}\right),$$

$$\frac{\gamma_0^2}{\delta_0^2}-\frac{\gamma_1^2}{\delta_1^2} = 2p(1-p)\left(\frac{\delta_1}{\delta_0}-\frac{\delta_0}{\delta_1}\right)+(1-p)^2\left(\frac{\delta_1^2}{\delta_0^2}-\frac{\delta_0^2}{\delta_1^2}\right) = (1-p)\left(\frac{\delta_1}{\delta_0}-\frac{\delta_0}{\delta_1}\right)\left(2p+(1-p)\left(\frac{\delta_1}{\delta_0}+\frac{\delta_0}{\delta_1}\right)\right),\quad (p_1\neq p_2)$$

$$\Delta = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(p_1^2p_2^2-p_1^2(1-p_2)^2\right)+\frac{\gamma_0^2}{\delta_0\delta_1}\left(p_1p_2(1-p_1)(1-p_2)-p_1p_2(1-p_1)(1-p_2)\right)+\frac{\gamma_1^2}{\delta_0\delta_1}\left((1-p_1)(1-p_2)\cdots\right)$$

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$$+\frac{\gamma_1\gamma_0}{\delta_1\delta_0}\left((1-p_1)^2(1-p_2)^2-(1-p_1)^2p_2^2\right) = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(-p_1^2+2p_1^2p_2+(1-p_1)^2(1-2p_2)\right) = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}(2p_2-1)\overbrace{\left(p_1^2-(1-p_1)^2\right)}^{(2p_1-1)},$$

$$T = p_1p_2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)+(1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0}+\frac{\gamma_0}{\delta_1}\right),\quad \frac{\partial}{\partial\gamma_0}\left(T/2\pm\left(T^2/4-\Delta\right)^{1/2}\right) = \frac{1}{2}\dot{T}\pm\frac{1}{2}\frac{T\dot{T}/2-\dot{\Delta}}{\left(T^2/4-\Delta\right)^{1/2}},$$

$$0=\dot{T}\left(T^2/4-\Delta\right)^{1/2}\pm\left(T\dot{T}/2-\dot{\Delta}\right),\quad \dot{T} = p_1p_2\left(\frac{1}{\delta_0}-\frac{1}{\delta_1}\right)+(1-p_1)(1-p_2)\left(\frac{1}{\delta_1}-\frac{1}{\delta_0}\right) = \left(\frac{1}{\delta_0}-\frac{1}{\delta_1}\right)(-1+p_1+p_2),\quad \dot{\Delta} = (1-2p_1)(1-2p_2)$$

$$\frac{1}{\delta_0\delta_1}(1-2\gamma_0),\quad \dot{T}^2\frac{T^2}{4}-\dot{T}^2\Delta = \frac{T^2\dot{T}^2}{4}-\dot{\Delta}\dot{T}T+\dot{\Delta}^2,\ \overset{(p_1=p_2)}{\Big|}\ \frac{\gamma_0\gamma_1}{\delta_0\delta_1}(2p-1)^2 = (2p-1)^2\frac{(1-2\gamma_0)}{\delta_0\delta_1}\left\{p^2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right)+(1-p)^2\right.$$

$$\left.\left(\frac{\gamma_1}{\delta_0}+\frac{\gamma_0}{\delta_1}\right)\right\}-\frac{(2p-1)^4}{\delta_0^2\delta_1^2}(1-2\gamma_0)^2\frac{\delta_0\delta_1}{(2p-1)(\delta_1-\delta_0)},\quad \gamma_0\gamma_1 = (1-2\gamma_0)\left\{p^2\left(\gamma_0\left(\frac{1}{\delta_0}-\frac{1}{\delta_1}\right)+\frac{1}{\delta_1}\right)+(1-p)^2\left(\frac{1}{\delta_0}+\gamma_0\left(\frac{1}{\delta_1}-\frac{1}{\delta_0}\right)\right)\right\}$$

$$-\frac{(2p-1)(1-2\gamma_0)^2}{\delta_1-\delta_0},\quad 0 = \gamma_0^2\left\{-\frac{(2p-1)\cdot 4}{\delta_1-\delta_0}-2p^2\left(\frac{1}{\delta_0}-\frac{1}{\delta_1}\right)-2(1-p)^2\left(\frac{1}{\delta_1}-\frac{1}{\delta_0}\right)+1\right\}$$

$|z + a_0| > |1 + 2a_0| - |a_0|$

$|e^{i\theta} + 2a_0 e^{i\theta} + a_0| > |1 + 2a_0| - |a_0|$

$|1 + 2a_0| > 2|a_0|$

$|1 + 3a_0|$

$|1 + 2|a_0| + a_0 e^{-i\theta}|$

$|1 + 2|a_0| + a_0| > |1 + |a_0|| \geq |a_0|$

$z^2 + a_1 z + a_0, \quad r = 1 + |a_1| + 2|a_0|$

$|z^2 + a_1 + a_1|a_1| + 2a_1|a_0| + a_0|$

$|z^2 + a_1 z + a_0|$

$1 + |a_1|^2 + 4|a_0|^2 + 2|a_1| + 4|a_0| + 4|a_0||a_1| + a_1 + a_1|a_1| + 2a_1|a_0| + |a_0|$

$$\mathbb{E}\left(\bar{W}^{-1} g(Y,\bar{A})\right) = \mathbb{E}\left(\bar{W}^{-1}\, \mathbb{E}\left(g(Y,\bar{A}) \mid \overline{AZLU}\right)\right)$$

product measure on state spaces

$$= \int \bar{W}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{azlu}\right) f_{\overline{AZLU}}(\overline{azlu})\, \mu_{\overline{AZLU}}(\overline{azlu})$$

$$= \int \bar{W}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) f_{\overline{AZLU}}(\overline{azlu})\, \mu_{\overline{AZLU}}(\overline{azlu})$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) (-1)^{1-z_T} f^{-1}_{Z_T \mid \overline{AZ}_{T-1}, \overline{LU}_T}(z_T, \overline{az}_{T-1}, \overline{lu}_T)\, \Delta_T^{-1}\, f_{\overline{AZLU}}(\overline{azlu})\, \mu_{\overline{AZLU}}(\overline{azlu})$$

$$= f_{A_T \mid \bar{A}_{T-1}, \overline{ZLU}} \cdot f_{Z_T \mid \overline{AZ}_{T-1}, \overline{LU}} \cdot f_{\overline{AZ}_{T-1}, \overline{LU}_T}$$

$$= f_{A_T \mid \bar{A}_{T-1}, \overline{ZLU}} \cdot f_{Z_T \mid \overline{AZ}_{T-1}, \bar{L}_T} \cdot f_{\overline{AZ}_{T-1}, \overline{LU}_T}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) (-1)^{1-z_T}\, \Delta_T^{-1}\, f_{A_T \mid \bar{A}_{T-1}, \overline{ZLU}}(a_T, \bar{a}_{T-1}, \overline{zlu})\, f_{\overline{AZ}_{T-1}, \overline{LU}_T} \cdot \mu_{\overline{AZLU}}\ \cancel{\mu_{\overline{AZLU}}(\overline{azlu})}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) \Delta_T^{-1} \left(\sum_{z_T \in \{0,1\}} (-1)^{1-z_t} f_{A_T \mid \bar{A}_{T-1}, \overline{ZLU}}(a_T, \bar{a}_{T-1}, z_T, \bar{z}_{T-1}, \overline{lu})\right) f_{\overline{AZ}_{T-1}, \overline{LU}} \cdot \mu_{\bar{A}, \bar{Z}_{T-1}, \overline{LU}}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) f_{\overline{AZ}_{T-1}, \overline{LU}}\ \mu_{\bar{A}, \bar{Z}_{T-1}, \overline{LU}}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}\right) f_{LU \mid \overline{AZLU}_{T-1}}\ f_{\overline{AZLU}_{T-1}}\ \mu_{\bar{A}, \bar{Z}_{T-1}, \overline{LU}}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \bar{a}_{T-1}, \overline{lu}\right) \cdot f_{LU \mid \overline{ALU}_{T-1}}\ f_{\overline{AZLU}_{T-1}}\ \mu_{\bar{A}, \bar{Z}_{T-1}, \overline{LU}}$$

$$= \int \bar{W}_{T-1}^{-1}\, \mathbb{E}\left(g(Y_{\bar{a}},\bar{a}) \mid \overline{alu}_{T-1}\right) f_{\overline{AZLU}_{T-1}}\ \mu_{\bar{A}, \overline{ZLU}_{T-1}}$$

$$= \int_{A} \mathbb{E}\left(\bar{W}_{T-1}^{-1}\, g(Y_{\bar{A}_{T-1}, a_T}, (\bar{A}_{T-1}, a_T))\right) \mu_{A_T}(a_T)$$

$$\left|\frac{a'}{b'}\right| \geq \frac{|(a'-\varepsilon)\wedge 0|}{|b'|+\varepsilon} \geq \frac{(|\tau|-\frac{\varepsilon}{|b|})\wedge 0}{1+\frac{\varepsilon}{|b|}} \qquad \frac{|\tau\delta-\varepsilon|}{|\delta+\varepsilon|} \qquad \frac{\tau-\frac{\varepsilon}{b}}{1+\frac{\varepsilon}{b}}$$

$\frac{a}{b}=\tau,\ b=\frac{\varepsilon}{\tau}$, $a=\tau b$

$a'=\tau b-\varepsilon$

$b'=b+\varepsilon$

$$\frac{a'-\varepsilon}{b'+\varepsilon}=\frac{\tau b-\varepsilon}{b+\varepsilon}=\frac{0}{\varepsilon(1+\frac{1}{\tau})}$$

$$\frac{p^2}{2}-2p+1 = (p-1)^2-\frac{p^2}{2}$$

$$4\left((1-\tfrac{1}{p})^2-\tfrac{1}{2}\right)=4\left(\tfrac{1}{2}-\tfrac{2}{p}+\tfrac{1}{p^2}\right)=2\left(1-\tfrac{4}{p}+\tfrac{2}{p^2}\right)$$

$$\frac{p^2}{4}\left(\delta_0^2+\left((2-\tfrac{2}{p})^2-2\right)\delta_0\delta_1+\delta_1^2\right)$$

$(p>\frac{1}{2})$
$$\frac{p^2}{4(2p-1)}<\frac{\delta_0\delta_1}{(\delta_0+\delta_1)^2}, \qquad 2\frac{\delta_0\delta_1}{(\delta_0+\delta_1)^2}<\frac{1}{2}\overset{?}{<}\frac{p^2}{2(2p-1)}$$

$$\frac{1}{2}\left(\frac{\delta_1}{\delta_0}+\frac{\delta_0}{\delta_1}\right)$$

$\cancel{p^2\leq 4p-2,\ -p^2+2(p-1)^2<0,\ (1-\frac{1}{p})^2<\frac{1}{2},\ \frac{1-p}{p}<\frac{1}{\sqrt2},\ p>(1+\frac{1}{\sqrt2})^{-1}=\frac{\sqrt2}{1+\sqrt2}}$

$$P(L_{t+1}=\ell\mid L_t=\ell)=\sum_a \mathbb{P}(L_{t+1}=\ell\mid A_t=a)\,\mathbb{P}(A_t=a\mid L=\ell)=\sum_a p_{AL}^{a=\ell}(1-p_{AL})^{a\neq\ell}\,p_{LA}^{a=\ell}(1-p_{LA})^{a\neq\ell}=\sum_a (p_{AL}p_{LA})^{a=\ell}((1-p_{AL})(1-p_{LA}))^{a\neq\ell}$$

$$=p_{AL}p_{LA}+(1-p_{AL})(1-p_{LA})$$

$$\mathbb{E}\left(\cdots\mathbb{E}\left(\mathbb{E}(\eta\mid\bar a_{T-1},\bar L_{T-1})\mid\bar a_{T-2},\bar L_{T-2}\right)\cdots\right)=0, \qquad \mathbb{E}\,\frac{\eta}{f(A_2\mid A_1,\bar L_2)f(A_1\mid L_1)}=\mathbb{E}\left(\frac{1}{f(A_1\mid L_1)}\,\mathbb{E}\left(\frac{\eta}{f(A_2\mid a_1,\bar L_2)}\,\Big|\,a_1,L_1\right)\right)$$

$$=\sum\mathbb{E}\left(\{\bar A=\bar a\}\frac{\eta}{f(a_2\mid a_1,\bar L_2)f(a_1,L_1)}\right)=\sum\mathbb{E}\left(\{A_1=a_1\}\frac{1}{f(a_1,L_1)}\,\mathbb{E}\left(\frac{\eta\{A_2=a_2\}}{f(a_2\mid a_1,\bar L_2)}\,\Big|\,a_1,\bar L_2\right)\right)=\sum\mathbb{E}\left(\{A_1=a_1\}\frac{1}{f(a_1\mid L_1)}\eta(a_2)\right)$$

$$\sum_{\bar a}\mathbb{E}\left(\{\bar A=\bar a\}\frac{\eta(\bar a,\bar L)}{\prod_t f(a_t\mid\bar a_{t-1},\bar L_t)}\right)=\sum_{\bar a}\mathbb{E}\left(\{\bar A_{T-1}=\bar a_{T-1}\}\frac{\eta(\bar a,\bar L)}{\prod_t\cdots}\,\mathbb{E}(\{A_T=a_T\}\mid\bar a_{T-1},\bar L_T)\right)=\sum_{\bar a}\mathbb{E}\left(\{\bar A_{T-1}=\bar a_{T-1}\}\frac{\mathbb{E}(\eta(\bar a,\bar L)\mid\bar a_{T-1},\bar L_{T-1})}{\prod_t^{T-1}f(a_t\mid\bar a_{t-1},\bar L_t)}\right)\cdots=0$$

$$\sum_{\bar a}\mathbb{E}\left(\{\bar A=\bar a\}\frac{\eta(\bar a,\bar L,\bar U)}{\prod_t\Delta_t(-1)^{1-z_t}f_{Z_t}(z_t\mid a_{t-1},z_{t-1},\overline{LU}_t)}\right)=\sum_{\bar a}\mathbb{E}\left(\mathbb{E}(\cdots\mid\bar A_{T-1}=\bar a_{T-1},\overline{LUZ})\right)=\sum_{\bar a}\mathbb{E}\left(\{\bar A_{T-1}=\bar a_{T-1}\}\frac{\eta}{\prod w}f(a_T\mid\overline{LUZ})^{\bar a_{T-1}}\right)$$

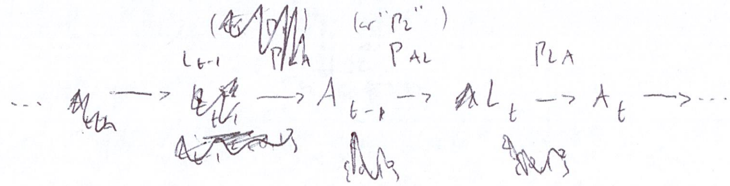
$$=\sum_{\bar a}\mathbb{E}\left(\mathbb{E}(\cdots\mid\bar a_{T-1},\bar z_{T-1},\overline{LU})\right)=\sum_{\bar a}\mathbb{E}\left(\{\bar A_{T-1}=\bar a_{T-1}\}\frac{\eta}{\prod^{T-1}w_t}\,\mathbb{E}\left(\frac{f(a_T\mid\overline{LUZ})}{(-1)^{1-z_T}\Delta_T f_{Z_T}(z_T\mid\cdots)}\,\Big|\,\bar a_{T-1},\bar z_{T-1},\overline{LU}\right)\right)=\sum_{\bar a}\mathbb{E}\left(\{\bar A_{T-1}=\bar a_{T-1}\}\frac{\mathbb{E}(\eta\mid a_{T-1},\overline{LU}_{T-1})}{\prod^{T-1}w_t}\right)$$

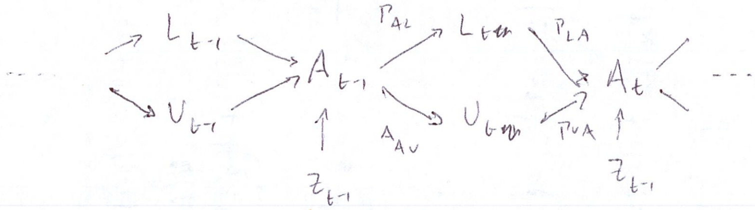
$\mathbb{E}$

$$\begin{vmatrix}p-\lambda & 1-p\\ 1-p & p-\lambda\end{vmatrix}=\lambda^2-2p\lambda-1+2p=$$

$$Y=m(\bar A)+\eta+\varepsilon, \qquad Y=\sum_a Y_a\{\bar A=\bar a\},$$

$$Y_a \perp ?\ A_t\mid \bar A_{t-1}=\bar a_{t-1},\bar L$$





	L = 0		L = 1	
	A = 0	A = 1	A = 0	A = 1
U = 1	$p_{01} \pm \delta_0$ p_{01}	$\overline{p_{01}} \mp \delta_0$ $\overline{p_{01}}$	$p_{11} \pm \delta_1$ p_{11}	$\overline{p_{11}} \mp \delta_1$ $\overline{p_{11}}$
U = 0	z=1 $p_{00} \pm \delta_0$ z=0 p_{00}	$\overline{p_{00}} \mp \delta_0$ $\overline{p_{00}}$	$p_{10} \pm \delta_1$ p_{10}	$\overline{p_{10}} \mp \delta_1$ $\overline{p_{10}}$

params p_{eu}, δ_e

$f(a|e,u,z=1) > 0$: $\delta_0 < \min(p_{01}, p_{00}, \overline{p_{01}}, \overline{p_{00}}) = \min(\tfrac{1}{2} - |p_{01} - \tfrac{1}{2}|, \tfrac{1}{2} - |p_{00} - \tfrac{1}{2}|)$

$= \tfrac{1}{2} - \max(|p_{01} - \tfrac{1}{2}|, |p_{00} - \tfrac{1}{2}|)$

$\delta_1 < \tfrac{1}{2} - \max(|p_{10} - \tfrac{1}{2}|, |p_{11} - \tfrac{1}{2}|)$

$\sum_a f(a|e,u,z) = 1$ (horizontal sums)

ICT: $\Delta(a|e,u) = f(a|e,u,z=1) - f(a|e,u,z=0) = \begin{cases} \delta_0, & e=0 \\ \delta_1, & e=1 \end{cases}$

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$$\frac{\delta_0^2-\delta_1^2}{\delta_0\delta_1(\delta_1-\delta_0)}$$

$$+\gamma\left\{\frac{4(2p-1)}{\delta_1-\delta_0}-2p^2\frac{1}{\delta_1}-2(1-p)^2\frac{1}{\delta_0}+p^2\left(\frac{1}{\delta_0}-\frac{1}{\delta_1}\right)+(1-p)^2\left(\frac{1}{\delta_1}-\frac{1}{\delta_0}\right)-1\right\}+$$

$$\left\{-\frac{2p-1}{\delta_1-\delta_0}+p^2\frac{1}{\delta_1}+(1-p)^2\frac{1}{\delta_0}\right\}=\gamma^2\left\{-\frac{4(2p-1)}{\delta_1-\delta_0}-2(1-2p)\frac{\delta_0-\delta_1}{\delta_0\delta_1}+1\right\}+\gamma\left\{\frac{4(2p-1)}{\delta_1-\delta_0}-1\right.$$

$$\left.-2p^2\left(\frac{1}{\delta_1}+\frac{1}{\delta_0}\right)-2(1-2p)\frac{1}{\delta_0}+(1-2p)\left(\frac{1}{\delta_1}-\frac{1}{\delta_0}\right)\right\}+\left\{p^2\left(\frac{1}{\delta_0}+\frac{1}{\delta_1}\right)+(1-2p)\left(\frac{1}{\delta_0}+\right.\right.$$

$$\left.\left.\frac{1}{\delta_1-\delta_0}\right)\right\}=\gamma^2\left\{2(2p-1)\left(\frac{\delta_0-\delta_1}{\delta_0\delta_1}-\frac{2}{\delta_1-\delta_0}\right)+1\right\}+\gamma\left\{2(2p-1)\left(\frac{2}{\delta_1-\delta_0}+\frac{1}{\delta_0}+\frac{1}{2\delta_0}-\frac{1}{2\delta_1}\right)-1-2p^2\left(\frac{1}{\delta_0}+\frac{1}{\delta_1}\right)\right\}$$

$$+\{\cdots\}$$

$\mathbb{E}(Y_a)=\mathbb{E}(\mathbb{E}(Y_a\mid L,A=a))=\mathbb{E}(\mathbb{E}(Y\mid L,A=a))=:m(a)$, $\mathbb{E}(\mathbb{E}(Y\mid L,A=a)-m(a))=0$, $g(L,A)=\mathbb{E}g(L,a)$ $=0$, $a\in\mathcal{A}$, $Y:=m(A)+\overset{\eta}{g}(L,A)$, $\mathbb{E}(Y_a\mid A=a,L)=\mathbb{E}(m(a)+g(L,a)\mid A=a,L)=m(a)+\overset{\eta}{g}(L,a)$, $\eta(L,a)=g_0^a(L)-m(a)$,

$Y:=m(\bar{A})+\eta(\bar{L},\bar{A})$, $\mathbb{E}(Y_{a_1,a_2}\mid a_1,a_2,\bar{L}_2)=\mathbb{E}(m(a_1,a_2)+\eta(\bar{L}_2,a_1,a_2)\mid\bar{a}_2,\bar{L}_2)=m(a_1,a_2)+\eta(\bar{L}_2,a_1,a_2)$

$\eta(\bar{L}_2,a_1,a_2):=\eta_1(\bar{L}_2,a_1)+m(a_1,0)-m(a_1,a_2)$, $\mathbb{E}(Y_{a_1,a_2}\mid a_1,L_1)=\mathbb{E}(\mathbb{E}(\quad\mid a_1,a_2,\bar{L}_2)\mid L_1,a_1)$

$=\mathbb{E}(\mathbb{E}(\quad\mid a_1,a_2,\bar{L}_2)\mid L_1,a_1)=\mathbb{E}(\eta_1(\bar{L}_2,a_1)+m(a_1,0)\mid L_1,a_1)=m(a_1,0)+\mathbb{E}(\eta_1(\bar{L}_2,a_1)\mid L_1,a_1)$,

$\mathbb{E}(\eta_1(\bar{L}_2,a_1)\mid L_1,a_1):=\eta_2(L_1)+m(0,0)-m(a_1,0)$, $\mathbb{E}(Y_a\mid A=a,L)=\mathbb{E}(Y\mid A=a,L)=\mathbb{E}(v(A,L)\mid A=a,$ $L)=h(L)$, $\mathbb{E}(u(A,L)-h(L)\mid A=a,L)=0$, $\mathbb{E}(\mathbb{E}(Y\mid A,L)-h(L)\mid A=a,L)=0$, $m(a,l)$,

$\overset{m(a)}{\mathbb{E}}(m(a,L))=\int m(a,l)f(l)\,dl$, $\mathbb{E}(Y\mid A=a,L)=m_a(L)$, $Y\mid A,L=\overset{m(A,L)}{m_A(L)}$, $Y=\overset{m(A,L)}{\cancel{m_A(L)}}+\varepsilon$,

$\cancel{\mathbb{E}(Y_{a_1}\mid\bar{L}_2)=m(a,\bar{L}_2)=\mathbb{E}}$ $\mathbb{E}(Y_{\bar{a}}\mid A=a,L_1)=\mathbb{E}(\mathbb{E}(Y_{\bar{a}}\mid A_1=a_1,\bar{L}_2)\mid L_1,A_1=a_1)=m(\bar{a},L_1)$

$\int v(\bar{a}_2,\bar{L}_2)f(l_2\mid L_1,a_1)=m_1(\bar{a}_2,L_1)=\mathbb{E}(m_2(\bar{a}_2,\bar{L}_2)\mid L_1,a_1)$, $m_2(\bar{a}_2,\bar{l}_2)=m_1(\bar{a}_2,L_1)+\varphi(\bar{a}_2,\bar{L}_2)-$

$\mathbb{E}[\varphi(\bar{a}_2,\bar{L}_2)\mid a_1,L_1]$, $\mathbb{E}(Y_1\mid\overset{a=1}{\bar{a}_1},L)=m(1)+L=\mathbb{E}(Y_1\mid\overset{a=1}{\bar{a}_2},L)$, $\cancel{\mathbb{E}(\mathbb{E}(Y\mid\bar{A},\bar{L})}\,\mathbb{E}(\mathbb{E}(Y\mid\bar{a}_2,\bar{L}_2)\mid a_1,L_1)$

$=\mathbb{E}(Y_{\bar{a}}\mid a_1,\bar{L}_1)$

$$\mathbb{E}\left(\frac{\eta(\bar{A},\bar{L})}{\prod_{t=1}f(A_t\mid\bar{A}_{t-1},\bar{L}_t)}\right)=\int\frac{\eta(\bar{a},\bar{l})}{\prod f(a_t\mid\bar{a}_{t-1},\bar{l}_t)}f(\bar{a},\bar{l})\,\mu(d\bar{a})\mu(d\bar{l})=\int\frac{1}{\prod\cdots}\prod_t f(a_t\mid\bar{a}_{t-1},\bar{l}_t)\prod_t f(l_t\mid\bar{a}_{t-1},\bar{l}_{t-1})$$

$$=\int\eta\prod_t f(l_t\mid\bar{a}_{t-1},\bar{l}_{t-1})=0$$

$\mathbb{E}(\cdots)$

concept para

long term followup of HIV patients

compliance percentage

outcome viral load

replicate analysis assuming SRA

DAG

ix of treatment, outcome

usual confounders - Miguel's papers for ART confounding

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$\mathbb{E}(V|U) = \mathbb{E}(V)$ $\Updownarrow$ $\cancel{\mathbb{E}(\mathbb{E}(V|U) - \mathbb{E}(V))}$

$\mathbb{E}(f(U)V) = \mathbb{E}f(U)\,\mathbb{E}V, \quad \mathbb{E}(f(U)\,\mathbb{E}(V|U)) = \mathbb{E}(V f(U)) = \mathbb{E}(V)\,\mathbb{E}(f(U))$

$\mathbb{E}\left(\{\bar A_{T-1} = \bar a_{T-1}\}\, \eta(\bar a, \overline{LU})\, w(\bar a, \bar L, \bar z_{T+1})\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_{T-1})\right) = \mathbb{E}\left(\{\bar A_{T-1} = \bar a_{T-1}\}\, \mathbb{E}(\eta(\bar a, \overline{LU}) | \overline{aLU}_{T-1})\, w f\right)$

$= \cancel{\mathbb{E}(\{\bar A_{T-1} = \bar a_{T-1}\}\, \mathbb{E}(\eta | \overline{aLU})\, \mathbb{E}(w f | \overline{aLU}))}$ $\mathbb{E}\left(\mathbb{E}(\eta | \overline{aLU}_{T-1})\, \mathbb{E}(\{\bar A_{T-1} = \bar a_{T-1}\} \cdot w \cdot f | \overline{aLU}_{T-1})\right)$

$= \mathbb{E}(f(V)\,\mathbb{E}(V|W))$

$\mathbb{E}(f(U)V) = \mathbb{E}(\mathbb{E}(f(U)|W)\,\mathbb{E}(V|W)), \Leftarrow \mathbb{E}(V|U,W) = \mathbb{E}(V|W), \quad \mathbb{E}(\mathbb{E}(V|U,W) f(U,W)) = \mathbb{E}(V f(U,W)),$

$= \mathbb{E}(f(V,W)\,\mathbb{E}(V|W))$

$\mathbb{E}(f(U,W)V) = \mathbb{E}(\cancel{\mathbb{E}(f(U)|W)}\,\mathbb{E}(f(V,W)|W)\,\mathbb{E}(V|W)) \Longleftrightarrow \mathbb{E}(V|U,W) = \mathbb{E}(V|W), \quad \mathbb{E}(\mathbb{E}(V|U,W) f(U,W))$

$\therefore \mathbb{E}(V f(U,W)) = \mathbb{E}(\mathbb{E}(V|W) f(V,W)), \quad U = \overline{ALU}_T, \quad W = \overline{ALU}_{T-1}, \quad V = \{\bar A_{T-1} = \bar a_{T-1}\}\, w(\bar a, \bar L, \bar z_T)\, f(a_T | \bar a_{T-1}, \overline{LUz})$

$\mathbb{E}(f(U,V)|U) = \mathbb{E} f(U,V), \quad \mathbb{E}(\{\bar A_{T-1} = \bar a_{T-1}\} w f | \overline{ALU}_T) = \mathbb{E}(\cdots | \overline{ALU}_{T-1}), \quad \sum_{z_T} \mathbb{E}(\{\bar A_{T-1} = \bar a_{T-1}\}\, w(\overline{AL}, \bar z_T)\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_{T-1})$

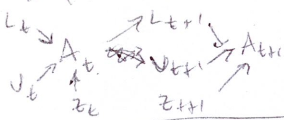
$\overline{ALU}_{T-1}, \bar z_T)\, f_{z_T}(z_T | \overline{ALU}_{T-1}) \perp U_T, \quad \sum_z w(\overline{AL}, \bar z_T)\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_T)\, f_z(z | \overline{AL})$

$\perp U_T, \quad \left(w = \frac{1}{\prod_t \Delta_t\, f_{z_t}\, (-1)^{1-z_t}}\right), \quad \sum_{z_{T-1}} \sum_{z_T} w f \cdot f_{z_T}(z_T | \ldots)\, f_{z_{T-1}}(\ldots | \ldots)$

$= \sum_{z_{T-1}} \cancel{f(a_T | \ldots)}\; w_{T-1}\,(f(a_T | \ldots z_T = 1) - f(a_T | \ldots z_T = 0))/\Delta_T\;\; f_{z_{T-1}}(\ldots | \ldots) = \ldots = 1$

$\mathbb{E}\left(\eta(\bar a_{t+1}, \overline{LU}_t)\, \{A_t = \bar a\}\, w(\bar a, \bar L, \bar z)\right) = \mathbb{E}\left(\eta(\bar a_t, \overline{LU}_t) \cdot \mathbb{E}(\{\bar A = \bar a\}\, w(\bar a, \bar L, \bar z) | \overline{aLU}_{t-1})\right)$

$\cdot\, \mathbb{E}(\{\bar A = \bar a\}\, w(\overline{aLz}) | \overline{aLU}_t) \perp U_t, L_t; \sum_{z_T} w(\overline{aLz})\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_{T-1})\, f_z(z | \overline{aLU})\, \varepsilon\, m\, \overline{ALU}_{T-1}$



$f_z(z | \overline{aLU}) = \prod_t f(z_t | \bar z_{t-1}, \overline{aLU}) = \prod_t f(z_t | \bar z_{t-1}, \overline{aLU}_t) = \prod_t f(z_t | \bar z_{t-1}, \overline{aL}_t)$

$\mathbb{E}\left(\{\bar A_t = \bar a_t\}\, \eta(\bar a, \bar L, \bar U) \cdot w(\bar a, \overline{Lz})\right) = \mathbb{E}\left(\{\bar A_{T-1} = \bar a_{T-1}\}\, \eta(\bar a, \overline{LU})\, w(\bar a, \overline{Lz})\, f(a_T | \bar a_{T-1}, \overline{LUz})\right)$

$w \perp f | \ldots$

$\mathbb{E}\left(\{\bar A_{T-1} = \bar a_{T-1}\}\, \eta(a, \overline{LU})\, w(\bar a, \bar L, \bar z_{T-1})\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_{T-1})\right)$

$\mathbb{E}\left(\{\bar A_{T-1} = \bar a_{T-1}\}\, \eta(\bar a, \bar L, \bar V) \sum_{z_T} \{w(\bar a, \bar L, \bar z_{T-1}, z_T)\, f(a_T | \bar a_{T-1}, \overline{LV}, \bar z_{T-1}, z_{T+1})\, f_{z_T}(z_T | \bar a_{T-1}, \bar z_{T-1}, \bar L)\}\right)$

$\mathbb{E}\left(\{\bar A_{t-1} = \bar a_{T-1}\}\, \eta(\bar a, \overline{LU}) \sum_{z_T} \{w(\bar a, \bar L, \bar z_{T-1}, z_T)\, f(a_T | \bar a_{T-1}, \overline{LU}, \bar z_{T-1}, z_{T+1})\}\right)$

$\eta = \mathbb{E}(Y | \overline{AZLU}) - m(\bar A) = \sum_{\bar a} (\mathbb{E}(Y_{\bar a} | \overline{aZLU}) - m(\bar a))\{\bar A = \bar a\}, \quad \int \eta(\overline{aZLU}) \prod_t f(LU_t | \overline{aLU}_{T-1}) = 0$

$Y = m(\bar A) + \eta(\overline{AZLU}) + \varepsilon \Rightarrow \mathbb{E}(Y_{\bar a}) = m(\bar a), \quad \eta = \sum_{\bar a} \{\bar A = \bar a\} \sum_{t=1}^{T} (\mathbb{E}(Y_{\bar a} | \overline{aZLU}_t) - \mathbb{E}(Y_{\bar a} | \overline{aZLU}_{t-1}))$

$= \sum_{\bar a} \{\bar A = \bar a\} \sum_{t=1}^{T} \{\mathbb{E}(Y_{\bar a} | \overline{aZLU}_t) - \mathbb{E}(\mathbb{E}(Y_{\bar a} | \overline{aZLU}_t) | \overline{aZLU}_{t-1})\} = \sum_{\bar a} \{\bar A = \bar a\} \sum_{t=1}^{T} \{\eta_t(\overline{aZLU}_t) - \mathbb{E}(\eta_t | \overline{aZLU}_{t-1})\},$

$\mathbb{E}(\{\bar A = \bar a\}\, \eta(\overline{aZLU}_t)\, w(\overline{aZL})) = \mathbb{E}(\{\bar A = \bar a\}\, \mathbb{E}(\eta | \overline{aLU}_{t-1})\, w(\overline{aZL})) = \mathbb{E}(\eta(\overline{aZLU}_t)\, \mathbb{E}(\{\bar A = \bar a\}\, w(\overline{aZL}) | \overline{aLU}_{t-1}))$

$\mathbb{E}(\{\bar A = \bar a\}\, w | \overline{aLU}_t, \bar z_{T-1}) = \{A = a\} z = \cancel{\ldots}\; \sum z f(a|z) f(z), \quad \mathbb{E}(\{A_t = a_t\} f(a_{t+1}|$

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$\mathbb{E}(\{\bar A=\bar a\}\,\eta(\overline{az}_{T-1},\overline{LU}_T)\,W(\overline{azL})) = \mathbb{E}(\eta(\overline{az}_{t-1},\overline{LU}_t)\,\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid \overline{aLU}_{t-1}))$, $U_T \perp \mathbb{E}(\{\bar A=\bar a\}$

$W(\overline{azL})\mid \overline{az}_{T-1},\overline{LU}_T)$ $\varepsilon\, m\overline{ALU}_{t-1}$, $U_T\perp \mathbb{E}(f(a_T\mid \bar a_{T-1}\overline{zLU})W(\overline{azL})\mid \overline{az}_{T-1},\overline{LU}_T) = \sum_{z_T} f(a_T\mid \overline{az}_{T-1},\overline{LU},z_T)\,W(\overline{aL},\bar z_{T-1},z_T)$

$f_{Z_T}(z_T\mid \overline{az}_{T-1},\overline{LU})$, $=\sum_{z_T} W(\overline{aL},\bar z_{T-1},z_T)\, f(a_T\mid\overline{az}_{T-1},\overline{LU},z_T)\, f_{Z_T}(z_T\mid\overline{az}_{T-1},\overline{LU})$, $U_{T-1}\perp \mathbb{E}[\{\bar A=\bar a\}$

$W(\overline{azL})\mid\overline{az}_{T-2},\overline{LU}_{T-1})$, $U_{T-1}\perp \mathbb{E}(W(\overline{azL})\,f(a_T, a_{T-1}\mid \bar a_{T-2},\bar z_{T-2},\overline{LU}_{T-1})\mid \overline{az}_{T-2},\overline{LU}_{T-1})$

$=\mathbb{E}(W(\overline{azL})\,f(a_T\mid\bar a_{T-1},\bar z_T,\overline{LU}_T)\,f(a_{T-1}\mid\overline{az}_{T-2},\overline{LU}_{T-1})\mid\overline{az}_{T-2},\overline{LU}_{T-1})$, $U_{T-1}\perp\mathbb{E}(W(\overline{azL})\{A_T=a_T\}$

$\varepsilon\, m\,\overline{ALU}_{T-1}$

$\{A_{T-1}=a_{T-1}\}\mid\overline{az}_{T-2},\overline{LU}_{T-1}) = \mathbb{E}(\mathbb{E}(W(\overline{azL})\{A_T=a_T\}\mid\overline{az}_{T-1},\overline{LU}_T)\,\{A_{T-1}=a_{T-1}\}\mid\overline{az}_{T-2},\overline{LU}_{T-1})$

$=\mathbb{E}(\omega(\overline{aLU}_{T-1})\,\{A_{T-1}=a_{T-1}\}\mid\overline{Az}_{T-2},\overline{LU}_{T-1}) = \mathbb{E}(\omega(\overline{aLU}_{T-1})\,\cancel{f(a_{T-1}\mid \overline{az}_{T-2},\overline{LU}_{T-1})\ \overline{az}_{T-2}},\overline{LU}_{T-1})$

$=\omega(\overline{aLU}_{T-1})\,f(a_{T-1}\mid\overline{Az}_{T-2},\overline{LU}_{T-1}) = \mathbb{E}(W(\overline{azL})\{A_T=a_T\}\mid\overline{az}_{T-1},\overline{LU}_T)\,f(a_{T-1}\mid\overline{az}_{T-2},\overline{LU}_{T-1})\ \varepsilon\, m\,\overline{LU}_{T-2}$,

$\mathbb{E}\{\{A_{T-1}=a_{T-1}\}\sum_{z_T}W(\overline{aL},\bar z_{T-1},z_T)\,f(a_T\mid\overline{az}_{T-1},\overline{LU},z_T)\,f_{Z_T}(z_T\mid\overline{az}_{T-1},\overline{LU})\mid\overline{az}_{T-2},\overline{LU}_{T-1}\}$

$\sum_{z_T}W(\overline{aL},\bar z_{T-1},z_T)\prod_t f(a_t\mid\bar a_{t-1},\overline{LUz}_t)\,f_{Z_T}(z_T\mid\overline{az}_{T-1},\overline{LU})$,

$\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid\overline{aLU}_t)\,\varepsilon m(\overline{aLU}_{t-1})$, $\cancel{\mathbb{E}(\{\bar A_{T-1}=\bar a_{T-1}\}f(a_T\mid\overline{azL}}$ $\mathbb{E}(\{\bar A_{T-1}=\bar z_{T-1}\}\,\eta(\ldots,\overline{LU}_T)\,f(a_T\mid\bar a_{T-1}\overline{zLU})$

$W(\overline{azL})) = \mathbb{E}(\eta_T(\overline{aLU})\,\mathbb{E}(\{\bar A_{T-1}=\bar a_{T-1}\}f(a_T\mid\bar a_{T-1},\overline{zLU})W(\overline{azL})\mid\overline{aLU}_{T-1}))$, $\sum_z\{\bar A_{T-1}=\bar a_{T-1}\}f(a_T\mid\bar a_{T-1},\bar z_T,\overline{LU})$

$W(\overline{azL})f_{Z_T}(z_T)\ldots\ \varepsilon\, m\,\overline{LU}_{T-1}$, $\cancel{\mathbb{E}(\{\bar A_{T-1}=\bar a_{T-1}\}\eta_t(\overline{aLU}_t)}$ $\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid\overline{aLU}_t)\ \varepsilon\, m\,\overline{LU}_{t-1}$,

$\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid\overline{az}_{t-1},\overline{LU}_t)\ \varepsilon\, m\,\overline{LU}_{t-1}$, $\sum_{z_T}W(z_T)f(a_T\mid z_T)f_{Z_T}(z_T)\perp\bar U$, $\mathbb{E}(\prod_t^T f(a_{t'}\mid a_{t'-1},\overline{zL},\bar U_t)$

$\{A_{t-1}=\bar a_{t-1}\}\,\eta(\overline{az}_{t-1},\overline{LU}_t)\,W(\overline{azL})) = \mathbb{E}\{\eta(\overline{az}_{t-1},\overline{LU}_t)\{\bar A_{t-1}=\bar a_{t-1}\}\,\mathbb{E}(\prod_t^T f(a_{t'}\mid\bar a_{t'-1},\overline{zL}_T\bar U_t)\,W(\overline{azL})\mid$

$\overline{aLU}_{t-1})\}$, $m\overline{LU}_{t-1}$ $\mathbb{E}(\prod_t^T f(a_{t'}\mid\ldots)W(\overline{azL})\mid\overline{az}_{t-1},\overline{LU}_t)$, $\cancel{\mathbb{E}(\{\bar A=\bar a\}\eta(\overline{az}_{t-1},\overline{LU}_t)\mathbb{E}(}$

$\mathbb{E}(\eta(\overline{az}_{t-1},\overline{LU}_t)\,\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid\overline{az}_{t-1},\overline{LU}_t)) = \mathbb{E}(\eta(\overline{az}_{t-1},\overline{LU}_t)\,\mathbb{E}(\{\bar A=\bar a\}W(\overline{azL})\mid\overline{az}_{t-1},\overline{LU}_{t-1})\mid\overline{aLU}_{t-1}))$,

$\mathbb{E}(\prod_{t'=t}^T f(a_{t'}\mid\bar a_{t'-1},\overline{zL}_T,\bar U_t)\,W(\overline{azL})\mid\overline{az}_{t-1},\overline{LU}_t)\ \varepsilon\, m\,\overline{LU}_{t-1}$, $(t=T-1)$, $\mathbb{E}(\prod_{T-1}^T f(a_{t'}\mid\ldots)W\mid\overline{az}_{T-2},\overline{LU}_{T-1})$

$=\mathbb{E}(\mathbb{E}(\ldots\mid\overline{az}_{T-1},\overline{LU}_T)\mid\overline{az}_{T-2},\overline{LU}_{T-1})$, $\cancel{\mathbb{E}(f(a_{T-1}\mid\bar a_{T-2},\overline{zL}_T}$ $\mathbb{E}(\prod_{t'=t}^T f(a_{t'}\mid\bar a_{t'-1},\overline{zL}_T,\bar U_t)\prod_{t'=t-1}^T W(\overline{azL}_{t'})\mid\overline{az}_{t-1},\overline{LU}_t$

$\varepsilon\, m\,\overline{LU}_{t-1}$, $\mathbb{E}(\mathbb{E}(f(a_{T-1}\mid\bar a_{T-2},\overline{zL}_T,\bar U_{T-1})\,f(a_T\mid\ldots)\,W(\overline{azL}_{T-1})\,W(\overline{azL}_T)\mid\overline{az}_{T-1},\overline{LU}_T)\mid\overline{az}_{T-2},\overline{LU}_{T-1})$,

$\mathbb{E}(\{\bar A=\bar a\}\prod_t W(\overline{azL}_t)\mid\overline{az}_{t-1},\overline{LU}_t)\ \varepsilon\, m(\overline{aLU}_{t-1})$, $\mathbb{E}(\mathbb{E}(\{\bar A=\bar a\}\tilde W(\overline{azL})\mid\overline{az}_{T-1},\overline{LU}_T)\mid\overline{az}_{T-2},\overline{LU}_{T-1}))$

$=\mathbb{E}(\{\bar A=\bar a\}\tilde W(\overline{azL})\mid\overline{az}_{T-1},\overline{LU}_T) = \mathbb{E}(\{\bar A_{T-1}=\bar a_{T-1}\}f(a_T\mid\bar a_{T-1},\overline{zLU}_T)\,\tilde W(\overline{azL})\mid\overline{az}_{T-1},\overline{LU}_T) = \{\bar A_{T-1}=\bar a_{T-1}\}\sum_{z_T}f(a_T\mid z_T)\,\tilde W(z_T)\,f_{Z_T}$

$\frac{w_{00L}}{w_{01}w_{10}} = 1 - \frac{1}{w_{01}} + \frac{w_{00}}{w_{01}}, \frac{1}{w_{10}} = \frac{w_{01}}{w_{00}} - \frac{1}{w_{00}} + 1,$ ⓐ $w_{01} = \frac{w_{00}}{w_{10}} + 1 - w_{00},$ ⓑ $\frac{w_{11}}{w_{10}} = \frac{w_{01}}{w_{00}}$

$h_0 w_{01}(p_{00}\frac{w_{00}}{w_{01}} + p_{01}) + h_1 w_{11}(\frac{w_{00}}{w_{01}} \underset{1-p_{00}}{p_{10}} + \underset{1-p_{01}}{p_{11}}) = c_h$

$h_0 (1 + w_{00}(\frac{1}{w_{10}} - 1)) (p_{00}\frac{w_{00}}{w_{01}} + p_{01}) + h_1 w_{11}(p_{10}\frac{w_{00}}{w_{01}} + p_{11}) = p_{00}\frac{w_{00}}{w_{01}}(-h_1 w_{11} + h_0 + h_0 w_{00}(\frac{1}{w_{10}}-1))$

$+\{h_0(1+w_{00}(\frac{1}{w_{10}}-1)) \overset{-h_1 w_{11}\}}{} p_{01} + h_1 w_{11}\frac{w_{00}}{w_{01}} + h_1 w_{11}$ ~~(1-p01)~~ $, \frac{w_{00}}{w_{10}} + 1 - w_{00} = w_{00}\frac{w_{11}}{w_{10}}, \frac{1}{w_{10}} + \frac{1}{w_{00}} - 1 = \frac{w_{11}}{w_{10}},$

$w_{10}+w_{11} = 1 + \frac{w_{10}}{w_{00}}$

$\frac{w_{00}}{w_{01}}(h_0 w_{01} p_{00} + h_1 w_{11} p_{10}) + p_{01}(h_0 w_{01} - h_1 w_{11}) + h_1 w_{11} = c_h$

$p_{00}(h_0 w_{00} - h_1 w_{10}) + p_{01}(h_0 w_{01} - h_1 w_{11}) + h_1 w_{10} + h_1 w_{11} = c_h$

$h_0(p_{00} w_{00} + p_{01} w_{01})(1 - \frac{h_1 w_{10}}{h_0 w_{00}}) + h_1(w_{10}+w_{11}) = c_h, (p_{00}w_{00}+p_{01}w_{01})(1-\frac{h_1 w_{10}}{h_0 w_{00}}) + \frac{h_1}{h_0}(1+\frac{w_{10}}{w_{00}}) = \frac{c_h}{h_0}$

$p_{00}w_{00} + p_{01}w_{01} + \frac{h_1}{h_0}\{1 + \frac{w_{10}}{w_{00}}(1 - p_{00}w_{00} - p_{01}w_{01})\} = c_h/h_0$

$q_{1L0} = \frac{p_{0LU}-p_{0L0}}{p_{0L1}-p_{0L0}}(q_{1L1} - \frac{q_{0L1}q_{1L0}}{q_{0L0}}) + \frac{q_{0LU}q_{1L0}}{q_{0L0}} = \frac{q_{1L0}}{q_{0L0}} + p_{0LU}\{\frac{q_{1L1} - \frac{q_{0L1}q_{1L0}}{q_{0L0}}}{p_{0L1}-p_{0L0}} - \frac{q_{1L0}}{q_{0L0}}\}$

$-\frac{p_{0L0}}{p_{0L1}-p_{0L0}}(\ldots), \quad p_{1LU} = (\frac{p_{0L0}-p_{0LU}}{p_{0L0}})(\frac{p_{0L0}p_{1L1} - p_{0L1}p_{1L0}}{p_{0L0}-p_{0L1}} + \frac{p_{0LU}p_{1L0}}{p_{0L0}-p_{0L0}})$

$= \frac{p_{0L0}-p_{0LU}}{p_{0L0}-p_{0L1}}(p_{1L1} - \frac{p_{0L1}p_{1L0}}{p_{0L0}}) + \frac{p_{0LU}p_{1L0}}{p_{0L0}} = p_{0LU}\{\frac{p_{1L0}}{p_{0L0}} - \frac{p_{1L1} - \frac{p_{0L1}p_{1L0}}{p_{0L0}}}{p_{0L0}-p_{0L1}}\}$

$+\frac{p_{0L0}}{p_{0L0}-p_{0L1}}(p_{1L1} - \frac{p_{0L1}p_{1L0}}{p_{0L0}}), \quad p_{0LU}\{\frac{q_{1L1} - \frac{q_{0L1}q_{1L0}}{q_{0L0}}}{p_{0L1}-p_{0L0}} - \frac{q_{1L0}}{q_{0L0}} + \frac{p_{1L0}}{p_{0L0}} \pm \frac{p_{1L1} - \frac{p_{0L1}p_{1L0}}{p_{0L0}}}{p_{0L0}-p_{0L1}}\}$

$= -\frac{q_{1L0}}{q_{0L0}} \mp \frac{p_{0L0}}{p_{0L1}-p_{0L0}}(q_{1L1} - \frac{q_{0L1}q_{1L0}}{q_{0L0}} + p_{1L1} \mp \frac{p_{0L1}p_{1L0}}{p_{0L0}}) + 1 = \frac{p_{1L0}-p_{0L0}}{1-p_{0L0}} + \frac{p_{0L0}}{p_{0L1}-p_{0L0}}($

$(1 - \frac{(1-p_{0L1})(1-p_{1L0})}{1-p_{0L0}} - \frac{p_{0L1}p_{1L0}}{p_{0L0}}) \quad \frac{1 - p_{0L0} - 1 + p_{0L1} + p_{1L0} - p_{0L1}p_{1L0}}{1-p_{0L0}}, =$

$\frac{p_{1L0}-p_{0L0}}{1-p_{0L0}} + \frac{p_{0L0}}{1-p_{0L0}} + \frac{p_{0L0}}{p_{0L1}-p_{0L0}}(\frac{p_{1L0}-p_{0L1}p_{1L0}}{1-p_{0L0}}) - \frac{p_{0L1}p_{1L0}}{p_{0L1}-p_{0L0}} = \frac{p_{1L0}}{1-p_{0L0}} \cdot \frac{p_{0L1}}{p_{0L1}-p_{0L0}}$

$-\frac{p_{0L0}p_{0L1}p_{1L0}}{(p_{0L1}-p_{0L0})(1-p_{0L0})} - \frac{p_{0L1}p_{1L0}}{p_{0L1}-p_{0L0}} = p_{1L0}p_{0L1}(\frac{1}{(1-p_{0L0})(p_{0L1}-p_{0L0})} - \frac{p_{0L0}}{(p_{0L1}-p_{0L0})(1-p_{0L0})} - \frac{1}{p_{0L1}-p_{0L0}})$

$= \frac{p_{1L0}p_{0L1}}{p_{0L1}-p_{0L0}}(\frac{1}{1-p_{0L0}} - \frac{p_{0L0}}{1-p_{0L0}} - 1) = 0$ $w_{00}p_0 + w_{01}p_1 = 1, \quad w_{10}(1-p_0) + w_{11}(1-p_1) = 1,$

$p_0 = -\frac{w_{01}}{w_{00}}p_1 + \frac{1}{w_{00}}, \quad = -\frac{w_{11}}{w_{10}}p_1 + \frac{1}{w_{10}} + 1 + \frac{w_{11}}{w_{10}}$

$\frac{w_{01}}{w_{00}} = \frac{w_{11}}{w_{10}}, \quad \frac{1}{w_{00}} = \frac{w_{11}}{w_{10}} + 1 - \frac{1}{w_{10}}$

$\frac{w_{10}}{w_{00}} = w_{11} + w_{10} - 1, \quad w_{10}(-1 + \frac{1}{w_{00}}) = w_{11} - 1$

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$\eta(\overline{ALU}_t) - \mathbb{E}(\eta(\overline{ALU})|\overline{ALU}_{t-1})$, $\mathbb{E}(\{\bar{A}=\bar{a}\}\eta(\overline{ALU}_t)\bar{w}(\overline{ALZ})) = \mathbb{E}(\{\bar{A}=\bar{a}\}\mathbb{E}(\eta(\overline{ALU}_t)|\overline{ALU}_{t-1})\bar{w}(\overline{ALZ}))$

$= \mathbb{E}(\eta(\overline{ALU}_t)\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{ALZ})|\overline{ALU}_{t-1}))$, $\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{ALZ})|\overline{ALU}_t) = \mathbb{E}(\ldots|\overline{ALU}_{t-1})$,

$\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{ALZ})|\overline{ALU}_T) = \{\bar{A}=\bar{a}\}_{T-1}\mathbb{E}(f(a_T|\overline{A}_{T-1}\overline{ZLU})\bar{w}(\overline{ALZ})|\overline{ALU}_T) = \{\bar{A}=\bar{a}\}_{T-1}\sum_{Z} w(\overline{ALZ}) f(a_T|$

$\overline{A}_{T-1}\overline{ZLU}) f_Z(z|\overline{ALU}) \in m(\overline{ALU}_{T-1}),\ldots, m(\overline{LU}_0)$, $\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{ALZ})|\overline{ALU}_{T-1}) = \mathbb{E}(\mathbb{E}(\ldots|\overline{ALU}_T)|\overline{ALU}_{T-1})$

$= \mathbb{E}(\{\bar{A}=\bar{a}\}_{T-1}\sum_z w(\overline{ALZ}) f(a_T|z) f_Z(z|\overline{AL})|\overline{ALU}_{T-1}) = \mathbb{E}(\{\bar{A}=\bar{a}\}_{T-1}\bar{w}'(\overline{AL}\ldots)|\overline{A}_{T-2}\overline{LU}_{T-1})$, $\mathbb{E}(\mathbb{E}(\{\bar{A}=\bar{a}\}_T\bar{w}(\overline{ALZ})|$

$\overline{ALU}_{T-1}, \bar{Z}_{T-1})|\overline{ALU}_{T-1,T}) = \mathbb{E}(\{\bar{A}=\bar{a}\}\bar{W}_{T-1}\sum_{z_T} f(a_T|\bar{a}_{T-1}, \bar{Z}_{T-1}, \overline{LU}) W_T(\overline{a_T L Z}_{T-1}, z_T) f_{Z_T}(z_T|\overline{LU}, \overline{AZ}_{T-1})|\overline{A}_{T-1}\overline{LU})$

$= \{\bar{A}=\bar{a}\}_{T-1}$, $\mathbb{E}(\{A=a\}_t W_t(\overline{aLZ}_t)|\overline{ALU}_t) \in m\,\overline{LU}_{t-1}$, $\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{aZL})|\overline{a}_{T-2}\overline{LU}_{T-1}) = \mathbb{E}(\{A=a\}_{T-1}\bar{W}_{T-1}\mathbb{E}(\{A_T=a_T\}W_T|\overline{a}_{T-1}\overline{LU}_T)|$

$\overline{a}_{T-2}\overline{LU}_{T-1}) = \mathbb{E}(\{\bar{A}=\bar{a}\}_{T-1}\bar{W}_{T-1}|\overline{a}_{T-2}\overline{LU}_{T-1})$, $\eta(\overline{AL}_t) - \mathbb{E}(\overline{AL}_t|\overline{AL}_{t-1}), \ldots|_{\bar{A}_t=\bar{a}_t} = \eta(\overline{aL}_t) - \mathbb{E}_\eta(\overline{AL}_t|\overline{aL}_{t-1})$

$\mathbb{E}(\eta(\overline{aL}_t)|\overline{aL}_{t-1})$, $\mathbb{E}(\{A=a\}_T W(\overline{aLZ})\eta(\overline{a}_{T-1}\overline{LU}_T)) = \mathbb{E}(\{A=a\}_T W(\overline{aLZ})\mathbb{E}(\eta(\overline{aLU}_{T-1}))) =$

$\mathbb{E}(\eta(\overline{a}_{T-1}\overline{LU})\mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{aLZ})|\overline{aLU}_{T-1}))$, $\mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{aLZ})|\overline{a}_{T-1}, \overline{LU}) \in m(\overline{LU}_{T-1})$,

$\mathbb{E}(\{A=a\}_{T-1} f(a_T|\bar{a}_{T-1}, \overline{LUZ})\bar{W}|\overline{a}_{T-1}\overline{LU}_T) = \{\bar{A}=\bar{a}\}_{T-1}\sum_Z f(a_T|\bar{a}_{T-1}, \overline{LU}, \bar{Z})W(\overline{aLZ}) \in m\,\overline{LU}_{T-1}$, $m(\overline{LU}_{T-2})$?

$\mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{aLZ})|\overline{a}_{T-2}\overline{LU}_{T-1}) = \mathbb{E}(\mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{aLZ})|\overline{a}_{T-1}\overline{LU})|\overline{a}_{T-2}\overline{LU}_{T-1}) = \mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{aLZ})|\overline{a}_{T-1}\overline{LU})$

$\mathbb{E}(\mathbb{E}(f(A_0, \bar{L}_2)|\bar{L}_1, \bar{A}_1)|L_0 A_0)$, $\mathbb{E}(f(X,Y)|X,Z) = \mathbb{E}(f(X,Y)|X)$,

$\mathbb{E}(f(X,Y)g(X_1)|X) = \mathbb{E}(f(X,Y)|X)\mathbb{E}(g(X_1)|X)$

$\eta(\overline{AZ}_{t-1}\overline{LU}_t) - \mathbb{E}(\eta(\overline{AZ}_{t-1}, \overline{LU}_t)|\overline{AZLU}_{t-1})$, $\mathbb{E}(\{\bar{A}=\bar{a}\}\bar{w}(\overline{ALZ})|\overline{AZ}_{t-1}, \overline{LU}_t) \in m(\overline{AZLU}_{t-1})$, $m(\overline{AZLU}_{T-1})$?

$\{\bar{A}_T=\bar{a}_T\}\mathbb{E}(f(a_T|\bar{A}_{T-1}, \overline{ZLU}_T) W_T(\overline{AZL}_T)|\overline{AZ}_{T-1}, \overline{LU}_T) = \bar{W}_{T-1}\{\bar{A}_{T-1}=\bar{a}_{T-1}\}\sum_{z_T} f(a_T|\bar{a}_{T-1}, z_T, \bar{Z}_{T-1}, \overline{LU}_T) W(\bar{a} z_T,$

$\bar{Z}_{T-1}, \bar{L}_T) f_{Z_T}(z_T|\bar{a}_{T-1}, \bar{Z}_{T-1}, \overline{LU}_T)$, $\mathbb{E}(\{A_T=a_T\}W_T|\overline{AZ}_{T-1}, \overline{LU}_T) \in m(\overline{AZLU}_{T-1})$,

$\sum_{a_t}\mathbb{E}(\{\bar{A}=\bar{a}\}\eta(\bar{A}_{s-1}\overline{LU}_s)W(\overline{ALZ})) = \sum_{a_t}\mathbb{E}(\{A=a\}\mathbb{E}(\eta|\overline{ALU}_{s-1})W)$

$= \sum_{a_t}\mathbb{E}(\eta(\bar{A}_{s-1}, \overline{LU}_s)\mathbb{E}(\{\bar{A}=\bar{a}\}W(\overline{ALZ})|\overline{ALU}_{s-1}))$, $\sum_{\bar{a}}\mathbb{E}(h(\bar{a})\eta(\bar{A}_{t-1}\overline{LU}_t)W(\overline{ALZ})) = \sum_{\bar{a}}\mathbb{E}(\eta(\bar{A}_{t-1}\overline{LU}_t)$

$\mathbb{E}(h(\bar{a})\{\bar{A}\}W(\overline{ALZ})|\overline{ALU}_{t-1}))$, $\mathbb{E}(h(\bar{A})\eta(\bar{A}_{t-1}\overline{LU}_t)W(\overline{ALZ})) = \mathbb{E}(\eta(\bar{A}_{t-1}\overline{LU}_t)\mathbb{E}(h(\bar{A})W(\overline{ALZ})|\overline{ALU}_{t-1}))$,

$\mathbb{E}(h(\bar{A})W(\overline{ALZ})|\bar{A}_{t-1}\overline{LU}_t) = \mathbb{E}(h(\bar{A})W(\overline{ALZ})|\overline{ALU}_{t-1})$, $\mathbb{E}(\{A_T=a_T\}W(\overline{ALZ})|\bar{A}_{T-1}\overline{LU}_T) = \sum_Z f(a_T|\bar{A}_{T-1}\overline{LUZ}_T)W(\bar{A}_{T-1},$

$a_T, \overline{LZ}) f_Z(\bar{z}|\ldots) \in m(\overline{ALU}_{T-1})$,

$\mathbb{E}(X|Y,Z) = \mathbb{E}(X|Y)$, $\mathbb{E}(X h(Z)|Y) = \mathbb{E}(X|Y)\mathbb{E}(h(Z)|Y)$, $\mathbb{E}(g(U)V|X,Y) = \mathbb{E}(g(U)V|X)$, $\mathbb{E}(g(U)V h(Y)|X) =$

$\mathbb{E}(g(U)V|X)\mathbb{E}(h(Y)|X)$

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$\boxed{W_{AZL} \\ P_{ZLU}}$

$W_{00L} P_{0LU} + W_{01L} P_{1LU} = 1 \qquad W_{00L} = P_{0LU}^{-1}(1 - W_{01L} P_{1LU})$

$W_{10L}\, q_{0LU} + W_{11L}\, q_{1LU} = 1 \qquad P_{0L0}^{-1}(1 - W_{01L} P_{1L0}) = P_{0L1}^{-1}(1 - W_{01L} P_{1L1})$

$$W_{01L} = \left(\frac{P_{1L1}}{P_{0L1}} - \frac{P_{1L0}}{P_{0L0}}\right)^{-1}\left(\frac{1}{P_{0L1}} - \frac{1}{P_{0L0}}\right) = \frac{P_{0L0} - P_{0L1}}{P_{0L0}P_{1L1} - P_{0L1}P_{1L0}}, \quad W_{00L} = P_{0L1}^{-1}\,\frac{P_{1L1}P_{0L1} - P_{0L1}P_{1L0}}{\cancel{P_{0L0} - P_{0L1}}}$$

$$= \frac{P_{1L1} - P_{1L0}}{P_{0L0}P_{1L1} - P_{0L1}P_{1L0}}, \quad W_{11L} = \frac{q_{0L0} - q_{0L1}}{q_{0L0}q_{1L1} - q_{0L1}q_{1L0}}, \quad W_{10L} = \frac{q_{1L1} - q_{1L0}}{q_{0L0}q_{1L1} - q_{0L1}q_{1L0}}$$

$$(1 - p_{0L0})(1 - p_{1L1}) - (1 - p_{0L1})(1 - p_{1L0}) = p_{0L0}p_{1L1} - p_{0L0} - p_{1L1} - p_{0L1}p_{1L0} + p_{0L1} + p_{1L0},$$

$$\frac{1}{W_{11L}} = \frac{p_{0L0}p_{1L1} - p_{0L1}p_{1L0} - p_{0L0} - p_{1L1} + p_{0L1} + p_{1L0}}{p_{0L1} - p_{0L0}} = 1 - \frac{1}{W_{01L}} + \frac{p_{1L0} - p_{1L1}}{p_{0L1} - p_{0L0}} = 1 - \frac{1}{W_{01L}} + \frac{W_{00L}}{W_{01L}},$$

$$\frac{1}{W_{10L}} = \frac{\quad}{p_{1L0} - p_{1L1}} = 1 - \frac{1}{W_{00L}} + \frac{p_{0L1} - p_{0L0}}{p_{1L0} - p_{1L1}} = 1 - \frac{1}{W_{00L}} + \frac{W_{01L}}{W_{00L}}, \quad \frac{W_{01L}}{W_{00L}} = \frac{W_{11L}}{W_{10L}},$$

$$\frac{W_{01L}}{W_{11L}} = -1 + W_{01L} + W_{00L} = \frac{W_{00L}}{W_{10L}}, \quad W_{01L} = (W_{00L} - 1)\left(\frac{1}{W_{11L}} - 1\right)^{-1}, \quad \frac{W_{00L}}{W_{10L}} = W_{00L} - 1 + W_{01L}$$

$$= W_{00L} - 1 + \frac{W_{11L}(W_{00L} - 1)}{1 - W_{11L}}, \quad \frac{W_{00L}}{W_{10L}} = W_{00L} - 1 + W_{00}\frac{W_{11}}{W_{10}}, \quad W_{00} = \left(\frac{1}{W_{10}} - 1 - \frac{W_{11}}{1 - W_{11}}\right)^{-1}\Big(-1$$

$$-\frac{W_{11}}{1 - W_{11}}\Big) = \left(\frac{1}{W_{10}} - \frac{1}{1 - W_{11}}\right)^{-1}\left(-\frac{1}{1 - W_{11}}\right) = \left(1 - \frac{1 - W_{11}}{W_{10}}\right)^{-1}, = \left(\frac{1}{W_{10}} - 1 - \frac{W_{11}}{W_{10}}\right)^{-1} = \Big(1 +$$

$$+ \frac{W_{11} - 1}{W_{10}}\Big)^{-1}, \quad W_{AZL} = 1,\ P_{ZLU} = q_{ZLU} = \tfrac{1}{2}. \quad P_{ZLU} = \mathbb{P}(A = 0 \mid ZLU) = p_{LU} + (-1)^{1-Z}\delta_{LU}, \quad W_{00L}\,P_{LU}$$

$$h_0 W_{00L} p_{LU} + h_0 W_{01L}(p_{LU} + \delta_{LU}) + h_1 W_{10L}(1 - p_{LU}) + h_1 W_{11L}(1 - p_{LU} - \delta_{LU}) = c_h$$

$$p_{LU}(W_{00L} + W_{01L}) + W_{01L}\delta_{LU} + \frac{h_1}{h_0}\{p_{LU}(-W_{10L} - W_{11L}) + W_{10L} + W_{11L}(1 - \delta_{LU})\} = \frac{c_h}{h_0}$$

$$p_{LU}\left(W_{00L} + W_{01L} - \frac{h_1}{h_0}(W_{10L} + W_{11L})\right) + \delta_{LU}\left(W_{01L} - \frac{h_1}{h_0}W_{11L}\right) + \frac{h_1}{h_0}(W_{10L} + W_{11L}) = \frac{c_h}{h_0},$$

$$p_{LU}(W_{00L} + W_{01L}) + \delta_{LU}W_{01L} = W_{01L}(p_{LU} + \delta_{LU}) + p_{LU}W_{00L} = \frac{c_{h_0,0}}{h_0}, \quad c_{h_0,0}/h_0 = e,$$

$$\left(p_{LU} = \tfrac{1}{2} + (-1)^{1-U}u_0\right), \delta_{LU} = \left(W_{01L} - \frac{h_1}{h_0}W_{11L}\right)^{-1}\left\{\left(\tfrac{1}{2} + (-1)^{1-U}u_0\right)^{-1}\left(\frac{h_1}{h_0}(W_{10L} + W_{11L}) - \right.\right.$$

$$\left.W_{00L} - W_{01L}\right) - \frac{h_1}{h_0}(W_{10L} + W_{11L}) + \frac{c_h}{h_0}\Big\}, \ (W_{AZL} = 1), \ = \left(1 - \frac{h_1}{h_0}\right)^{-1}\left\{\left(\tfrac{1}{2} + (-1)^{1-U}u_0\right)^{-1} 2\left(\frac{h_1}{h_0} - 1\right) - 2\frac{h_1}{h_0} + \frac{c_h}{h_0}\right\}$$

$$= \frac{2}{(-1)^{1-U}u_0 - \frac{1}{2}} + \frac{c_h - 2h_1}{h_1 - h_0} \Bigg| \quad P_{0L0}^{-1}(1 - W_{01L}P_{1L0}) = P_{0LU}^{-1}(1 - W_{01L}P_{1LU}), \quad \frac{P_{0L0} - P_{0L1}}{P_{0L0}P_{1L1} - P_{0L1}P_{1L0}} =$$

$$\frac{P_{0L0} - P_{0LU}}{P_{0L0}P_{1LU} - P_{0LU}P_{1L0}}, \quad \frac{P_{0L1} - P_{0L0}}{q_{0L0}q_{1L1} - q_{0L1}q_{1L0}} = \frac{P_{0LU} - P_{0L0}}{q_{0L0}q_{1LU} - q_{0LU}q_{1L0}}, \quad q_{1LU} = \left(\frac{P_{0LU} - P_{0L0}}{q_{0L0}}\right)\left(\frac{q_{0L0}q_{1L1} - q_{0L1}q_{1L0}}{P_{0L1} - P_{0L0}} + \frac{q_{0LU}q_{1L0}}{P_{0LU} - P_{0L0}}\right)$$

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$\mathbb{E}(\{A_\tau = a_\tau\} w(\overline{ALZ})$, $\mathbb{E}(h(\bar A)\eta(\overline{AZ}_{t-1}, \overline{LU}_t) w(\overline{ALZ})) = \mathbb{E}(h(\bar A) w(\overline{ALZ}) \mathbb{E}(\eta | \overline{AZLU}_{t-1})) = \mathbb{E}(\eta(\overline{AZ}_{t-1}, \overline{LU}_t)$

$\mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{AZLU}_{t-1}))$, $\mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{AZ}_{t-1}, \overline{LU}_t) = \mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{AZLU}_{t-1})$,

$\bar w_{\tau-1} \mathbb{E}(\{A_\tau = a_\tau\} w_\tau | \overline{AZ}_{\tau-1}, \overline{LU}) = \bar w_{\tau-1} \mathbb{E}(f(a_\tau | \overline{AZLU}) w_\tau | \ldots) = \bar w_{\tau-1} \sum_{z_\tau} f(a_\tau | z_\tau \ldots) w_\tau(z_\tau) f(z_\tau | \ldots)$,

$\sum_{z_\tau} f(a_\tau | z_\tau \ldots) w_\tau(z_\tau) f(z_\tau | \ldots) \in m(\overline{AZLU}_{\tau-1})$,

$p_{00} w_{00} + p_{01} w_{01} = \alpha$, $p_{10} w_{10} + p_{11} w_{11} = \beta$, $w_{00} - p_{10} w_{00} + w_{01} - p_{11} w_{01} = \alpha$, $p_{10} =$

$w_{00}^{-1}(w_{00} - \alpha + w_{01} - p_{11} w_{01})$, $p_{11} = (w_{11} - \frac{w_{10}}{w_{00}} w_{01})^{-1} (\beta - \frac{w_{10}}{w_{00}}(w_{00} - \alpha + w_{01})) \in m(ALZ)$,

$\frac{w_{01}}{w_{11}} = \frac{w_{00}}{w_{10}}$, $\frac{w_{00}}{w_{01}} = \frac{w_{10}}{w_{11}}$, $(\alpha = 0)$ $\frac{p_{00}}{p_{01}} = -\frac{w_{01}}{w_{00}} = -\frac{w_{11}}{w_{10}}$, $(1 - p_{00}) -$

$(1 - p_{01}) \frac{p_{00}}{p_{01}} = \frac{\beta}{w_{10}} = 1 - \frac{p_{00}}{p_{01}} = \frac{\beta}{w_{10}}$, $p_{00} - p_{01} = -\frac{\beta}{w_{10}} p_{01}$, $\frac{p_{00}}{p_{01}} = 1 - \frac{\beta}{w_{10}}$,

$\alpha = p_{01}(w_{01} + w_{00} - \beta \frac{w_{00}}{w_{10}})$, $\beta = \frac{w_{10}}{w_{00}} w_{01} + w_{10} = w_{11} + w_{10}$, $\frac{1 - p_{10}}{1 - p_{11}} = -\frac{w_{11}}{w_{10}}$,

$\beta = w_{10}(1 + (1 - p_{11}) \frac{w_{11}}{w_{10}}) + p_{11} w_{11} = w_{10} + w_{11}(1 - p_{11}) + p_{11} w_{11} = w_{11} + w_{10}$

$p_{00} + p_{01} w = p_{10} + p_{11} w = 1 - p_{00} + w - w p_{01}$, $w = \frac{p_{00} - p_{10}}{p_{11} - p_{01}} = \frac{2p_{00} - 1}{2p_{11} - 1}$

$p_{00} + p_{01} w = \alpha$, $p_{10} + p_{11} w = \beta$, $1 - p_{00} + (1 - p_{01}) w = \beta$, $1 + w = \alpha + \beta$,

$\mathbb{E}(Y_{\bar a} | A = \bar a, \overline{ZLU}) = \mathbb{E}(Y_{\bar a} | \bar a_{\tau-1} \overline{LU}) = \mathbb{E}(Y_{\bar a} | \overline{aZ}_{\tau-1}, \overline{LU})$, $\mathbb{E}(- | \overline{aZ}_{\tau-1} \overline{LU}_{\tau-1}) = \mathbb{E}(Y_{\bar a} | \overline{aZLU}_{\tau-1})$

$= \mathbb{E}(Y_{\bar a} | \overline{aLU}_{\tau-1})$, $s \le t$, $Z_s \perp\!\!\!\perp (Y_{\bar a}, \underline{L_{t+1}}, \underline{U_{t+1}}) | \overline{a} \overline{LU}_t$, $Z_s \perp\!\!\!\perp Y_{\bar a} | \bar a_t, \overline{LU}_{t+1}$,

$\mathbb{E}(\eta | \overline{aLU}_{\tau-1}) = \mathbb{E}(\eta | \overline{aLU}_{\tau-1}, Z_s)$, $\xi(\overline{AZ}_{\tau-1} \overline{LU}_\tau) - \mathbb{E}(\xi | \overline{ALU}_{\tau-1}, Z_s)$, $\mathbb{E}(h(\bar A) w(\overline{ALZ}) \xi(\overline{AZ}_{\tau-1} \overline{LU}))$

$= \mathbb{E}(\mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{ALU}_{\tau-1}, Z_s) \xi(\overline{AZ}_{\tau-1} \overline{LU}))$, $\mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{AZ}_{\tau-1}, \overline{LU}) = \mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{ALU}_{\tau-1}, Z_s)$

$\sum_{z_\tau} f(a_\tau | a_{\tau-1}, z_\tau, \bar z_{\tau-1}, \overline{LU}) w(\overline{aLZ}_\tau) f_{z_\tau}(z_\tau | \overline{aZ}_{\tau-1} \overline{LU}) \in m(\overline{aLU}_{\tau-1}, Z_s)$, $\in m(\overline{LU}_{\tau-1})$,

~~$\mathbb{E}(h(\bar A) w(\overline{ALZ}) | \overline{AZ}_{\tau-1}$~~ $\mathbb{E}(\{A = \bar a\} w | \overline{aZ}_{t-1} \overline{LU}_t) \in m(\overline{LU}_{\tau-1})$, $\mathbb{E}(\mathbb{E}(\{\} w | \overline{aZ}_{\tau-1} \overline{LU}_\tau) | \overline{aZ}_{\tau-2}, \overline{LU}_{\tau-1}) \in$

$m(\overline{LU}_{\tau-2})$, $\mathbb{E}(Y_{\bar a} | \overline{aZLU})$, $\mathbb{E}(\mathbb{E}(Y_{\bar a} | \overline{aZ}_{\tau-1} \overline{LU}) | \overline{aLU}_{\tau-1}) = \mathbb{E}(Y_{\bar a} | \overline{aLU}_{\tau-1}) = \mathbb{E}(Y_{\bar a} | \overline{aZLU}_{\tau-1})$, $\eta(\overline{AZLU})$

$= \sum_{\bar a} \{\bar A = \bar a\} (\mathbb{E}(Y_{\bar a} | \overline{aZLU}) - m(\bar a)) = \sum_{\bar a} \sum_{t=1}^{\tau} (\mathbb{E}(Y_{\bar a} | \overline{aZLU}_t) - \mathbb{E}(Y_{\bar a} | \overline{aZLU}_{t-1}))$

$$\mathbb{E}\left(h(\bar{A})\, w(\overline{AZ}) \mid AZ_{t-1}, LU_t \right) = \mathbb{E}\left(h(\bar{A})\, w(\overline{AZ}) \mid \overline{ALU}_{t-1}, \bar{Z}_s \right),\; s \le t-1, \; = \mathbb{E}\left(h(\bar{A})\, w(\overline{AZ}) \mid \overline{ALU}_{t-1} \right)$$

$$\ldots \mathbb{E}\left(hw \mid AZ_{T-1}, \underset{\ell U_T}{\overset{''}{LU_T}} \right) \mid A_{T-2} \underset{\ell U_{T-1}}{\overset{''}{LU_{T-1}}} \mid A_{T-3} \underset{\ell U_{T-2}}{\overset{''}{LU_{T-2}}} \cdots = c$$

$$\int \mathbb{E}\left(hw \mid AZ_{T-1}, \underset{\ell U_T}{\overset{''}{LU_T}} \right) f\left(a_{T-1} \mid A_{T-2}, \ell u_{T-1}\right) f\left(a_{T-2} \mid a_{T-3}, \ell u_{T-2}\right) \cdots f\left(a_1 \mid \ell u_1\right)\, d\bar{a}_{T-1} = c$$

$$\int h(a_T, \bar{A}_{T-1})\, w(a_T, \bar{A}_{T-1}, \overline{LU}, \bar{Z}_{T-1}, Z_T)\, f_{a_T}(a_T \mid \bar{A}_{T-1}, \overline{ZLU}_T)\, f_{Z_T}(z_T \mid \bar{A}_{T-1}, \overline{LU}_T, \bar{Z}_{T-1})\, da_T\, dz_T$$

$$\mathbb{E}\left(\{A=a\}\, Y \mid A \right) = \sum_a \{A=a\}\, \mathbb{E}\left(\{A=a\}\, Y \mid a \right) = \{A=a\}\, \mathbb{E}(Y \mid a) = \{A=a\}\, \mathbb{E}(Y \mid A)$$

$$\mathbb{E}\left(\bar{a}W \mid aZ_{T-2}\, LU_{T-1} \right) = \mathbb{E}\left(\bar{a}W \mid AZ_{T-2}\, LU_{T-1} \right) \Big|_{A_{T-2}=a_{T-2}} = \mathbb{E}\left(\mathbb{E}\left(\bar{a}W \mid AZ_{T-1}, LU_T \right) \Big| AZ_{T-2}\, LU_{T-1} \right) \Big|_{\cancel{A_{T-2}=a_{T-2}}\; a_{T-2}}$$

$$= \mathbb{E}\left(\{\bar{A}_{T-1} = \bar{a}_{T-1}\}\, \mathbb{E}\left(\{\bar{A}_{T} = \bar{a}_{T}\}\, w \mid aZ_{T-1}, \overline{LU}_T \right) \Big| AZ_{T-2}\, LU_{T-1} \right) \Big|_{a_{T-2}} =$$

$$\mathbb{E}\left(\{\bar{A}_{T-1} = \bar{a}_{T-1}\}\, \mathbb{E}\left(\{\bar{A}_{T} = \bar{a}_{T}\}\, w \mid \overline{aZLU_{T-1}} \right) \Big| AZ_{T-2}\, LU_{T-1} \right) \Big|_{a_{T-2}}$$

$$= \cancel{\mathbb{E}\left(\mathbb{E}\left(\{\bar{A}_T = \bar{a}_T\} \right) w \mid \overline{AZLU_{T-1}} \right) \mid AZ_{T-2}\, LU_{T-1}}$$

$$= \mathbb{E}\left(\{\bar{A}_T = \bar{a}_T\}\, W \mid aLU_{T-1} \right) \mathbb{E}\left(\{\bar{A}_{T-1} = \bar{a}_{T-1}\} \mid \underset{a_{T-2}}{Z_{T-2}}\, LU_{T-1} \right)$$

$$\{A_{T-2} = a_{T-2}\}\; \mathbb{P}\left(A_{T-1} = a_{T-1} \mid aZ_{T-2}, LU_{T-1} \right) \in m(LU_{T-2})$$

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$\frac{w_{10}}{w_{00}} = \frac{w_{01}}{w_{00}} w_{10} - w_{10} - 1$, $\frac{1}{w_{00}} + \frac{1}{w_{10}} = \frac{w_{01}}{w_{00}} - 1$, $1 + \frac{w_{00}}{w_{10}} = w_{01} - w_{00}$, $w_{01} - w_{00} - 1 = \frac{1}{w_{11} - w_{10} - 1}$, $1 = \frac{1}{w_{01} - w_{00}} + \frac{1}{w_{11} - w_{10}}$, $1 = \frac{1}{w_{00}} + \frac{1}{k w_{10}}$, $\frac{1}{w_{00}} + \frac{1}{w_{10}} =$ $r = r(\ell)$, $w_{00} = w_{10} = \frac{2}{\ell}$, $r = 2$, $w_{01} = w_{11} = \frac{4}{\ell}$,

$p_1 = w_{01}^{-1}(1 - w_{00} p_0) = \frac{\ell}{4}\left(1 - \frac{2}{\ell} p_0\right)$, $\cancel{h_0 w_{00} p_0 +}$ $E(\{A=0\} h_0 w_{0Z} | L U) + E(\{A=1\} h_1 w_{1Z} | L U) =$

$h_0(w_{00} p_0 + w_{01} p_1) + h_1(w_{10}(1-p_0) + w_{11}(1-p_1)) = h_0(w_{00} p_0 + w_{01} p_1) - h_1(w_{10} p_0 + w_{11} p_1) = c_h - h_1(w_{10} + w_{11}) = h_0\left(\frac{2}{\ell} p_0 + 1 - \frac{2}{\ell} p_0\right) - h_1\left(\frac{2}{\ell} p_0 + 1 - \frac{2}{\ell} p_0\right) = c_h - h_1 \frac{6}{\ell}$, $w_{11} - w_{10} = \frac{w_{01} - w_{00}}{w_{01} - w_{00} - 1}$

$\frac{w_{10}}{w_{00}} =? \frac{1}{w_{01} - w_{00} - 1}$, $w_{01} - w_{00} - 1 =? \frac{w_{00}}{w_{10}} = \frac{w_{01}}{w_{11}}$, $= \frac{w_{11}}{w_{10}} = \frac{w_{01}}{w_{00}}$, $w_{01} = w_{00} r(\ell)$, $w_{11} = w_{10} r(\ell)$, $\frac{w_{10}}{w_{00}} = w_{10} r(\ell) + w_{10} - 1$,

$\frac{1}{w_{10}} = r(\ell) + 1 - \frac{1}{w_{00}}$, $w_{00} = \frac{2}{\ell}$, $r(\ell) = 2$, $w_{10} = \left(3 - \frac{\ell}{2}\right)^{-1} = \frac{2}{6 - \ell}$, $p_1 = \frac{\ell}{4}\left(1 - \frac{2}{\ell} p_0(\ell, v)\right)$, $\underbrace{h_0\left(\frac{2}{\ell} p_0 + 1 - \frac{2}{\ell} p_0\right)}_{= h_0 + h_1' = c_h} + h_1\left(\frac{2}{6-\ell}(1-p_0) + \frac{4}{6-\ell}\left(1 - \frac{\ell}{4} + \frac{1}{2} p_0\right)\right) = h_0 + \frac{h_1}{6-\ell}(2 - 2p_0 + 4 - \ell + 2p_0)$, $\frac{\ell}{2-\rho} - \frac{2}{2-\rho} \frac{4}{2-\rho} -$

$\cancel{\frac{2}{2-\rho} - 1 + 4}$, $\frac{2}{2-\rho}(1-p_0) + \frac{4}{2-\rho}\left(1 - \frac{\ell}{4} + \frac{p_0}{2}\right)$, $w_{01}^{-1}(1 - w_{00} p_0) = \cancel{\frac{w_{00}}{w_{01}}(\ldots 2 p_0)} - \frac{w_{10}}{w_{11}}\left(p_0 + \frac{1}{w_{10}} + 1 - \frac{w_{11}}{w_{10}}\right)$

$= -\frac{w_{10}}{w_{11}} p_0 - \frac{1}{w_{11}} - \frac{w_{10}}{w_{11}} + 1$, $\cancel{p_0\left(\frac{w_{10}}{w_{11}} - \frac{w_{00}}{w_{01}}\right)}$ $\frac{1}{w_{01}} = -\frac{1}{w_{11}} - \frac{w_{10}}{w_{11}} + 1$, $\frac{w_{11}}{w_{01}} = -1 - w_{10} + w_{11}$,

$\cancel{-\frac{1}{2} p_0 - \frac{2-\ell}{4} + \frac{1}{2} = -\frac{1}{2} p_0 + \frac{\ell}{4} = \frac{\ell - 2p_0}{4}}$, $p_1 + \frac{p_0}{2} + \frac{2-\ell}{4} + \frac{1}{2}$, $\frac{2}{2-\rho}(1-p_0) + \cancel{\frac{4\ell}{2-\rho}\left(1 - \frac{2p_0}{\ell}\right) = \frac{1}{2-\rho}(2 - 2p_0 + \ell)}$,

$\cancel{2 \frac{\ell}{4}\left(1 - \frac{2p_0}{\ell}\right) - \frac{2-\ell}{2} + 2^{-1} = -\frac{\ell}{2} + p_0 - 1 + \frac{\ell}{2} + 2^{-1} = p_0}$

$h_0((w_{00} + w_{01}) p_0 + w_{01} \delta) = c$, $\frac{w_{11}}{w_{10}} = \frac{\beta}{\alpha}$, $w_{10}\left(\frac{1}{\alpha} - 1\right) = w_{11} - 1 = \frac{\beta}{\alpha} w_{10} - 1$,

$w_{10} = \left(\frac{1}{\alpha} - 1 - \frac{\beta}{\alpha}\right)^{-1} = \frac{\alpha}{1 - \beta - \alpha}$, $w_{11} = \frac{\beta}{1 - \beta - \alpha}$, $\alpha p_0 + \beta p_1 = 1$, $\frac{\alpha}{1-\beta-\alpha}(1-p_0) + \frac{\beta}{1-\beta-\alpha}(1-p_1) = \frac{\alpha + \beta - 1}{1 - \beta - \alpha}$

(34)

$$\mathbb{P}(L=l \mid U=u) = \sum_{a_{-1}\in 0,1} \mathbb{P}(L=l \mid U=u, A_{-1}=a_{-1})\, \mathbb{P}(A_{-1}=a \mid U=u) = \sum_a \frac{\mathbb{P}(A=a, L=l, U=u)}{\mathbb{P}(U=u \mid A=a)\,\mathbb{P}(A=a)} \frac{\mathbb{P}(U=u \mid A=a)}{P(U=u)}$$

$$\cdot \mathbb{P}(A=a) = \sum_a \frac{\frac{1}{2}\mathbb{P}(A=a \mid L=l, U=u)}{\mathbb{P}(U=u \mid A=a)\cdot\frac{1}{2}} \cdot \frac{\mathbb{P}(U=u \mid A=a)}{\frac{1}{2}} = \frac{1}{2}\sum_a \mathbb{P}(A=a \mid L=l, U=u) = \frac{1}{2}\sum_a \mathbb{P}(A=a \mid L=l) =$$

$$\sum_u \mathbb{P}(A=a \mid L=l, U=u)\,\mathbb{P}(U=u \mid L=l) = \frac{1}{2}\sum_u \mathbb{P}(A=a \mid L=l, U=u) \cdot \cancel{\sum_a \mathbb{P}(\ldots)}, \quad \mathbb{E}\,\frac{A(L-\mathbb{E}(L \mid A))}{f(A \mid L)} =$$

$$\mathbb{E}\left(\frac{A(L-\mathbb{E}(L \mid A=1))}{f(A=1 \mid L)}\right) = \mathbb{E}\left(\cancel{\mathbb{E}(A=1 \mid L)(\ )}\right) = \cancel{\mathbb{E}(L-\mathbb{E}(L \mid A=1))} = \mathbb{E}(L_0-\mathbb{E}(L_0 \mid A_0=1)), \quad \mathbb{E}\left(\{A_1=a_1\}\{A_2=a_2\}\right.$$

$$\left.\frac{(L_2-\mathbb{E}(L_2 \mid A_1))}{f(A_2 \mid L_2)\,f(A_1 \mid L_1)}\right) = \mathbb{E}\left(\mathbb{E}(\ldots \mid \bar{L}_2, A_1)\right) = \mathbb{E}\left(\{A_1=a_1\}\frac{L_2-\mathbb{E}(L_2 \mid A_1)}{f(A_1 \mid L_1)}\right) = \mathbb{E}\left(\mathbb{E}(\ldots \mid L_1, A_1)\right)$$

$$= \mathbb{E}\left(\{A_1=a_1\}\frac{\mathbb{E}(L_2 \mid A_1)-\mathbb{E}(L_2 \mid A_1)}{f(A_1 \mid L_1)}\right), \quad \mathbb{E}(L \mid A=a) = \sum_b \mathbb{E}(L \mid A=a, B=b)\,\mathbb{P}(B=b) = (1-p_B)\,\mathbb{E}(L \mid A_L=a, B=0)$$

$$\cancel{(1-p_B)\,\mathbb{E}(L \mid A_U=a)} \quad p_B \quad \mathbb{E}(L \mid A_U^{B=1}=a) = (1-p_B)\, p_L^{A_L=1}(1-p_L)^{A_L=0} + p_B \cdot \frac{1}{2},$$

$$\gamma_0+\gamma_1 = p_L\delta_0+(1-p_L)\delta_1+p_L\delta_1+(1-p_L)\delta_0 = \delta_0+\delta_1, \quad \frac{\gamma_0^2}{\delta_0^2}+\frac{\gamma_1^2}{\delta_1^2} = (\delta_0^2+\delta_1^2+\gamma_1^2-2\delta_0\delta_1-2\delta_0\gamma_1-2\delta_1\gamma_1)/\delta_0^2+\frac{\gamma_1^2}{\delta_1^2}$$

$$\cancel{=1+\frac{2\gamma_1^2+\delta_0^2}{\delta_1^2}-2\delta_0^{-2}(\delta_0\delta_1+\delta_0\gamma_1+\delta_1\gamma_1)}, \quad \cancel{\frac{\gamma_0^2}{\delta_1^2}+\frac{\gamma_1^2}{\delta_0^2} = 1+\frac{2\gamma_1^2}{\delta_0^2}} \qquad \mathbb{E}\left(\frac{(-1)^{1-z_t}}{\Delta(a_t \mid \bar{A}\bar{Z}_{T-1}, \bar{L}\bar{U}_T)\, f_z(z_T \mid \bar{A}\bar{Z}_{T-1}, \bar{L}\bar{U}_T)}\right)$$

$$= \mathbb{E}\left(\mathbb{E}(\ldots \mid \bar{A}\bar{Z}_{T-1}, \bar{L}\bar{U}_T)\right) =$$

$g: A \to \mathbb{R}$, $g: a \mapsto \inf_{b \in B} d(a,b)$, $g \overset{?}{\in} C^0$, $\inf(\operatorname{Im} g) = 0 \Rightarrow \{a_n\} \subset A$, $\{b_n\} \subset B$, $d(a_n, b_n) \to 0$,

$\Rightarrow a_n \in B = \bar{B}$. $(g \in ? C^0)$ $b \in B: d(a,b) < g(a) + \varepsilon$, $|a - a'| < \delta \Rightarrow d(a', b) < \delta + g(a) + \varepsilon$,

$|g(a) - g(a')| < \delta + \varepsilon$ #1 // $Z_n := \{z \in \mathbb{C} : f^{(n)}(z) = 0\}$, $|Z_n| = \aleph_0$, $|\bigcup_n Z_n| = \aleph_0$,

$\Rightarrow n : Z_n = \mathbb{C}$ #2 // $g \equiv 0 \Rightarrow f \equiv 0$, $(g \not\equiv 0)$ $z \in Z(g) \Rightarrow r$, $(D(z;r)/z) \cap Z(g) = \emptyset$, $z' \in$

$D(z,r)$, $z' \neq z \Rightarrow \left|\frac{f}{g}(z')\right| \leq 1$, $g_0 : \frac{f}{g}(z) := g_0$, $\frac{f}{g}\big|_{D(z,r)} \in H(D(z,r))$, $\frac{f}{g} = c$ #3 //

$\left|\frac{f(z)}{A + B|z|^k}\right| \leq 1$, $f \in H(D(0,r))$, $r > 0$, $f^{(n)}(0) \leq \frac{n!(A + Br^k)}{r^n}$

$= \frac{n!A}{r^n} + Br^{k-n} \xrightarrow[r \to \infty]{} 0$ $\Leftarrow k < n$, $c_n = 0$, $f(z) = \sum_{j=0}^{k} c_j z^j$ #4 // $\int_\Gamma \frac{(f - f_n)(w)}{w - z} dw = (f$

$- f_n)(z)$ $\cdot \operatorname{Ind}_\gamma(z) = (f - f_n)(z) \int_{\partial D} \frac{1}{w - z} dw \xrightarrow[n \to \infty]{} 0$, $\int_{\partial D} \frac{(f - f_n)(w) - (f - f_n)(z)}{w - z} dw$

$\to 0$, $\frac{(f - f_n)(w) - (f - f_n)(z)}{w - z} \xrightarrow{z \to w} (f - f_n)'(w)$, $\sup_{w \in \partial D} \sup_{z \in \bar{D}} \left|\frac{(f - f_n)(w) - (f - f_n)(z)}{w - z}\right| < M$,

$(f - f_n)(w) - (f - f_n)(z) = \int_{z \to w} (f - f_n)(\xi) d\xi = \int_0^1 (w - z)(f - f_n)(\xi(t)) dt$, $\xi(t) = z + (w - z)t$

$f(w) - f(z) = \int_{z \to w} f(\xi) d\xi = (w - z) \int_0^1 f(\xi(t)) dt$, $\int_\Gamma \frac{f(w)}{w - z} dw = \int_\Gamma \left(\frac{f(z)}{w - z} + \int_0^1 f'(\xi(t)) dt\right) dw$

$= f(z) \operatorname{Ind}_\gamma(z) + \int_\Gamma \int_0^1 f'(\xi(t)) dt\, dw$, $\int_\Gamma \frac{f(w)}{w - z} = \int_\Gamma \frac{f(w) - f(z)}{w - z} + \int_\Gamma \frac{f(z)}{w - z}$

$D(1,1) = \{1 + re^{i\theta} : 0 \leq r < 1, 0 \leq \theta < 2\pi\}$, $1 + r\cos\theta + ir\sin\theta = (1 + 2r\cos\theta + r^2)^{1/2} \exp\left(i \tan^{-1} \frac{r\sin\theta}{1 + r\cos\theta}\right)$, $\Omega = \{x + iy :$

$x = \log(1 + r^2 + 2r\cos\theta)^{1/2}$, $y = \tan^{-1} \frac{r\sin\theta}{1 + r\cos\theta}$, $0 \leq r < 1$, $0 \leq \theta \leq 2\pi\}$, $1 + r_0^2 + 2r_0\cos\theta_0 = 1 + r_1^2 + 2r_1\cos\theta_1$,

$r_0 + 2\cos\theta_0 = r_1 + 2\cos\theta_1$, $\tan^{-1} \frac{r_0\sin\theta_0}{1 + r_0\cos\theta_0} - \tan^{-1} \frac{r_1\sin\theta_1}{1 + r_1\cos\theta_1} = 2\pi k$, $-\frac{\pi}{2} < \tan^{-1} \frac{r\sin\theta}{1 + r\cos\theta} < \frac{\pi}{2}$, $r_0\sin\theta_0 +$

$r_0 r_1 \sin\theta_0 \cos\theta_1 = r_1 \sin\theta_1 + r_0 r_1 \sin\theta_1 \cos\theta_0$, $e^{x + iy} = e^{x' + iy'} \Rightarrow x = x'$, $y = y' + 2\pi k$, $\sup\{|y - y'| :$

$x + iy, x' + iy' \in \Omega\} = \frac{\pi}{2} - -\frac{\pi}{2}$. $\Omega_k = \{x + i(y + 2\pi k) : x + iy \in \Omega\}$. $\frac{\log(w) - \log(z)}{w - z} = \frac{\log w - \log z}{\exp\log w - \exp\log z}$

$\xrightarrow[w \to z]{} \frac{1}{\exp\log z} = \frac{1}{z} \Leftarrow \log w \xrightarrow[w \to z]{} \log z$. $n! a_n = \left(\frac{1}{z}\right)^{(n)}\Big|_{z=1} = (-1)^n n! z^{-n-1}\Big|_{z=1} = (-1)^n n!$, $a_n = (-1)^n$,

$\frac{1}{z} = \sum_1^\infty n c_n (z-1)^{n-1} = \sum_0^\infty (n+1) c_{n+1} (z-1)^n$, $c_{n+1} = \frac{(-1)^n}{n+1}$, $c_n = \frac{(-1)^{n-1}}{n}$ $(n > 0)$, $c_0 = \log 1 = 0$ #6 //

$\frac{\partial}{\partial z} \int_\Gamma \frac{f(w)}{w - z} dw = \lim_{\xi \to z} \frac{1}{\xi - z} \int_\Gamma \left(\frac{f(w)}{w - \xi} - \frac{f(w)}{w - z}\right) dw = \lim_\xi \int_\Gamma f(w) \left(\frac{\frac{1}{w - \xi} - \frac{1}{w - z}}{\xi - z}\right) dw$, $\Leftarrow \sup_\Gamma |f(w)| < \infty$,

$\sup_w \sup_{|z - \xi| < \delta} \frac{\frac{1}{w - \xi} - \frac{1}{w - z}}{\xi - z} < \infty$

$\hat f(t) = \begin{cases}\pi, & |t|<1\\ \pi/2, & |t|=1 \\ 0 & |t|>1\end{cases}$, $\int \hat f(t) e^{itx}\,dt = 2\int_0^\infty \hat f(t)\cos(tx)\,dt = 2\int_0^1 \pi\cos tx = 2\pi \frac{\sin x}{x}$.

$|R(x)| < c/x^2$, $\int_{-\infty}^\infty R(x) = \lim_{A\to\infty}\int_{-A}^A R(x)\{|x|<A\}$, $\int_{\Gamma_A} R(z) = \int_{-A}^A R(x) + \int_0^\pi R(Ae^{it})Aie^{it}\,dt$,

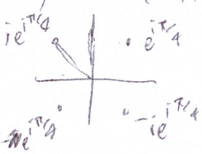
$|R(Ae^{it})Aie^{it}| = A\left|\frac{c}{A^2}e^{2it}\right| = \frac{c}{A}$, $\int_{\Gamma_A} R(z) \xrightarrow[A\to\infty]{} \int_{-\infty}^\infty R(x) = 2\pi i \sum_{a:Q(a)=0} \mathrm{Res}(R;a)$.

$z^4+1 = (z^2+i)(z^2-i) = (z+i\sqrt i)(z-i\sqrt i)(z+\sqrt i)(z-\sqrt i) = (z+ie^{i\pi/4})(z-ie^{i\pi/4})(z+e^{i\pi/4})(z-e^{i\pi/4})$, $\frac{P(z)}{\prod_1^m (z-r_j)} = \sum_{j=0}^\infty c_j (z-r_0)^j$,

$\frac{P(z)}{\prod_0^m(z-r_j)} = \sum_0^\infty c_j(z-r_0)^{j-1} = \sum_{-1}^\infty c_{j+1}(z-r_0)^j$, $\mathrm{Res}\left(\frac{P(z)}{\prod_0^m(z-r_j)}; r_0\right) = c_0 = \frac{P(r_0)}{\prod_1^m(r_0-r_j)}$, $\frac{1}{2\pi i}\int_{-\infty}^\infty \frac{x^2}{x^4+1}$

$\frac{(ie^{i\pi/4})^2}{-2ie^{i\pi/4}(-ie^{i\pi/4}+e^{i\pi/4})(-ie^{i\pi/4}-e^{i\pi/4})} + \frac{(ie^{i\pi/4})^2}{2ie^{i\pi/4}(ie^{i\pi/4}+e^{i\pi/4})(ie^{i\pi/4}-e^{i\pi/4})} + \cancel{\frac{1}{(-i+ie^{i\pi/4})(-i-ie^{i\pi/4})(-2i)} + \frac{-1}{(i+ie^{i\pi/4})(i-ie^{i\pi/4})(2i)}}$

$\frac{(e^{i\pi/4})^2}{(e^{i\pi/4})^3(-1+i)(-1-i)(-1-1)} + \frac{(e^{i\pi/4})^2}{(e^{i\pi/4})^3(1+i)(1-i)\cdot 2}$,



, $\frac{1}{2\pi i}\int_{-\infty}^\infty \frac{x^2}{x^4+1} = \frac{-e^{i\pi/2}}{2ie^{i\pi/4\cdot 3}(i+1)(i-1)}$

$+\frac{e^{i\pi/2}}{2e^{i3\pi/4}(1-i)(1+i)} = -\frac{1}{2e^{i3\pi/4}(-2)} + \frac{i}{4e^{i3\pi/4}} = \frac{1+i}{4e^{i3\pi/4}} = \frac{1+i}{4(-1/\sqrt2+i/\sqrt2)} = \frac{\sqrt2}{4}\frac{i+1}{i-1} = -\frac{\sqrt2}{4}\cdot i = \frac{\sqrt2}{4i}$ #8 //

$\mathrm{Res}(f;i) = \frac{e^{-t}}{2i}$, $\int_{-\infty}^\infty \frac{e^{itx}}{1+x^2} = \pi e^{-t}$ $(t>0)$, $\int_{-\infty}^\infty \hat f(t)e^{itx} = \int \pi e^{-t+itx}\,dt = \frac{\pi}{ix-1}e^{t(ix-1)}\Big|_{-\infty}^\infty$, $e^{itRe^{i\theta}}$

$= e^{-tR^2\sin\theta} e^{itR\cos\theta} \xrightarrow[R\to\infty]{} 0 \Leftarrow 0\le\theta\le\pi,\ t>0$, $\mathrm{Res}(f;-i) = \frac{e^t}{+2i}$, $\int \frac{e^{itx}}{1+x^2} = +\pi e^t$ $(t<0)$, $(t=0)$ $\int_{-\infty}^\infty \frac{1}{1+x^2}$

$\operatorname{atan}(x)\Big|_{-\infty}^\infty = \pi$, $\hat f(t) = \begin{cases} e^{-t}/2 & (t>0)\\ 1/2 & (t=0) \\ +e^{t}/2 & (t<0)\end{cases}$, $\int_{-\infty}^\infty \hat f(t)e^{itx} = \int_{-\infty}^0 +\frac12 e^{t+itx} + \int_0^\infty \frac12 e^{-t+itx} =$

$\frac12\left(+\frac{1}{1+ix}e^{t(1+ix)}\Big|_{-\infty}^0 + \frac{1}{ix-1}e^{t(ix-1)}\Big|_0^\infty\right) = \frac12\left(\frac{1}{1+ix} - \frac{1}{ix-1}\right) = \frac{1}{x^2+1}$ #9 // $g(x)=$

$\sum_{j=0}^\infty c_j(z-a)^j$, $f(x) = \frac{g(x)}{(z-a)^m} = \sum_{j=0}^\infty c_j (z-a)^{j-m} = \sum_{j=-m}^\infty c_{j+m}(z-a)^j$, $\mathrm{Res}(f;a) = c_{m-1} = \frac{1}{(m-1)!}g^{(m-1)}(0)$,

$\frac{1}{2\pi i}\int_\gamma \frac{e^z-e^{-z}}{z^4} = \mathrm{Res}(e^z-e^{-z};0) = \frac{1}{3!}(e^z+e^z)\Big|_{z=0} = \frac13$ #10 // $(z-\alpha)(z-1/\alpha) = z^2 - z(1/\alpha+\alpha)+1$, $\Big|_{z=e^{i\theta}}$

$= \cos 2\theta + i\sin 2\theta - (\cos\theta + i\sin\theta)(1/\alpha+\alpha)+1$, $\int_\Gamma \frac{1}{(z-\alpha)(z-1/\alpha)} = \int_0^{2\pi} \frac{ie^{i\theta}}{e^{2i\theta} - e^{i\theta}(1/\alpha+\alpha)+1}\,d\theta = \int_0^{2\pi}\frac{i\alpha\,d\theta}{\alpha e^{i\theta} - 1 - \alpha^2 + \alpha e^{-i\theta}}$

$= \int \frac{i\alpha\,d\theta}{2\alpha\cos\theta - \alpha^2 - 1} = -i\alpha\int\frac{d\theta}{1+\alpha^2-2\alpha\cos\theta}$, $(|\alpha|>1)$ $\int_\Gamma \frac{1}{(z-\alpha)(z-1/\alpha)} = 2\pi i\,\mathrm{Res}\left(\frac{1}{(z-\alpha)(z-1/\alpha)}; 1/\alpha\right) = \frac{2\pi i}{1/\alpha - \alpha}$

$= \frac{2\pi i\alpha}{1-\alpha^2}$, $(|\alpha|<1)$ $\int_\Gamma \cdots = 2\pi i\,\mathrm{Res}(\cdots;\alpha) = \frac{2\pi i}{\alpha-1/\alpha} = \frac{2\pi i\alpha}{\alpha^2-1}$, $\int_0^{2\pi}\frac{d\theta}{1+\alpha^2-2\alpha\cos\theta} = \frac{1}{\alpha^2-1}$ $(|\alpha|>1)$,

$\frac{1}{1-\alpha^2}$ $(|\alpha|<1)$ #11 // $-4\sin^2 z\, e^{itz} = (e^{iz}-e^{-iz})^2 e^{itz} = (e^{(2+t)iz} - 2e^{itz} + e^{(t-2)iz}) =$

Rudin #3

$$\left(e^{(t+2)iRe^{i\theta}} - 2e^{itRe^{i\theta}} + e^{(t-2)iRe^{i\theta}}\right) = \left(e^{i(t+2)R\cos\theta} e^{-(t+2)R\sin\theta} - 2e^{itR\cos\theta} e^{-tR\sin\theta}\right.$$

$$\left.+ e^{i(t-2)R\cos\theta} e^{-(t-2)R\sin\theta}\right) = e^{itR\cos\theta} e^{-tR\sin\theta}\left(e^{i2R\cos\theta} e^{-2R\sin\theta} - 2 + e^{-i2R\cos\theta} e^{2R\sin\theta}\right),$$

$\sin\theta > 0 \Rightarrow t-2 > 0$, $\sin\theta < 0 \Rightarrow t+2 < 0$, $(t=0)$ $\left(\frac{\sin z}{z}\right)^2 = -\frac{1}{4z^2}(e^{iz} - e^{-iz})^2 = -\frac{1}{4z^2}\left(e^{2iz} + e^{-2iz} - 2\right) = -\frac{1}{4z^2}\left(e^{2i(x+iy)}\right.$

$\left.+ e^{-2i(x+iy)} - 2\right) = -\frac{1}{4}\left(e^{-2y} e^{i2x} + e^{2y} e^{i(-2x)} - 2\right) / ((x^2-y^2) + i2xy)$, $\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 e^{itx} = 2\int_0^\infty \left(\frac{\sin x}{x}\right)^2 \cos(tx)$,

$\int_{-A}^{A}\left(\frac{\sin z}{z}\right)^2 e^{itz} = \int_{\Gamma_A} z^{-2}\left(e^{iz(t+2)} - 2e^{itz} + e^{iz(t-2)}\right)/-4\,dz = -\frac{i}{4}\int_{-\pi}^{0} A^{-1} e^{-i\theta} e^{ite^{i\theta}A}\left(e^{i2e^{i\theta}A} - 2\right.$

$\left.+ e^{-2ie^{i\theta}A}\right) = -\frac{Ai}{4}\int_{-\pi}^{0} e^{iA(t\cos\theta - \theta)} e^{-At\sin\theta}\left(e^{i2A\cos\theta} e^{-2A\sin\theta} + e^{-i2A\cos\theta} e^{2A\sin\theta} - 2\right),$

~~$\xrightarrow[A\to\infty]{} 0 \Leftarrow t\sin\theta > 0,\ t < 0,$~~ $-4\int_{-A}^{A} = \int_{\Gamma_A} \ldots = \varphi(t+2) - 2\varphi(t) + \varphi(t-2)$,

$\varphi(s) := \int_{\Gamma_A} z^{-2} e^{isz}$, $\varphi(s) = \int_{-\pi}^{0} iA^{-1} e^{-i\theta} e^{isAe^{i\theta}}\,d\theta = \int_{-\pi}^{0} iA^{-1} e^{i(sA\cos\theta - \theta)} e^{-sA\sin\theta}\,d\theta \xrightarrow[A\to\infty]{} 0$

$\Leftarrow sA\sin\theta > 0,\ s < 0$, $\varphi(s) = -\int_0^\pi iA^{-1} e^{i(sA\cos\theta-\theta)} e^{-sA\sin\theta}\,d\theta + 2\pi i\,\mathrm{Res}(z^{-2}e^{isz}; 0) \xrightarrow[A\to\infty]{(s>0)} 2\pi i\,\mathrm{Res}(\ )$

$= -2\pi s$, $-4\int_{-\infty}^{\infty} =$ ~~$(0<t<2)$~~ $(|t|>2)\ 0$, $(0<t<2)\ -2\pi(2-t)$, $(-2<t<0)\ -2\pi(t+2)$,

$$\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 e^{itx} = \begin{cases} \frac{\pi}{2}(t+2), & -2<t<0 \\ 0 & |t|>2, \\ \frac{\pi}{2}(2-t) & 0<t<2 \end{cases}$$

~~$\int f(t) e^{itx} = \frac{\pi}{2}\int_{-2}^{2}(t+2)(\cos tx + i\sin tx) = \frac{\pi}{2}\int (t+2)\cos tx$~~

~~$\frac{\pi}{2}\int t\cos tx + \frac{\pi}{2}\int ti\sin tx = \pi \cdot \frac{1}{x}\sin tx\Big|_{-2}^{2} + i\frac{\pi}{2}\left(t\left(-\frac{1}{x}\cos tx\right)\Big|_{-2}^{2} + \int_{-2}^{2}\frac{1}{x}\cos tx\right) = \frac{\pi}{x}2\sin 2x +$~~

~~$i\frac{\pi}{2}\left(4\cos 2x + \frac{1}{x^2}2\sin 2x\right)$~~, $\frac{2}{\pi}\int f(t)e^{itx} = \int_{-2}^{0}(t+2)e^{itx}\,dt + \int_0^2(2-t)e^{itx}\,dt = \left(\int_{-2}^{0} + \int_2^0\right)te^{itx}\,dt$

$+2\int_{-2}^{2} e^{itx}\,dt = \frac{t}{ix}e^{itx}\Big|_{-2}^{0} + \Big|_2^0 - \frac{1}{ix}\int_{-2}^{0} + \int_2^0 e^{itx} + 4\int_0^2 \cos tx = \frac{2}{ix}e^{-i2x} - \frac{2}{ix}e^{i2x} + \frac{1}{x^2}e^{itx}\Big|_{-2}^{0} + \Big|_2^0$

$+ \frac{4}{x}\sin tx\Big|_0^2 = -\frac{2}{ix}2i\sin 2x + \frac{4}{x}\sin 2x + \frac{1}{x^2}\left(2 - e^{-i2x} - e^{i2x}\right) =$ ~~$\frac{2x\cos 2x}{x^2}$~~ ~~$\frac{2(\cos^2 x - \sin^2 x)}{x^2}$~~ ~~$= \frac{2 - 4\sin^2 x}{x^2}$~~

$= \frac{2 - 2\cos 2x}{x^2} = \frac{4\sin^2 x}{x^2}$ // #12 $\int_{-A}^{0}\frac{dx}{1+x^3} = -\int_A^0 \frac{dx}{1-x^3}$, $\int_0^R e^{\frac{2\pi}{n}}(1+r^n)^{-1}\,dr$, $\int_\Gamma \frac{1}{1+z^n}$

$= 2\pi i\,\mathrm{Res}\left(\frac{1}{1+z^n}; e^{i\pi/n}\right) \sim \left(\int_0^R \frac{1}{1+x^n}\right)\left(1 - e^{i2\pi/n}\right)$, $\int_0^\infty \frac{1}{1+x^n} = \frac{2\pi i\,\mathrm{Res}((1+z^n)^{-1}; e^{i\pi/n})}{1 - e^{i2\pi/n}}$, $(1+z^n)^{-1}$

$= \left(\prod_{j=1}^{n/2}(z - e^{i\frac{\pi}{n}j})(z - e^{-i\frac{\pi}{n}j})\right)^{-1}$ $(2|n)$, $= \left((z+1)\prod_{j=1}^{\frac{1}{2}(n-1)}(z - e^{i\frac{\pi}{n}j})(z - e^{-i\frac{\pi}{n}j})\right)^{-1}$ $(2\nmid n)$, $\mathrm{Res} = \left((e^{i\frac{\pi}{n}} - e^{-i\frac{\pi}{n}}\right)\cdot$

$\prod_{j=2}^{n/2}\left(e^{i\pi/n} - e^{i\frac{\pi}{n}j}\right)\left(e^{i\pi/n} - e^{-i\frac{\pi}{n}j}\right)\Big)^{-1} = \left((2i\sin\tfrac{\pi}{n})e^{i\frac{\pi}{n}(\frac{n}{2}-1)}\prod_{j=1}^{\frac{n}{2}-1}(1 - e^{i\frac{\pi}{n}j})(1 - e^{-i\frac{\pi}{n}j})\right)^{-1} = \left((2i\sin\tfrac{\pi}{n})\cdot ie^{-i\frac{\pi}{n}}\prod_{j=1}^{\frac{n}{2}-1}\left(1 - 2\cos\tfrac{\pi}{n}j\right.\right.$

$\left.\left.+1\right)\right)^{-1} = -e^{i\frac{\pi}{n}}\left(2\sin\tfrac{\pi}{n}\, 2^{\frac{n}{2}-1}\prod_{j=1}^{\frac{n}{2}-1}\left(1 - \cos\tfrac{\pi}{n}j\right)\right)^{-1}$, $\mathrm{Res} = \lim_{z\to e^{i\pi/n}} \frac{z - e^{i\pi/n}}{1+z^n} = \lim\,(nz^{n-1})^{-1}$

Rudin #4

$\text{Res} = \frac{1}{n e^{i\pi\frac{n-1}{n}}} = \frac{-1}{n} e^{\frac{i\pi}{n}}$, $\int \frac{1}{1+x^n} = \frac{-2\pi i e^{i\pi/n}}{n(1-e^{i2\pi/n})} = \frac{-2\pi i}{n(e^{-i\pi/n} - e^{i\pi/n})} = \frac{2\pi i}{2 i \sin \pi/n} = \frac{\pi}{n \sin \pi/n}$ #13

$f(z) - f(z_0) = (z-z_0)(f'(z_0) + \varepsilon(z))$, $h(z) - h(z_0) = g(w) - g(w_0) = (z-z_0)(h'(z_0) + \eta(z_0))$,

$\frac{g(w)-g(w_0)}{w-w_0} = \frac{h'(z_0)+\eta(z)}{f'(z_0)+\varepsilon(z)} \xrightarrow[w\to w_0]{} \left(\frac{h'}{f'}\right)(z_0) \Leftarrow f^{-1} \in C_0(\operatorname{Im} f)$. $f \in H(\Omega_1)$, $g = c$, $h = c$. #14

$f(w) = (w-w_0)^m \sum_{j=0}^\infty c_j (w-w_0)^j =: (w-w_0)^m f_0(w)$, $f_0(w_0) \neq 0$, $g(z) = (\varphi(z) - \varphi(z_0))^m f_0(\varphi(z))$, $f_0(\varphi(z_0)) =$

$f_0(w_0) \neq 0$, $= (\varphi'(z_0) + \varepsilon(z))^m (z-z_0)^m f_0(\varphi(z)) =: (z-z_0)^m f_1(\varphi(z))$, $f_1(\varphi(z_0)) = (\varphi'(z_0))^m f_0(w) \neq 0$,

$\varphi'(z_0) + \varepsilon(z) = \frac{\varphi(z) - z_0}{z - z_0} \in \mathcal{H}(\Omega_1)$. $\varphi'(z) = (z-z_0)^k \psi(z)$, $\psi(z_0) \neq 0$, $g(z) = ((z-z_0)^k \psi(z) + \varepsilon(z))^m$

$(z-z_0)^m f_0(z)$ $= (z-z_0)^{km} \psi(z)^m \left(1 + \frac{\varepsilon(z)}{(z-z_0)^k \psi(z)}\right)^m (z-z_0)^m f_0(z) = (z-z_0)^{(k+1)m} \left\{ \left(\psi(z) + \frac{\varepsilon(z)}{(z-z_0)^k}\right)^m \right.$

$\left. f_0(z) \right\} =: (z-z_0)^{(k+1)m} f_2(z)$, $f_2(z_0) \neq 0$, $\psi + \frac{\varepsilon(z)}{(z-z_0)^k} = \frac{\varphi(z) - \varphi(z_0)}{(z-z_0)^{k+1}} = \frac{\varphi'(z) + \varepsilon(z)}{(z-z_0)^k}$,

$\varepsilon(z) = \frac{\varphi(z) - \varphi(z_0)}{z - z_0} - \varphi'(z_0)$, $(z-z_0)\varepsilon(z) = \sum_{j=0} c_j (z-z_0)^j - c_0 - (z-z_0) \sum_{j=0}^\infty (j+1) c_{j+1} (z-z_0)^j = \sum_{j=1}^\infty (1-j) c_j (z-z_0)^j$,

$\varepsilon(z) = \sum_{j=1}^\infty (1-j) c_j (z-z_0)^{j-1} = \sum_{j=0}^\infty -j c_{j+1} (z-z_0)^j$, ~~$\varphi(z) = \sum_{j=m}^\infty c_j (z-z_0)^j$, $c_0 = \dots = c_{m-1} = 0$~~ $\varphi'(t) = \sum_{j=0}^\infty (j+1) c_{j+1}$

$(z-z_0)^j$, $c_1 = \dots = c_m = 0$, $\frac{\varepsilon(z)}{(z-z_0)^m} \in \mathcal{H}(\Omega_1)$, $f_2 \in \mathcal{H}(\Omega_1)$ #15 $g(z,w) := \frac{\varphi(z,t_0) - \varphi(w,t_0)}{z-w}$

$(z \neq w)$, $= \frac{\partial \varphi}{\partial z}(z, t_0)$ $(z = w)$, $t_0 \in X$, $g: \Omega \times \Omega \to \mathbb{C}$, $\overset{(10.29)}{\Rightarrow} g \in C_0(\Omega \times \Omega)$, $N :=$

$\sup_{w,z \in K\times K} |g(z,w)|$, $\left| \frac{\varphi(z,t) - \varphi(w,t)}{z-w} \right| \leq \left| \frac{\varphi(z,t_0) - \varphi(w,t_0)}{z-w} \right| + \left| \varphi(z,t) - \varphi(z,t_0) - \varphi(w,t) + \varphi(w,t_0) \right|$

$\leq N + \left| \frac{\varphi(z,t) - \varphi(w,t) - (\varphi(z,t_0) - \varphi(w,t_0))}{z-w} \right| \{|z-w| > \delta\} + |\cdots| \{|z-w| < \delta\} \leq N + \frac{2M}{\delta}$

$+ |\cdots| \{|z-w| < \delta\}$, $\left\{ \frac{\varphi(z,t) - \varphi(w,t)}{z-w} \right\} = |\varphi'(w,t) + \varepsilon(z,t)| \leq \frac{M}{\delta} + \sup_{|z-w|<\delta} |\varepsilon(z,t)|$,

$\varphi(z,t) = \sum_{j=0}^\infty c_j^{(t)} (z-w)^j$, $\frac{\varphi(z,t) - \varphi(w,t)}{z-w} = \sum_{j=1}^\infty c_j^{(t)} (z-w)^{j-1}$, $\varepsilon(z,t) = \sum_{j=1}^\infty \{ c_j^{(t)} (z-w)^{j-1}$

$- j c_j^{(t)} (z-w)^{j-1} \} = \sum_{j=2}^\infty (1-j) c_j^{(t)} (z-w)^{j-1}$, $\left| \sum_{j=1}^\infty c_j^{(t)} (z-w)^{j-1} \right| \leq \left| \sum_{j=1}^\infty \frac{\varphi^{(j)}(w,t)}{j!} (z-w)^{j-1} \right| \leq$

$\sum_{j=1}^\infty \frac{M}{\delta^j} |(z-w)^{j-1}|$, $\left| \sum_{j=1}^\infty c_j^{(t)} (z-w)^{j-1} \right| \leq \sum_{j=1}^\infty j |c_j^{(t)} (z-w)^{j-1}|$, $\sum_{j=2} |(1-j) c_j^{(t)} (z-w)^{j-1}|$

$\leq |z-w| \sum_{j=0}^\infty |(j+1) c_{j+2}^{(t)}| |z-w|^j \leq |z-w| \sum_{j=0}^\infty |j+2||j+1| |c_{j+2}^{(t)} (z-w)^j| = |z-w| \sum_{j=1} |(j+1)(j) c_{j+1}^{(t)}$

$(z-w)^{j-1}|$, $|z-w| \sum_{j=2}^\infty |(j-1) c_j^{(t)} (z-w)^{j-2}| = |z-w| \sum_{1} |j c_{j+1}^{(t)} (z-w)^{j-1}|$, $\sum_j |c_j| = \sum_j \left| \frac{\varphi^{(n)}(z,t)}{j!} \right|$

$\leq \sum_j \frac{\|\varphi(z,t)\|_\infty}{\delta^j} =: M_0 < \infty$, $\sup_{|z-w|<\delta} \left| \frac{\varphi(z,t) - \varphi(w,t)}{z-w} \right| \leq \delta \sum_{j=1}^\infty c_j^{(t)} \leq \delta M_0$ #16

Rudin #5

$tz=-1$, $\Omega_\delta := \mathbb{C} \setminus \bigcup_{y \le -1} B(y;\delta)$, $f \in H(\Omega_\delta)$, $f \in H(\mathbb{C} \setminus (-\infty,-1])$. $e^{t(x+iy)} = e^{tx} e^{ity}$,

$\Omega_\delta := \{z : \operatorname{Re} z \le 0\}$, $\left|\frac{e^{tz}}{1+t^2}\right| \le e^{|\operatorname{Re} z|}$, $\Omega_M : \{z : |\operatorname{Re} z| < M\}$, $h \in H(\Omega_M)$,

$h \in H(\mathbb{C})$ #17 // $\frac{f'(z)}{f(z)} z^p = \frac{m(a)}{z-a} z^p + \frac{h'(z)}{h(z)} z^p$, $\frac{h'}{h} z^p \in H(D(a,r))$, $(z-a)\frac{m(z-a)^{m-1}}{(z-a)^m} z^p$

$\xrightarrow[z\to a]{} m a^p = \operatorname{Res}_a\left(\frac{f'}{f} z^p\right)$, $\frac{1}{2\pi i}\int_\gamma \frac{f'}{f} z^p \, dz = \sum_a \operatorname{Res}\left(\frac{f'}{f} z^p, a\right) = \sum_a m(a) a^p$. $\frac{1}{2\pi i}\int_\gamma \frac{f'}{f} \varphi(z)\, dz$

$= \sum_a m(a)\varphi(a)$ #18 // $\left(\frac{f'}{f} - \frac{g'}{g}\right)\left(\frac{1}{n}\right) = 0$, $\frac{f'}{f} - \frac{g'}{g} \equiv 0$, $f'g = fg'$, $\left(\frac{f}{g}\right)' = 0$, $f = c \cdot g$ #19 //

ch 11

$(u_x v + u v_x)_x + (u_y v + u v_y)_y = u_{xx} v + 2u_x v_x + u v_{xx} + u_{yy} v + 2u_y v_y + u v_{yy} = 2(u_x v_x + u_y v_y) := 0$, $u_x^2 + u_y^2$

$= 0$, $u_x = u_y = 0$, $u = c$. $|f|^2 = u^2 + v^2$, $2(u u_x + v v_x)_x + 2(u u_y + v v_y)_y = 2(u_x^2 + u u_{xx} + v_x^2 +$

$v v_{xx} + u_y^2 + u u_{yy} + v_y^2 + v v_{yy}) = 2(u(u_{xx} + u_{yy}) + v(v_{xx} + v_{yy}) + u_x^2 + v_x^2 + u_y^2 + v_y^2) = 2(u\mathcal{L}(u) + v\mathcal{L}(v) + |f_x|^2$

$+ |f_y|^2) = 2(|f_x|^2 + |f_y|^2) := 0$, $f_x = f_y = 0$, $f = c$ #1 // $u_{xx} + u_{yy} = v_{xx} + v_{yy} = 0$,

$f^2 = u^2 - v^2 + i(2uv)$, $2(u u_x - v v_x)_x + 2(u u_y - v v_y)_y = 2(u_x^2 + u u_{xx} - v_x^2 - v v_{xx} + u_y^2 + u u_{yy} - v_y^2 - v v_{yy})$

$= 2(u_x^2 + u_y^2 - v_x^2 - v_y^2) = 0$, $u_x^2 + u_y^2 = v_x^2 + v_y^2$, $(u_x v + u v_x)_x + (u_y v + u v_y)_y = 2u_x v_x + 2u_y v_y = 0$,

$-\frac{u_x}{u_y} = \frac{v_y}{v_x}$, $u_y^2\left(1 + \frac{v_y^2}{v_x^2}\right) = v_x^2 + v_y^2$, $u_y^2(v_x^2 + v_y^2) = v_x^2(v_x^2 + v_y^2)$, $u_y = \pm v_x$, $u_x = \mp v_y$ #2 //

$\begin{vmatrix} u_{xx} & u_{xy} \\ u_{yx} & u_{yy} \end{vmatrix} = -u_{xx}^2 - u_{xy}^2 < 0$,  #3 // $u = \operatorname{Re} f$, $u_{xxx} + u_{xyy}$, u_{xyy}

$= u_{yxy} = u_{yyx}$, $\mathcal{L}(u_x) = (u_{xx} + u_{yy})_x = 0$. $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{r}$, $r_x = \cos\theta$, $r_y = \sin\theta$, $\theta = \tan^{-1}\frac{y}{x}$, $\frac{y}{x} = \tan\theta$, $-\frac{y}{x^2} = \sec^2\theta \frac{\partial\theta}{\partial x}$,

$\frac{\partial\theta}{\partial x} = -\frac{y}{x^2}\cos^2\theta = -\frac{1}{x}\tan\theta\cos^2\theta = -\frac{\sin\theta}{r}$, $\frac{1}{x} = \sec^2\theta \frac{\partial\theta}{\partial y}$, $\theta_y = \frac{1}{x}\cos^2\theta$, $\theta_y = \frac{\cos\theta}{r}$, $\mathcal{L}(u(r,\theta)) = (u_r r_x + u_\theta \theta_x)_x$

$+ (u_r r_y + u_\theta \theta_y)_y = u_{rr} r_x^2 + u_r r_{xx} + u_{\theta\theta}\theta_x^2 + u_\theta \theta_{xx} + \ldots$, $r_{xx} = \frac{r - x r_x}{r^2} = \frac{r - x^2/r}{r^2} = \frac{1}{r} - \frac{x^2}{r^3}$

$= \frac{\sin^2\theta}{r}$, $r_{yy} = \frac{\cos^2\theta}{r}$, $\theta_{xx} = \frac{-\cos\theta\, \theta_x r + \sin\theta\, r_x}{r^2} = r^{-2}\left(\frac{\sin\theta\cos\theta}{} + \sin\theta\cos\theta\right) = \frac{2\sin\theta\cos\theta}{r^2}$, $\theta_{yy} = \frac{-\sin\theta\,\theta_y r - \cos\theta\, r_y}{r^2}$

$= -r^{-2}\, 2\sin\theta\cos\theta$, $\mathcal{L}(u(r,\theta)) = u_{rr}\cos^2\theta + u_r\frac{\sin^2\theta}{r} + u_{\theta\theta}\frac{\sin^2\theta}{r^2} + u_\theta\left(\frac{2\sin\theta\cos\theta}{r^2}\right) + u_{rr}\sin^2\theta + u_r\frac{\cos^2\theta}{r} + u_{\theta\theta}\frac{\cos^2\theta}{r^2}$

$+ u_\theta\left(\frac{-2\sin\theta\cos\theta}{r^2}\right) = u_{rr} + u_r/r + u_{\theta\theta}/r^2$, $\mathcal{L}(P_r(\theta - t))$, $\frac{\partial}{\partial r} = -2r(1 - 2r\cos(\theta-t) + r^2) - (1-r^2)(2r - 2\cos(\theta-t))/$

$(\ )^2 = \{r^2(4\cos(\theta-t) - 2\cos(\theta-t)) + r(-2-2) + 2\cos(\theta-t)\}/(\ )^2 = (2(r^2+1)\cos(\theta-t) - 4r)/(1 - 2r\cos + r^2)^2$,

$u_\theta := -(1-r^2)2r\sin(\theta-t)/(\ )^2$, $u_{\theta\theta} = -(1-r^2)2r\cos(\theta-t)/(\ )^2 + 2(1-r^2)4r^2\sin^2(\theta-t)/(\ )^3$,

Rudin #6 $\quad P_r = \frac{1-r^2}{1-2r\cos(\theta-t)+r^2}$

$$u_{\theta\theta} = \frac{-(1-r^2)2r\cos\cdot(\ \ ) + 2(1-r^2)4r^2\sin^2}{(\ \ )^3} = -2r(1-r^2)\{\cos - 2r\cos^2 + r^2\cos + 4r^2\sin^2\}/(\ \ )^3$$

$$= -2r(1-r^2)\{(1+r^2)\cos - 2r - 2r\sin^2\}/(\ \ )^3, \quad u_{rr} = \frac{4r\cos-4}{(\ \ )^2} - 2(2(r^2+1)\cos-4r)(-2\cos+2r)/(\ \ )^3$$

$$u_{rr} + u_{\theta\theta}/r^2 = \frac{4r\cos-4}{(\ \ )^2} - 2\,(1-r^2)\{(\tfrac1r + r)\cos - 2 - 2\sin^2\} + (2(r^2+1)\cos - 4r)($$

$$-2\cos+2r) = \cos\{(1-r^2)(\tfrac1r + r) + 2(r^2+1)2r + 8r\} - 2(1-r^2)(\sin^2+1) - 8r^2 - 4(r^2+1)\cos^2 = \{\tfrac1r - r^3 + 4r^3$$

$$\tfrac1r + 3r^3 + 12r$$

$$+12r + 4\}\cos - 2(1-r^2)(2-\cos^2) - 8r^2 - 4(r^2+1)\cos^2 = (\tfrac1r + 3r^3 + 12r)\cdot\cos - 4 - 4r^2 + (-2-6r^2)\cos^2$$

$$4r\cos - 4 + 2(r+\tfrac1r)\cos - 4 = (6r + \tfrac2r)\cos - 8, \quad 0 = ?\ ((6r+\tfrac2r)\cos - 8)\cdot(1-2r\cos + r^2) - 2(\tfrac1r + 3r^3 + 12r$$

$$)\cos + 8 + 8r^2 + (4+12r^2)\cos^2 = \cos^2(-12r^2 - 4 + 4 + 12r^2) + \cos\cdot\{16r + (6r + \tfrac2r)(1+r^2) - \tfrac2r$$

$$-6r^3 - 24r\} + 8(1+r^2) - 8(1+r^2) = \cos\cdot\{16r + 8r + 6r^3 + \tfrac2r - \tfrac2r - 6r^3 - 24r\}$$

$= 0.$

$\log|f| = \log\sqrt{u^2+v^2}$, $\frac{\partial}{\partial x} = \frac{uu_x + vv_x}{u^2+v^2}$, $\frac{\partial}{\partial y} = \frac{uu_y+vv_y}{u^2+v^2}$, $\frac{\partial^2}{\partial x^2} = (u_x^2 + uu_{xx} + v_x^2 + vv_{xx})/(u^2+v^2)$

$-2(uu_x + vv_x)^2/(u^2+v^2)^2$, $\Delta = (u^2+v^2)^{-1}(u_x^2 + v_x^2 + u_y^2 + v_y^2) - 2(u^2+v^2)^{-2}(u^2u_x^2 + v^2v_x^2 + 2uvu_xv_x$

$+ u^2u_y^2 + v^2v_y^2 + 2uvu_yv_y)$, $\quad (u^2+v^2)^2\cdot\Delta = (u^2+v^2)(u_x^2+v_x^2+u_y^2+v_y^2) - 2((uu_x+vv_x)^2 + (uu_y+vv_y)^2)$

$= -u^2u_x^2 + u^2(v_x^2+v_y^2) - u^2u_y^2 + v^2(u_x^2+u_y^2) - v^2v_x^2 - v^2v_y^2 - 4uv(u_xv_x + u_yv_y)$

$= u^2(u_y^2 + u_x^2 - u_x^2) - u^2u_y^2 + v^2(u_x^2 + u_y^2 - u_y^2 - u_x^2) - 4uv(-u_xu_y + u_yu_x) = 0.$ $\quad A_n :=$

$\Omega \cap \{u+iv \in \mathbb{C} : 2\pi n \le v < 2\pi(n+1)\}$, $f_n : A_n \to \mathbb{C}$, $f_n : z \mapsto e^z \in \mathcal{H}(A_n)$, $f_n^{-1} \in \mathcal{H}(e^{A_n})$,

$\log|f| = \mathrm{Re}\, f_n^{-1}$ #5 $\quad \varphi\circ(f\bar f) = \varphi(u^2(z) + v^2(z))$, $2\partial = \varphi'(f\bar f)(2uu_x + 2vv_x + i2uu_y + i2vv_y)$,

$4\bar\partial\partial = \varphi''(f\bar f)(2uu_x + 2vv_x + i2uu_y + i2vv_y)(2uu_x+2vv_x) + 2\varphi'(f\bar f)(u_x^2 + uu_{xx} + v_x^2 + vv_{xx} + i(u_xu_y + uu_{yx} + v_xv_y + vv_{xy}))$

$-i\varphi''(f\bar f)(2uu_y + 2vv_y)(2uu_x + 2vv_x + i2uu_y + i2vv_y) - i2\varphi'(f\bar f)(u_yu_x + uu_{xy} + v_yv_x + vv_{xy} + iu_y^2 + iuu_{yy}$

$+ iv_y^2 + ivv_{yy}) = \varphi''(f\bar f)2(uu_x + vv_x + iuu_y + ivv_y)2(uu_x + vv_x - iuu_y - ivv_y) + 2\varphi'(f\bar f)\{u_x^2 + uu_{xx}$

$+ v_x^2 + vv_{xx} + u_y^2 + uu_{yy} + v_y^2 + vv_{yy} + i(u_xu_y + uu_{xy} + v_xv_y + vv_{xy} - u_xu_y - uu_{xy} - v_yv_x - vv_{xy})\} =$

$4\varphi''(f\bar f)\,|uu_x + vv_x + i(uu_y + vv_y)|^2 + 2\varphi'(f\bar f)(u_x^2 + uu_{xx} + v_x^2 + vv_{xx} + u_y^2 + uu_{yy} + v_y^2 + vv_{yy})$, $\quad \partial\varphi\circ|f|^2$

$= \frac12\varphi'(|f|^2)\left(\frac{\partial}{\partial x}|f|^2 + i\frac{\partial}{\partial y}|f|^2\right) = \varphi'(|f|^2)\,\partial|f|^2$, $\quad \partial f(z)g(z) = \frac12(f_xg + fg_x - if_yg - ifg_y)$

$= \partial f\cdot g + f\,\partial g$, $\quad \bar\partial\partial\varphi\circ|f|^2 = \bar\partial\varphi'(|f|^2)\,\partial|f|^2 + \varphi'(|f|^2)\,\bar\partial\partial|f|^2 = \varphi''(|f|^2)\,\partial|f|^2\,\bar\partial|f|^2 + \varphi'(|f|^2)\,\bar\partial\partial|f|^2$

Rudin #7

$\partial\bar\partial|f|^2 = \partial(uu_x+vv_x+iuu_y+ivv_y) = \frac12\big(u_x^2+uu_{xx}+v_x^2+vv_{xx}+i(u_xu_y+uu_{xy}+v_xv_y+vv_{xy})$

$-i(u_yu_x+uu_{xy}+v_xv_y+vv_{xy})+u_y^2+uu_{yy}+v_y^2+vv_{yy}\big) = \frac12\big(u_x^2+v_x^2+uu_{xx}+vv_{xx}+u_y^2+uu_{yy}+v_y^2+vv_{yy}\big)$

$=\frac12\big(u_x^2+u_y^2+uu_{xx}-vu_{yx}+u_y^2+uu_{yy}+u_x^2+vu_{xy}\big) = u_x^2+u_y^2+\frac{u}{2}(u_{xx}+u_{yy}) = u_x^2+u_y^2$

$=u_x^2+v_x^2 = |u_x+iv_x|^2 = |f_x|^2$, $|f'|^2 = |\partial f|^2 = |\frac12(u_x+iv_x-iu_y+v_y)|^2 = |u_x+iv_x|^2$, $\partial\bar\partial|f|^2 = |f'|^2$

$\partial\bar\partial\,\varphi\circ|f|^2 = \varphi''(|f|^2)\,\partial|f|^2\,\bar\partial|f|^2+\varphi'(|f|^2)|f'|^2 = \big(\varphi''(|f|^2)\,\partial|f|^2\,\bar\partial|f|^2/|f'|^2+\varphi'(|f|^2)\big)|f'|^2$,

$|f|^2 = ?\ \frac{\partial|f|^2\,\bar\partial|f|^2}{|f'|^2} = |f'|^{-2}\,|\partial|f|^2|^2 = |f'|^{-2}(|f|\,\partial|f|)^2 = (|f|/|f'|)^2(\partial|f|)^2$, $\cancel{|f'|^2 = ?\ (\partial|f|)^2}$

$|f|^2 = ?\ |f'|^{-2}\,\|uu_x+vv_x-ivv_y-iuu_y\|^2 = |f'|^{-2}\,\|uu_x-vu_y-ivu_x-iuu_y\|^2$,

$|f'|^{-2}\,|2|f|\,\partial|f||^2 = 4(|f|/|f'|)^2\,|\partial|f||^2 = \frac{|f'|^2}{4} = ?\ |\partial|f||^2 = 1$, $|f'|^{-2}\,\|u^2u_x^2+v^2u_y^2-$

$2uvu_xu_y+v^2u_x^2+u^2u_y^2+2uvu_xu_y|^2 = |f'|^{-2}\,|u_x^2|f|^2+u_y^2|f|^2| = |f|^2$. $\frac14\Delta(|f|^\alpha) = \partial\bar\partial(f\bar f)^{\alpha/2}$

$=|f'|^2\big(\frac\alpha2|f|^{2(\frac\alpha2-1)}+|f|^2\frac\alpha2(\frac\alpha2-1)|f|^{2(\frac\alpha2-2)}\big) = |f'|^2\frac\alpha2\big(|f|^{\alpha-2}+(\frac\alpha2-1)|f|^{\alpha-2}\big) = |f'|^2\frac\alpha2|f|^{\alpha-2}\cancel{(1+(\frac\alpha2-1))|f|}$

$=|f'|^2\frac\alpha2|f|^{\alpha-2}\frac\alpha2$. $\Delta(g\circ h) = (g'(h)h_x)_x+(g'(h)h_y)_y = g''(h)h_x^2+g'(h)h_{xx}+g''(h)h_y^2+g'(h)h_{yy}$

$=g''(h)(h_x^2+h_y^2)+g'(h)\Delta h$, $h\in\mathcal{H}(\Omega)$, $h_x^2+h_y^2 = (u_x-iu_y)^2+(u_y+iu_x)^2 =$

$u_x^2+u_y^2-u_y^2-u_x^2 = 0$, $\Delta(g\circ f) = \big(g_u(f)u_x+g_v(f)v_x\big)_x+\big(g_u(f)u_y+g_v(f)v_y\big)_y =$

$(g_{uu}(f)u_x+g_{uv}(f)v_x)u_x+g_u(f)u_{xx}+(g_{vu}(f)u_x+g_{vv}(f)v_x)v_x+g_v(f)v_{xx}$

$+(g_{uu}(f)u_y+g_{uv}(f)v_y)u_y+g_u(f)u_{yy}+(g_{vu}(f)u_y+g_{vv}(f)v_y)v_y+g_v(f)v_{yy}$

$=g_{uu}(f)(u_x^2+u_y^2)+g_{uv}(f)(2u_xv_x+2u_yv_y)+g_{vv}(f)(v_x^2+v_y^2)+g_u(f)(u_{xx}+u_{yy})$

$+g_v(f)(v_{xx}+v_{yy}) = ((\Delta g)\circ f)(u_x^2+u_y^2)+2g_{uv}(f)(-u_xu_y+u_yv_x)$

$=((\Delta g)\circ f)\,|f'|^2$ #7 // $\pi r^2u(0) = \iint_{D(0,r)}u(x,y)\,dx\,dy = \int_0^{2\pi}\int_0^r u(\rho,\theta)\rho\,d\rho\,d\theta$, $2\pi r\,u(0) =$

$\int_0^{2\pi}u(r,\theta)\,r\,d\theta$, $u(0) = \frac{1}{2\pi}\int_0^{2\pi}u(r,\theta)\,d\theta$ #9 // $\int\frac{1}{t+i\varepsilon}-\int\frac{1}{t-i\varepsilon} = -\int\frac{2i\varepsilon}{t^2+\varepsilon^2} = \frac{-2i}{\varepsilon}\int\frac{dt}{1+(\frac t\varepsilon)^2}$

$=-2i\tan^{-1}(\frac t\varepsilon)\Big|$, $\int_a^b\frac{\varphi(t)}{t-(x+i\varepsilon)}-\int\frac{\varphi(t)}{t-(x-i\varepsilon)} = \int_a^b\varphi(t)\frac{2i\varepsilon}{(t-x)^2+\varepsilon^2} = \frac{2i}{\varepsilon}\int_a^b\frac{\varphi(t)}{1+(\frac{t-x}{\varepsilon})^2}$, $|\ldots|$

$\le\frac2\varepsilon\|\varphi\|_\infty\int_a^b\frac{dt}{1+(\frac{t-x}{\varepsilon})^2} = 2\|\varphi\|_\infty\tan^{-1}(\frac{t-x}{\varepsilon})\Big|_a^b = 2\|\varphi\|_\infty\left|\tan^{-1}\frac{b-x}{\varepsilon}-\tan^{-1}\frac{a-x}{\varepsilon}\right| \to \begin{cases}0, & x\notin[a,b]\\ 2\pi\|\varphi\|_\infty, & x\in(a,b)\end{cases}$

Rudin #8

$\int_a^b \frac{\varphi(t)\varepsilon}{(t-x)^2+\varepsilon^2} = \int_{a-x}^{b-x} \frac{\varphi(t+x)\varepsilon}{t^2+\varepsilon^2}$, $-\delta < t < \delta \Rightarrow \varphi(x)-\eta < \varphi(t+x) < \varphi(x)+\eta$, $\int_{-\delta}^{\delta} \frac{\varphi(t+x)\varepsilon}{t^2+\varepsilon^2} < \int \frac{(\varphi(x)+\eta)\varepsilon}{t^2+\varepsilon^2}$

$< \frac{1}{\varepsilon}(\varphi(x)+\eta\varepsilon) \int_{-\delta}^{\delta} \frac{1}{1+(t/\varepsilon)^2} = (\varphi(x)+\eta\varepsilon)\left(\tan^{-1}\left(\delta/\varepsilon\right) - \tan^{-1}\left(-\delta/\varepsilon\right)\right) \xrightarrow[\varepsilon\to 0]{} \pi(\varphi(x)+\eta)$, $\lim \int_a^b \frac{\varphi(t)}{t-z} = \begin{cases} 2\pi i\,\varphi(x), & x\in(a,b) \\ 0, & x\notin(a,b) \end{cases}$

$(\varphi \in L^1)$ $0 < t < \delta \Rightarrow R-\eta < \varphi(t+x) < R+\eta$, $R := \lim_{t\to 0^+} \varphi(t+x)$, $\int_0^{\delta} \frac{\varphi(t+x)\varepsilon}{t^2+\varepsilon^2} < (R+\eta)\left(\tan^{-1}\left(\delta/\varepsilon\right)\right.$

$\left.-\tan^{-1} 0\right) = (R+\eta)\tan^{-1}\left(\delta/\varepsilon\right) \xrightarrow[\varepsilon\to 0]{} \frac{\pi}{2}(R+\eta)$, $\int_{-\delta}^{0} \to \frac{\pi}{2}(L+\eta)$, $\int_{-\delta}^{\delta} \to \pi\left(\frac{L+R}{2}+\eta\right)$,

$\int_a^b \frac{\varphi(t)}{t-z}$ $\lim_{\varepsilon\to 0}[\ldots] = \begin{cases} \frac{L+R}{2} & x\in(a,b) \\ 0 & x\notin[a,b] \end{cases}$ // #10 $f(x) = u(x) + iv(x)$,

$\cancel{A}\ \mathbb{P}(B=0 \mid A=a) = \mathbb{P}(A^L=a \mid B=0)\,\mathbb{P}(B=0) / \mathbb{P}(A=a) = (1-q)\,\mathbb{P}(A^L=a) / \mathbb{P}(A=a) = 1-q$

$$\mathbb{P}(U_{t+1}=u, L_{t+1}=l, Z_{t+1}=z \mid A_t=a) = \frac{1}{2}\sum_b \mathbb{P}(\cdots \mid A_t=a, \cancel{B_t=b) \cdot \frac{1}{2}})$$

$$= (1-q)\,\mathbb{P}(U_{t+1}=u)\,\mathbb{P}(Z_{t+1}=z)\,\mathbb{P}(L_{t+1}=l \mid A^L_t=a) + q\,\mathbb{P}(L_{t+1}=l)\,\mathbb{P}(Z_{t+1}=z)\,\mathbb{P}(U_{t+1}=u \mid A^U_t=a)$$

$$= \frac{1}{4}\,\mathbb{P}(A=a \mid L=l, U=u)$$

$|v+w| = 2\left|\cos\frac{\theta_{vw}}{2}\right| = 2\sqrt{\frac{1}{2}|\cos\theta_{vw}+1|} = \sqrt{|2\cos\theta+2|} \ge \sqrt{(2+\sqrt{3})}$, $\cos\theta \ge \frac{\sqrt{3}}{2}$, $-2\cos\theta-2 \le -2-\sqrt{3}$, $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$,

$-\frac{\pi}{6} \le 2\pi\frac{j}{1997} \le \frac{\pi}{6}$, $-\frac{1997}{12} \le j \le \frac{1997}{12} = 166\frac{5}{12}$, $|j_v - j_w| \le 166$, $1997\cdot 332 \Big/ \binom{1997}{2} = \frac{1997\cdot 332\cdot 2!\cdot 1995!}{1997!}$

$= \frac{332\cdot 2}{1996} = \frac{166}{499} = \frac{2\cdot 83}{499}$ #906 // $1 - \frac{365\cdots(365-n+1)}{365^n}$ #907 // $\sum_{j=0}^{3}(2j+3)!\,(2(3-j)+3)!$, $\binom{6}{3}\cdot 3\cdot\sum_{j=0}^{3}$ —

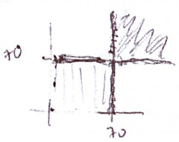
$= \binom{6}{3}\cdot 3\cdot\left(2\cdot 9! + 4!7! + 6!5! + 8!3!\right) = 6\cdot5\cdot4\cdot3\left(9!/3 + 4\cdot 7! + 6!\cdot5\cdot4 + 8!\right)$, $\frac{1}{12!}(\cdots) = 360\left(\frac{1}{12\cdot11\cdot10\cdot3} + \frac{4}{12\cdots8}\right.$

$\left. + \frac{5\cdot4}{12\cdots6} + \frac{1}{12\cdots9}\right) = \frac{1}{11} + \frac{4}{11\cdot3\cdot8} + \frac{5\cdot4}{11\cdot3\cdot8\cdot7\cdot6} + \frac{1}{11\cdot3} = \frac{1}{11}\left(1 + \frac{1}{6} + \frac{1}{3} + \frac{5}{252}\right) = \frac{1}{11}\cdot\frac{126\cdot3+5}{252} = \frac{383}{11\cdot252}$ #908 //

$\frac{(m-1)(n-m)}{(m-1)^2}$ #909 // $\cancel{\binom{n}{2}2^{-3} + 2^{-3}\binom{n}{3} = 2^{-3}\left(\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6}\right) = 2^{-4}\,\frac{n+1}{3} = \frac{n(n^2-1)}{48}}$ $\binom{3}{2}2^{-3} + \binom{3}{3}2^{-3} = \frac{1}{2}$ #915 //

$\mathbb{P}(C|P) = \frac{\mathbb{P}(P|C)\mathbb{P}(C)}{\mathbb{P}(P|C)\mathbb{P}(C) + \mathbb{P}(P|\bar{C})\mathbb{P}(\bar{C})} = \frac{6/1000}{6/1000 + 693/10^4} = \frac{6}{6+69.3} = 6\cdot\frac{10}{753} = \frac{20}{251}$ #916 // $\cancel{p(5,2) = \frac{1}{2}p(4,2) + \frac{1}{2}p}$

$p\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$ $p(5,2) =$



$\mathbb{P}(x\wedge y \ge 70) = \mathbb{P}\big(((x\ge70)\vee(y\ge70))\setminus(x\vee y\ge70)\big) + \mathbb{P}(x\ge y\ge70) = \mathbb{P}(x\ge70 \vee y\ge70) + \mathbb{P}(x\ge y\ge70) - c = a+b-c$ #918 //

$5\alpha = \sum_{j=0}^{n-1} \binom{n}{j} 5^{n-j}(-1)^j = 4^n - (-1)^n = 4^n + 1$, $\alpha = \frac{1}{5}(4^n+1)$, $\sum_{j=0}^{n-1} 4^j$, $\sum_{j=0}^{2k} 4^j$

$\overset{(5)}{\equiv} 1 + \cancel{\sum_{j=1}^{2k}(-1)^j}$ $\alpha = \frac{1}{5}(4\cdot 16^k+1)$, $k \in \mathbb{N}$, $4\cdot 5 \underset{(7)}{\equiv} -1$, $2^k \underset{(7)}{\not\equiv} 5$, $\alpha \underset{(3)}{\equiv} 1$,

$1 + 4^{2k+1} = 1-(-4)^{2k+1} = (-5)\sum_{j=0}^{2k}(-4)^j$, $\alpha = \sum_{j=0}^{2k}(-4)^j$, $k \geq 2$, $= 205 + \sum_{j=5}^{2k+1}(-4)^j$

$= 205 - 64\sum_{j=0}^{2k+1}(-4)^j = 205 + 64\frac{1}{5}(4^{2(k+1)}-1)$, $\overset{(7)}{\equiv} 2 + 3(2^{k+1}-1) \not\equiv 0$, $\overset{(3)}{\equiv}$, $(k=0)$ $397 = 205 - 1024$

$+4096 = 205 - 1024\sum_{j=0}^{2k+1}(-4)^j = 205 + 1024\cdot\frac{1}{5}(16^{k+1}-1)$, $k \geq 0$, 3277, $205\cdot 16^{k+1} - \frac{1}{5}(16^{k+1}-1)$, $\frac{1}{5}(1024\cdot 16^{k+1}$

$+1) = \frac{1}{5}(2^{14+4k}+1) =$

$\frac{1}{5}(4\cdot 16^{k+3}+1)$, $(\sum_{j=0}^{k-1} 4^{2j})(-3) + 4^{2k}$

$= -3\sum^{k-1} 16^j + 16^k$, $1 + 3\sum_0^{k-1} 4^{2j+1}$

$\prod(x-r_j)^2 + \prod(x-\bar{r}_j)^2 = -2P(x) + Q_2^2(x)$

$= 2P(x) + Q_1^2(x)$, $-4P(x) = Q_1^2(x) - Q_2^2(x) = -|Q_1(x)|^2 - Q_2^2(x)$ #82 //

$\prod(x-r_j) + \prod(x-\bar{r}_j) = Q_2(x)$ $= Q_2(x)$

$\prod(x-r_j) - \prod(x-\bar{r}_j)$ $~~= 2i\,\mathrm{Im}(r_1 r_2) + 2i\,\mathrm{Im}(r_1 r_2) = c~~$ $= Q_1(x)$

$\prod(x-r_j) - \prod(x-\bar{r}_j)$

$\prod(x-r_j)^2 + \prod(x-\bar{r}_j)^2 = 2P(x) + c^2$

$= -2P(x) + d^2$

$= \prod(x-\bar{r}_j) - \prod(x-\bar{r}_j)$, $(x-r_1)(x-\bar{r}_1)(x-r_2)(x-\bar{r}_2)$

$\mathrm{Re}\, Q_1 = 0$ $(x^2 - 2\mathrm{Re}\, r_1 x + |r_1|^2)(x^2 - 2\mathrm{Re}\, r_2 x + |r_2|^2)$

$x^4 - 2(\mathrm{Re}\, r_1 + \mathrm{Re}\, r_2)x^3 + (|r_1|^2 + |r_2|^2 + 4\mathrm{Re}\, r_1 + \mathrm{Re}\, r_2)x^2 - 2(|r_1|\mathrm{Re}\, r_2 + |r_2|\mathrm{Re}\, r_1)x + |r_1 r_2|^2$

$(ax^2+bx+c)^2 + (dx^2+ex+f)^2$

$= (a^2+d^2)x^4 + (b^2+e^2)x^2 + c^2 + f^2 + 2(ab+de)x^3 + 2(ac+df)x^2 + 2(bc+ef)x$

$= (a^2+d^2)x^4 + 2(ab+de)x^3 + (b^2+e^2+2(ac+df))x^2 + 2(bc+ef)x + c^2 + f^2$

$(p=1,\ q=2)$ $(A+B)$, $1+2B$

$C + BC = I$, $BC = B$, $C = 1$, $B' = 0 \Rightarrow B = 0$

$~~B^{q-1}C = B^{q-1}~~$ $~~CAB = C,\ B^{q-1}~~$

$B^{q-1}C = B^{q-1}$, $~~C = I,\ B = 0~~$

$\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix} = \begin{pmatrix}1&0\\0&0\end{pmatrix}$

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{matrix}1&0\\0&1\end{matrix}, \begin{matrix}1&0\\0&-1\end{matrix} = \begin{matrix}1&0\\0&-1\end{matrix}$

$AC + BC = I$

$C + A^{p-1}BC = A^{p-1}$

$B^{q-1}C = B^{q-1}A^{p-1}$

$B^{q-1}CA = B^{q-1}$

$A^{pq} - B^{pq} = (A-B)(A^{pq-1} + A^{pq-2}B + \dots + AB^{pq-2} + B^{pq-1})$

$= (A-B)(A^{-1} + A^{-2}B + \dots + A^{pq-1-(q-1)}B^{q-1})$

$= (A-B)(A^{-1} + A^{-2}B + \dots + A^{-q}B^{q-1})$

$A^{pq} - (-B)^{pq} = (A+B)(A^{-1} - A^{-2}B + \dots + (-1)^{q-1}A^{-q}B^{q-1})$

$I =$

$2^x + 3^x - 2^{2x} - 3^{2x} + (2\cdot 3)^x$

$(2^x + 3^x - \frac{1}{2})^2 = 2^{2x} + 3^{2x} + \frac{1}{4} + 2(2\cdot 3)^x - 2^x - 3^x$

$2^{2x} + 3^{2x} - 2^x - 3^x \geq 3(2\cdot 3)^x - \frac{1}{4}$, $\frac{3}{4}$, $6^x \leq 1$, $-\frac{3}{4}$ #95 //

$6^x \leq \frac{1}{4}$, $x \leq -\log 4/\log 6 < 0$, $x = 0 \Rightarrow 2+3-4+6-9 \leq |1|-|1|-1$

$a^2 - a + b - b^2 = (a+b)(a-b) + (a-b) = (a+b-1)(a-b)$, $a^2 + b < 0 \Rightarrow a^4 < b^2$,

$b^2 + a < 0$, $b^4 < a^2$, $(a+b+\frac{1}{2})^2 = a^2 + b^2 + \frac{1}{4} + 2ab + a + b$, ≥ 0,

$\geq -(2ab + \frac{1}{4}) \geq -5/4$

$(-1 \leq a, b \leq 0)$ $|a| < |b| \Rightarrow b^2 + a \geq b^2 + b$, $b^2 > a^2$

$b^2 + a \geq a^2 + a$, $b^2 + a \geq \min(a^2+a)$

$\geq \min(a^2+a) = -\frac{1}{4} \Leftarrow (a+\frac{1}{2})^2 \geq 0$ #94 //