ROCKETBOOK EVERLAST

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(do not microwave this notebook)

$$\hat{\Theta} = \sum_{ij} \varphi_{ij} / M^2 = \sum_{i<j} \varphi_{ij} \Big/ \binom{M}{2} = \sum_{ij} \varphi_{ij} \Big/ M^2 .$$

$$V(x_i) = \sum_j \varphi_{ij} / M, \quad V(y_j) = \sum_i \varphi_{ij} / M = \quad V(x_i) = \varphi_{i\cdot}/M, \quad V(y_j) = \varphi_{\cdot j}/M$$

$$S_x = \sum_i (V(x_i) - \hat{\theta})^2, \quad S_y = \sum_j (V(y_j) - \hat{\theta})^2$$

$$S_{xy} = \sum_k (V(x_k) - \hat{\theta})(V(y_k) - \hat{\theta})$$

$$\hat{\sigma}^2 = S_x/M^2 + S_y/M^2 + 2S_{xy}/M^2 = [M(M-1)]^{-1} \sum_j (V(x_j) + V(y_j) - 2\hat{\theta})^2$$

$$\theta_{-i} = \sum_{j \neq i, k \neq i} \varphi_{jk} \Big/ (M-1)^2, \quad \hat{\theta}_j = M\hat{\theta} - (M-1)\theta_{-j}, \quad \hat{\sigma}^2 = (M(M-1))^{-1} \sum_j (\hat{\theta}_j - \bar{\hat{\theta}})^2,$$

$$\bar{\hat{\theta}} = M\hat{\theta} - \frac{M-1}{M} \sum_j \theta_{-j}, \quad (M-1)^2 \theta_{-i} = \sum_{jk} \varphi_{jk} - \sum_j \varphi_{ij} - \sum_i \varphi_{ij} + \varphi_{ii}$$

$$= M^2\hat{\theta} - \varphi_{i\cdot} - \varphi_{\cdot i} + \varphi_{ii}, \quad \sum_j \theta_{-j} = \frac{M^3}{(M-1)^2}\hat{\theta} - \frac{2M^2\hat{\theta}}{(M-1)^2} + \frac{\operatorname{diag}\varphi}{(M-1)^2},$$

$$\bar{\hat{\theta}} = \hat{\theta}\left(M - \frac{M^2}{M-1} + \frac{2M}{M-1}\right) - \frac{\operatorname{diag}\varphi}{M(M-1)}, \quad 1 - \frac{M-2}{M-1} = \frac{1}{M-1},$$

$$\bar{\hat{\theta}} = \frac{1}{M-1}\left(M\hat{\theta} - \operatorname{diag}\varphi / M\right), \quad \hat{\theta}_j - \bar{\hat{\theta}} = M\hat{\theta} - (M-1)\theta_{-j} - \frac{M}{M-1}\hat{\theta} + \frac{\operatorname{diag}\varphi}{M(M-1)}$$

$$= \frac{\hat{\theta}M(M-2)}{M-1} + \frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{1}{M-1}(M^2\hat{\theta} - \varphi_{j\cdot} - \varphi_{\cdot j} + \varphi_{jj})$$

$$= -\frac{2M}{M-1}\hat{\theta} + \frac{\operatorname{tr}\varphi}{M(M-1)} + \frac{\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}}{M-1}, \quad \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}) = 2M^2\hat{\theta} - \operatorname{tr}\varphi,$$

$$M\left(\frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)^2 + 2\left(\frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)\frac{2M^2\hat{\theta} - \operatorname{tr}\varphi}{M-1} + \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2}$$

$$= \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2} - M\left(\frac{\operatorname{tr}\varphi - 2M^2\hat{\theta}}{M(M-1)}\right)^2 = M(M-1)\hat{\sigma}^2_{jk}, \quad \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat{\theta}\right)^2$$

$$= M^{-2} \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj} + \varphi_{jj} - 2M\hat{\theta})^2, \quad \sum (\varphi_{jj} - 2M\hat{\theta})^2 = \operatorname{tr}\varphi - 4M\hat{\theta}\operatorname{tr}\varphi$$

$$+ 4M^3\hat{\theta}^2, \quad \sum (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})(\varphi_{jj} - 2M\hat{\theta}) =$$

$$-2M\hat{\theta}(2M^2\hat{\theta} - \operatorname{tr}\varphi) + \sum \varphi_{j\cdot}\varphi_{jj} + \sum \varphi_{\cdot j}\varphi_{jj} - \operatorname{tr}\varphi, \ -\operatorname{tr}\varphi - 4M^3\hat{\theta}^2 + 2\sum \varphi_{jj}($$

$$\varphi_{j\cdot} + \varphi_{\cdot j}) + \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2, \quad \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{M^2} - \frac{\operatorname{tr}\varphi}{M^3} - 4M\hat{\theta}^2$$

$$+ \frac{2}{M^2} \sum \varphi_{jj}(\varphi_{j\cdot} + \varphi_{\cdot j}) = M(M-1)\hat{\sigma}^2_0$$

$$M(M-1)\hat\sigma^2_{osu} = \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat\theta\right)^2 = M^{-2}\left(\sum_j \varphi_{j\cdot}^2 + \sum_j \varphi_{\cdot j}^2 + 2\sum \varphi_{\cdot j}\varphi_{j\cdot}\right)$$

$$-4\hat\theta/M \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j}) + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1}$$

$$- 4\hat\theta/M \cdot 2M^2\hat\theta + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\dots)\mathbb{1} - 4M\hat\theta^2 =$$

$$M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4/M\,\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} \qquad \frac{1}{(M-1)^2} - \frac{1}{M^2} = \frac{2M-1}{(M(M-1))^2}$$

$$0 \doteq ? \quad \frac{2M-1}{(M(M-1))^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \frac{2}{(M-1)^2}\mathbb{1}^T(\varphi + \varphi^T - I/2$$

$$-\frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T + \operatorname{diag}\varphi\,\mathbb{1}^T/2M)\cdot\operatorname{diag}\varphi$$

$$\frac{M-1/2}{M^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \mathbb{1}^T(\varphi + \varphi^T - \frac{I}{2} - \frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T +$$

$$\operatorname{diag}\varphi\,\mathbb{1}^T/2M)\operatorname{diag}\varphi \overset{(1)}{=} \frac{1}{2}(M-1)^3 M(\sigma^2_{jk} - \sigma^2_{osu})$$

$$(\varphi = \varphi^T) \quad \mathbb{1}^T\varphi(I - \frac{1}{M}\mathbb{1}\mathbb{1}^T)\varphi\mathbb{1} = \sum s_j^2 - \frac{1}{M}(\sum s_j)^2 = (M-1)\operatorname{var}(\mathbb{1}^T\varphi) = \operatorname{var}(\mathbb{1}^T\varphi^T)$$

$$\le \frac{M}{2}\left((M - M/2)^2 + (M/2)^2\right) = \frac{M^3}{4}$$

$$\mathbb{1}^T(\varphi^T\varphi - \varphi\mathbb{1}\mathbb{1}^T\varphi/M)\mathbb{1} = \mathbb{1}^T(\varphi^T - \varphi\mathbb{1}\mathbb{1}^T/M)\varphi\mathbb{1}, \quad \mathbb{1}^T\varphi \cdot \mathrm{Proj}_{\mathbb{1}^\perp}\varphi\mathbb{1}$$

$$= \mathbb{1}^T\varphi(\vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec c}), \quad (\vec c, \vec r) - (\sum \varphi_{ij})^2/M,$$

$$(\vec c, \vec r) = \sum_j (\sum_i \varphi_{ij})(\sum_i \varphi_{ji}) = \sum_j \sum_{i,k} \varphi_{ij}\varphi_{jk}, \quad \sum_{i,k}\sum_j \varphi_{ij}\varphi_{jk} - \sum_{ik}\sum_{jl}\varphi_{ij}\varphi_{lk}/M$$

$$= \sum_{ikj}(\varphi_{ij}\varphi_{jk} - \sum_l \varphi_{ij}\varphi_{lk}/M) = \sum_{ikj}\varphi_{ij}(\varphi_{jk} - \varphi_{\cdot k})$$

$$M(M-1)\sigma^2_{osu} = M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - 2M\hat\theta)^2 \qquad \varphi_{ij} = 1 - \varphi_{ji}, \qquad \varphi_{\cdot j} = \sum_i \varphi_{ij}$$

$$= \sum_i (1 - \varphi_{ji}) = M - \varphi_{j\cdot}, \qquad M^{-2}\sum_j (M - 2M\hat\theta)^2 = M - 4M\hat\theta + 4M\hat\theta^2 = M(1 - 2\hat\theta)^2,$$

$$\sigma^2_{osu} = \frac{(1-2\hat\theta)^2}{M-1}$$

$$M(M-1)\sigma^2_{jk} = \frac{M^3}{(M-1)^2} - M\left(\frac{-2M^3\hat\theta}{M(M-1)}\right)^2 = \frac{M^3}{(M-1)^2} - \frac{4M^3\hat\theta^2}{(M-1)^2} = \frac{M^3}{(M-1)^2}(1 - 4\hat\theta^2),$$

$$\sigma^2_{jk} = \frac{M^2}{(M-1)^3}(1 - 4\hat\theta^2)$$

$$\varphi_{\cdot j} = \sum_i \mathbb{P}(X_i < Y_j) = M - \sum_i \mathbb{P}(X_i > Y_j)$$

$$M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - \operatorname{tr}\varphi/M^2 - 4M\hat\theta^2 + \frac{2}{M^2}\sum \varphi_{jj}(\varphi_{j\cdot} + \varphi_{\cdot j}) = ?$$

$$(M-1)^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - M^{-1}\left(\frac{\operatorname{tr}\varphi - 2M^2\hat\theta}{(M-1)}\right)^2$$

$$\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} + \varphi_{jj})^2, \quad \sum_j \varphi_{j\cdot}^2 = \sum_j(\sum_i \varphi_{ji})^2 = \sum_j\left(\sum_i \varphi_{ji} + 2\sum_{i<k}\right.$$

$$\left.\varphi_{ji}\varphi_{jk}\right) = M^2\hat\theta + 2\sum_{i<k}(\varphi_{\cdot i}, \varphi_{\cdot k}), \quad \sum_j \varphi_{j\cdot}^2 = \mathbb{1}^T\varphi^T\varphi\mathbb{1}, \quad \sum_j \varphi_{\cdot j}^2 = \mathbb{1}^T\varphi\varphi^T\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{\cdot j} = \sum_j(\sum_i \varphi_{ji})(\sum_k \varphi_{kj}) = \sum_j\sum_{i,k}\varphi_{ji}\varphi_{kj} = \sum_{i,k}\sum_j \varphi_{ji}\varphi_{kj} = \mathbb{1}^T\varphi\varphi\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{jj} = \sum_i\sum_j \varphi_{ji}\varphi_{jj} = \mathbb{1}^T\varphi\operatorname{diag}\varphi, \quad \sum_j \varphi_{\cdot j}\varphi_{jj} = (\operatorname{diag}\varphi)^T\varphi\mathbb{1},$$

$$\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} + \operatorname{tr}\varphi - 2\mathbb{1}^T(\varphi + \varphi^T)\cdot$$

$$\operatorname{diag}\varphi = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T)\operatorname{diag}\varphi,$$

$$\operatorname{tr}\varphi - 2M^2\hat\theta = \operatorname{tr}\varphi - 2\mathbb{1}^T\varphi\mathbb{1}, \quad (M-1)^{-2}\left\{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - (\operatorname{tr}\varphi)^2/M -\right.$$

$$\left.4M^3\hat\theta^2 + 4M\hat\theta\operatorname{tr}\varphi\right\}, \quad \hat\theta^2 = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\varphi\mathbb{1}/M^4, \quad \hat\theta\operatorname{tr}\varphi = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\operatorname{diag}\varphi,$$

$$(M-1)^{-2}\left\{\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4M^{-1}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T - 2/M\,\varphi\mathbb{1}\mathbb{1}^T\right.$$

$$\left.+ \operatorname{diag}\varphi\cdot\mathbb{1}^T/2M)\operatorname{diag}\varphi\right\} = M(M-1)\hat\sigma^2_{jk}$$

$$\mathbb{E}(\delta_{L_1} \mid L_0 = \ell_0) = p\,\delta_{\ell_0} + (1-p)\,\delta_{1-\ell_0}, \quad \mathbb{E}(\gamma_{A_0}\delta_{L_1} \mid L_0 = \ell_0)$$

$$= p_1p_2\gamma_{\ell_0}\delta_{\ell_0} + p_1(1-p_2)\gamma_{\ell_0}\delta_{1-\ell_0} + (1-p_1)p_2\gamma_{1-\ell_0}\delta_{1-\ell_0} + (1-p_1)(1-p_2)\gamma_{1-\ell_0}\delta_{\ell_0}$$

$$(\gamma_{\ell_0} = \gamma_{1-\ell_0} = 1) \; = \delta_{\ell_0}(1-p_1-p_2+2p_1p_2) + \delta_{1-\ell_0}(p_1+p_2-2p_1p_2), \quad p_1(p_2-1)$$

$$+p_2(p_1-1)+1, \quad p = p_1p_2 + (1-p_1)(1-p_2), \quad 1-p = p_1+p_2-2p_1p_2$$

$$T^2 = p_1^2p_2^2\left(\frac{\gamma_0^2}{\delta_0^2} + \frac{\gamma_1^2}{\delta_1^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right) + (1-p_1)^2(1-p_2)^2\left(\frac{\gamma_0^2}{\delta_1^2} + \frac{\gamma_1^2}{\delta_0^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right) + 2p_1p_2(1-p_1)(1-p_2).$$

$$\left(\frac{\gamma_0^2}{\delta_0\delta_1} + \frac{\gamma_0\gamma_1}{\delta_0^2} + \frac{\gamma_0\gamma_1}{\delta_1^2} + \frac{\gamma_1^2}{\delta_0\delta_1}\right), \quad \alpha \quad p_1p_2(1-p_1)(1-p_2)(\cdots) - (2p_1-1)(2p_2-1)\frac{\gamma_0\gamma_1}{\delta_0\delta_1},$$

$$\frac{p_1^2p_2^2}{2} + \frac{(1-p_1)^2(1-p_2)^2}{2} - (2p_1-1)(2p_2-1) = \frac{1}{2}\Big(p_1^2p_2^2 + 1 + p_1^2 + p_2^2 + p_1^2p_2^2 - p_1 - p_2 + p_1p_2 + p_1p_2$$

$1-p_1-p_2+p_1p_2$

$$-p_1^2p_2 - p_1p_2^2 - \ldots + 4p_1 + 4p_2 - 2\Big) = \frac{1}{2}\Big(2p_1^2p_2^2 + p_1^2 + p_2^2 - 6p_1p_2 + 3p_1 + 3p_2 - p_1^2p_2$$

$$-p_1p_2^2 - 1\Big) \qquad (p_1 = p_2 =: p_0) \quad p = p_0^2 + (1-p_0)^2, \quad 1-p = 2p_0(1-p_0), \quad 2p_0 - 1 = p_0^2 - (1-p_0)^2,$$

$$\cancel{\ldots = p(\delta_0+\delta_1) + (1-p)\delta_0\delta_1} \quad 2p-1 = p_0^2 + (1-p_0)(1-p_0-2p_0), \quad 2p-1 = (p_0 - (1-p_0))^2 = (2p_0-1)^2,$$

$$\alpha = \frac{1}{4}(1-p)^2\left(\frac{\gamma_0^2}{\delta_0\delta_1} + \frac{\gamma_0\gamma_1}{\delta_0^2} + \frac{\gamma_0\gamma_1}{\delta_1^2} + \frac{\gamma_1^2}{\delta_0\delta_1}\right) - (2p-1)\frac{\gamma_0\gamma_1}{\delta_0\delta_1} = \frac{1}{4}(\quad) + \frac{(1-2p)}{4}\Big(\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\cdot 4$$

$$+\frac{\gamma_0^2}{\delta_0\delta_1} + \frac{\gamma_0\gamma_1}{\delta_0^2} + \frac{\gamma_0\gamma_1}{\delta_1^2} + \frac{\gamma_1^2}{\delta_0\delta_1}\Big) = \frac{1}{4}(\quad) + \frac{(1-2p)}{4}\left(\gamma_0\gamma_1\left(\frac{1}{\delta_0} + \frac{1}{\delta_1}\right)^2 + \frac{1}{\delta_0\delta_1}(\gamma_0+\gamma_1)^2\right),$$

$$\alpha = \frac{1}{4}(1-p)^2\left(\gamma_0\gamma_1\left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right)^2 + \frac{1}{\delta_0\delta_1}(\gamma_0-\gamma_1)^2\right) + (1-p)^2\frac{\gamma_0\gamma_1}{\delta_0\delta_1} - (2p-1)\frac{\gamma_0\gamma_1}{\delta_0\delta_1} = \frac{1}{4}(1-p)^2(\ldots)$$

$$+\frac{\gamma_0\gamma_1}{\delta_0\delta_1}(p^2 - 4p + 2)$$

$$p(1-p)(\gamma_0+\gamma_1) \approx ? \; p^2\gamma_0 + (1-p)^2\gamma_1 = (p\sqrt{\gamma_0} - (1-p)\sqrt{\gamma_1})^2 + 2p(1-p)\sqrt{\gamma_0\gamma_1}, \quad p(1-p)(\sqrt{\gamma_0} - \sqrt{\gamma_1})^2$$

$$\approx ? \; (p\sqrt{\gamma_0} - (1-p)\sqrt{\gamma_1})^2, \quad \frac{1-2p}{1-p}\gamma_0 + \frac{2p-1}{p}\gamma_1 \approx ? \; 0, \quad \frac{\gamma_0}{1-p} \approx ? \; \frac{\gamma_1}{p} \quad 46a$$

$$P = \begin{pmatrix} p_1p_2\gamma_0/\delta_0 + (1-p_1)(1-p_2)\gamma_1/\delta_0 & p_1(1-p_2)\gamma_0/\delta_1 + (1-p_1)p_2\gamma_1/\delta_1 \\ p_1(1-p_2)\gamma_1/\delta_0 + p_2(1-p_1)\gamma_0/\delta_0 & p_1p_2\gamma_1/\delta_1 + (1-p_1)(1-p_2)\gamma_0/\delta_1 \end{pmatrix}$$

$$4p_1p_2(1-p_1)(1-p_2) - (2p_1-1)(2p_2-1) = -4p_1^2p_2 - 4p_1p_2^2 + 4p_1^2p_2^2 + 2p_1 + 2p_2 - 1$$

$p_1 = p_{LA},\ p_2 = p_{AL}$

$P_{11} \cdot P_{22} = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(p_1^2 p_2^2 + \overset{1-p_1-p_2+p_1p_2}{(1-p_1)^2(1-p_2)^2}\right) + p_1 p_2 (1-p_1)(1-p_2)(\gamma_0^2+\gamma_1^2)/\delta_0\delta_1,$

$P_{11}P_{22} - \Delta = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(2p_1^2p_2^2 + p_1^2 + p_2^2 + 1 - p_1 - p_2 + p_1p_2 + p_1p_2 - p_1^2 p_2 - p_1 p_2^2 - 4p_1p_2 + 2p_1 + 2p_2 - 1\right) + p_1p_2\overset{p_1^2p_2^2 - p_1^2p_2 - p_1p_2^2 + p_1p_2}{(1-p_1)(1-p_2)}(\gamma_0^2+\gamma_1^2)/\delta_0\delta_1,$ $2p_1^2p_2^2 + p_1^2 + p_2^2 + p_1 + p_2 - 2p_1p_2 - p_1^2p_2 - p_1p_2^2 = p_1p_2(p_1p_2 - p_1 - p_2) + (p_1-p_2)^2 + p_1 + p_2 = p_1p_2(1-p_1)(1-p_2) + p_1^2p_2^2 + p_1^2 + p_2^2 + p_1 + p_2 - 3p_1p_2,$ $2p_1^2p_2^2 + p_1^2 + p_2^2 - p_1^2p_2 - p_1p_2^2 = p_1p_2(2p_1p_2 - p_1 - p_2) + p_1^2 + p_2^2 = 2p_1^2p_2^2 + p_1^2(1-p_2) + p_2^2(1-p_1) \geq 0,$ $P_{11}P_{22} - \Delta = \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left\{(p_1-p_2)^2 + 2p_1^2p_2^2 + p_1^2(1-p_2) + p_2^2(1-p_1)\right\} + p_1p_2(1-p_1)(1-p_2)(\gamma_0^2+\gamma_1^2)/\delta_0\delta_1,$ $p_1^2(1-p_2+p_2^2) + p_2^2(1-p_1+p_1^2) = p_1^2(p_2-1)^2 + p_1^2p_2 + p_2^2(1-p_1)^2 + p_1p_2^2 = p_1^2(1-p_2)^2 + p_2^2(1-p_1)^2 + p_1p_2(p_1+p_2),$ $\frac{T^2}{4} - \Delta = \frac{1}{4}\left(p_1p_2\left(\frac{\gamma_0}{\delta_0} - \frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0} - \frac{\gamma_0}{\delta_1}\right)\right)^2 +$

$\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left\{\overset{(p_1-p_2)^2 + p_1p_2(1-p_1)(1-p_2) + p_1^2p_2^2 + p_1^2 + p_2^2 - p_1p_2}{(p_1-p_2)^2 + 2p_1^2p_2^2 + p_1^2(1-p_2) + p_2^2(1-p_1)}\right\} + p_1p_2(1-p_1)(1-p_2)(\gamma_0^2+\gamma_1^2)/\delta_0\delta_1,$ $= 2(p_1-p_2)^2 + p_1p_2\{(1-p_1)(1-p_2) + 1 + p_1p_2\}$

$\lambda \leq \frac{1}{2}\left\{p_1p_2\left(\frac{\gamma_0}{\delta_0} + \frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_0}{\delta_1} + \frac{\gamma_1}{\delta_0}\right)\right\} + \frac{1}{2}\left|p_1p_2\left(\frac{\gamma_0}{\delta_0} + \frac{\gamma_1}{\delta_1}\right) - (1-p_1)(1-p_2)\left(\frac{\gamma_0}{\delta_1} + \frac{\gamma_1}{\delta_0}\right)\right| + \sqrt{\frac{\gamma_0\gamma_1}{\delta_0\delta_1}}\sqrt{\overset{(a)}{\{(p_1-p_2)^2 + 2p_1^2p_2^2 + p_1^2(1-p_2) + p_2^2(1-p_1)\}}} + \sqrt{\frac{\gamma_0^2+\gamma_1^2}{\delta_0\delta_1}}\sqrt{\{p_1p_2(1-p_1)(1-p_2)\}}$

$\sqrt{\left\{p_1p_2(1-p_1)(1-p_2)\left(\frac{\gamma_0^2+\gamma_1^2}{\delta_0\delta_1} + \gamma_0\gamma_1\left(\frac{1}{\delta_0^2} + \frac{1}{\delta_1^2}\right)\right)\right\}} = \sqrt{\left\{p_1p_2(1-p_1)(1-p_2)\left(\frac{(\gamma_0+\gamma_1)^2}{\delta_0\delta_1} + \frac{\gamma_0\gamma_1}{(\delta_0+\delta_1)^2} - 4\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right)\right\}},$ $\frac{\partial}{\partial p_1}(a) = 2(p_1-p_2) + 4p_1p_2^2 + 2p_1(1-p_2) - p_2^2,$ $p_1(2 + 4p_2^2 + 2(1-p_2)) = 2p_2 + p_2^2$

$(p_1 = p_2) = p$ $\frac{T^2}{4} - \Delta = \frac{1}{4}\left(p^2\left(\frac{\gamma_0}{\delta_0} - \frac{\gamma_1}{\delta_1}\right) + (1-p)^2\left(\frac{\gamma_1}{\delta_0} - \frac{\gamma_0}{\delta_1}\right)\right)^2 + \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left\{2p^4 + \overset{2p^2(1-p)^2 - 2p^3}{2p^2(1-p)}\right\}$

$+ \frac{\gamma_0^2+\gamma_1^2}{\delta_0\delta_1}p^2(1-p)^2,$ ...

$2p_1^2p_2^2 + p_1^2 + p_2^2 + 1 - 2p_1 - 2p_2 + 4p_1p_2 - 2p_1^2p_2 - 2p_1p_2^2 - 4p_1p_2 + 2p_1 + 2p_2 - 1 = 2p_1^2p_2^2 + p_1^2 + p_2^2 - 2p_1^2p_2 - 2p_1p_2^2$ $= 2p_1^2p_2(p_2-1) + 2p_1p_2^2(p_1-1) + p_1^2 + p_2^2 = (p_1 - p_1p_2)^2 + (p_2 - p_1p_2)^2 = p_1^2(1-p_2)^2 + p_2^2(1-p_1)^2,$ $P_{11}P_{22} - \Delta$

$= \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(p_1^2(1-p_2)^2 + p_2^2(1-p_1)^2\right) + p_1(1-p_2)p_2(1-p_1)\frac{\gamma_0^2+\gamma_1^2}{\delta_0\delta_1}$

vd Vaart ch 7

$p_\theta = e^{-\theta}\,\theta^x/x!$, $s_\theta = e^{-\theta/2}\dfrac{\theta^{x/2}}{\sqrt{x!}}$, $\frac{\partial}{\partial\theta}\log p_\theta = -1 + x/\theta$, $I_\theta = \mathbb{E}_\theta(-1+x/\theta)^2$

$= 1 - 2 + \theta^{-2}(\theta + \theta^2) = -1 + 1/\theta + 1 = 1/\theta$ #1 // $p_\theta = f(x-\theta)$, $\dot{\ell}_\theta = -\frac{f'}{f}(x-\theta)$,

$I_\theta = \mathbb{E}_\theta\left(\frac{f'}{f}\right)^2 = \int f'(x-\theta)^2/f(x-\theta)\,dx = \int f'(x)^2/f(x)$, $f(x-\theta) = (2\pi\sigma^2)^{-1/2}\exp\left(\left(\frac{x-\theta}{\sigma}\right)^2\right.$

$\left.\cdot -\frac{1}{2}\right)$, $\cancel{= \varphi((x-\theta)/\sigma)}$, $\cancel{I_\theta = \int \frac{1}{\sigma^2}\varphi'((x-\theta)/\sigma)^2/\varphi((x-\theta)/\sigma)}$ $I_\theta = \int \varphi'(x)^2/\varphi(x)$

$= \int \left(\frac{x}{\sigma^2}\right)^2 \varphi_\sigma(x)\,dx = \sigma^{-4}\sigma^2 = \sigma^{-2}$. $p_\theta(x) = (2b)^{-1}\exp(-b^{-1}|x-\theta|)$, I_θ

$= \int p_\theta(x)\, b^{-2}\,dx = 1/b^2$. $p_\theta(x) = s^{-1} e^{-s^{-1}(x-\theta)}\left(1 + e^{-s^{-1}(x-\theta)}\right)^{-2}$, $\frac{\partial}{\partial\theta}p_\theta$

$= s^{-1}p_\theta(x) - 2e^{-s^{-1}(x-\theta)}/s\; p_\theta(x)\left(1+e^{-s^{-1}(x-\theta)}\right)^{-1}$ $\cancel{= p_\theta(x)/s\,(1 + 2e^{-s^{-1}(x-\theta)})}$, $\cancel{I_\theta = s^{-2}\int p(x)}$

$\cancel{(1 + 2e^{-s^{-1}(x-\theta)})^2\,dx = s^{-1}(1 + 4\mathbb{E}_s e^{-s^{-1}x} + 4\mathbb{E}_s e^{-s^{-1}2x})}$, $\cancel{\frac{1}{s}\int e^{-x/s} \cdot e^{-x/s}}$

$\cancel{(1+e^{-x/s})^2\,dx = \int \frac{e^{-2x}}{(1+e^{-x})^2}\,dx = \int \frac{dx}{(1+e^x)^2} = \int_0^\infty \frac{1}{x}\frac{1}{(1+x)^2}\,dx = \infty \ldots}$,

$\cancel{p(x) = e^{-x}/(1+e^{-x})^2}$, $\cancel{\dot{p}_\theta = -e^{-x}/(1+e^{-x})^2 - 2}$, $= p_\theta(x)\left(\frac{1}{s} - \frac{2}{s}\frac{e^{-s^{-1}(x-\theta)}}{1+e^{-s^{-1}(x-\theta)}}\right)$

$= s^{-1}p_\theta(x)\dfrac{1-e^{-s^{-1}(x-\theta)}}{1+e^{-s^{-1}(x-\theta)}}$, $\cancel{I_\theta = \int s^{-1}(1+e^{-s^{-1}(x-\theta)})^{-2}}\; p(x)\,dx$, $p(x) = e^{-x}/(1+e^{-x})^2$,

$\dot{p} = -e^{-x}/(1+e^{-x})^2 + 2e^{-x}/(1+e^{-x})^3 \cdot e^{-x} = -p_\theta(x) + p_\theta(x)\cdot\frac{2e^{-x}}{1+e^{-x}} = p_\theta(x)\cdot\frac{e^{-x}-1}{1+e^{-x}}$,

$I_\theta = \int\left(\frac{1-e^{-x}}{1+e^{-x}}\right)^2 p_\theta(x) = \int \frac{e^{-x}(1-e^{-x})^2}{(1+e^{-x})^4}$, $\int \frac{e^{-x}}{(1+e^{-x})^4} = -\frac{1}{(1+e^{-x})^3} = -1$,

$\int \frac{e^{-2x}}{(1+e^{-x})^4} = \int \frac{1}{(e^{x/2}+e^{-x/2})^4} = \frac{1}{16}\int_{-\infty}^\infty \frac{1}{\cosh^4 x}$, $\int_0^\infty \frac{(1-x)^2}{(1+x)^4}\,dx = \int_0^{\pi/2} 2\tan x\cdot\sec^2 x\cdot\sec^{-8}x$

$(\cos^2 - \sin^2)^2\sec^4 = 2\int \sin x\cos^2 2x \cos x\,dx = 2\left(-\frac{1}{2}\cos^2 x\cos^2 2x\Big|_0^{\pi/2} - 2\int \cos^2 x\cos 2x\sin 2x\right)$

$= 1 - 8\int(\cos^5\sin - \cos^3\sin^3) = 1 - 8\left(-\frac{1}{6}\cos^6\Big|_0^{\pi/2} - \int\cos^3\sin^3\right) = -\frac{1}{3} + 8\int\cos^3\sin^3$,

$\cos^2\frac{1}{4}\sin^4\Big|_0^{\pi/2} + \frac{1}{2}\int\cos\sin^5 = \frac{1}{12}\sin^6\Big|_0^{\pi/2} = 1/12$, $I_\theta = -1/2 + 2/3 = 1/6$, $I_{\theta,s} = \frac{1}{6s}$ #2 //

$p_\theta(x) = f(x-\theta)$, $\dot{l}_\theta = -\frac{f'(x-\theta)}{f(x-\theta)}$, $I_\theta = \int \frac{f'(x-\theta)^2}{f(x-\theta)} dx = \int \frac{f'(x)^2}{f(x)} dx$, $p_\theta(x) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$,

$f'(x) = \frac{1}{\pi} \frac{-2x}{(1+x^2)^2}$, $\pi I_\theta = \int \frac{4x^2}{(1+x^2)^3} dx = x \frac{-1}{(1+x^2)^2}\Big|_{-\infty}^{\infty} + \int \frac{1}{(1+x^2)^2} = \int_{-\pi/2}^{\pi/2} \sec^2 \cos^4$

$= \int \cos^2 = \int \frac{1}{2}(\cos 2x + 1) = \frac{\pi}{2} + \frac{1}{4} \sin 2x \Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{1}{2}$, $I_\theta = \frac{1}{2}\left(1 + \frac{1}{\pi}\right)$, $I_{\theta,\sigma} = \frac{1}{\sigma^2} I_\theta$ #3 //

$p_{\mu,\sigma} = f\left(\frac{x-\mu}{\sigma}\right)/\sigma$, $\dot{l} = \sigma f\left(\frac{x-\mu}{\sigma}\right)^{-1} \left(-f'\left(\frac{x-\mu}{\sigma}\right)/\sigma^2, f'\left(\frac{x-\mu}{\sigma}\right)/\sigma \left(\frac{\mu - x}{\sigma^2}\right) - f\left(\frac{x-\mu}{\sigma}\right)/\sigma^2\right)$,

$\int \sigma f\left(\frac{x-\mu}{\sigma}\right)^{-1} \left\{ f f'/\sigma^4 + (f')^2/\sigma^3 \left(\frac{x-\mu}{\sigma^2}\right) \right\} dx = \int \left\{ \frac{f'}{\sigma^3} + \frac{(f')^2}{f\sigma^4} \left(\frac{x-\mu}{\sigma^2}\right) \right\}$

$= \int_{-\infty}^{\infty} (f')^2/f\sigma^4 \cdot x \, dx$, $f(x) = f(-x)$, $f'(x) = \lim \delta^{-1}(f(x+\delta) - f(x)) = \lim \delta^{-1}($

$f(-x-\delta) - f(-x)) = -\lim \delta^{-1}(f(-x) - f(-x-\delta)) = -f'(-x)$, $f''(x) = \lim_\delta \delta^{-1}(f'(x+\delta) - f'(x))$

$= \lim \delta^{-1}(f'(-x) - f'(-x-\delta)) = f''(-x)$, $\int x f''/f = 0$ #4 // $p_{\theta+h_n/\sqrt{n}} = (2\pi)^{-1/2} e^{-\frac{1}{2}(x-\theta)^2}$,

$\dot{l}_\theta = x - \theta$, $\log \prod \frac{p_{\theta+h/\sqrt{n}}}{p_\theta}$ ~~$= \sum \frac{1}{2}(2\theta x - \theta^2) = \theta n \bar{x} - \frac{n}{2}\theta^2$~~ $\frac{1}{\sqrt{n}} \sum h \dot{l}(x_j) - \frac{1}{2} I_\theta h^2$

$= n^{-1/2} hn(\bar{x} - \theta) - h^2/2$, $\log \prod \frac{p_{\theta+h_n/\sqrt{n}}}{p_\theta} = \frac{1}{2} \sum \left\{ (x_j - \theta)^2 - \left(x_j - \theta - \frac{h_n}{\sqrt{n}}\right)^2 \right\} = \frac{1}{2} \sum \left\{ 2 \frac{h_n}{\sqrt{n}}(x_j - \right.$

$\left.\theta) - h_n^2/n \right\} = h_n n^{-1/2} n(\bar{x} - \theta) - h_n^2/2$, $o_{p_\theta}(1) = n^{1/2}(\bar{x} - \theta)(h - h_n) + \frac{1}{2}(h^2 - h_n^2)$ #5 //

$\sqrt{f(x-h)} - \sqrt{f(x)} = \int_0^1 \frac{\partial}{\partial u} \cancel{f(x-uh)} du = \int_0^1 h \operatorname{sgn}(x-uh) f(x-uh) \, du$, ~~$\left(\int_0^1 h \operatorname{sgn}(x-uh) f(x-uh) \, du\right)^2$~~

~~$\leq \int_0^1 h^2 f^2(x-uh) \, du$, $\int \left(t^{-1}\left(\sqrt{f(x-ht)} - \sqrt{f(x)}\right)\right)^2 d\mu \leq t^{-2} \int \int_0^1 t^2 h^2 f^2(x-uth) du \, d\mu$~~

~~$= \int_0^1 \frac{t}{3} h^2 \int \frac{\left(\frac{\partial}{\partial(ut)} f(x-uth)\right)^2}{f(x-uth)}$~~ $\sqrt{f(x-h)} - \sqrt{f(x)} = \int_0^1 \frac{\partial}{\partial u} \sqrt{f(x-uh)} \, du = \int_0^1 \frac{\operatorname{sgn}(x-uh) \cdot h \cdot f(x-uh)}{2\sqrt{f(x-uh)}}$

$= \int_0^1 \frac{h}{2} \operatorname{sgn}(x-uh) \sqrt{f(x-uh)} \, du$, $\left(\int_0^1 \frac{h}{2} \operatorname{sgn}(x-uh) \sqrt{f(x-uh)} \, du\right)^2 \leq \int_0^1 \frac{h^2}{4} f(x-uh) \, du$, $\int \left(t^{-1}\left(\right.\right.$

$\left.\left.\sqrt{f(x-th)} - \sqrt{f(x)}\right)\right)^2 d\mu \leq \int t^{-2} \int_0^1 \frac{(ht)^2}{4} f(x-uth) \, du \, d\mu = \int \frac{h^2}{4} \int_0^1 \frac{\left(\frac{\partial}{\partial\theta} f(x-uth)\right)^2}{f(x-uth)} du \, d\mu$

$= \int_0^1 I_{uth} \frac{h^2}{4} \xrightarrow[t \to 0]{} \frac{1}{4} h^2 I_0$, $t^{-1}\left(\sqrt{f(x-th)} - \sqrt{f(x)}\right) \xrightarrow[t \to 0]{} \frac{h}{2} \operatorname{sgn}(x-uh) \sqrt{f(x-h)}$, $\int (\cdots)^2 d\mu$

$= h^2/4 = \frac{h^2}{4} I_0$ #6 // $p_\theta = f\left(\frac{x-\mu}{\sigma}\right)/\sigma$, $\dot{p}_\theta = \left(-f'\left(\frac{x-\mu}{\sigma}\right)/\sigma^2, f'\left(\frac{x-\mu}{\sigma}\right)\left(\frac{\mu - x}{\sigma^3}\right) - f\left(\frac{x-\mu}{\sigma}\right)/\sigma^2\right)$,

$\dot{l}_\theta = \left(-f'/\sigma f, -f'/f \left(\frac{x-\mu}{\sigma^2}\right) - 1/\sigma\right)$ #7 //

$P_{\theta+h}(\{p_\theta=0\}) = \int_{\theta-h}^{\theta} p_{\theta-h} = \int_{\theta-h}^{\theta} f_k(x-(\theta-h))\,dx = \int_0^h f_k(x)\,dx = \int_0^h \frac{1}{\Gamma(k)} x^{k-1} e^{-x}\,dx$

$\geq \frac{e^{-h}}{\Gamma(k)}\int_0^h x^{k-1}\,dx = \frac{e^{-h}h^k}{\Gamma(k+1)} = o(h^{k-\varepsilon}) \Rightarrow k>2$, $\int\{(x-h)^{(k-1)/2} e^{-(x-h)/2} -$

$x^{(k-1)/2} e^{-x/2} - h\cdot\frac12\left((k-1)x^{k-2}e^{-x} + x^{k-1}e^{-x}\right)/\left(x^{k-1}e^{-x}\right)^{1/2}\}^2\,dx = \int\{\cdots$

$-\frac h2\left(x^{(k-1)/2}e^{-x/2} - (k-1)x^{(k-3)/2}e^{-x/2}\right)\}^2\,dx = \int x^{k-1}e^{-x}\{(1-\frac hx)^{(k-1)/2}e^{h/2} - 1$

$-\frac h2\left(1-(k-1)\cdot\frac1x\right)\}^2\,dx \le c\cdot\int\{\cdots\}^2\,dx = c\int\{(1-\frac hx)^{\frac{k-1}{2}}e^{h/2} - (1+\frac h2)$

$+\frac h2(k-1)\cdot\frac1x\}^2\,dx = c\cdot\int\{(1-\frac hx)^{k-1}e^h + (1+\frac h2)^2 + \frac{h^2}{4}(k-1)^2\cdot\frac1{x^2} - 2(1-\frac hx)^{\frac{k-1}{2}}(1+\frac h2)e^{h/2}$

$+ he^{h/2}(k-1)(1-\frac hx)^{\frac{k-1}{2}}\cdot\frac1x\cdots$, $\{\cdots\} = (1-\frac hx)^{(k-1)/2} - 1 + o(\frac h2) + \frac h2(k-1)\cdot\frac1x = (\frac{k-1}{2})(-\frac hx)$
$(k>2)$

$+o(\frac hx) + o(\frac h2) + \frac h2(k-1)\cdot\frac1x = o(\frac hx)+o(\frac h2)$, $\int\{\cdots\}^2\,dx = \int x^{k-1}e^{-x}\left(o(\frac hx)+o(\frac h2)\right)^2$

$dx = o(h^2)$ // $Z_t\sim N(0,1)$, ~~var~~ $Var(X_t) = \theta^2 Var(X_{t-1}) + \sigma^2$, $var(X_t) = \frac{\sigma^2}{1-\theta^2}$,

$cov(X_t,X_{t+1}) = \theta v$, $cov(X_t,X_{t+2}) = cov(X_t, \theta X_{t+1}+Z_{t+2}) = \theta^2 v$, $(\overbrace{Var(X_1,\ldots,X_n)}^{\text{"}\Sigma_\theta\text{"}})_{ij}$

$= v\theta^{|i-j|}$, $\log\frac{dP_{n,\theta+r_n^{-1}h_n}}{dP_{n,\theta}} = \log|\Sigma_\theta|^{1/2}/|\Sigma_{\theta+r_n^{-1}h_n}|^{1/2} - \left(x^t(\Sigma^{-1}_{\theta+r_n^{-1}h_n} - \Sigma^{-1}_\theta)x\right)\cdot\frac12$,

$(\theta+u)^{|i-j|} = \sum_{m=0}^{|i-j|}\binom{|i-j|}{m}\theta^m u^{|i-j|-m}$, $(\Sigma+u)^{-1} = \Sigma^{-1}(1+u\Sigma^{-1})^{-1} = \Sigma^{-1}\sum_j(-u\Sigma^{-1})^j$

$= \Sigma^{-1} - u\Sigma^{-2} + u^2\Sigma^{-3} - \ldots$, $\Sigma_{\theta+r_n^{-1}h_n} = \Sigma_\theta + r_n^{-1}h_n M + o(r_n^{-1}h_n)$, $u = \Sigma_{\theta+r_n^{-1}h_n} -$

$\Sigma_\theta = \underbrace{(\cdots)}_{M_1} r_n^{-1}h_n + (\cdots) r_n^{-2}h_n^2 + \ldots$, $(\theta+u)^{|i-j|} - \theta^{|i-j|} = \theta^{|i-j|}\left((1+\frac u\theta)^{|i-j|} - 1\right)$

$=\theta^{|i-j|}\left(\frac u\theta + \binom{|i-j|}{2}(\frac u\theta)^2 + o((\frac u\theta)^2)\right)$, $\underbrace{\Sigma_{\theta+u}-\Sigma_\theta}_{\text{"A"}} = M_1\cdot u + M_2\cdot u^2 + \ldots$, $(\Sigma+A)^{-1} -$

$\Sigma^{-1} = -\Sigma^{-1}A\Sigma^{-1} + \Sigma^{-1}A^2\Sigma^{-2} - \ldots = -\Sigma^{-1}M\Sigma^{-1}u - \Sigma^{-1}M_2\Sigma^{-1}u^2 + \Sigma^{-1}M_1^2\Sigma^{-2}u^2$

$+o(u^2)$, $u := r_n^{-1}h_n := n^{-1/2}h_n,\ldots$ $\frac12\sum x_j^2\left(\frac1{\sigma^2} - \frac1{\sigma^2+r_n}\right) = \frac12\sum x_j^2\frac{r_n}{\sigma^2(\sigma^2+r_n)}$,

$\dot\ell_\sigma = -\frac{1}{2\sigma^2} + \frac{1}{2\sigma^4}(x-\mu)^2$, $\log\frac{P_{n,\theta+r_n^{-1}h_n}}{dP_{n,\theta}} = \frac12\log|\Sigma_\theta| - \frac12\left(\log|\Sigma_\theta| + (r_n^{-1}h_n)\frac{\partial/\partial\theta|\Sigma_\theta|}{|\Sigma_\theta|} + o(r_n^{-1}h_n)\right)$

$-\frac12\left(x^T\left(r_n^{-1}h_n\frac{\partial}{\partial\theta}\Sigma_\theta^{-1} + (r_n^{-1}h_n)^2\frac{\partial^2}{\partial\theta^2}\Sigma_\theta^{-1} + o(r_n^{-2}h_n^2)\right)x\right)$, $\Sigma_{\theta+u} = \Sigma_\theta + O(u)$,

$\Sigma^{-1}_{\theta+u} = \Sigma_\theta^{-1} + L + o(u)$, $1 = (\Sigma_\theta^{-1} + L + o(u))(\Sigma_\theta + uM_1 + o(u))$, $L = u\Sigma_\theta^{-1}M_1\Sigma_\theta^{-1}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{pmatrix}$$

Project Euler

$E(n) = E(0) = 0$, $(0<j<n)$, $E(j) = 1 + \frac{1}{9}(E(j-1) + E(j) + E(j+1)) + \frac{1}{9}(E(j) + E(j+1) + E(j+2)) + \frac{1}{9}(E(j-2) + E(j-1) + E(j)) = 1 + \frac{2}{9}E(j-1) + \frac{1}{3}E(j) + \frac{2}{9}E(j+1) + \frac{1}{9}(E(j+2) + E(j-2))$, $\cancel{E(n-1) = 1 + \frac{2}{9}E(n-2) + \frac{1}{3}E(n-1)}$, $\frac{2}{3}E(j) = 1 + \frac{2}{9}(E(j-1) + E(j+1)) + \frac{1}{9}(E(j+2) + E(j-2))$ $(j \neq n-1)$, $E(n-1) = E(1) = 1 + \frac{1}{9}(E(2) + E(1) + E(0)) + \frac{1}{9}(E(1) + E(2) + E(3)) + \frac{1}{9}(E(0) + E(1) + E(2)) = 1 + E(1)\cdot\frac{4}{9} + E(2)\cdot\frac{2}{9}$, $\frac{5}{9}E(1) = 1 + \frac{2}{9}E(2)$

~~[illegible]~~ Wharton

$\delta_0\delta_1(P_{11}P_{22}-\Delta) = \gamma_0\gamma_1\left(p_1^2(1-p_2)^2 + p_2^2(1-p_1)^2\right) + (\gamma_0^2+\gamma_1^2)\,p_1(1-p_1)\,p_2(1-p_2)$

$= \gamma_0\gamma_1\big(\overbrace{p_1(1-p_2) + p_2(1-p_1)}^{p_1+p_2-2p_1p_2}\big)^2 + (\gamma_0 - \gamma_1)^2 p_1(1-p_1)p_2(1-p_2)$, $(p_1=p_2=p_0)$ $\delta_0\delta_1(P_{11}P_{22}-\Delta) = \gamma_0\gamma_1 4p_0^2(1-p_0)^2 + (\gamma_0-\gamma_1)^2 p_0^2(1-p_0)^2 = p_0^2(1-p_0)^2(\gamma_0+\gamma_1)^2$, $\frac{\partial}{\partial p_1} = 2\gamma_0\gamma_1\big(p_1(1-p_2)+p_2(1-p_1)\big)(1-2p_2) + (\gamma_0-\gamma_1)^2 p_2(1-p_2)(1-2p_1) := 0$, $2\gamma_0\gamma_1\big(p_1(1-p_2)+\cancel{p_2(1-p_1)}\,p_2(1-p_1)\big) = -(\gamma_0-\gamma_1)^2 p_2(1-p_2)$, $p_1(1-p_1) = p_2(1-p_2)$, $\cancel{p_1=p_2,\ p_1=}\ p_1\in\{p_2, 1-p_2\}$, $-p_1^2+p_1-p_2(1-p_2)=0$, $\pm\frac{1}{2}\left(-1\pm\sqrt{(1-4p_2(1-p_2))}\right) = \frac{1}{2}\left(-1\pm(p_2-\frac{1}{2})\cdot 2\right)$,

$(p_1=1-p_2)$ $\delta_0\delta_1(P_{11}P_{22}-\Delta) = \gamma_0\gamma_1\left(p_1^4+(1-p_1)^4\right) + (\gamma_0^2+\gamma_1^2)\,p_1^2(1-p_1)^2$

$= \gamma_0\gamma_1\big(\overbrace{p_1^2-(1-p_1)^2}^{(2p_1-1)^2=4p_1^2-4p_1+1}\big)^2 + (\gamma_0+\gamma_1)^2 p_1^2(1-p_1)^2 = \gamma_0\gamma_1 + 4\gamma_0\gamma_1\, p_1(1-p_1) + (\gamma_0+\gamma_1)^2 p_1^2(1-p_1)^2$, $\frac{\partial}{\partial p_1} = 4\gamma_0\gamma_1(2p_1-1) + (\gamma_0+\gamma_1)^2\left(2p_1(1-p_1)^2 - 2p_1^2(1-p_1)\right)$

$= 4\gamma_0\gamma_1(2p_1-1) + (\gamma_0+\gamma_1)^2\, 2p_1(1-p_1)(1-2p_1) = 0$, $\hat{p}_1(1-\hat{p}_1) = \frac{2\gamma_0\gamma_1}{(\gamma_0+\gamma_1)^2}$

$= \left(1+\frac{1}{2}\left(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}\right)\right)^{-1}$, $\delta_0\delta_1(P_{11}P_{22}-\Delta) \le \gamma_0\gamma_1(2p_1-1)^2 + \frac{4\gamma_0^2\gamma_1^2}{(\gamma_0+\gamma_1)^2}$,

$p_1 = -\frac{1}{2}\left(-1\pm\sqrt{1-8\gamma_0\gamma_1/(\gamma_0+\gamma_1)^2}\right) = -\frac{1}{2}(-1\pm\sqrt{\ }$, $(2p_1-1)^2 = 4p_1^2-4p_1+1$

$= 4p_1(p_1-1)+1$; $(2p_1-1)^2 = -\frac{8\gamma_0\gamma_1}{(\gamma_0+\gamma_1)^2}+1$, $P_{11}P_{22}-\Delta \le \gamma_0\gamma_1/\delta_0\delta_1$, $\frac{T^2}{4}-\Delta$

$= \frac{1}{4}\left\{p_1p_2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0}-\frac{\gamma_0}{\delta_1}\right)\right\}^2 + \frac{\gamma_0\gamma_1}{\delta_0\delta_1}\cancel{\left(p_1^2(1-p_2)^2+p_2^2(1-p_1)^2\right)}$

$\cancel{\gamma_0^2+\gamma_1^2}$ $+\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\left(p_1(1-p_2)+p_2(1-p_1)\right)^2 + \frac{(\gamma_0-\gamma_1)^2}{\delta_0\delta_1}p_1(1-p_1)p_2(1-p_2) \le \frac{1}{4}\{\cdots\}^2$

$+\frac{\gamma_0\gamma_1}{\delta_0\delta_1}$, $\sqrt{\frac{T^2}{4}-\Delta} \le \frac{1}{2}\left|p_1p_2\left(\frac{\gamma_0}{\delta_0}-\frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0}-\frac{\gamma_0}{\delta_1}\right)\right| + \sqrt{\frac{\gamma_0\gamma_1}{\delta_0\delta_1}}$,

$\lambda \le \frac{1}{2}\left(p_1p_2\left(\frac{\gamma_0}{\delta_0}+\frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0}+\frac{\gamma_0}{\delta_1}\right)\right) + \frac{1}{2}|\cdots| + \sqrt{\frac{\gamma_0\gamma_1}{\delta_0\delta_1}} \le$

$p_1p_2\left(\frac{\gamma_0}{\delta_0}\vee\frac{\gamma_1}{\delta_1}\right) + (1-p_1)(1-p_2)\left(\frac{\gamma_1}{\delta_0}\vee\frac{\gamma_0}{\delta_1}\right) + \sqrt{\frac{\gamma_0\gamma_1}{\delta_0\delta_1}}$, $2p_1p_2-p_1-p_2+2 = p_1(p_2-1)+p_2(p_1-1)+2$

I seem to have forgotten $p_1=1-p_2$, already using this in first inequality for $P_{11}P_{22}-\Delta$

$\frac{1}{\delta_0^2}-\frac{1}{\delta_1^2}=\alpha$, $\delta_1^2\beta^2-\frac{1}{\delta_1^2}=\alpha$, $\delta_1^2=\frac{1}{2}\left(\frac{\alpha}{\beta^2}\pm\sqrt{\left(\frac{\alpha^2}{\beta^4}+\frac{4}{\beta^2}\right)}\right)$:

$\frac{1}{\delta_0\delta_1}=\beta$, $\delta_0=\frac{1}{\delta_1\beta}$, $\delta_1^4-\frac{\alpha}{\beta^2}\delta_1^2-\frac{1}{\beta^2}=0$

516

$$ESS = \Sigma(y_j - \hat\beta_0 - \hat\beta_1 x_j)^2 = \Sigma(y_j - (\bar y - \hat\beta_1 \bar x) - \hat\beta_1 x_j)^2 = \Sigma[y_j - \bar y - \hat\beta_1 (x_j - \bar x)]^2$$

$$= \Sigma(y_j - \bar y)^2 - 2\hat\beta_1 \Sigma(y_j - \bar y)(x_j - \bar x) + \hat\beta_1^2 S_{xx} = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = S_{yy} - \hat\beta_1 S_{xy}$$

$$r^2 = \left(\frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}\right)^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = (S_{yy} - ESS)/S_{yy} = 1 - \frac{ESS}{S_{yy}}$$

$$\hat\beta_1 \Big/ \sqrt{\frac{ESS}{(n-2)S_{xx}}} = \frac{S_{xy}}{\sqrt{S_{xx}}} \sqrt{\frac{n-2}{S_{yy} - r^2 S_{yy}}} = r\sqrt{\frac{n-2}{(1-r^2)}}$$

z-test, $z \sim N(0,1)$ t-test.. χ^2

H_A | RR

$\bar x \pm z_{\alpha/2} \hat{\mathrm{var}}(\bar x)$ $\bar x$, $\bar x_1 - \bar x_2$

p-val

$\hat p_1 = \frac{17}{56}$, $\hat p_2 = \frac{49}{130}$, $H_A: p_2 - p_1 > 0$, RR: $[z_{.005}, \infty)$, $\frac{\hat p_2 - \hat p_1}{(\hat p_1(1-\hat p_1)/112 + \frac{\hat p_2(1-\hat p_2)}{260})^{1/2}}$,

$\frac{358 - 280}{s/\sqrt n} \sim t_q$, $358 \pm t_{.005} \frac{s}{\sqrt n}$, $\frac{(n-1)S^2}{\sigma_0^2} \overset{H_0}{\sim} \chi^2_{n-1}$, $\frac{S_1^2}{S_2^2} \overset{H_0}{\sim} F^{n_1-1}_{n_2-1}$, $\frac{7(.018)}{.01} > \chi^2_{.05}$

$\frac{2.7}{.017} \sim F^{14}_{14}$

$$\frac{L(\theta_0)}{L(\theta_A)} = \frac{e^{-\lambda_0}\frac{\lambda_0^{\Sigma x_j}}{\Pi!}}{e^{-\lambda_A}\lambda_A^{\Sigma x_j}} = e^{\lambda_A - \lambda_0}\left(\frac{\lambda_0}{\lambda_A}\right)^{\Sigma x_j} < k, \quad \Sigma x_j > k',$$

$f_{\mu_0} = (2\pi\sigma^2)^{-1/2} \exp(-\frac{1}{2}\Sigma(x_j - \mu_0)^2)$, $\frac{f_{\mu_0}}{f_{\mu_1}} = \exp(\frac{1}{2}\Sigma\{(x_j - \mu_a)^2 - (x_j - \mu_0)^2\}) = \exp(\frac{1}{2}$

$(2(\mu_0 - \mu_a)n\bar x + \mu_a^2 - \mu_0^2)) < k$, $(\mu_0 - \mu_a)\bar x < k'$, $\bar x > k'$, $\alpha = \mathbb{P}_0(\bar x > k) =$

$\mathbb{P}\left(\frac{\bar x - 7}{\sigma/\sqrt n} > \frac{k - 7}{\sigma/\sqrt n}\right)$, $k = 7 + z_\alpha \frac{\sigma}{\sqrt n}$

<u>Whartner</u>

$\frac{1}{\delta_0^2} - \frac{1}{\delta_1^2} = \alpha$, $\frac{1}{\delta_0\delta_1} = \beta$, $\frac{1}{\delta_0^2} + \frac{1}{\delta_1^2} = \left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right)^2 + \frac{2}{\delta_0\delta_1} = \left(\frac{\alpha}{\frac{1}{\delta_0}+\frac{1}{\delta_1}}\right)^2$

$+2\beta$, $u^2 - v^2 = \alpha$, $uv = \beta$, $u^2 - \beta^2/u^2 = \alpha$, $u^4 - \alpha u^2 - \beta^2 = 0$,

$u^2 = \frac{1}{2}\left(\alpha \pm (\alpha^2 + 4\beta^2)^{1/2}\right)$, $u^2 + v^2 = u^2 + \frac{\beta^2}{u^2}$, $u^2 \frac{\cdot 2}{4}\left(\alpha^2 - \alpha^2 - 4\beta^2\right) = -\beta^2/u^2$,

$v^2 = \frac{1}{2}\left(-\alpha \pm (\alpha^2 + 4\beta^2)^{1/2}\right)$, $[2\alpha^2 + 4\beta^2 \pm 2\alpha(\alpha^2+4\beta^2)^{1/2})$ $\cancel{u^2+v^2}/4$

$+\beta^2]/u^2 = \left(\alpha^2/2 + 2\beta^2 \pm \frac{\alpha}{2}(\alpha^2+4\beta^2)^{1/2}\right)/u^2$, $\frac{p^2}{4}\alpha^2 + (1-p)^2\beta^2$,

$(\alpha^2+4\beta^2)^{1/2}(\alpha^2+4\beta^2\pm\alpha)$

$\alpha^2 + 4\beta^2 \pm \alpha(\alpha^2+4\beta^2)^{1/2}$, $\frac{\alpha}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}(\alpha^2+4\beta^2)^{1/2}$, $(\alpha^2+4\beta^2 \pm \alpha(\alpha^2+4\beta^2)^{1/2})(\alpha \mp$

$(\alpha^2+4\beta^2)^{1/2}) = \cancel{\alpha(\alpha^2+4\beta^2) \pm \alpha^2}$ $\overset{(\alpha^2+\alpha)}{\alpha(\alpha^2+4\beta^2)} + (\alpha^2+4\beta^2)^{1/2}\cancel{(\alpha^2+2\beta^2)\cdot 2}$

$(\pm\alpha^2 \mp \alpha^2 \mp 4\beta^2) = (\alpha^2+\alpha)(\alpha^2+4\beta^2) \mp 4\beta^2(\alpha^2+4\beta^2)^{1/2} = \beta^2(u^2+v^2)$,

$u^2+v^2 = \pm(\alpha^2+4\beta^2)^{1/2}$, $\lambda = \pm\frac{p}{2}(\alpha^2+4\beta^2)^{1/2} + \left(\frac{p^2}{4}\alpha^2 + (1-p)^2\beta^2\right)^{1/2} = \frac{p}{2}(\alpha^2+4\beta^2)^{1/2}$

$+\left(\frac{p^2}{4}(\alpha^2+4\beta^2) + \beta^2 - 2p\beta^2\right)^{1/2} = \frac{p}{2}(\alpha^2+4\beta^2)^{1/2}\left(1 + \left(1 + \frac{\beta^2(1-2p)}{\frac{p^2}{4}(\alpha^2+4\beta^2)}\right)^{1/2}\right) = \frac{p}{2}(\alpha^2+4\beta^2)^{1/2}$.

$\left(1 + \left(1 + \frac{1-2p}{\frac{p^2}{4}\left(\frac{\alpha^2}{\beta^2}+4\right)}\right)^{1/2}\right)$ $(p = 1/2)$ $\lambda = \frac{1}{2}(\alpha^2+4\beta^2)^{1/2}$

<u>stabilized</u> $p_1p_2 + (1-p_1)(1-p_2) = 2p_1p_2 - p_1 - p_2 + 1 = \frac{1}{2}(2p_1-1)(2p_2-1) + \frac{1}{2}$, $(\sqrt{\cdots})^2$

$= p_1^2p_2^2\underbrace{(u_{00}^2 + u_{11}^2)}_{\alpha} + (1-p_1)^2(1-p_2)^2\underbrace{(u_{01}^2+u_{10}^2)}_{\beta} - \underbrace{u_{00}u_{11}}_{\gamma}$, $u_{00}u_{11} = u_{01}u_{10}$, $(u_{00}+u_{11})^2$

$= \alpha + 2\gamma$, $u_{01}+u_{10} = (\beta+2\gamma)^{1/2}$, $\lambda = \frac{1}{2}\left(p_1p_2(\alpha+2\gamma)^{1/2} + (1-p_1)(1-p_2)(\beta+2\gamma)^{1/2}\right)$

$+\left\{p_1^2p_2^2\alpha + (1-p_1)^2(1-p_2)^2\beta - \gamma\right\}^{1/2}$, $p_1^2p_2^2\left(\frac{\gamma_0^2}{\delta_0^2} + \frac{\gamma_1^2}{\delta_1^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right) + \bar{p}_1^2\bar{p}_2^2\left(\frac{\gamma_0^2}{\delta_1^2} + \frac{\gamma_1^2}{\delta_0^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right)$

$p_1^2p_2^2(\gamma_0^2u^2 + \gamma_1^2v^2 + 2\gamma_0\gamma_1\beta) + \bar{p}_1^2\bar{p}_2^2(\gamma_0^2v^2 + \gamma_1^2u_2 + 2\gamma_0\gamma_1\beta)$, $\bar{p}_1^2\bar{p}_2^2 =$

$(p_1^2 - 2p_1 + 1)(p_2^2 - 2p_2 + 1)$, $p^2 + (1-p)^2 = 2p^2 - 2p + 1$, $4p^2 - 4p + 1 = (2p-1)^2 = 2(p^2+(1-p)^2) - 1$,

$\gamma_0\delta_0 + \gamma_1\delta_1 = \alpha$, $\gamma_0\delta_1 + \gamma_1\delta_0 = \beta$, $\delta_0 = \gamma_0^{-1}(\alpha - \gamma_1\delta_1)$, $\delta_1 = \gamma_0^{-1}\left(\beta - \frac{\gamma_1}{\gamma_0}(\alpha - \gamma_1\delta_1)\right)$,

$= \frac{1}{\gamma_0}\left(\beta - \frac{\gamma_1}{\gamma_0}\alpha\right) + \frac{\gamma_1^2}{\gamma_0^2}\delta_1$, $\delta_1 = \left(1 - \frac{\gamma_1^2}{\gamma_0^2}\right)^{-1}\frac{1}{\gamma_0}\left(\beta - \frac{\gamma_1}{\gamma_0}\alpha\right) = \frac{1}{\gamma_0^2 - \gamma_1^2}\cdot \beta\gamma_0 - \alpha\gamma_1$

$\delta_1 = \dfrac{\beta\gamma_0 - \alpha\gamma_1}{\gamma_0^2 - \gamma_1^2}$, $\delta_0 = \gamma_0^{-1}(\alpha - \gamma_1\delta_1) = (\gamma_0^2 - \gamma_1^2)^{-1}\gamma_0^{-1}(\alpha\gamma_0^2 - \alpha\gamma_1^2 - \beta\gamma_0\gamma_1 + \alpha\gamma_1^2)$

$= \dfrac{\alpha\gamma_0 - \beta\gamma_1}{\gamma_0^2 - \gamma_1^2}$, $\alpha\beta\gamma_0^2 - \alpha^2\gamma_0\gamma_1 - \beta^2\gamma_0\gamma_1 + \alpha\beta\gamma_1^2 = \alpha\beta(\gamma_0^2 + \gamma_1^2) - \gamma_0\gamma_1(\alpha^2 + \beta^2)$

$u^2 - v^2 = \alpha$, $uv = \beta$, $u^2 = u^2(\alpha,\beta)$, $v^2 = v^2(\alpha,\beta)$, $(u^2 = 1/\delta_0,\ v^2 = 1/\delta_1,\ \beta = uv = \frac{1}{\delta_0\delta_1})$

$(p_1 = 1 - p_2)$ $\lambda = \dfrac{p_1\bar{p}_1}{2}\left(\dfrac{\gamma_0}{\delta_0} + \dfrac{\gamma_1}{\delta_1} + \dfrac{\gamma_1}{\delta_0} + \dfrac{\gamma_0}{\delta_1}\right) + \Big\{ \dfrac{p_1^2\bar{p}_1^2}{4}\left(\dfrac{\gamma_0}{\delta_0} - \dfrac{\gamma_1}{\delta_1} + \dfrac{\gamma_1}{\delta_0} - \dfrac{\gamma_0}{\delta_1}\right)^2 + \dfrac{1}{\delta_0\delta_1}(\gamma_0\gamma_1(2p_1 - 1)^2$

Abc

$+ (\gamma_0 + \gamma_1)^2 p_1^2\bar{p}_1^2 \Big\}^{1/2} = \pm\dfrac{p_1\bar{p}_1}{2}(\alpha^2 + 4\beta^2)^{1/2}(\gamma_0 + \gamma_1) + \Big\{\dfrac{p_1^2\bar{p}_1^2}{4}\alpha(\gamma_0 + \gamma_1) + \beta^2$

$g(\gamma)\Big\}^{1/2}$, $\dfrac{\partial T}{\partial p_1} = p_2(c_{00} + c_{11}) - (1 - p_2)(c_{01} + c_{10})$, $\dfrac{\partial T}{\partial p_2} = p_1(c_{00} + c_{11}) - (1 - p_1)(c_{01} + c_{10})$

$T(p_1, p_2) = T(p_2, p_1)$, $(p_1 - p_2)(c_{00} + c_{11} + c_{01} + c_{10}) = 0$, $p_1 = p_2$, $p_1(c_{00} + c_{11}) = (1 - p_1)(c_{01}$

$+ c_{10})$, $\dfrac{p_1}{1 - p_1} = \dfrac{c_{01} + c_{10}}{(c_{00} + c_{11}}$, $p_1 = \sigma\left(\log\left(\dfrac{c_{01} + c_{10}}{c_{00} + c_{11}}\right)\right) = \left(\dfrac{c_{01} + c_{10}}{c_{00} + c_{11}}\right)\left(\dfrac{c_{00} + c_{11} + c_{10} + c_{01}}{c_{00} + c_{11}}\right)^{-1} = \dfrac{c_{01} + c_{10}}{c_{00} + c_{11} + c_{10} + c_{01}}$

$a_1p_1^2p_2^2 + a_2p_1^2p_2 + a_3p_1p_2^2 + a_4p_1p_2 + a_5p_1 + a_6p_2 + a_7$, $\dfrac{\partial^2}{\partial p_1^2} = 2a_1p_2^2 + 2a_2p_2$,

$\dfrac{\partial^2}{\partial p_1\partial p_2} = 4a_1p_1p_2 + 2a_2p_1 + 2a_3p_2 + a_4$, $\dfrac{\partial^2}{\partial p_1^2}\cdot\dfrac{\partial^2}{\partial p_2^2} = 4p_2(a_1p_2 + a_2)p_1(a_1p_1 + a_3) =$

$4p_1p_2(a_1^2p_1p_2 + a_1a_3p_2 + a_1a_2p_1 + a_2a_3)$, $\Delta^2 = -12a_1^2p_1^2p_2^2 - 12a_1a_3p_1p_2^2 - 12a_1a_2p_1^2p_2$

$- 4a_2a_3p_1p_2 - 8a_1a_4p_1p_2 - 4a_2a_4p_1 - 4a_3a_4p_2$, $\gamma_0u_0 + \gamma_1u_1 = \dfrac{\alpha}{2}(\gamma_0 - \gamma_1) \pm \frac{1}{2}(\alpha^2 + 4\beta^2)^{1/2}$

$(\gamma_0 + \gamma_1)$, $\gamma_1u_0 + \gamma_0u_1 = \frac{1}{2}\alpha(\gamma_1 - \gamma_0) + \frac{1}{2}(\alpha^2 + 4\beta^2)^{1/2}(\gamma_0 + \gamma_1)$, $p_1p_2 + (1 - p_1)(1 - p_2) = 2p_1p_2 - p_1 - p_2 + 1$,

$\delta_0\delta_1 T = T/\beta = \pm\dfrac{\alpha}{2}(\gamma_1 - \gamma_0)(p_1p_2 - (1 - p_1)(1 - p_2)) + \frac{1}{2}(\alpha^2 + 4\beta^2)^{1/2}(\gamma_0 + \gamma_1)(p_1p_2 + (1 - p_1)(1 - p_2))$