ROCKETBOOK EVERLAST

everlast.getrocketbook.com

(do not microwave this notebook)

$$\hat{\Theta} = \sum_{i,j} \varphi_{ij} / M^2 = \sum_{i<j} \varphi_{ij} \Big/ \binom{M}{2} = \sum_{i,j} \varphi_{ij} \Big/ M^2 .$$

$$V(x_i) = \sum_j \varphi_{ij}/M, \quad V(y_j) = \sum_i \varphi_{ij}/M \quad V(x_i) = \varphi_{i\cdot}/M, \; V(y_j) = \varphi_{\cdot j}/M$$

$$S_x = \sum_i (V(x_i) - \hat{\theta})^2, \quad S_y = \sum_j (V(y_j) - \hat{\theta})^2$$

$$S_{xy} = \sum_k (V(x_k) - \hat{\theta})(V(y_k) - \hat{\theta})$$

$$\hat{\sigma}^2 = S_x/M^2 + S_y/M^2 + 2S_{xy}/M^2 = [M(M-1)]^{-1} \sum_j (V(x_j) + V(y_j) - 2\hat{\theta})^2$$

$$\theta_{-i} = \sum_{j\neq i, k\neq i} \varphi_{jk} / (M-1)^2, \quad \hat{\theta}_j = M\hat{\theta} - (M-1)\theta_{-j}, \quad \hat{\sigma}^2 = (M(M-1))^{-1} \sum_j (\hat{\theta}_j - \bar{\hat{\theta}})^2,$$

$$\bar{\hat{\theta}} = M\hat{\theta} - \frac{M-1}{M} \sum_j \theta_{-j}, \quad (M-1)^2 \theta_{-i} = \sum_{jk} \varphi_{jk} - \sum_j \varphi_{ij} - \sum_i \varphi_{ij} + \varphi_{ii}$$

$$= M^2\hat{\theta} - \varphi_{i\cdot} - \varphi_{\cdot i} + \varphi_{ii}, \quad \sum_j \theta_{-j} = \frac{M^3}{(M-1)^2}\hat{\theta} - \frac{2M^2\hat{\theta}}{(M-1)^2} + \frac{\operatorname{diag}\varphi}{(M-1)^2},$$

$$\bar{\hat{\theta}} = \hat{\theta}\left(M - \frac{M^2}{M-1} + \frac{2M}{M-1}\right) - \frac{\operatorname{diag}\varphi}{M(M-1)}, \quad 1 - \frac{M-2}{M-1} = \frac{1}{M-1},$$

$$\bar{\hat{\theta}} = \frac{M}{M-1}\left(\hat{\theta} - \operatorname{diag}\varphi/M\right), \quad \hat{\theta}_j - \bar{\hat{\theta}} = M\hat{\theta} - (M-1)\theta_{-j} - \frac{M}{M-1}\hat{\theta} + \frac{\operatorname{diag}\varphi}{M(M-1)}$$

$$= \frac{\hat{\theta}M(M-2)}{M-1} + \frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{1}{M-1}\left(M^2\hat{\theta} - \varphi_{j\cdot} - \varphi_{\cdot j} + \varphi_{jj}\right)$$

$$= -\frac{2M}{M-1}\hat{\theta} + \frac{\operatorname{tr}\varphi}{M(M-1)} + \frac{\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}}{M-1}, \quad \sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}) = 2M^2\hat{\theta} - \operatorname{tr}\varphi,$$

$$M\left(\frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)^2 + 2\left(\frac{\operatorname{tr}\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)\frac{2M^2\hat{\theta} - \operatorname{tr}\varphi}{M-1} + \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2}$$

$$= \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2} - M\left(\frac{\operatorname{tr}\varphi - 2M^2\hat{\theta}}{M(M-1)}\right)^2 = M(M-1)\hat{\sigma}^2_{jk}, \quad \sum_j\left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat{\theta}\right)^2$$

$$= M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj} + \varphi_{jj} - 2M\hat{\theta})^2, \quad \sum(\varphi_{jj} - 2M\hat{\theta})^2 = \operatorname{tr}\varphi - 4M\hat{\theta}\operatorname{tr}\varphi$$

$$+ 4M^3\hat{\theta}^2), \quad \sum(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})(\varphi_{jj} - 2M\hat{\theta}) =$$

$$-2M\hat{\theta}(2M^2\hat{\theta} - \operatorname{tr}\varphi) + \sum\varphi_{j\cdot}\varphi_{jj} + \sum\varphi_{\cdot j}\varphi_{jj} - \operatorname{tr}\varphi, - \operatorname{tr}\varphi - 4M^3\hat{\theta}^2 + 2\sum\varphi_{jj}($$

$$\varphi_{j\cdot} + \varphi_{\cdot j}) + \sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2, \quad \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{M^2} - \frac{\operatorname{tr}\varphi}{M^2} - 4M\hat{\theta}^2$$

$$+ \frac{2}{M^2}\sum\varphi_{jj}(\varphi_{j\cdot} + \varphi_{\cdot j}) = M(M-1)\hat{\sigma}^2_{oso}$$

$$M(M-1)\hat\sigma^2_{osu} = \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat\theta\right)^2 = M^{-2}\left(\sum_j \varphi_{j\cdot}^2 + \sum_j \varphi_{\cdot j}^2 + 2\sum \varphi_{\cdot j}\varphi_{j\cdot}\right)$$

$$-\frac{4\hat\theta}{M}\sum_j(\varphi_{j\cdot}+\varphi_{\cdot j}) + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1}$$

$$-\frac{4\hat\theta}{M}\cdot 2M^2\hat\theta + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\dots)\mathbb{1} - 4M\hat\theta^2 =$$

$$M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4/M\,\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} \qquad \frac{1}{(M-1)^2} - \frac{1}{M^2} = \frac{2M-1}{(M(M-1))^2}$$

$$0 \doteq ? \quad \frac{2M-1}{(M(M-1))^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \frac{2}{(M-1)^2}\mathbb{1}^T(\varphi + \varphi^T - I/2$$

$$-\frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T + \operatorname{diag}\varphi\,\mathbb{1}^T/2M)\cdot\operatorname{diag}\varphi$$

$$\frac{M-1/2}{M^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \mathbb{1}^T(\varphi + \varphi^T - \frac{I}{2} - \frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T +$$

$$\operatorname{diag}\varphi\,\mathbb{1}^T/2M)\operatorname{diag}\varphi \overset{(1)}{=} \frac{1}{2}(M-1)^3M(\sigma^2_{jk} - \sigma^2_{osu})$$

$$(\varphi = \varphi^T)\quad \mathbb{1}^T\varphi(\cdot\cdot I - \frac{1}{M}\mathbb{1}\mathbb{1}^T)\varphi\mathbb{1} = \sum s_j^2 - \frac{1}{M}(\sum s_j)^2 = (M-1)\operatorname{var}(\mathbb{1}^T\varphi) = \operatorname{var}(\mathbb{1}^T\varphi^T)$$

$$\leq \frac{M}{2}\left((M - M/2)^2 + (M/2)^2\right) = \frac{M^3}{4}$$

$$\mathbb{1}^T(\varphi^T\varphi - \varphi\mathbb{1}\mathbb{1}^T\varphi/M)\mathbb{1} = \mathbb{1}^T(\varphi^T - \varphi\mathbb{1}\mathbb{1}^T/M)\varphi\mathbb{1}, \quad \mathbb{1}^T\varphi\cdot\operatorname{Proj}_{\mathbb{1}^\perp}\varphi\mathbb{1}$$

$$= \mathbb{1}^T\varphi(\vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec c}), \quad (\vec c, \vec r) - (\sum\varphi_{ij})^2/M,$$

$$(\vec c, \vec r) = \sum_j(\sum_i \varphi_{ij})(\sum_i \varphi_{ji}) = \sum_j\sum_{i,k}\varphi_{ij}\varphi_{jk}, \quad \sum_{i,k}\sum_j \varphi_{ij}\varphi_{jk} - \sum_{ik}\sum_{jl}\varphi_{ij}\varphi_{lk}/M$$

$$= \sum_{i,k,j}(\varphi_{ij}\varphi_{jk} - \sum_l \varphi_{ij}\varphi_{lk}/M) = \sum_{i,k,j}\varphi_{ij}(\varphi_{jk} - \varphi_{\cdot k})$$

$$M(M-1)\sigma^2_{osu} = M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - 2M\hat\theta)^2, \quad \varphi_{\cdot j} = 1 - \varphi_{ji}, \quad \varphi_{\cdot j} = \sum_i \varphi_{ij}$$

$$= \sum_i(1 - \varphi_{ji}) = M - \varphi_{j\cdot}, \quad M^{-2}\sum_j(M - 2M\hat\theta)^2 = M - 4M\hat\theta + 4M\hat\theta^2 = M(1-2\hat\theta)^2,$$

$$\sigma^2_{osu} = \frac{(1-2\hat\theta)^2}{M-1}$$

$$M(M-1)\sigma^2_{jk} = \frac{M^3}{(M-1)^2} - M\left(\frac{-2M^2\hat\theta}{M(M-1)}\right)^2 = \frac{M^3}{(M-1)^2} - \frac{4M^3\hat\theta^2}{(M-1)^2} = \frac{M^3}{(M-1)^2}(1 - 4\hat\theta^2),$$

$$\sigma^2_{jk} = \frac{M^2}{(M-1)^3}(1 - 4\hat\theta^2)$$

$$\sum \varphi_{\cdot j} = \sum_i \mathbb{P}(X_i < Y_j) = M - \sum_i \mathbb{P}(X_i > Y_j)$$

$$M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - \operatorname{tr}\varphi/M^2 - 4M\hat\theta^2 + \frac{2}{M^2}\sum\varphi_{jj}(\varphi_{j\cdot} + \varphi_{\cdot j}) = ?$$

$$(M-1)^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - M^{-1}\left(\frac{\operatorname{tr}\varphi - 2M^2\hat\theta}{(M-1)}\right)^2$$

$$\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} + \varphi_{jj})^2, \quad \sum_j \varphi_{j\cdot}^2 = \sum_j(\sum_i \varphi_{ji})^2 = \sum_j(\sum_i \varphi_{ji} + 2\sum_{i<k}$$

$$\varphi_{ji}\varphi_{jk}) = M^2\hat\theta + 2\sum_{i<k}(\varphi_{i}, \varphi_{k}), \quad \sum_j \varphi_{j\cdot}^2 = \mathbb{1}^T\varphi^T\varphi\mathbb{1}, \quad \sum_j \varphi_{\cdot j}^2 = \mathbb{1}^T\varphi\varphi^T\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{\cdot j} = \sum_j(\sum_i \varphi_{ji})(\sum_k \varphi_{kj}) = \sum_j\sum_{i,k}\varphi_{ji}\varphi_{kj} = \sum_{i,k}\sum_j \varphi_{ji}\varphi_{kj} = \mathbb{1}^T\varphi\varphi\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{jj} = \sum_i\sum_j \varphi_{ji}\varphi_{jj} = \mathbb{1}^T\varphi\operatorname{diag}\varphi, \quad \sum_j \varphi_{\cdot j}\varphi_{jj} = (\operatorname{diag}\varphi)^T\varphi\mathbb{1},$$

$$\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} + \operatorname{tr}\varphi - 2\mathbb{1}^T(\varphi + \varphi^T)\cdot$$

$$\operatorname{diag}\varphi = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T)\operatorname{diag}\varphi,$$

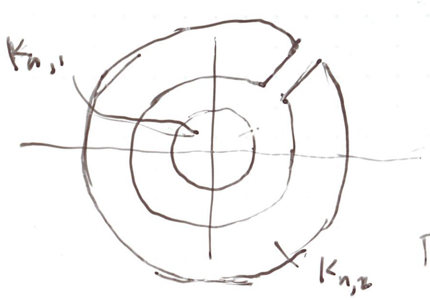
$$\operatorname{tr}\varphi - 2M^2\hat\theta = \operatorname{tr}\varphi - 2\mathbb{1}^T\varphi\mathbb{1}, \quad (M-1)^{-2}\left\{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - (\operatorname{tr}\varphi)^2/M -\right.$$

$$\left.4M^3\hat\theta^2 + 4M\hat\theta\operatorname{tr}\varphi\right\}, \quad \hat\theta^2 = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\varphi\mathbb{1}/M^4, \quad \hat\theta\operatorname{tr}\varphi = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\operatorname{diag}\varphi,$$

$$(M-1)^{-2}\left\{\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4M^{-1}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T - 2/M\,\varphi\mathbb{1}\mathbb{1}^T\right.$$

$$\left.+ \operatorname{diag}\varphi\cdot\mathbb{1}^T/2M)\operatorname{diag}\varphi\right\} = M(M-1)\hat\sigma^2_{jk}$$

Rudin ch 13, cont



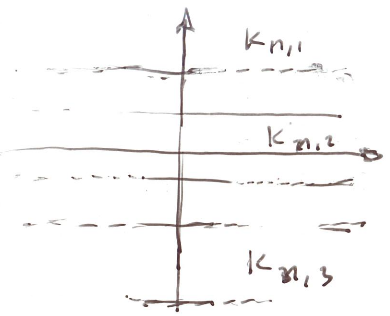
$\overbrace{}^{K_{n,1}}$ $\overbrace{}^{K_{n,2}}$

$K_n := \bar{D}(0, \frac{1}{n}) \cup \{ re^{i\theta} : \frac{1}{n} + \frac{1}{n^2} \le r \le n,\ \theta \le \sum_{j=1}^{n-1} \frac{1}{n^2} \frac{1}{2\pi} \}$

$\theta \notin \frac{1}{2\pi} (\sum_{j=1}^{n-1} \frac{1}{j^2}, \sum_{j=1}^{n} \frac{1}{j^2}) \}$, $P_n(z)$: $z \in K_n \Rightarrow$

$|P_n(z) - f_n(z)| < \frac{1}{n}$, $f_n(z) := \begin{cases} 0, & z \in K_{n,1} \\ 1, & z \in K_{n,2} \end{cases}$

$P_n(z) \xrightarrow[n\to\infty]{} \begin{cases} 1, & z = 0 \\ 0, & z \ne 0 \end{cases}$, $\tilde{P}_n(z) := P_n(z)/P_n(0)$ // #3



$K_n := \{ |\mathrm{Im}\, z| \le \frac{1}{n} \} \cup \{ \frac{1}{n} + \frac{1}{n^2} \le \mathrm{Im}\, z \le n \} \cup \{ -\frac{1}{n} - \frac{1}{n^2} \ge$

$\mathrm{Im}\, z \ge -n \} =: K_{n,1} \cup K_{n,2} \cup K_{n,3}$, $f_n(z) = \begin{cases} 1, & z \in K_{n,1} \\ 0, & z \in K_{n,2} \\ -1, & z \in K_{n,3} \end{cases}$

$P_n(z)$: $|f - P_n|_{K_n} < \frac{1}{n}$ #4 // $a \in \partial\Omega \Rightarrow a \notin K_n$ #7 //

$A_n := A \cap \{ n < |z| \le n+1 \}$, $|A_n| < \infty$, $Q_n := \sum_{a \in A_n} P_a(z)$,

$Q_n \in H(\bar{D}(0,n))$, $\sum_{j=0}^{\infty} a_j z_j$: $|\sum a_j z_j - Q_n|_{\bar{D}(0,n)} \to 0$, m : $|\sum_{j=1}^{m} a_j z_j -$

$Q_n(z)|_{\bar{D}(0,n)} < 2^{-n}$, $\tilde{Q}_n(z) := \sum_{j=1}^{m} a_j z_j$, $f(z) := Q_1 + \sum_{j=2}^{\infty} (Q_j(z) - \tilde{Q}_j(z))$,

$(z \in \bar{D}(0,n))$, $|\sum_{j=n}^{\infty} (Q_j(z) - \tilde{Q}_j(z))|_{\bar{D}(0,n)} \le 2^{-n+1}$, ~~$f \in H(\bar{D}$~~

$\sum_{j=n}^{\infty} (Q_j(z) - \tilde{Q}_j(z)) \in H(\bar{D}(0,n))$, $f(z) - \sum_{j=1}^{n-1} Q_j(z) \in H(\bar{D}(0,n))$ #8 //

$n_k \xrightarrow[k\to\infty]{} \infty$, $z^{2n_k} \to 0$, $k_0 : k > k_0 \Rightarrow z^{2n_{k-1}} < 5^{-1}$, $5^k z^{2n_{k-1}} < 1$, $\sum_{k_0}^{\infty} 5^k z^{n_k}$

$< \sum (5 z^{n_{k-1}})^k < \infty$. $|z| = 1 - \frac{1}{n_m}$, ~~$|h(z)| \le \sum_1^{\infty} 5^k (\frac{n_m - 1}{n_m})^{n_k}$~~, $(z = \frac{n_m - 1}{n_m})$

$\sum_1^{\infty} 5^k (\frac{n_m - 1}{n_m})^{n_k} = 5^m \sum_1^{\infty} 5^{k-m} (\frac{n_m - 1}{n_m})^{2kn_{k-1}}$, $\lim_{m\to\infty} \sum_k 5^{k-m} (\frac{n_m - 1}{n_m})^{2kn_{k-1}} > 0$?

$(1 - \frac{1}{n_m})^{n_m} \to e^{-1}$, $(|z| = 1 - \frac{1}{n_m})$ $\sum_{k=1}^{m-1} 5^{k-m} (\frac{n_m - 1}{n_m})^{n_k - n_m} < \sum_{k=1}^{m-1} 5^{k-m} (\frac{n_m}{n_m - 1})^{n_m}$

$= (e + o_m(1)) \sum_{k=1}^{m-1} 5^{-m} \frac{1 - 5^m}{1 - 4} \le \frac{1}{3} (e + o_m(1))$, $\sum_{k=m+1}^{\infty} 5^{k-m} (\frac{n_m - 1}{n_m})^{n_k - n_m}$

$< \sum (5 \frac{n_m - 1}{n_m})^{k-m} (\frac{n_m - 1}{n_m})^{n_{k-1} - kn_m}$ $5^{-m} \sum_{k=1}^{\infty} 5^k (1 - \frac{1}{n_m})^{n_{k+m} - n_m} = \sum_{k=1}^{\infty} 5^k (\frac{1}{e} + o(1))^{\frac{n_{k+m}}{n_m} - 1}$

$= o(1)$, $|\sum_{k=1}^{\infty} 5^k z^{n_k}| \ge 5^m |z|^m - \sum_{j \ne m} 5^j |z|^j \ge 5^m \cdot c$ //

5.16

$\int_0^\infty \frac{y}{\theta} e^{-y/\theta} = -y e^{-y/\theta} \Big|_0^\infty + \int e^{-y/\theta} = -\theta e^{-y/\theta} \Big|_0^\infty = \theta$, $\int \frac{y^2}{\theta} e^{-y/\theta}$

$= -y\theta e^{-y/\theta} \Big|_0^\infty + \int \theta e^{-y/\theta} = -\theta^2 e^{-y/\theta} \Big|_0^\infty = \theta^2$

$\sum (Y_j^2 - 2\bar{Y} Y_j + \bar{Y}^2) = \sum Y_j^2 - n\bar{Y}^2$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $(\frac{n}{\sigma^2})^2 \mathrm{Var}\, S^2 = 2(n-1)$,

$\mathrm{Var}\, S^2 = \frac{2\sigma^4}{n-1}$, $\mathrm{Var}(Y_1^2 - 2Y_1Y_2 + Y_2^2)$, $Y_1 - Y_2 \sim N(0, 2\sigma^2)$, $\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}} \sim Z$,

~~$1 = \frac{1}{2\sigma^2} \mathrm{Var}(Y_1 - Y_2)$~~, $\mathrm{Var}((\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}})^2) \sim \mathrm{Var}\, \chi^2(1) = 2$, $\mathrm{Var}(Y_1 - Y_2)^2$

$= 8\sigma^4$,

$\int_0^1 \theta y^\theta = \frac{\theta}{\theta+1} y^{\theta+1} \Big|_0^1 = \frac{\theta}{\theta+1}$

$2 E Y_i^2 = 2\lambda(1+\lambda) - 2\lambda^2 = 2\lambda$, $\mathrm{Var}((Y_i - Y_j)^2)$

$\frac{\theta+1}{\theta} y^{\theta+1} \Big|_0^1 = \frac{\theta+1}{\theta}$, $\frac{\theta+1}{\theta+2} y^{\theta+2} \Big| = \frac{\theta+1}{\theta+2} = \bar{X}$, $\theta(1 - \bar{X}) = 2\bar{X} - 1$, $\hat{\theta} = \frac{2\bar{X} - 1}{1 - \bar{X}} \xrightarrow{p} \theta$

$\sigma^2 + \mu^2 = \bar{X^2}$, $-\frac{y}{p^2} - (n-y)\frac{1}{(1-p)^2}$, $e^{-x/\theta}/\theta$, $\theta^{-n} e^{-\theta^{-1} \sum x}$, $-n \log \theta$

$-\sum x/\theta$, $-\frac{n}{\theta} + \frac{\sum x}{\theta^2}$, $\hat{\theta} = \bar{x}$, $(\frac{1}{2\theta+1})^n \{X_{(n)} \le 2\theta + 1\}$, $\hat{\theta} = \frac{1}{2}(X_{(n)} - 1)$,

$\frac{1}{2\theta+1} \int_0^{2\theta+1} y^2$, $\frac{1}{3}(2\theta+1)^2 - \frac{1}{4}(2\theta+1)^2 = \frac{1}{12}(2\theta+1)^2$, $\theta^{-2n} \prod y_j e^{-\sum y/\theta}$, $-2n \log\theta + \sum y_j - \sum y/\theta$,

$-\frac{2n}{\theta} + \frac{\sum y}{\theta^2}$, $\hat{\theta} = \bar{Y}/2$, 9.38

a) $Y_1, \dots, Y_n$ iid $\sim N(\mu, \sigma^2)$, pdf $f(y) = (2\pi\sigma^2)^{-1/2} \exp\left(-\left(\frac{y-\mu}{\sigma}\right)^2/2\right)$,

joint pdf

$$\prod_{j=1}^{n} f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}(y_j-\mu)^2\right)$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}(y_j^2 - 2\mu y_j + \mu^2)\right)$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum y_j^2 + \frac{\mu}{\sigma^2}\sum y_j - \frac{n\mu^2}{2\sigma^2}\right)$$

$$f(y_1, \dots, y_n) = \prod f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(\frac{\mu}{\sigma^2}\sum y_j - \frac{n\mu^2}{2\sigma^2}\right) \exp\left(-\frac{1}{2\sigma^2}\sum y_j^2\right)$$

$$= \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(\frac{n\mu}{\sigma^2}\bar{y} - \frac{n\mu^2}{2\sigma^2}\right)}_{\text{depends on } y_1,\dots,y_n \text{ only through } \bar{y}} \underbrace{\exp\left(-\frac{1}{2\sigma^2}\sum y_j^2\right)}_{\text{does not depend on target } \mu}$$

b)

$$f(y_1, \dots, y_n) = \prod f(y_j) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}(y_j-\mu)^2\right)}_{\text{depends on } y_1,\dots,y_n \text{ only through } \sum(y_j-\mu)^2} \cdot \underbrace{1}_{\text{does not depend on target } \sigma^2}$$

c)

$$f(y_1, \dots, y_n) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum y_j^2 + \frac{\mu}{\sigma^2}\sum y_j - \frac{n\mu^2}{2\sigma^2}\right)}_{\text{depends on } y_1,\dots,y_n \text{ only through } (\sum y_j^2, \sum y_j)} \cdot \underbrace{1}_{\text{does not depend on } (\mu, \sigma^2)}$$

9.39

$Y_1, \dots, Y_n \sim Poi(\lambda)$, pmf is $f_Y(j) = e^{-\lambda} \frac{\lambda^j}{j!}$ for $j = 0, 1, \dots$

also $\sum_{j=1}^{n} Y_j \sim Poi(n\lambda)$, pmf is then $f_{\Sigma Y}(j) = e^{-n\lambda} \frac{(n\lambda)^j}{j!}$.

Then $\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \mid \Sigma Y_j = \Sigma y_j)$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \text{ and } \Sigma Y_j = \Sigma y_j)}{\mathbb{P}(\Sigma Y_j = \Sigma y_j)}$$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n)}{\mathbb{P}(\Sigma Y_j = \Sigma y_j)} = \frac{\prod_{j=1}^{n} f_Y(y_j)}{f_{\Sigma Y}(\Sigma y_j)}$$

$$= \prod_{j=1}^{n} \left(e^{-\lambda} \frac{\lambda^{y_j}}{y_j!} \right) \Big/ e^{-n\lambda} \frac{(n\lambda)^{\Sigma y_j}}{(\Sigma y_j)!} = e^{-n\lambda} \frac{\lambda^{\Sigma y_j}}{\prod y_j!} \Big/ e^{-n\lambda} \frac{(n\lambda)^{\Sigma y_j}}{(\Sigma y_j)!}$$

$$= \frac{(\Sigma y_j)!}{\prod y_j!}$$ which doesn't depend on λ

9.40 $Y_1, \dots, Y_n \sim Rayleigh(\theta)$, pdf is $f_Y(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta}$ for $y > 0$,

so $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\frac{2y_j}{\theta}\right) e^{-y_j^2/\theta} = \left(\frac{2}{\theta}\right)^n \left(\prod y_j\right) e^{-\frac{\Sigma y_j^2}{\theta}}$

$$= \underbrace{\left(\frac{2}{\theta}\right)^n e^{-\Sigma y_j^2/\theta}}_{\text{depends on } y_1, \dots, y_n \text{ only through } \Sigma y_j^2} \cdot \underbrace{\left(\prod y_j\right)}_{\text{doesn't depend on } \theta}$$

9.41

$Y_1, \dots, Y_n \sim Weibull(\alpha)$, pdf is $f_Y(y) = \frac{1}{\alpha} m y^{m-1} e^{-y^m/\alpha}$ for $y > 0$

so $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\frac{1}{\alpha} m y_j^{m-1} e^{-y_j^m/\alpha}\right) = \left(\frac{m}{\alpha}\right)^n \left(\prod y_j\right)^{m-1} e^{-\Sigma y_j^m/\alpha}$

$$= \underbrace{\left(\frac{m}{\alpha}\right)^n e^{-\Sigma y_j^m/\alpha}}_{\text{dependence on sample thru } \Sigma y_j^m} \cdot \underbrace{\left(\prod y_j\right)^{m-1}}_{\text{no } \alpha \text{ dependence}}$$

9.42 — HW

9.43

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha y_j^{\alpha-1}/\theta^\alpha\right) = \underbrace{\left(\frac{\alpha}{\theta^\alpha}\right)^n \left(\prod y_j\right)^{\alpha-1}}_{\text{depends on sample thru } \prod y_j} \cdot \underbrace{1}_{\text{no } \alpha \text{ dependence}}$$

9.44

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha \beta^\alpha y_j^{-(\alpha+1)}\right) = \underbrace{(\alpha\beta^\alpha)^n \left(\prod y_j\right)^{-(\alpha+1)}}_{} \cdot \underbrace{1}_{\text{ditto}}$$

9.45 $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(a(\theta) b(y_j) e^{-c(\theta) d(y_j)}\right)$

$$= \underbrace{a(\theta)^n e^{-c(\theta) \Sigma d(y_j)}}_{\text{depends on sample thru } \Sigma d(y_j)} \cdot \underbrace{\prod b(y_j)}_{\text{no } \theta \text{ dependence}}$$

9.46 pdf is $f_{Y_i}(y) = \frac{1}{\beta} e^{-y/\beta}$ which has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \beta$, $c(\theta) = 1/\beta$, $a(\theta) = 1/\beta$, $b(y) = 1$.

9.47 pdf $\alpha y^{\alpha-1}/\theta^{\alpha} = \theta^{-\alpha} \alpha \exp((\alpha - 1)\log y)$ has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$, $c(\theta) =$

$\alpha - 1$, $a(\theta) = \theta^{-\alpha} \cdot \alpha$. So by 9.45 $\sum_j d(y_j) = \sum \log y_j$

is sufficient for α. No contradictions since $\sum \log y_j$

$= \log(\prod y_j)$ and "log" is one-to-one on $\mathbb{R}^+$

9.48 pdf $\alpha \beta^{\alpha} y^{-(\alpha+1)} = \alpha \beta^{\alpha} \exp(-(\alpha+1)\log y)$ has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$,

$c(\theta) = \alpha + 1$, $a(\theta) = \alpha \beta^{\alpha}$, $b(y) = 1$. Suff. stat. is

then $\sum d(y_j) = \sum \log y_j = \log(\prod y_j)$.

Rudin ch12

$z(t): [0,1] \mapsto ta + (1-t)b$, $|(t-1)(a-b)||t(a-b)||z-c| = t|t-1||a-b|^2|\overset{\alpha}{b-c} + t\overset{\beta}{(a-b)}|$

$\propto t(1-t)(|\alpha_r + t\beta_r|^2 + |\alpha_i + t\beta_i|^2)^{1/2}$, $\frac{\partial}{\partial t} = (1-2t)(\cdots)^{1/2} + t(1-t)(\cdots)^{-1/2}$.

$(\beta_r(\alpha_r + t\beta_r) + \beta_i(\alpha_i + t\beta_i)) = 0$, $(1-2t)|b-c+t(a-b)|^2 + t(1-t)(\beta_r(\alpha_r$

$+ t\beta_r) + \beta_i(\alpha_i + t\beta_i)) = (1-2t)(t^2(\beta_r^2 + \beta_i^2) + 2t(\alpha_r\beta_r + \alpha_i\beta_i) + \alpha_r^2 + \alpha_i^2) +$

$t(1-t)(t(\beta_r^2 + \beta_i^2) + \alpha_r\beta_r + \alpha_i\beta_i) = t^3 \cdot 2|\beta|^2 + t^2(4(\alpha,\beta) - |\beta|^2 - |\beta|^2 + (\alpha,\beta))$

$+ t(2|\alpha|^2 - 2(\alpha,\beta) - (\alpha,\beta)) - |\alpha|^2 = t^3 2|\beta|^2 + t^2(5(\alpha,\beta) - 2|\beta|^2) + 2t(|\alpha|^2 - (\alpha,\beta)) - |\alpha|^2$,

$(c = 0, a = i, b = 1, \alpha = 1, \beta = i - 1)$ $4t^3 + t^2(-5-4) + 2t(1-1) - 1 = 4t^3 - 9t^2 - 1$

$= t^3(-2|\beta|^2 + |\beta|^2) + t^2(|\beta|^2 - 3(\alpha,\beta) - |\beta|^2) + t(2(\alpha,\beta) - 2|\alpha|^2 - (\alpha,\beta)) + |\alpha|^2$

$= |\beta|^2 t^3 + 3(\alpha,\beta)t^2 - 2t((\alpha,\beta) - 2|\alpha|^2) + |\alpha|^2$, $2t^3 - 3t^2 + 6t - 1$

$= \cancel{t^3(-2|\beta|^2 - |\beta|^2)}$ $(1-2t)(|a|^2 + t^2|\beta|^2 + 2t\,\mathrm{Re}(a\bar{\beta})) + t(1-t)(t|\beta|^2 + \mathrm{Re}(a\bar{\beta}))$

$= t^3(-3|\beta|^2) + t^2(2|\beta|^2 - 5\,\mathrm{Re}(a\bar{\beta})) + t(3\,\mathrm{Re}(\alpha\bar{\beta}) - 2|\alpha|^2) + |\alpha|^2$, $(\alpha = 1, \beta = i - 1)$

$t^3(-6) + t^2(4 + 5) + t(-3-2) + 1$, $-\frac{3}{4} + \frac{9}{4} - \frac{5}{2} + 1 = 0$, $\alpha = x$, $\beta = a - x$,

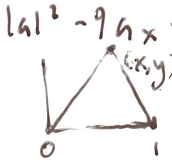
$2|\beta|^2 - 5\,\mathrm{Re}\,\alpha\bar{\beta} = 2(|a|^2 + x^2 - 2x\,\mathrm{Re}\,a) - 5(-x^2 + x\,\mathrm{Re}\,a) = 7x^2 - 9\,\mathrm{Re}\,a\,x + 2|a|^2$,

$3\,\mathrm{Re}(\alpha\bar{\beta}) - 2|\alpha|^2 = -3x^2 + 3\,\mathrm{Re}\,a\,x - 2x^2 = -5x^2 + 3\,\mathrm{Re}\,a\,x$, $-3(|a|^2 + x^2$

$-2x a_x)t^3 + (7x^2 - 9a_x x + 2|a|^2)t^2 + (-5x^2 + 3a_x x)t + x^2$ $(x = 1, a = i)$

$-3(2)t^3 + 9t^2 - 5t + 1$, $-3(|a|^2 - 2a_x + 1)t^3 + (2|a|^2 - 9a_x + 7)t^2$

$+ (3a_x - 5)t + 1 = -3(|a| - 1)^2 t^3 + (2|a| - 7)(|a| - 1)t^2 +$



$\max_{0 \le t \le 1} t^2(1-t)^2((x-t)^2 + y^2)$, $= (t^4 - 2t^3 + t^2)((x-t)^2 + y^2)$, $\frac{\partial}{\partial t} = (4t^3 - 6t^2 + 2t)((x-t)^2 + y^2)$

$-(t^4 - 2t^3 + t^2)2(x-t)$, $(2t^2 - 3t + 1)(t^2 - 2xt + x^2 + y^2) + t(1-t)^2 t(t-x) = 0$

$(2t-1)(t-1)$, $(2t-1)(t^2 - 2xt + x^2 + y^2) + t(t-1)(t-x) = 3t^3 + t^2(-4x - 1 - x - 1)$

$+ t(2(x^2+y^2) + 2x + x) - (x^2+y^2) = 3t^3 - (5x+2)t^2 + (2(x+y)^2 - x)t - (x^2+y^2)$, $\frac{3}{8} - \frac{9}{8} + 1 - \frac{1}{4} = 0$,

$\frac{5}{3a} - \frac{5^2}{9a^2} = \frac{2}{9}(x^2+y^2) + \frac{x}{3} - \frac{25x^2 + 20x + 4}{81} = \frac{1}{81}(-7x^2 + 7x) + \frac{2}{9}y^2 - \frac{4}{81} \cdots$ $\cancel{\max \prod_j |t - a_j|}$

$\max_{z \in \Delta} \prod_{j=1}^{3} (z - a_j) = \max_{z \in \partial\Delta} \prod (z - a_j) = \frac{s^2}{4} \cdot \frac{\sqrt{3}}{2} s = \frac{\sqrt{3}}{8} s^3$

$f'(i) \le \frac{1! \cdot 1}{1} = 1$, $f(z) = \frac{1}{1+z-i}$, $|1+z-i| \ge 1Re$, $Re^{i\theta} + R^{-1}e^{-i\theta} = (R+\frac{1}{R})\cos\theta$

$+i(R-\frac{1}{R})\sin\theta$, $R^2+\frac{1}{R^2}+2(\cos^2-\sin^2) = R^2+\frac{1}{R^2}+2\cos 2\theta$, $z-\frac{1}{z}$, $R^2+\frac{1}{R^2}-2\cos 2\theta$,

$R^{\frac{1}{2}}+\frac{1}{R^{\frac{1}{2}}}+2\cos\frac{\theta}{2}$, $z^{1/4}+z^{-1/4} = \frac{z^{1/2}+1}{z^{1/4}} = z^{-1}(z^{5/4}+z^{3/4})$, $z^{\alpha}+z^{-\alpha} =$

$z^{-\alpha}(z^{2\alpha}+1)$, $\frac{1}{(z-a)(z-b)}$, $\frac{1}{(z-a)^2(z-b)} + \frac{1}{(z-a)(z-b)^2}\Big|_{z=i} = (i-a)^{-2}(i-b)^{-1}+(i-a)^{-1}(i-b)^{-2}$

$= \frac{2i-a-b}{(i-a)^2(i-b)^2}$, $\frac{2i-(x_1-iy+x_2-i\frac{1}{y})}{(i-x_1+iy)^2(i-x_2+i\frac{1}{y})^2} = \frac{-(x_1+x_2)+i(2+y+\frac{1}{y})}{(-x_1+i(1+y))^2(-x_2+i(1+\frac{1}{y}))^2}$,

$((x_1+x_2)^2+16)^{1/2} \overset{?}{\le} (x_1^2+4)(x_2^2+4)$; $2+y+\frac{1}{y} \ge ? \ (x_1^2+(1+y)^2)(x_2^2+(1+\frac{1}{y})^2) \ge$

$(2+y+\frac{1}{y})^2 \ldots$

$f(x)=0 \Rightarrow |f(x)| \le \min_{y\in\bar D}|f(y)|$, $|f(x)| \ne 0$, $|f(x)| \le \inf_{y\in D}|f(y)| \Rightarrow f(y)\ne 0$,

$y\in D$, $|\frac{1}{f}(x)| \ge \sup_{y\in D/2}|\frac{1}{f}(y)|$, $\frac{1}{f} = \text{const.}$ #3// $f(x)\ne 0$, $x\in D \Rightarrow \max_{\bar D}|f(x)|$

$= \max_{\partial D}|f(x)| = c = \min_{\bar D}|f(x)|$, $|f(x)| = c$, $x\in\bar D$, $u^2(x,y)+v^2(x,y) = c$, uu_x+vv_x

$= uu_y+vv_y = 0$, $\frac{v_x}{u_x} = \frac{v_y}{u_y}$, $-\frac{u_y}{u_x} = \frac{u_x}{u_y}$, $u_x^2 = -u_y^2$, $u_x = v_y = 0$,

$u_y = v_y = 0$, $f(x) = \text{const}$, —·— #4a//

$m,n > n_0 \Rightarrow \varepsilon > \max_{z\in\partial D}|f_n(z)-f_m(z)| \ge \max_{z\in D}|\cdots|$ #5// $N(x) := \max_{\mathrm{Re}\, z = x}|fh_\varepsilon|$,

$|y| > \frac{B}{\eta\varepsilon} \Rightarrow |fh_\varepsilon| \le \frac{B}{\varepsilon}\cdot\frac{\eta\varepsilon}{B} = \eta$; $R := \{(x,y): a\le x\le b, -\frac{B}{\eta\varepsilon}\le y\le\frac{B}{\eta\varepsilon}\}$,

$x_0: \{x<x_0\}\cap R \Rightarrow |f(z)| < \eta$, $|f(z)h_\varepsilon| < \eta$, $x<x_0 \Rightarrow N(x) \le \eta$,

$N(x) \underset{x\to a^+}{\to} 0$, $a<x<\xi<b \Rightarrow N(\xi) \le N(x)^{\frac{b-\xi}{b-x}} N(b)^{\frac{\xi-x}{b-x}} \underset{x\to a^+}{\to} 0$, $N(x)\equiv 0$,

$h_\varepsilon(z)\ne 0$, $z\in\Omega$, $f\equiv 0$ #7// $M(r)^{\log\frac{b}{a}} \le ? \ M(a)^{\log\frac{b}{r}} M(b)^{\log\frac{r}{a}}$, $M(a) = M(b)$

$= 1 \Rightarrow M(r)\le 1$, $\sup_{\mathrm{Re}\, z = x}\log|f(z)| = \log\sup_{\mathrm{Re}\, z=x}|f(z)|$, $(\frac{b}{a})^{\log M(r)} \le ? \ (\frac{b}{r})^{\log M(a)}(\frac{r}{a})^{\log M(b)}$

$(f(z) = z)$ $r^{\log\frac{b}{a}} \le ? \ a^{\log\frac{b}{r}} b^{\log\frac{r}{a}}$, $\log b\log r - \log a\log r \le ? \ \log a\log b - \log a\log r$

$\log r \le ? \ + \log b\log r - \log b\log a$,

$|z|^{\alpha} = \exp(\alpha\log|z|)$, $e^{\beta z} \pm e^{-\beta z} = e^{\beta x}e^{i\beta y} \pm e^{-\beta x}e^{-i\beta y} = (e^{\beta x}\pm e^{-\beta x})\cos\beta y$

$+i(e^{\beta x}\mp e^{-\beta x})\sin\beta y$, $|\cdots|^2 = e^{2\beta x}+e^{-2\beta x} \pm 2\cos\beta y \mp 2\sin\beta y = 2\cosh 2\beta x$

$\pm 2\cos(\beta y+\frac{\pi}{4}) \ne 0$, $x > x_0 \Rightarrow |e^{\beta z}+e^{-\beta z}|^2 > \delta > 0$, $\beta y+\frac{\pi}{4} = \frac{(2k+1)\pi}{2}$, $y =$

$\beta^{-1}\frac{\pi}{2}(2k+\frac{1}{2})$, $k\in\mathbb{Z}$, $Y := \{\beta^{-1}\frac{\pi}{2}(2k+\frac{1}{2}) : k\in\mathbb{Z}\}$, $e^{\beta z}+e^{-\beta z} =$

$\mathrm{Im}\, z\in Y \Rightarrow e^{\beta z}+e^{-\beta z} = 2\cosh 2\beta x$, $|e^{\beta i z}+e^{-\beta i z}|^2 = 2\cosh 2\beta y +$

$2\cos(\beta x+\frac{\pi}{4})$, $|e^z+e^{iz}| = |e^x e^{iy}+e^{-y}e^{ix}|$, $|e^x e^{iy}+e^{-x}e^{-iy}+e^{-y}e^{ix}+e^{y}e^{-ix}|$

$\ge |e^{|x|}$, $z \mp iz = x+y+i(y-x)$, $|e^{z-iz}| = e^{x+y}$, $e^{z-iz}+e^{iz-z} = e^{x+y}e^{i(y-x)}+e^{-x-y}e^{i(x-y)}$

$= (e^{x+y}+e^{-x-y})\cos(x-y)+i(\cdots)$,… $z^{1/n} \in ? \ H(\mathbb{C}\setminus 0)$, $Re^{i\theta} \mapsto R^{1/n}e^{\frac{i\theta}{n}} \in ?$

$H(\mathbb{C}\setminus 0)$, $(z\in\Pi)$ $n: 1-\frac{1}{n} > \alpha$, $z^{1-1/n} := R^{1-1/n}e^{i(1-1/n)\theta}$, $\mathrm{Re}\, z^{1-1/n} =$

$R^{1-1/n}\cos((1-\frac{1}{n})\theta) > R^{1-1/n}\cos((1-\frac{1}{n})\frac{\pi}{2}) =: R^{1-1/n}\delta > 0$, $|\exp(-\varepsilon z^{1-1/n})|$

$= \exp(-\varepsilon\,\mathrm{Re}\, z^{1-1/n}) < \exp(-\varepsilon R^{1-1/n}\delta) < 1$ $(z\in\Pi)$, $h_\varepsilon(z) := e^{-\varepsilon z^{1-1/n}}$,

$z\in\Pi$, $fh_\varepsilon \in H(\Pi)$, $|fh_\varepsilon| \underset{(\mathrm{Re}\, z=0)}{<} 1$, $|fh_\varepsilon| < A\exp(|z|^{\alpha}-\varepsilon\,\mathrm{Re}\, z^{1-1/n})$

$\underset{|z|=R\to\infty}{\to} 0$), $R_0: |z|>R_0 \Rightarrow |fh_\varepsilon| < 1$, $\max_{z\in\Pi\cap\{|z|\ge R_0\}}|fh_\varepsilon| = \max_{z\in\partial(\cdots)}|fh_\varepsilon|$

$= 1$, $|fh_\varepsilon| \le 1$ $(z\in\Pi, \varepsilon>0)$, $|f|\le 1$. $(\alpha=1)$ $f(z) := e^z$,

$|f(z)| = e^{\mathrm{Re}\, z} \le e^{|z|}$, $|f(x)| = e^x \underset{x\to\infty}{\to} \infty$.

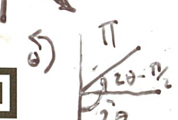
$(\frac{\pi}{2}+\theta)\alpha < \frac{\pi}{2}$, $\alpha < \frac{\frac{\pi}{2}}{\frac{\pi}{2}+\theta}$ #9

$|e^{niy}| \le 1$, $|f(iy)e^{niy}| \le 1$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| = |e^{nr\cos\alpha}f(re^{i\alpha})| = \exp(nr\cos\alpha$

$+\log|f(re^{i\alpha})|) = \exp(r(n\cos\alpha+r^{-1}\log|f(re^{i\alpha})|))$, $n\cos\alpha+r^{-1}\log|f(re^{i\alpha})| \to -\infty$,

$\frac{1}{A} := \max_r \exp(r(n\cos\alpha+r^{-1}\log|f(re^{i\alpha})|))$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| \le A$.

$\alpha < \frac{\frac{\pi}{2}}{\frac{\pi}{2}-\theta}$, $\alpha<\beta<\frac{\frac{\pi}{2}}{\frac{\pi}{2}-\theta}$, $\mathrm{Re}\, z^{\beta} = r^{\beta}\cos(\beta\arg z) \ge r^{\beta}\overbrace{\cos(\beta(\frac{\pi}{2}-\theta))}^{>0}$;



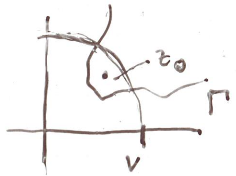
$z \mapsto (e^{-i\theta}z)^{\beta}$, $\mathrm{Re}(e^{-i\theta}z)^{\beta} > 0 \Leftarrow z\in\Pi$, $h_\varepsilon(z) := \exp(-\varepsilon(e^{-i\theta}z)^{\beta}) < 1$,

$|fh_\varepsilon| < A\exp(|z|^{\alpha}-\varepsilon r^{\beta}\cos(\beta(\frac{\pi}{2}-\theta))) \underset{|z|=r\to\infty}{\to} 0$. $|e^{nz}| < Ae^{|z|^{\alpha}}$ #10//

$(\Omega = \mathbb{C})$ $\Gamma = \emptyset$, $M = ?$ $(x_0 \notin \Omega)$ $\sup_{z\in\Omega} |f(z)| = \sup_{z \in \Omega - x_0} |f(z+x_0)|$, $0 \notin$

$\Omega - x_0$ $(0 \notin \Omega)$ $r := \inf_{z\in\Omega} |z|$, $r \neq 0$, $\sup_{z\in\Omega} |f(z)| = \sup_{z \in r\Omega} |f(z/r)|$,

$r\Omega \cap U \neq \emptyset$.



$\left|\frac{f(z_0)^n}{z_0}\right| \le \max_{\Gamma\cap V} \left|\frac{f(z)^n}{z}\right| \vee \max_{r=v} \left|\frac{f(z)^n}{z}\right| \le$

$M^n \vee \frac{B^n}{v}$, $\max(M^n, \frac{B^n}{v}) \xrightarrow[v\to\infty]{} M^n$, $|f(z_0)|^n \le |z_0| M^n$, $n \in \mathbb{Z}$,

$|f(z_0)| \le M$ #11 // $\operatorname{diam} E_n < \infty \Rightarrow \|f\|_{E_n} < \|f\|_{\partial E_n} = n$, $E_n = \emptyset$; $\|f\|_{E_n} < n+1$

$\Rightarrow \|f\|_{E_n} \le \|f\|_{\partial E_n} = n$, $E_n \neq \emptyset$;

$\gamma: \gamma(1-\frac{1}{n}) \in E_n$, $|\gamma(1-\frac{1}{n})$

$- \gamma(1-\frac{1}{n+1})| > 1$ #12 // $e^x \xrightarrow[x\to\infty]{} \infty$, $e^{-x} \xrightarrow[x\to\infty]{} 0$, $\operatorname{Re}\gamma \to \infty \Rightarrow e \to \infty$, $\operatorname{Re}\gamma \to -\infty$

$\Rightarrow e^\gamma \to 0$, $\operatorname{Re}\gamma \not\to \infty \Rightarrow \operatorname{Im}\gamma \to 0$, $e^\gamma \not\to L$. $e^{iz} - e^{-iz} = e^{-y+ix} - e^{y-ix}$

$= (e^y - e^{-y})\cos x + i(e^y + e^{-y})\sin x$, $|e^{iz} - e^{-iz}|^2 = e^{2y} + e^{-2y} - 2 \xrightarrow[|y|\to\infty]{} \infty$

$(|y| \not\to \infty)$ $x \to \infty$, $e^{iz} - e^{-iz} \not\to L \in \mathbb{C} \cup \infty$ #13 // $g(z) := f(1/z)$, Thm 10.21,

$(c \in \mathbb{C}: \tilde{g}(0) := c, \tilde{g}(z) := g(z), z \neq 0, \tilde{g} \in H(\mathbb{C}))$ $z_n \to \infty$, $g(\frac{1}{z_n}) \to c$,

$f(z_n) \to c$, $f = \text{const}$; $(g(z) = \sum_{j=-m}^{\infty} c_j z^j)$ $z_n \to 0 \Rightarrow c \in \mathbb{C}: g(z_n) - \sum_{j=-m}^{-1} c_j z_n^j$

$\to c$, $f(w_n) - \sum_{j=1}^{m} c_{-j} w_n^j \xrightarrow[w_n\to\infty]{} c$, $f(w_n) - \sum_{j=1}^{m} c_{-j} w^j = \text{const.}$, $f(w_n) =$

$\sum_{j=0}^{m} \tilde{c}_j w^j$, $\operatorname{Im} f = \mathbb{C}$; $(\operatorname{cl}(\tilde{g}(D(0;r))) = \mathbb{C})$ $c \in \mathbb{C}$, $z_n: z_n \to 0$, $g(z_n) \to c$,

$f(1/z_n) \to c$ #14 //

Rudin ch 13

$|A| = \infty$, $\lim_{r\to 0^+} |D(\infty, r) \cap A| > 0$, $\infty \in \operatorname{cl} A \Rightarrow |A| < \infty$, $f(z) - \sum_{j=1}^{|A|} R_j$

$\in H(\mathbb{C})$. #1 //

$\varphi: D(\frac{1}{2}, \frac{1}{2}) \to V$, $\varphi, \varphi^{-1} \in C'$,

$\varphi: 1 + \alpha v(\theta)$

$\mapsto 1 + (\cos\theta + \frac{\alpha}{2}) v(\theta)$, $0 < \alpha < 2\cos\theta$, $\bar{v}(\theta) := -1 - \frac{1}{\cos\theta}$, $v(\theta) := \bar{v}(\theta)/|\bar{v}(\theta)|$, $\pi/2 < \theta < 3\pi/2$.

$f(z) := \frac{1}{z - \frac{1}{2}}$, $\inf_U |f| = \frac{2}{3}$, $\sup_\Omega |P - f| < ?\ \frac{1}{3}$, $|(z-\frac{1}{2})P - 1| < \frac{1}{2}$, $\sup_{c(\Omega)} |(z-\frac{1}{2})P - 1| \le \frac{1}{2}$,

$\sup_{D(\frac{1}{2},\frac{1}{2})} |(z-\frac{1}{2})P - 1| \le \sup_{\partial D(\frac{1}{2},\frac{1}{2})} |\cdots| = \frac{1}{2}$, $((z-\frac{1}{2})P - 1)|_{z=\frac{1}{2}} = 1$ #2 //

5/6

1] $.95 = \mathbb{P}(L < \mu < U) = \mathbb{P}(X - L > X - \mu > X - U)$, $X - \mu \sim N(0,1)$,

$X - L = z_{.025}$, $L = X - z_{.025}$, $X - U = -z_{.025}$, $U = X + z_{.025}$ //

2] $.56 \pm z_{.005}\sqrt{\frac{.56 \cdot .44}{100^3}} = .56 \pm z_{.005} \frac{4}{1000}\sqrt{154}$

3] a) $\mathbb{E}X - \mu = 0$ b) $1 - 2\Phi(-1) = 2\Phi(1) - 1$ c) $\mathbb{P}(|X_1 - \mu| > \varepsilon) \ge$

$\mathbb{P}(|X_1 - \mu| \ge 1) = 2\Phi(1) - 1 \not\to 0$

4] a) $f_\mu(x_1, \dots, x_n) = \prod_{j=1}^n f_\mu(x_j) = (2\pi)^{-n/2} \exp(-\frac{1}{2}\sum_{j=1}^n (x_j - \mu)^2) =$

$(2\pi)^{-n/2} \exp(-\frac{1}{2}\sum x_j^2 + \mu\sum x_j + \mu^2/2) = \underbrace{(2\pi)^{-n/2} \exp(\mu n \bar{X} + \mu^2/2)}_{\text{depends on data through } \bar{X}} \underbrace{\exp(-\frac{1}{2}\sum x_j^2)}_{\text{doesn't depend on } \mu}$

b) $\mathbb{E}(\bar{Y}^2) = \operatorname{Var}\bar{Y} + (\mathbb{E}(\bar{Y}))^2 = \frac{1}{n} + \mu^2$, $\mathbb{E}(\bar{Y}^2) - \frac{1}{n} = \mu^2$

c) LLN: $\bar{Y} \xrightarrow[p]{} \mu$, so $\bar{Y}^2 \xrightarrow[p]{} \mu^2$ since $x \mapsto x^2$ is continuous.

5] $1 - \alpha = \mathbb{P}(L < \sigma^2 < U) = \mathbb{P}\left(\frac{(n-1)S^2}{L} > \frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)S^2}{U}\right)$,

$\frac{(n-1)S^2}{L} := \chi^2_{n-1, \frac{1}{2}(1-\alpha)}$, $L = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{1}{2}(1-\alpha)}}$, $U = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}$.

6] a) $(-1, \infty)$ b) $\mathbb{E}Y = \int_0^1 y(\theta+1)y^\theta\, dy = \frac{\theta+1}{\theta+2} := \bar{Y}$, $\hat{\theta}(1 - \bar{Y}) = 2\bar{Y} - 1$,

$\hat{\theta}_{MOM} = \frac{2\bar{Y} - 1}{1 - \bar{Y}}$ c) $(\theta+1)^n (\prod y_j)^\theta$, $n\log(\theta+1) + \theta\log\sum Y_j$, $\frac{n}{\theta+1} + \log\sum Y_j$

$= 0$, $\hat{\theta}_{MLE} = -\frac{n}{\log\sum Y_j} - 1$ d) $\hat{\theta}^2_{MLE}$ e)

7.84] $\theta^{-2n}\prod y_j e^{-\sum y_j/\theta}$, $-2n\log\theta - \sum y_j/\theta$, $-\frac{2n}{\theta} + \sum y_j/\theta^2 = 0$, $\hat{\theta} = \sum y_j/2n$,

$\mathbb{E}Y = \theta^{-2}\int y^2 e^{-y/\theta} = \theta^{-2}\{-\theta e^{-y/\theta} y^2|_0^\infty + 2\theta\int y e^{-y/\theta}\} = 2\theta$

$\cancel{(1-p)^{\sum y_j - n}}$

$\frac{2^n \prod y_j}{\theta^n} e^{-\sum y_j^2/\theta}$, $-n\log\theta - \sum y_j^2/\theta$, $-\frac{n}{\theta} + \sum y_j^2/\theta^2$, $\hat\theta = \frac{1}{n}\sum y_j^2$,

$\int y^2 e^{-y^2/\theta} = y\left(-\frac{\theta}{2}e^{-y^2/\theta}\right)\Big|_0^\infty + \int \frac{\theta}{2} y e^{-y^2/\theta} = \left(\frac{\theta}{2}\right)^2$

$p^n(1-p)^{\sum y_j - n}$, $n\log p + (\sum y_j - n)\log(1-p)$, $\frac{n}{p} - \frac{\sum y_j - n}{1-p} = 0$

$0 = n(1-p) - p(\sum y_j - n) = n - p(\sum y_j - n + 1)$, $p = \frac{n}{\sum y_j - n + 1}$

$\prod_{j=1}^{n} (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}(x_j-\mu)^2\right) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum (x_j-\mu)^2\right)$,

$\frac{L(\mu_{10})}{L(\mu_5)} = \exp\left(\frac{1}{2\sigma^2}\sum\{(x_j-\mu_5)^2 - (x_j-\mu_{10})^2\}\right) = \exp\left(\frac{1}{2\sigma^2}\{-2x_j\mu_5\right.$

$\left. + 2x_j\mu_{10} + \mu_5^2 - \mu_{10}^2\}\right) < k$, $2x_j(\mu_0 - \mu_a) + \mu_a^2 - \mu_0^2 < 2\sigma^2 \log k$,

$\exp\left(\frac{1}{2\sigma^2}\{2n\bar{x}(\mu_0-\mu_a) + n(\mu_a^2-\mu_0^2)\}\right) < k$, $\bar{X} < \frac{1}{2n\cdot 5}\{2\sigma^2\log k -$

$n(\mu_a^2 - \mu_0^2)\}$, $P_0(\bar{X} < k) = \alpha = P\left(Z < \frac{k-\mu_0}{\sqrt{\sigma^2/n}}\right)$, $k = \mu_0 + z_{1-\alpha}\, 5/\sqrt{n}$

$= 10 - z_\alpha \frac{5}{\sqrt{n}}$, $(2\theta^3)^{-n}\left(\prod y_j^2\right) e^{-\theta^{-1}\sum y_j}$, $\frac{L(\theta_0)}{L(\theta_a)} = \left(\frac{\theta_a}{\theta_0}\right)^{3n} e^{-\sum y_j(1/\theta_0 - 1/\theta_a)} < k$,

$-(\sum y_j)(1/\theta_0 - 1/\theta_a) < k$, $\sum y_j > k$, $y_j \sim \Gamma(3,\theta)$, $\sum y_j \sim \Gamma($

$P(Y_1+Y_2<c) = \int_0^1 F_2(c-y_1) = \int_0^1 (c-y_1) = c - \frac{1}{2}$, $P(Y_1+Y_2<1+c) = c + \int_c^1 F(1+c-y)$

$= c + \frac{1}{2}\cancel{(1+c)} \ldots 1 - c^2 - \frac{1}{2} + \frac{1}{2}c^2 = c + \frac{1}{2} - \frac{1}{2}c^2$, $f = 1-c$, $\frac{1}{2}(\theta+1)^2$,

Andreescu

$a^3 = n+3$, $b^3 = n^2+3n+3$, $b^3 - a^3 = n^2+2n = n(n+2)$, $b^3 + a^3 = n^2+4n+6 = (n+2)^2+2$

$b^3 = (n+3)(n+1) - n = (n+2)(n+1)+1$, $b^3 = a^3(n+1) - n$, $b^3 = n^2+2n+a^3$,

$n^2+2n = b^3 - a^3 = (b-a)(b^2+ab+a^2) = n(n+2)$,

$(2|n)$ $2\nmid a$, $2\nmid b$ $(2\nmid n)$ $2|a$, $2\nmid b$, $\frac{n+2}{n} + \frac{2}{n(n+2)}$,

$b^3 = a^6 - 6a^3 + 9 + 3a^3 - 6 = a^6 - 3a^3 + 3 = (a^3-3)(a^3-1) + a^3 = (a^3-2)^2$

$+ a^3 - 1$, $b^3 - a^3 = (a^3-3)(a^3-1) = (a^3-3)(a-1)(a^2+a+1)$

$2\nmid a \Rightarrow 2\nmid b$, $2|a \Rightarrow 2\nmid b$, $2|(n^2+3n) = n(n+3)$, $b^3 - 3 = a^3(a^3-3)$,

$b^3 = (n+2)(n+1)+1$, $(b-1)(b^2+b+1) = (n+1)(n+2)$, $a^3 = m$, $b^3 = m^2 - 6m + 9 + 3m - 9 + 3$

$= m^2 - 3m + 3$, $a^3 = m+1$, $b^3 = m^2 - 4m + 4 + 3m - 6 + 3 = m^2 - m + 1$, n^2+6n+9,

$c^3 - b^3 = 3n+6 = 3(n+2)$, $(n+3)^4 = n^4 + 4n^3\cdot 3 + 6n^2\cdot 9 + 4\cdot 27n + 81$, $n^4 + 9n^2 + 9$

$+ (3n^3 + 3n^2 + 9n)2 = n^4 + 6n^3 + 15n^2 + 18n + 9$, $n^3 + 6n^2 + 12n + 9$, $(n+1)^3 =$

$n^3 + 3n^2 + 3n + 1$, $(n+2)^3 = n^3 + 6n^2 + 12n + 8 \ldots$

$A^p = I$, $B^q = 0$, $A^{p-k} = A^{-k}$, $A^k = A^{-(p-k)}$, $(q=2, p=1)$ $(I+B)^2$

$= I + 2B$, $3B + I$, $A^{+p} = I$; $(A+I)^{-1} = \sum(-A)^j$

$5^{5^{n+2}} + 5^{5^{n+1}} - 5^{5^{n+1}} - 5^{5^n} = 5^{5^{n+2}} - 5^{5^n} = 5^{5^n}(5^{5^n\cdot 24} - 1)$, $(n=0)$ $25\cdot 625 + 6$

$= 15631 = 7\cdot 2233 = 49\cdot 319 = 7^2\cdot 11\cdot 29$, $5^{5^n\cdot 2^3\cdot 3} - 1$ $(n=0)$ $5^5 + 6 = 3131$

$= 31\cdot 101$, $5^2 - 1 = 24$, $124 = 31\cdot 4 = 5^3 - 1$, $5^{5^n\cdot 2^3\cdot 3} - 1 = (5^3-1)\sum_{j=0}^{5^n\cdot 8 - 1} 5^j$,

$31 \mid (5^{5^{n+2}} - 5^{5^n})$ #64 // $S = \sum_{j=0}^{n-1} (-1)^j \binom{n}{j} 5^{n-j-1}$, $5S = \sum_{j=0}^{n} \binom{n}{j}(-1)^j 5^{n-j} - (-1)^n$,

$5S - 1 = 6^n$, $S_n = \frac{1}{5}(6^n + 1)$, $(n=5)$ $36\cdot 216 + 1 = 7777 = 7\cdot 1111 = 7\cdot 11\cdot 101$

$216 \cdot 36 = 1296 + 648 = 7776$

$5S_k - 1 = 4^n$, $(n=5)$ $4^n + 1 = 1025 = 25\cdot 41$, $S_5 = 5\cdot 41$, S_{2k+3}

$- S_{2k+1} = \frac{1}{5}(4^{2k+3} - 4^{2k+1}) = \frac{4^{2k+1}}{5}\cdot 15 = 3\cdot 4^{2k+1}$, 4^{2k+1}, $4^{2k+1} + 1 \overset{(5)}{\equiv} 4 + 1 \overset{(5)}{\equiv} 0$

$4^{2^{2k+1}} + 1 = ?\ 5p \overset{(3)}{=} 2p$

$(a^2b + b^2c + c^2a)(bc^2 + ca^2 + ab^2) \geq (3abc)^2$ // #103 $\sum_{j=1}^{n} a_j^4$ $\geq (\sum a_j)(\sum \frac{a_j^4}{n})$

$\geq (\sum \frac{a_j^{5/2}}{\sqrt{n}})^2$, $(\sum a_j^{5/2})^2 \geq ?\ n^2$, $\sum a_j^4 \geq ?\ \sum a_j$, $n = \sum a_j \leq n^{3/4}(\sum a_j^4)^{1/4}$

$n \leq \sum a_j^3 \leq n^{1/2}(\sum a_j^4)^{1/2}$, $\sum a_j^4 \geq n$ // $\leq \sum a_j^3$ / $(\sum f_j)(\sum f_j x_j^2) \geq (\sum f_j x_j)^2$,

$(\sum f_j)(\sum f_j x^2) \geq (\sum \sqrt{f_j} \cdot \sqrt{f_j}\, x)^2$ #106 // $\sum_{j}^{n} k_j = 5n-4$, $1 = \sum \frac{1}{k_j}$, $1 \leq \sum k_j^3 \sum k_j^{-4}$

$1 \leq \sum k^{-3}$, $5n-4 \leq (\sum \frac{1}{k_j})^2 (\sum k_j^3)^2 = (\sum k_j^3)^2$, $\sum_j \frac{1}{k_j^3} \geq \frac{1}{5n-4}$, $\frac{1}{8} \geq \frac{1}{5n-4}$,

$0 \leq 5n^2 - 4n - 8$, $16 + 20c$, $n^2 \leq \sum k_j^2 \sum k_j^{-2}$, $n^2 = (\sum k^{1/2} k^{-1/2})^2 \leq \sum k \sum \frac{1}{k} =$

$5n-4$, $(n-1)(n-4) \leq 0$, $1 \leq n \leq 4$, $(n=1)\ k_1 = k_2$ $(n=2)\ k_1 = k_2 = 2$ $(n=3)\ k_1 = k_2 =$

$k_3 = 3$, $\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{2}$, $2(k_1 + k_2) = k_1 k_2$, $(n=4)\ k_1 = k_2 = k_3 = k_4 = \frac{1}{k_1} = \frac{1}{k_2} = \frac{1}{k_3} = \frac{1}{k_4}$,

$k_j = 4$ #107 // $\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b} \geq ?\ \frac{1}{\cos(a-b)} = \frac{1}{\cos a \cos b + \sin a \sin b}$, $1 \leq (\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b})$

$(\cos a \cos b + \sin a \sin b) \geq$ $\sin^2 a + \sin^2 b$ #111 //

$(P(x) = (x - r_1)(x - \bar{r}_1)(x - r_2)(x - \bar{r}_2)(x - r_3)$, $35 = -|r_1 r_2|^2 r_3$, -10

$= \bar{r}_1 |r_2|^2 r_3 + r_1 |r_2|^2 r_3 + |r_1|^2 \bar{r}_2 r_3 + |r_1|^2 r_2 r_3 + |r_1 r_2|^2 = 2\,\mathrm{Re}\, r_1 \cdot |r_2|^2 r_3$

$+ 2\,\mathrm{Re}\, r_2\, |r_1|^2 r_3 + |r_1 r_2|^2$, $\frac{2}{7} = \frac{2\,\mathrm{Re}\, r_1}{|r_1|^2} + \frac{2\,\mathrm{Re}\, r_2}{|r_2|^2} + \frac{1}{r_3}$, $\frac{1}{7} = \frac{\mathrm{Re}\, r_1}{|r_1|^2} + \frac{\mathrm{Re}\, r_2}{|r_2|^2} + \frac{1}{2r_3}$

$\frac{2}{7} = \sum \frac{1}{r_j}$ also ...

$(n^{n+1}(n-1)^2)^{1/2} \leq \frac{1}{2}(n^{n+1} + n^2 - 2n + 1)$, $\leq \frac{1}{2}(n^{n+1}(n-1) + n - 1)$, $n^n - 1 = (n-1)\sum_{j=0}^{n-1} n^j$

$\geq (n-1)\, n (\prod_{j=0}^{n-1} n^j)^{1/n} = (n-1)\, n^{1+\frac{n-1}{2}} = (n-1)\, n^{\frac{n+1}{2}}$ #122 // $1 + (r_1 \cdots r_n)^{1/n} \leq ?\ ((1+r_1)\cdots(1+r_n))^{1/n}$,

$\log(1 + (\prod r_j)^{1/n}) \leq ?\ \frac{1}{n}\sum \log(1 + r_j) \geq (\prod \log(1 + r_j))^{1/n}$, $\log(1 + (\prod r_j)^{1/n}) \leq \log(1 + \bar{r})$

$= \log(\frac{1}{n}\sum(1 + r_j)) \geq \frac{1}{n}\sum \log(1 + r_j)$ $\log(1 + r_1)\log(1 + r_2) \geq ?\ (\log(1 + \sqrt{r_1 r_2}))^2$,

$\frac{1}{n}\sum \log\log(1 + r_j) \geq ?\ \log\log(1 + (\prod r_j)^{1/n})$, $1 + \sqrt{r_1 r_2} \leq ?\ ((1 + r_1)(1 + r_2))^{1/2}$, $\sum_{j=0}^{n} \binom{n}{j} (\prod r_j)^{j/n}$

$\leq ?\ \sum_{j=0}^{n} \sum_{|S|=j} r_S \geq \sum_{j=0}^{n} (\prod r_j)^{\frac{\binom{n-1}{j-1}}{\binom{n}{j}}} \binom{n}{j} = \sum_{j=0}^{n} (\prod r_j)^{j/n} \binom{n}{j}$ #123

Wooldridge

$\hat{y}_2 = \mathbb{E}(zz^T)^{-1}\mathbb{E}(zy^T)$, $\hat{v}_2 = y_2 - \Pi_z(y_2)$, $\overset{\text{"}x\text{"}}{(z_1, y_2)} - \Pi_{\hat{v}_2}(z_1, y_2)$, $\Pi_{\hat{v}_2}(z_1, y_2)$

$= \hat{v}_2\, \mathbb{E}(\hat{v}_2 \hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2 (z_1, y_2)^T)$, $\Pi_{\ddot{x}}(y) = \mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\mathbb{E}(\ddot{x}y)$, $\mathbb{E}(\ddot{x}\ddot{x}^T) =$

$= \mathbb{E}(xx^T) - \mathbb{E}(xx^T \Pi_{\hat{v}_2} \hat{v}_2^T) - \mathbb{E}(\hat{v}_2 \Pi_{\hat{v}_2} xx^T) + \mathbb{E}(\hat{v}_2 \Pi_{\hat{v}_2} xx^T \Pi_{\hat{v}_2} \hat{v}_2^T) =$

$\mathbb{E}(xx^T) - \mathbb{E}(xx^T \Pi_{\hat{v}_2} y_2^T) + \mathbb{E}(xx^T \Pi_{\hat{v}_2} y_2^T \Pi_z z^T) - \mathbb{E}(y_2 \Pi_{\hat{v}_2} xx^T) + \mathbb{E}(z \Pi_z y_2 \Pi_{\hat{v}_2} xx^T)$

$+ \mathbb{E}(y_2 \Pi_{\hat{v}_2} xx^T \Pi_{\hat{v}_2} y_2^T) - \mathbb{E}(y_2 \Pi_{\hat{v}_2} xx^T \Pi_{\hat{v}_2} y_2^T \Pi_z z^T) - \mathbb{E}(z \Pi_z y_2 \Pi_{\hat{v}_2} xx^T \Pi_{\hat{v}_2} y_2^T)$

$+ \mathbb{E}(z \Pi_z y_2 \Pi_{\hat{v}_2} xx^T \Pi_{\hat{v}_2} y_2^T \Pi_z z^T)$, $\Pi_{\ddot{x}} y = ?\ \Pi_{\hat{x}} y$, $\Pi_{\hat{x}} y = \mathbb{E}(\hat{x}\hat{x}^T)^{-1}\mathbb{E}(\hat{x}y)$,

$\hat{x} = \Pi_z x = \bar{\Pi}_z x$, $\Pi_{\hat{x}} y = \mathbb{E}(\Pi_z \mathbb{E}(xx^T)\bar{\Pi}_z)^{-1}\,\bar{\Pi}_z \mathbb{E}(xy)$,

$P_{\hat{x}} y = \mathbb{E}(y\hat{x}^T)\mathbb{E}(\hat{x}\hat{x}^T)^{-1}\hat{x}$, $\hat{x} = P_z x = \mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z$, $P_{\hat{x}} y = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}$

$\{\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T)\}\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z = \mathbb{E}(P_z y \cdot x^T)\,\mathbb{E}(P_z x \cdot x^T)^{-1}$

$\cdot P_z x$, $P_{\ddot{x}} y = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\ddot{x}$, $\ddot{x} = x - P_x \hat{v}_2$, $\hat{v}_2 = \bar{P}_z y_2$,

$\bar{P}_{\hat{v}_2} x = x - \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$, $\hat{v}_2 = y_2 - P_{y_2} z = y_2 - \mathbb{E}(y_2 \ldots$

$\mathbb{E}(zz^T)^{-1} z$, $\mathbb{E}(y\hat{v}_2^T P_x^T) = \mathbb{E}(y(y_2^T - z^T \mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T)))\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2 x^T)$, $x = \binom{z_1}{y_2}$

$P(x|\hat{v}_2) = \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$, $\mathbb{E}(\binom{z_1}{y_2}(y_2 - z^T \mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2))) = \binom{0}{\mathbb{E}y_2^2 - \mathbb{E}y_2 P(y_2|z)}$

$\mathbb{E}(y_2^2) - \mathbb{E}(y_2 P(y_2|z)) = \mathbb{E}y_2^2 - \mathbb{E}(y_2 z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2) = \mathbb{E}y_2^2 - \mathbb{E}(y_2 z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)^{-1}$

$\mathbb{E}(y_2^2) = \mathbb{E}(y_2^2) - \mathbb{E}(P(y_2|z)^2) = \mathbb{E}\hat{v}_2^2$, $\mathbb{E}(x\hat{v}_2) = \binom{0}{\mathbb{E}\hat{v}_2^2}$, $P(x|\hat{v}_2) = \binom{0}{\hat{v}_2}$

$\ddot{x} = x - P(x|\hat{v}_2) = \binom{z_1}{y_2 - \hat{v}_2}$, $(\hat{\delta}_1, \hat{\alpha}_1) = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}$ $= (\mathbb{E}yz_1^T,$

$\mathbb{E}yy_2 - \mathbb{E}y\hat{v}_2)\begin{pmatrix}\mathbb{E}z_1 z_1^T & \mathbb{E}z_1 y_2 \\ \mathbb{E}y_2 z_1^T & \mathbb{E}(y_2 - \hat{v}_2)^2\end{pmatrix}^{-1} = ?\ \mathbb{E}(P(y|z) \cdot x^T) \cdot \mathbb{E}(P(x|z) \cdot x^T)^{-1}$

$\mathbb{E}(P(y|z) \cdot x^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(z \cdot (z_1^T, y_2^T)) = \mathbb{E}(yz^T)[I_{|z_1| \times |z_1|} : \mathbb{E}(zz^T)^{-1}$

$\mathbb{E}(zy_2^T)] = \mathbb{E}(yz_1^T) : \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T) = (\mathbb{E}yz_1^T, \mathbb{E}(yP(y_2|z)^T)) = (\mathbb{E}yz_1^T, \mathbb{E}(y(y_2 - \hat{v}_2)))$ ✓,

$\mathbb{E}(P(x|z) \cdot x^T) = \mathbb{E}(P(x|z)P(x|z)^T) = \mathbb{E}(\binom{z_1}{y_2 - \hat{v}_2}^{\otimes 2})$ #5.1 // $\hat{x} = P(x|z)$

$= \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}z$, $\hat{\beta}_1 = \mathbb{E}(y\hat{x})\mathbb{E}(\hat{x}^2)^{-1} = \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}\mathbb{E}(yz)\mathbb{E}(z^2)\mathbb{E}(xz)^{-2}\mathbb{E}(z^2)$

$\beta_1 = \mathbb{E}(yz)/\mathbb{E}(xz)$, $\hat\beta_1 = \sum yz/\sum xz = (\sum xz)^{-1}(\beta_0 \sum z + \beta_1 \sum xz + \sum zu) = \beta_0 \bar z/\overline{xz}$

$+\beta_1 + \sum xu/\sum xz$, $\sqrt{N}(\hat\beta_1 - \beta_1) = \beta_0 \sqrt{N}\bar z/\overline{xz} + \sqrt{N}\,\overline{xu}/\overline{xz} \rightsquigarrow \frac{\beta_0}{\sigma_{xz}^2} N(0, \ldots)$ $\sqrt{N}(\hat\beta_1 - \beta_1)$

$= \frac{\sqrt{N}\,\overline{zu}}{\overline{zx}} \rightsquigarrow \sigma_{zx}^{-1} N(0, \operatorname{Var} zu) = N(0, \sigma_z^2\sigma_u^2/\sigma_{zx}^2) = N(0, \rho_{zx}^{-2}\sigma_u^2/\sigma_x^2)$ #5.10 //

$\tilde y_2 = P(y_2|z_2) = \mathbb{E}(y_2 z_2^T)\mathbb{E}(z_2 z_2^T)^{-1} z_2$, $(\tilde\delta_1^T, \tilde\alpha_1) = \mathbb{E}(y_1(z_1^T, \tilde y_2))\mathbb{E}\left(\binom{z_1}{\tilde y_2}(z_1^T, \tilde y_2)\right)^{-1}$

$= \mathbb{E}\left\{(\delta_1^T, \alpha_1)\binom{z_1}{y_2}(z_1^T, \tilde y_2) + u_1(z_1^T, \tilde y_2)\right\}\mathbb{E}\begin{pmatrix} z_1 z_1^T & z_1 \tilde y_2 \\ z_1^T \tilde y_2 & \tilde y_2^2\end{pmatrix}^{-1} = (\delta_1^T, \alpha_1)\mathbb{E}\left(\binom{z_1}{y_2}(z_1^T, \tilde y_2)\right)$

$\mathbb{E}\left(\binom{z_1}{\tilde y_2}^{\otimes 2}\right)^{-1}$, $\mathbb{E}(\tilde y_2^2) = \mathbb{E}(y_2 \tilde y_2)$, $\mathbb{E}(y_2 z_1) \neq \mathbb{E}(\tilde y_2 z_1)$ #5.11 //

$\begin{matrix}\mathbb{E}x_{-k}x_{-k}^T & \mathbb{E}x_{-k}z^T \\ \mathbb{E}z x_{-k}^T & \mathbb{E}zz^T\end{matrix}$, $\begin{matrix}\mathbb{E}x_{-k}x^T \\ \mathbb{E}z x^T\end{matrix} = \begin{matrix}\mathbb{E}x_{-k}x_{-k}^T & \mathbb{E}x_{-k}x_k \\ \mathbb{E}z x_{-k}^T & \mathbb{E}z x_k\end{matrix}$, $(\delta, \theta) = \mathbb{E}(x_k(x_{-k}^T, z^T))\mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}$

$\theta = 0 \Rightarrow \mathbb{E}((x_k x_{-k}^T, x_k z^T)) \perp \mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[, k:(k+M)]$,

$(\mathbb{E}x_{-k}x_{-k}^T, \mathbb{E}x_{-k}z^T) \perp \mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[, k:(k+M)]$, $\operatorname{rank}\begin{pmatrix}\mathbb{E}x_{-k}x^T \\ \mathbb{E}zx^T\end{pmatrix} = K-1$.

$\begin{pmatrix}\mathbb{E}x_{-k}x_k \\ \mathbb{E}zx_k\end{pmatrix} \in \operatorname{sp}\begin{pmatrix}\mathbb{E}x_{-k}x_{-k}^T \\ \mathbb{E}zx_{-k}^T\end{pmatrix} \Rightarrow \begin{pmatrix}\mathbb{E}x_{-k}x_k \\ \mathbb{E}zx_k\end{pmatrix} \perp \mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[, k:(k+M)]$, $\theta = 0$ #5.12 //

$\beta_1 = \operatorname{Cov}(y,z)/\operatorname{Cov}(x,z) = (\mathbb{E}(yz) - \mathbb{E}y\mathbb{E}z)/(\mathbb{E}(xz) - \mathbb{E}x\mathbb{E}z) \doteq$

$(\sum y\{z=1\} - \sum y \cdot \sum z/n)/(\sum x\{z=1\} - \sum x \cdot \sum z/n)$, $\mathbb{E}yz = \beta_0\mathbb{E}z + \beta_1\mathbb{E}xz$,

$\mathbb{E}\left(\binom{1}{x}(1\ z)\right)\mathbb{E}^{-1}\begin{pmatrix}1 & z \\ z & z^2\end{pmatrix} = \begin{pmatrix}1 & \mathbb{E}z \\ \mathbb{E}x & \mathbb{E}xz\end{pmatrix}\begin{pmatrix}1 & \mathbb{E}z \\ \mathbb{E}z & \mathbb{E}z^2\end{pmatrix}^{-1} = (\operatorname{Var} z)^{-1}\begin{pmatrix}1 & \mathbb{E}z \\ \mathbb{E}x & \mathbb{E}xz\end{pmatrix}\begin{pmatrix}\mathbb{E}z^2 & -\mathbb{E}z \\ -\mathbb{E}z & 1\end{pmatrix}$,

$\mathbb{E}\left(\binom{1}{z}(1\ x)\right)^{-1}\mathbb{E}\binom{y}{yz} = \mathbb{E}\begin{pmatrix}1 & x \\ z & xz\end{pmatrix}^{-1}\mathbb{E}\binom{y}{yz} = (\operatorname{Cov}(x,z))^{-1}\mathbb{E}\begin{pmatrix}xz & -x \\ -z & 1\end{pmatrix}\mathbb{E}\binom{y}{yz}$,

$\hat\beta_1 = \frac{\mathbb{E}yz - \mathbb{E}y\mathbb{E}z}{\mathbb{E}xz - \mathbb{E}x\mathbb{E}z}$, $\frac{1}{n}(\sum y \sum z) = \frac{1}{n}(\sum y\{z=1\} + \sum_{j\neq k} y_j z_k)$, $\sum y_j(\{z_j=1\} - \bar z)$,

$\sum y_j(\{z_j=1\} - \{z_j=0\}) = 2\sum y_j\{z_j=1\} - \sum y_j$, $\sum y\{z=1\} - \bar y \sum\{z=1\}$, $\sum y\{z=1\}$

$-\bar z \sum y(\{z=1\} + \{z=0\}) = (1-\bar z)\sum y\{z=1\} - \bar z\sum y\{z=0\} = n\bar z(1-\bar z)\bar y_1 - \bar z n(1-\bar z)\bar y_0$ #5.13 //

$L(y|z) = L(\sum_{j=1}^{K} \beta_j x_j + u|z) = \sum_{j=1}^{K-1}\beta_j x_j + \beta_K L(x_K|z)$, $L(x_K|z) = \sum^{K-1}\delta_j x_j + \sum\theta_j z_j$,

$\theta \neq 0$, $\operatorname{rank}\mathbb{E}(zx^T) = K$ #5.14 //

$\frac{dP_n}{dQ_n} = \exp(\frac{1}{2}(x-\mu_n)^2 - \frac{1}{2}x^2) = \exp(-x\mu_n + \frac{\mu_n^2}{2})$, $\mathbb{P}_{Q_n}(\frac{dP_n}{dQ_n} \le c) = \mathbb{P}_{Q_n}(X \ge -\frac{1}{\mu_n}(\log c$

$-\frac{\mu_n^2}{2})) = \mathbb{P}(z > -\frac{\log c}{\mu_n} + \frac{\mu_n}{2})$, $\frac{\partial}{\partial c} = \varphi(-\frac{\log c}{\mu_n} - \frac{\mu_n}{2})/c\mu_n$, $\mathbb{P}_{Q_n}(\frac{dP_n}{dQ_n} > 0)$

$\xrightarrow{n\to\infty} \mathbb{P}_{Q_\infty}(\exp(-x\mu_\infty + \frac{\mu_\infty^2}{2}) > 0) = 1 \Leftarrow -x\mu_\infty + \frac{\mu_\infty^2}{2} > -\infty$ wp $1 - Q_\infty$,

$x\mu_\infty + \frac{\mu_\infty^2}{2} \underset{Q_\infty}{\sim} z\mu_\infty - \frac{\mu_\infty^2}{2} > 0$ wp 1 $\Leftarrow \mu_n \in [-K, K]$. $\mathbb{E}_{Q_n}\frac{dP_n}{dQ_n}$

$= \int \exp(-x\mu_n + \frac{\mu_n^2}{2})(2\pi)^{-1/2}\exp(-\frac{1}{2}(x-\mu_n)^2)\,dx = (2\pi)^{-1/2}\int\exp(-\frac{x^2}{2}) = 1$ #1 //

$\frac{dP_n}{dQ_n} = (2\pi/n)^{-1/2}\exp(\frac{1}{2}n(\ldots(x-\theta_n)^2 - x^2)) = \ldots\exp(\ldots\theta_n^2 - 2x\theta_n\ldots/2n)$, $\mathbb{P}_{Q_n}(\frac{dP_n}{dQ_n} < \delta)$

$(\theta_n = O(n^{-1/2}))$

$= \mathbb{P}_{Q_n}((\theta_n^2 - 2x\theta_n)\ldots < \log(\ldots)) = \mathbb{P}(N(-\frac{\theta_n^2}{2n}, \frac{\theta_n^2}{n^2}) < \log(\delta\sqrt{\frac{2\pi}{n}}))$

$= \Phi(\{\log(\delta\sqrt{\frac{2\pi}{n}}) + \frac{\theta_n^2}{2n}\}/\ldots) = \cancel{\Phi(O(\ldots)(\log(\delta\ldots) + o(1)))} \to 0$

$= \Phi(O(n^{\ldots})\log(\delta\ldots))$ $\frac{dP_n}{dQ_n} = (\ldots)^{-1}\exp(\frac{n}{2}(\theta_n^2 - 2x\theta_n))$, $\mathbb{P}_{Q_n}(\frac{dP_n}{dQ_n} < \delta)$

$= \mathbb{P}(N(-\theta_n^2\frac{n}{2}, n\theta_n^2) < \log(\ldots\delta)) = \Phi(\{\log(\ldots\delta) + \theta_n^2\frac{n}{2}\}/\sqrt{n}\theta_n)$

$= \Phi(O(\ldots)\{\log(\ldots\delta) + \ldots\})$ ... $\frac{\log(\ldots\delta)}{\sqrt{n}}$ ... $\to 0$,

$\delta \to 0 \to 0$, $\frac{dP_n}{dQ_n} \overset{Q_n}{\rightsquigarrow} V \Rightarrow \mathbb{P}(V=0) = 0$, $\mathbb{P}_{P_n}(\frac{dQ_n}{dP_n} < \delta)$:

$\mathbb{P}(N(-\frac{\theta_n^2}{2}n, n\theta_n^2) < \log\delta) \xrightarrow{\delta\to 0, n\to\infty} 0$, $\frac{dQ_n}{dP_n} \overset{P_n}{\rightsquigarrow} W \Rightarrow \mathbb{P}(W=0) = 0$, $P_n \triangleleft\triangleright Q_n$,

$\theta_n \neq O(n^{-1/2})$, $\ldots \to \infty$ $\limsup \theta_n n^{1/2} = \infty$, $\mathbb{P}_{Q_n}(\frac{dP_n}{dQ_n} < \delta) = \Phi(\frac{\log\delta}{\sqrt{n}\theta_n} + \frac{\theta_n\sqrt{n}}{2})$

$\xrightarrow{n_k} 1 \Leftarrow \theta_{n_k} n_k^{1/2} \xrightarrow{k\to\infty} \infty$ #2 // $\frac{dP_n}{dQ_n} = \frac{n}{n+1}\{[0,1]\} \overset{Q_n}{\rightsquigarrow} V\ldots$, $\mathbb{P}_{Q_n}(\ldots V < \delta)$

$\lim_{k\to\infty} \mathbb{P}_{Q_{n_k}}(\frac{dP_{n_k}}{dQ_{n_k}} < \delta) = \lim_{k\to\infty}\{\frac{n_k}{n_k+1} < \delta\} = 0 \Leftarrow \delta < \frac{1}{2}$, $Q_n \triangleleft P_n$. $\frac{dQ_n}{dP_n} = \frac{n+1}{n}\{[0,1]\}$,

$\mathbb{P}_{P_n}(\frac{dQ_n}{dP_n} < \delta) = \mathbb{P}_{P_n}(\{0 \le x \le 1\}\frac{\delta n}{n+1}) = \mathbb{P}_{P_n}(x \notin [0,1]) = \frac{1}{n+1} \to 0$, $P_n \triangleleft Q_n$ #3 //

$P_n(A) \to 0 \Rightarrow Q_n(A) = P_n(A) + Q_n(A) - P_n(A) \le P_n(A) + \|P_n - Q_n\|_{TV} \to 0$ #4 //

$P_n(0) = 1-\varepsilon$, $P_n(1) = \varepsilon$, $Q_n(0) = \varepsilon$, $Q_n(1) = 1-\varepsilon$, $\|P_n - Q_n\| = 1 - 2\varepsilon$, $P_n(A) \to 0 \Rightarrow A \cap \{0,1\}$

$= \emptyset$, $Q_n(A) = 0$ #5 // $P_n(0) = \frac{1}{2} = P_n(1)$, $Q_n(0) = 1$ //

Andreescu

$a+b+c=2 \geq 3(abc)^{1/3}$, $(1-a)(1-b)(1-c) = 1-a-b-c+ab+bc+ac-abc =$

$-1+\alpha$, $0 \leq ? \ (1-a)(1-b)(1-c)$, $a+b>c$, $1-a-b<1-c$, $(1-a)(1-b)(1-c)$

$= \alpha - 1 \leq \left(\tfrac{1}{3}\cancel{(a+b+c)}\right)^3 \cancel{= \tfrac{8}{27}}, \ \cancel{\alpha \leq \tfrac{35}{27}}$ $\left(\tfrac{1}{3}(3-a-b-c)\right)^3 = \tfrac{1}{27}$, $\alpha \leq \tfrac{28}{27}$.

$\alpha \geq ab+bc+ac - \left(\tfrac{1}{3}(a+b+c)\right)^3 = ab+bc+ac-\tfrac{8}{27}$, $(1-a)(1-b)(a+b-1)$

$=(b-1)(a-1)(b+a-1)$, $0 \leq ? \ (1-a)(1-b)(1-c)$, $a>1 \Rightarrow b+c = 2-a < 1$,

$a > b+c$, $a, b, c < 1$ #124 // $365^{-25}\prod_{j=1}^{25}(365-j) \leq ? \ \tfrac{1}{2}$, $\left(\tfrac{1}{25}\sum(365-j)\right)^{25} \leq ?$

$365^{25}/2$, $365 - \tfrac{25\cdot 26}{50} \leq ? \ 365/2^{1/25}$, $\cancel{352/365}$ $2^{1/25} \leq ? \ \tfrac{365}{352} = \tfrac{13}{11\cdot 32}+1$

$\geq 1+\tfrac{1}{32} \approx 1.02$, $\tfrac{13}{352} \geq \tfrac{13}{390} = \tfrac{1}{30}$, $(1.03)^{25} \geq ! \ 2$, $1.03^2 = 1.0609$, 1.12, 1.25,

1.56, $9: 1.2035$, $\tfrac{156}{124}$ ... #125/ $(k=3)$ $1, 5 = 1+(2-1)+k$, $12 = 5+1+2k$, 13

$3\cdot 4 = 22$, $a_{n-1} \underset{(k)}{\equiv} n-1$, $a_n - a_{n-1} \underset{(k)}{\equiv} 1$, $a_n - a_{n-1} = k(n-1)+1$, $a_n = \sum_{m=1}^{n} k(m-1)+1$

$= n + \tfrac{n(n-1)}{2}k$ #697 // $f(0)+2f(f(0)) = 5$, $f(f(0)) = \tfrac{1}{2}(5-f(0)) = f\left(\tfrac{5-2f^2(0)}{\cdots}\right)$, $5-f(0) \neq 0$,

$2 \mid (5-f(0))$, $f(0) \in \{1,3,5\}$ $(f(0)=1)$ $f(1) = 2$ ..., $f(n) = f(f(n))$

$= \tfrac{1}{2}(3n+5-f(n))$, $f(2) = f^2(1) = 3$, $f(3) = f^2(2)$ ... $f(n+1) = f(f(n)) = \tfrac{1}{2}(3n+5-(n+1))$

$= n+2$, $f(n) = n+1$ $(f(0)=3)$ ... $f^2(\ldots)$ ... $f(3)) \notin \mathbb{N}$

$(f(0)=5)$ $f(5)=0$, $f(1)$: ... #701 // $0 = f(n)+3f(0)f(n)$,

$f(0) = -\tfrac{1}{3}$, $f(1) = 0$, ... $\Rightarrow f(1) = 0$, $f(n^2) =$

$2f(n)+3f(n)^2 = (3f(n)$ ... $f(2n) = f(2)+f(n)+3f(2)f(n)$,

$\dfrac{f(mn)}{f(n)} = 1 + \dfrac{f(m)}{f(n)} + \ldots \geq 0 \Rightarrow f(n) \geq 1$, $f(6) \geq 3+6 = 9$, $f(4)$

$= 2f(2)+3f(2)^2$, $f(8)$: $f(4)+f(2)+3f(4)f(2) = 3f(2)+3f(2)^2+6f(2)^2+9f(2)^3$

$= 3f_2 + 9f_2^2 + 9f_2^3$, $f(n f^{-1}(1)) = 1 + 4f(n)$

causal

$p(1-p) = \tfrac{1}{4} - (p-\tfrac{1}{2})^2$, $1 + 4\dfrac{p_2(1-p_2)}{p_1(1-p_1)}$, $1 + \dfrac{1-4(p_2-\tfrac{1}{2})^2}{\tfrac{1}{4} - (p_1-\tfrac{1}{2})^2}(p_1-\tfrac{1}{2})^2$, $1+4$ 

$1 + \dfrac{1-4(p_2-\tfrac{1}{2})^2}{\tfrac{1}{4a} - 1}$, $1 + \dfrac{p_2(1-p_2)}{p_1(1-p_1)}$, $4p_2(1-p_2) = 4(p_2-\tfrac{1}{2})^2 + \dfrac{p_2(1-p_2)}{p_1(1-p_1)}$

$T = p(\delta_0+\delta_1)$, $\Delta = p^2\delta_0\delta_1 - (1-p)^2\delta_0\delta_1 = \delta_0\delta_1(2p-1)$, $\tfrac{T^2}{4} - \Delta = \tfrac{p^2}{4}(\delta_0^2+\delta_1^2+2\delta_0\delta_1)$

$-\delta_0\delta_1(2p-1)$, $\tfrac{T^2}{4} - \Delta = (1-p)^2\delta_0\delta_1 + \tfrac{p^2}{4}(\delta_0^2+\delta_1^2+2\delta_0\delta_1) - p^2\delta_0\delta_1 = (1-p)^2\delta_0\delta_1$

$+ \tfrac{p^2}{4}(\delta_0-\delta_1)^2 = (\delta_0-\delta_1)^2\left(\tfrac{p^2}{4} + (1-p)^2\dfrac{\delta_0\delta_1}{\delta_0^2 - 2\delta_0\delta_1+\delta_1^2}\right) = (\delta_0-\delta_1)^2\left(\tfrac{p^2}{4} + (1-p)^2 \cdots\right)$

$\left(\tfrac{\delta_0}{\delta_1} + \tfrac{\delta_1}{\delta_0} - 2\right)^{-1}\Big)$, $\tfrac{T^2}{4} - \Delta = \tfrac{p^2}{4}(\delta_0+\delta_1)^2\left(1 - \tfrac{4}{p^2}(2p-1)\dfrac{1}{\delta_0/\delta_1 + \delta_1/\delta_0 + 2}\right)$, $y = x+\delta$, $1 - \tfrac{x}{x+\delta}$

$<? \ \tfrac{x+\delta}{x} - 1 = \tfrac{\delta}{x}$, $1 <? \ \dfrac{x^2+x\delta+\delta x}{x(x+\delta)} = 1 + \dfrac{\delta}{x+\delta}$, $\inf_{\delta_0,\delta_1} \delta_0/\delta_1 + \delta_1/\delta_0 = 2$, $\tfrac{T^2}{4} - \Delta \in \tfrac{p^2}{4}(\delta_0+\delta_1)^2$

$(1 - \ldots)$ $\tfrac{T}{4} \cdot \tfrac{p^2}{4}(\delta_0+\delta_1)^2$ $\tfrac{T^2}{4} - \Delta \leq \tfrac{p^2}{4}(\delta_0+\delta_1)^2(1 - \tfrac{2}{p} + \tfrac{1}{p^2})$

$\tfrac{p^2}{4}(\delta_0-\delta_1)^2 + (1-p)^2\delta_0\delta_1$

vdVaart ch6 cont

$P(A_n) \to 0$, $Q(A_n) \not\to 0 \Rightarrow \{A_{n_k}\}_k$, $\delta > 0$: $Q(A_{n_k}) > \delta$, $\{A_{n_{k_j}}\}_j$, $P(A_{n_{k_j}}) < 2^{-j}$, $B_m := \bigcup_{j=m}^{\infty} A_{n_{k_j}}$, $P(B_m) \leq 2^{-m+1} \xrightarrow[m\to\infty]{} 0$, $B_m \downarrow B_\infty$, $0 = \lim P(B_m) = P(\lim B_m) = Q(\lim B_m) = \lim Q(B_m)$, $Q(B_m) > \delta$, #7 // $\mathbb{P}_{P}\left(\frac{dQ}{dP} > 0\right) = \mathbb{P}_{P}(dQ > 0) = 1 - \mathbb{P}_P(dQ = 0) = 1$ #8 //

Andreescu

$(mx+ny)^2 + 10(jx+ky)^2 = (m^2+10j^2)x^2 + (2mn+20jk)xy + (n^2+10k^2)y^2$, $mn = -10jk$, $m^2+10j^2 = 1$, $n^2+10k^2 = 10$, $n^2+10k^2 = 10(m^2+10j^2)$, $2mn+20jk = 0$, $mn = -10jk$

$n^2+10 = 10m^2+100$, $j=k=1$, $m=-1, n=10$; $m^2+10j^2 = u^2$, $n^2+10k^2 = 10u^2$, $n^2u^2 - 10j^2n^2 = 10j^2k^2 = n^2(u^2-10j^2)$, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, … 1, 4, 9, 5, 6; $3z^2 = \_\{3, 2, 7, 5, 8\}$, $x^2+10y^2 = \_\{1, 4, 9, 5, 6\}$, $\cap = \{5\}$, $5|x, 5|y, 5|z$ #706 / $(2x+2y)(2x-2y) = z^2 - t^2 = (z+t)(z-t)$, $4|(z^2-t^2)$, $x^2-y^2 = \frac{1}{4}(z^2-t^2)$, $24y^2 = 5z^2 - t^2$, $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} = -\frac{1}{24}\begin{pmatrix} 1 & -5 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} z^2 \\ t^2 \end{pmatrix}$

$24x^2 = 5t^2 - z^2$, $24y^2 = 5z^2 - t^2$, $5|(z-x)(z+x)$, ~~…~~ $(z = x+5k, t = y+5j)$

$5y^2 = 10kx + 25k^2$, $y^2 = 2kx + 5k^2$, $x^2 = 2jy + 5j^2$, $(z-x)(z+x) = 5y_3^2 = y_1y_2$, $z = \frac{1}{2}(y_1+y_2)$, $x = \frac{1}{2}(y_2-y_1)$; $(t-y)(t+y) = 5x^2 = x_1x_2$, $t = \frac{1}{2}(x_1+x_2)$, $y = \frac{1}{2}(x_2-x_1)$, $4y^2 = x_1^2+x_2^2-10x^2 = \frac{4}{5}(z^2-x^2)$, $-\frac{46}{5}x^2 + x_1^2 + x_2^2 = t^2$, $5|x$, $5|z$, $5|t$, $25\left(\frac{x}{5}\right)^2 + 5y^2 = 25\left(\frac{z}{5}\right)^2$, $5x_{(1)}^2 + y^2 = 5z_{(1)}^2$, $x^2 + 5y_{(1)}^2 = t_{(1)}^2$, $5|?y$, $25|(z^2-x^2)$, $5|y$, $\left(\frac{x}{5}\right)^2 + 5\left(\frac{y}{5}\right)^2 = \left(\frac{z}{5}\right)^2$, $5\left(\frac{x}{5}\right)^2 + \left(\frac{y}{5}\right)^2 = \left(\frac{t}{5}\right)^2$ #707 // $p|x \Rightarrow p|(x-y)(x+y)$, $p|y$, $x=y$ #708 // $x_j^2, x_{j+1}^2, x_{j+2}^2, \ldots$, $x_{j+1}^2 - x_j^2 = d$, $x_{j+1} - x_j = \frac{d}{x_j + x_{j+1}} \to 0$ #709 // $n^3 - 4n^2 + 1 + 5n^2 - 1 + 4 = n^3 + n^2 + 3 + (1-2n)\cdot(1+2n)$, $(2n-1)|(n^3+n^2+3)$, $2n^3 - n^3 - n^2 - 2n^2 + 4 = n^2(2n-1) - n^3 - 2n^2 + 4$, $(2n-1)|(n^3+2n^2-4)$, $(2n-1)|(n^2-7)$; $n^3 - 2n^2 - n^2 - n + n + 4 = -n(2n-1) + n^3 - n^2 - n + 4$, $(2n-1)|(n^3-n^2-n+4)$, $(2n-1)|(2n^2+n-1) = (2n-1)(n+1)$, $n^2 = \_\{1, 4, 9, 6, 5, 0\}$; $n^2 - 7 = \_\{4, 7, 2, 9, 8, 3\}$ $(2n-1)|(4n^2-1)$, $(2n-1)|(n^3-17)$; $(2n-1)|(5n^2-4)$, $(2n-1)|(-27)$, $2n-1 \in \{1, 3, 9\}$ #724 // $1!\,2!\cdots100! = 1^{100}\cdot2^{99}\,3^{98}\cdots99^2\cdot100^1 = x^2(2\cdot4\cdot6\cdots100) = x^2\,50!\,2^{50} = y^2\,50!$ #725 // $a_j = m+j$, $m|a_k \Rightarrow (a_j, a_k) = m$, $(a_j, a_{2j}) = j \Rightarrow j|m$, $(a_j, a_m) = j$, $a_j = b_j \cdot j$, $(j,k) = (b_j\cdot j, b_k\cdot k) \Rightarrow (b_j, b_k) = 1$ $b_{j_0} \neq 1$?, $(a_{j_0}, a_{b_{j_0}}) = (j_0 b_{j_0}, b_{j_0} b_{j_0}) = (j_0, b_{j_0})$, $b_{j_0}|j_0$, ~~$j_0 \nmid b_{j_0}$, $a_j = j^2$?~~ ~~$(a_j, a_{j^2}) = j \neq (j^2, j^4)$~~ // $(a_{j_0}, a_{b_{j_0}^2}) = (j_0 b_{j_0}, b_{j_0}^2 \cdot c)$, $b_{j_0}^2 | j_0, \ldots, b_{j_0} = 1$ #726 //

IV

$\frac{1}{2}\frac{\partial T}{\partial \gamma} \pm \left(\frac{1}{2}\frac{\partial T}{\partial \gamma} - \frac{\partial D}{\partial \gamma}\right)\frac{1}{2}\left(T^2/4 - D\right)^{-1/2} = 0$, $T = p^2\left(\gamma^2/\delta_0 + (1-\gamma)^2/\delta_1\right) + (1-p)^2\left(\gamma^2/\delta_1 + (1-\gamma)^2/\delta_0\right)$,

$\frac{\partial T}{\partial \gamma} = 2p^2\left(\gamma/\delta_0 - (1-\gamma)/\delta_1\right) + 2(1-p)^2\left(\gamma/\delta_1 - (1-\gamma)/\delta_0\right)$, $\frac{1}{2}(p^4 + (1-p)^4) \cdot (2p-1)^2$

$T = p^2\left(\gamma/\delta_0 + (1-\gamma)/\delta_1\right) + (1-p)^2\left(\gamma/\delta_1 + (1-\gamma)/\delta_0\right)$, $\frac{\partial T}{\partial \gamma} = p^2\left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right) + (1-p)^2\left(\frac{1}{\delta_1} - \frac{1}{\delta_0}\right)$

$= \left(\frac{1}{\delta_0} - \frac{1}{\delta_1}\right)(2p-1)$, $T^2 = p^4\left(\frac{\gamma^2}{\delta_0^2} + \frac{(1-\gamma)^2}{\delta_1^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right) + (1-p)^4\left(\frac{\gamma^2}{\delta_1^2} + \frac{(1-\gamma)^2}{\delta_0^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right)$

$+ 2p^2(1-p)^2\left(\frac{\gamma^2}{\delta_0\delta_1} + \frac{(1-\gamma)^2}{\delta_0\delta_1} + \gamma_0\gamma_1\left(1/\delta_0^2 + 1/\delta_1^2\right)\right)$ $(\delta_0 = \delta_1)$ $T = p^2\delta^{-2}(\gamma_0^2 + \gamma_1^2)(2p^2$

$-2p+1)$, $D = (2p-1)^2\delta^{-4}(\gamma_0\gamma_1)^2$, $(w_0\gamma_0 + w_1\gamma_1)/\delta_0 + (w_0\gamma_1 + w_1\gamma_0)/\delta_1$

$= ((w_0 - w_1)\gamma_0 + w_1)/\delta_0 + ((w_1 - w_0)\gamma_0 + w_0)/\delta_1$

$\mathbb{P}(A_{t+1} = a \mid L_t = \ell, U_t = u) = \sum_b p_B^b(1-p_B)^{1-b}\,\mathbb{P}(A_{t+1} = a \mid L_t = \ell, U_t = u, B_{t+1} = b) = p_B\,\mathbb{P}(A^L_{t+1} = a \mid L_t = \ell) + (1-p_B)\,\mathbb{P}(A^U_{t+1} = a \mid$

$U_t = u) = p_B\, p_{LA}^{a=\ell}(1-p_{LA})^{a\neq\ell} + (1-p_B)\, p_{UA}^{a=u}(1-p_{UA})^{a\neq u}$, $\mathbb{P}(L_{t+1} = \ell, U_{t+1} = u \mid$

$A_t = a)$

32aa

$= p_B\,\mathbb{P}(L_{t+1} = \ell, U_{t+1} = u \mid A^L_t = a, B_t = 1) + (1-p_B)\,\mathbb{P}(L_{t+1} = \ell, U_{t+1} = u / A^U_t = a,$

$B_t = 0)$ $= \frac{1}{2}p_B\, p_{AL}^{a=\ell}(1-p_{AL})^{a\neq\ell} + \frac{1}{2}(1-p_B)\, p_{AU}^{a=u}(1-p_{AU})^{a\neq u}$, $\mathbb{P}(L_{t+1} = \ell \mid A_t = a)$

$= p_B\,\mathbb{P}(L_{t+1} = \ell \mid A^L_t = a) + (1-p_B)\,\mathbb{P}(L_{t+1} = \ell) = p_B\, p_{AL}^{a=\ell}(1-p_{AL})^{a\neq\ell} + \frac{1}{2}(1-p_B)$,

$\mathbb{P}(A_t = a \mid L_t = \ell) = p_B\, p_{LA}^{a=\ell}(1-p_{LA})^{a\neq\ell} + (1-p_B)/2$

$P(L_t = \ell \mid L_{t-1} = \ell) = \sum_a \mathbb{P}(L_t = \ell / A_{t-1} = a)\,\mathbb{P}(A_{t-1} = a \mid L_{t-1} = \ell)$

$\delta_j^{-2} \mapsto \delta_j$

$T = p(1/\delta_0^2 + 1/\delta_1^2)$, $\Delta = p^2/\delta_0^2\delta_1^2 - (1-p)^2/\delta_0^2\delta_1^2 = (2p-1)/\delta_0^2\delta_1^2$, $T^2/4 - \Delta =$

$\frac{1}{4}\left(p^2(\delta_0^2 + \delta_1^2) + 2\delta_0\delta_1\right) - (2p-1)\delta_0\delta_1 = \frac{p^2}{4}(\delta_0^2 + \delta_1^2) + \delta_0\delta_1(p^2/2 - 2p + 1)$

$= \frac{p^2}{4}(\delta_0 - \delta_1)^2 + \delta_0\delta_1(p^2 - 2p + 1) = \frac{p^2}{4}(\delta_0 - \delta_1)^2 + \delta_0\delta_1(p-1)^2$ $(\delta_0 = \delta_1)$

$\frac{1}{2}p\,\cancel{\left(1/\delta_0^2 + 1/\delta_1^2\right)} \pm \frac{p}{\delta_0^2} \pm (1-p)/\delta_0^2 = \frac{1}{\delta_0^2}$, $(2p-1)\frac{1}{\delta_0^2}$

$1/\delta_0 + 1/\delta_1 = s$, $T = p^2((\gamma_0 - \gamma_1)/\delta_0 + \gamma_1 s) + (1-p)^2((\gamma_1 - \gamma_0)/\delta_0 + \gamma_0 s)$

$= (\gamma_0 - \gamma_1)\delta_0^{-1}(2p-1) + s(p^2\gamma_1 + (1-p)^2\gamma_0) =$

Andreescu

$n^a-1 = (n^{(a,b)}-1)(1+n+\dots+n^{a'-1})$, $a' := \frac{a}{(a,b)}$, $n^{(a,b)}-1 \mid (n^a-1, n^b-1)$. $d \mid (n^a-1, n^b-1)$,

$d \mid (n^a-n^b) = (n^{(a,b)a'} - n^{(a,b)b'}) = (n^{a'}-n^{b'})(n^{a'((a,b)-1)} + n^{a'((a,b)-2)}n^{b'} + \dots + n^{b'((a,b)-1)})$

$= (n^{a'}-n^{b'})(n^{a-a'} + n^{a-2a'+b'} + n^{a-3a'+2b'} + \dots + n^{b-b'})$, $n^a-1 = (n^g-1)\sum_{j=0}^{a'-1} n^{jg}$

$n^b-1 = (n^g-1)\sum_{j=0}^{b'-1} n^{jg}$, $(b>a)$ $n^b-1-(n^a-1) = (n^g-1)\sum_{j=a'}^{b'-1} n^j$, $\sum_{b'}^{b'+a'-1}$

$(a',b')=1$, $a'x+b'y=1$, $\frac{1+n+n^2}{1+n+n^2+n^3+n^4}$, $\frac{1+n+n^2}{1+n}$, $1 = (\sum^{5-1} - n^2\sum^{3-1})(-n) + \sum^{3-1} = -n\sum^{5-1} +$

$(n^2+1)\sum^{3-1}$, $S_{a'-1} := \sum_{j=0}^{a'-1} n^j$, $\alpha S_{a'-1} + \beta S_{b'-1} = ?\ 1$, $\alpha,\beta\in\mathbb{Z}$, $(a'x - b'y = 1)$

$(\sum_{j=0}^{x} n^{ja'}) S_{a'-1} = (\sum_{j=0}^{x} n^{ja'})\sum_{j=0}^{a'-1} n^j = \sum_{j=0}^{a'x} n^j$, $(\sum_{j=0}^{y} n^{jb'}) S_{b'-1} = \sum_{j=0}^{b'y} n^j = \sum_{j=0}^{a'x-1} n^j$,

$(\sum_{j=0}^{x} n^{ja'}) S_{a'-1} - n(\sum_{j=0}^{y} n^{jb'}) S_{b'-1} = 1$ #727 // $g=(a,b)$, $a=ga'$, $b=gb'$, $(a',b')=1$,

$(2\nmid a', 2\nmid b')$, $2^a+1 = (2^g+1)\sum_{j=0}^{a'-1} 2^{jg}$, $2^b+1 = (2^g+1)\sum_{j=0}^{b'-1} 2^{jg}$ (#727) $\Rightarrow \alpha,\beta\in\mathbb{Z}$,

$\alpha\sum_{j=0}^{a'} 2^j + \beta\sum_{j=0}^{b'} 2^j = 1$, $2^g+1 = \alpha(2^a+1)+\beta(2^b+1)$, $(2^a+1, 2^b+1) \mid (2^g+1)$

$(2\nmid a',\ b' = 2^k b'',\ 2\nmid b'')$ $2^a+1 = (2^g+1)\sum_{j=0}^{a'-1}(-2^g)^j$, $2^b+1 = (2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{g2^k})^j$

$\alpha,\beta\in\mathbb{N}?: \alpha(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j = (2^g+1)\{\beta\sum_{j=0}^{a'-1}(-2^g)^j - 1\}$, $a^{2^{k+1}}$ 2^k.

$= \sum_{j=0}^{k-1} 2^j + 1 = \sum_{j=1}^{k-1} 2^j + 2$, $(\alpha := (2^g+1))$ $?\beta: (2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j = \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1$,

LHS $= \sum_{j=0}^{b''-1}(-1)^j 2^{2^k g(j+1)} + \sum_{j=0}^{b''-1}(-1)^j(2^{2^k g})^j = -\sum_{j=1}^{b''}(-1)^j 2^{2^k g j} + \sum_{j=0}^{b''-1}(-1)^j(2^{2^k g})^j$

$= 1 - (-2^{2^k g})^{b''} = ?\ \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1$, $(-2^{2^k g})^{b''} = ?\ \beta\sum_{j=0}^{a'-1}(-2^g)^j$,

$?\alpha',\beta: \{-2^b - 1\}\alpha' = \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1 = \beta\frac{2^a+1}{2^g+1} - 1$, $\alpha'(2^b+1)(2^g+1) = ?$

$\beta(2^a+1) - 1$, $\alpha'(2^{b+g} + 2^g(1+2^{b-g})+1) = ?\ \beta 2^a + \beta - 1$; $2^g\{\alpha'(1+2^{b-g}+2^g)$

$-\beta 2^{a-g}\} = ?\ \beta - 1 - \alpha'$,

516

$L(\mu) = (2\pi\sigma^2)^{-n/2}\exp(-\frac{1}{2\sigma^2}\sum(x_j-\mu)^2)$, $\frac{L(\mu_0)}{L(\mu_a)} = \exp(\frac{1}{50}\sum\{(x_j-\mu_a)^2 - (x_j-\mu_0)^2\})$

$= \exp(\frac{1}{50}\sum(2x_j(\mu_0-\mu_a) + \mu_a^2 - \mu_0^2)) < k$, $2(\mu_0-\mu_a)n\bar{X} + n(\mu_a^2-\mu_0^2) < k$

$\bar X < k$, $\alpha = \mathbb{P}_{\mu=10}(\bar X < k) = \mathbb{P}(Z < \frac{k-10}{5/\sqrt n})$, $k = 10 - \frac{5}{\sqrt n} z_{.025}$, $\mathbb{P}_{\mu=5}(\bar X <$

$10 - \frac{5}{\sqrt n}z_\alpha) = \mathbb{P}(Z < \frac{5 - 5/\sqrt n\, z_\alpha}{5/\sqrt n}) = \mathbb{P}(Z < \sqrt n - z_\alpha)$, $\frac{7(.018)}{.01} > \chi^2_{.05}$

$(\frac{\sigma_a}{\sigma_0})^n\exp(\sum(x_j-\mu)^2(\frac{1}{2\sigma_a^2} - \frac{1}{2\sigma_0^2})) < k$, $\sum(x_j-\mu)^2(\frac{1}{2\sigma_a^2}-\frac{1}{2\sigma_0^2}) < k$

$\sum(x_j-\mu)^2 > k$, $\chi^2(n) > k = \chi^2_\alpha$, $\theta^{2N_1}(2\theta(1-\theta))^{N_2}(1-\theta)^{2N_3}$

$\frac{\theta_0}{\theta_a}$ $= 2^{N_2}\theta^{2N_1+N_2}(1-\theta)^{N_2+2N_3}$, $L(\theta) = (\frac{\theta_0}{\theta_a})^{2N_1+N_2}(\frac{1-\theta_0}{1-\theta_a})^{N_2+2N_3} < k$

treatment

first sentence to bullets $(2N_1+N_2)\log(\frac{\theta_0}{\theta_a}) + (N_2+2N_3)\log(\ ) < k$

$2N_1+N_2 > k$

stick with DAGs not linear a's $L(\lambda) = e^{-n\lambda}\frac{\lambda^{n\bar x}}{\prod x_j!}$, $\frac{L(\lambda_0)}{L(\lambda_a)}$

alternate slide

at top of slide $= e^{n(\lambda_a-\lambda_0)}(\frac{\lambda_0}{\lambda_a})^{n\bar x} < k$, $\bar x\log(\ ) < k$, $\bar x > k$,

$\alpha = \mathbb{P}(\sum x_j > k)$, $\binom{124}{76}p^{76}(1-p)^{48}\binom{147}{53}p^{53}(1-p)^{94}\dots = (\ )p^2(1-p)^2$, $(2\pi)^{-3n/2}(\sigma_1^2\sigma_2^2$

$\sigma_3^2)^{-n/2}\exp(-\frac{1}{2\sigma_1^2}\sum^{n}(x_j-\mu_1)^2 - \frac{1}{2\sigma_2^2}\sum^{n}(y_j-\mu_2)^2 - \frac{1}{2\sigma_3^2}\sum^{n}(w_j-\mu_3)^2)$, $L(\theta_0) = (2\pi)^{-3n/2}\sigma^{-3n}$

$\exp(-\frac{1}{2\sigma^2}(\sum^1 + \sum^2 + \sum^3))$, $\log = -\frac{3}{2}\log\sigma^2 - \frac{1}{2\sigma^2}(\sum^1+\sum^2+\sum^3)$,

$\frac{\partial\log}{\partial\sigma^2} = -\frac{3}{2\sigma^2} + \frac{1}{2\sigma^4}(\sum\dots) = 0$, $\sigma^{-2} = 3/(\sum\dots)$, $\hat\sigma^2 = \frac13(\sum+\dots)$, $\frac{\partial\log}{\partial\mu_1} = \frac{1}{\sigma^2}\sum(x_j-\mu_1) = 0$

$\mu_1 = \bar X$, $L(\theta) =$

$\frac{1}{\|u\|^2}(u,b) = u/2$, $\lambda(A) = 1$, $A^2 = uu^Tuu^T = A$ // $uu^T = \begin{pmatrix} 4/9 & 4/9 & 2/9 \\ 4/9 & 4/9 & 2/9 \\ 2/9 & 2/9 & 1/9 \end{pmatrix}$, $P = \begin{pmatrix} 5/9 & -4/9 & -2/9 \\ -4/9 & 5/9 & -2/9 \\ -2/9 & -2/9 & 8/9 \end{pmatrix}$

$Pb = (5/9, -4/9, -2/9)$, $\lambda(P) = \{0, 1, 1\}$, $P^2 = I - 2uu^T + uu^Tuu^T = P$ //

$H = \begin{pmatrix} 1/9 & 1/9 & 5/9 \\ 1/9 & 1/9 & 5/9 \\ 5/9 & 5/9 & 7/9 \end{pmatrix}$, $\lambda(H) = \{1, 1, -1\}$, $H^2 = I - 4uu^T + 4uu^Tuu^T = I$ // $R(A) = c\cdot(1,1)$,

$N(A) = (1,1,2)^\perp$, $R(A^T) = \{(2,1,1)\}$, $N(A^T) = (1,1)^\perp = c\cdot(1,-1)$ //

$A^TA = wv^Tvw^T = (v^Tv)ww^T$, $A^TAw = \|v\|^2\|w\|^2 w$ // $I = BB^{-1} = I - (c+1)vw^T$

$+ cvw^Tvw^T = I + (-c-1+cw^Tv)vw^T$, $c(w^Tv - 1) = 1$, $c = (w^Tv-1)^{-1}$ //

$BB^{-1} = I - cvw^TA^{-1} - vw^TA^{-1} + cvw^TA^{-1}vw^TA^{-1} = I + (-c-1)vw^TA^{-1} +$

$cw^TA^{-1}vvw^TA^{-1} = I + vw^TA^{-1}(-c-1+cw^TA^{-1}v)$, $c = (w^TA^{-1}v - 1)$.

$v = w = (1,0)$, $c = \bar{a}_{11} - 1$, $c\begin{pmatrix}\bar{a}_{11} \\ \bar{a}_{21}\end{pmatrix}(\bar{a}_{11}\ \bar{a}_{12})$ 6.85 //

$\lambda(A) = 0$, $\{1,-1\}$, $\begin{pmatrix}1 & -1 \\ 1 & -1\end{pmatrix} = \begin{pmatrix}1 & 0 \\ -1 & 0\end{pmatrix}\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}(\ ) = \begin{pmatrix}0 & 1 \\ 0 & -1\end{pmatrix}(\ 1\ 1)$, $\begin{pmatrix}a & b \\ c & d\end{pmatrix}\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}\begin{pmatrix}d & -b \\ -c & a\end{pmatrix}\frac{1}{ad-bc}$

$= \begin{pmatrix}0 & a \\ 0 & c\end{pmatrix}\begin{pmatrix}d & -b \\ -c & a\end{pmatrix}\frac{1}{ad-bc} = \begin{pmatrix}-ac & a^2 \\ -c^2 & ac\end{pmatrix}\cdot\frac{1}{\Delta}$, $a^2 = ac$, $a = c$, $\begin{matrix}-1 & 1 \\ -1 & 1\end{matrix}\ \frac{1}{d-b}$

$d - b = -1$, $M = \begin{pmatrix}1 & b \\ 1 & b-1\end{pmatrix}$, $\begin{pmatrix}1 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & -1 \\ 1 & 1\end{pmatrix}\cdot -1 = \begin{pmatrix}0 & -1 \\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & -1 \\ 1 & 1\end{pmatrix} = \begin{matrix}-1 & -1 \\ -1 & -1\end{matrix}$

$\begin{pmatrix}-a & a \\ -a & a\end{pmatrix}\cdot\frac{1}{d-b}$, $a = c = -1$, $d = 1$, $b = 0$, $\begin{pmatrix}-1 & 0 \\ -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 \\ 1 & -1\end{pmatrix}\cdot -1 = \begin{pmatrix}0 & 1 \\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 \\ 1 & -1\end{pmatrix} = A$ 6.21 //

$A^T = (S^T)^{-1}\Lambda S^T$, $(S^T)^{-1}$ 6.22 // $A^T(1,1,1) = 0$, $N(A^T) = \{(1,1,1)\}$, $R(A^T) = \{(1,0,-1),$

$(1,-1,0)\}$ 6.25 // $\begin{pmatrix}3 & 0 \\ 0 & 5\end{pmatrix} = \begin{pmatrix}1 & 0 \\ -4/3 & 1\end{pmatrix}A$, $A = \begin{pmatrix}1 & 0 \\ 4/3 & 1\end{pmatrix}\begin{pmatrix}3 & 0 \\ 0 & 5\end{pmatrix}$, $\begin{pmatrix}3 & 0 \\ 0 & 5\end{pmatrix} = A\begin{pmatrix}1 & 0 \\ -4/5 & 1\end{pmatrix}$,

$\begin{pmatrix}3 & -15/4 \\ 4 & 0\end{pmatrix} = A\begin{pmatrix}1 & -5/4 \\ 0 & 1\end{pmatrix}$, $\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}A = \begin{pmatrix}4 & 5 \\ 3 & 0\end{pmatrix}$, $\begin{pmatrix}r_1q_{11} & r_2q_1 + r_3q_2 \\ r_1q_{21} & \end{pmatrix} = A$, $q_1 \propto \begin{pmatrix}3 \\ 4\end{pmatrix}$,

$q_2 \propto \begin{pmatrix}3 \\ 4\end{pmatrix}^\perp \propto \begin{pmatrix}1 \\ -3/4\end{pmatrix}$, $q_1 = \begin{pmatrix}3 \\ 4\end{pmatrix}/5$, $q_2 = \begin{pmatrix}1 \\ -3/4\end{pmatrix}/\frac{5}{4} = \begin{pmatrix}4/5 \\ -3/5\end{pmatrix}$, $r_1 = 5$, $r_2\begin{pmatrix}3/5 \\ 4/5\end{pmatrix} + r_3\begin{pmatrix}4/5 \\ -3/5\end{pmatrix}$

$= \begin{pmatrix}0 \\ 5\end{pmatrix}$, $r_2 = -\frac{4}{3}r_3$, $\left(-\frac{16}{5} - \frac{3}{5}\right)r_3 = 5$, $r_3 = \frac{-25}{19}$, $r_2 = \frac{100}{57}$, $A = \begin{pmatrix}3/5 & 4/5 \\ 4/5 & -3/5\end{pmatrix}\begin{pmatrix}5 & \frac{100}{57} \\ 0 & -\frac{25}{19}\end{pmatrix}$

$A^TA = \begin{pmatrix}25 & 20 \\ 20 & 25\end{pmatrix} = \begin{pmatrix}b_1 & b_2 \\ b_2 & b_3\end{pmatrix}^2 = \begin{pmatrix}b_1^2 + b_2^2 & b_1b_2 + b_2b_3 \\ \cdot & b_2^2 + b_3^2\end{pmatrix}$, $b_1^2 = b_3^2$, $b_1 = b_3$, $b_1^2 + b_2^2 = 25$,

$2b_1b_2 = 20$, $b_1^2 + \frac{100}{b_1^2} = 25$, $b_1^4 - 25b_1^2 + 100 = 0$, $b_1^2 = \frac{1}{2}(25 \pm 15) = 20, 5$, $B = \begin{pmatrix} \sqrt{\ } & \sqrt{5} \\ & \sqrt{5}\end{pmatrix}$

$Q = AB^{-1} = \begin{pmatrix}3 & 0 \\ 4 & 5\end{pmatrix}\begin{pmatrix}\sqrt{5} & -2\sqrt{5} \\ \cdot & \sqrt{5}\end{pmatrix}/-15 = \begin{pmatrix}-1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5}\end{pmatrix}$ // $(3-\lambda)(5-\lambda) =$, $4 + 5v_2 = 3v_2$ $(1,-2)$,

$(0,1)$, $A = \begin{pmatrix}1 & 0 \\ -2 & 1\end{pmatrix}\begin{pmatrix}3 & \\ & 5\end{pmatrix}\begin{pmatrix}1 & 0 \\ 2 & 1\end{pmatrix}$, $\begin{pmatrix}0 & 1 \\ 1 & -2\end{pmatrix} = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & -2 \\ 0 & 1\end{pmatrix}$, $S^{-1} = \begin{pmatrix}1 & 2 \\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$,

$(25-\lambda)^2 - 400 = \lambda^2 - 50\lambda + 225$, $\frac{1}{2}(50 \pm 40) = 45, 5$, $25 + 20q = 45$, $q = 1$, $q_1 = (1,1)$,

$q_2 = (1,-1)$, $A^TA = \begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}45 & \\ & 5\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}$, $\cancel{A = \begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}3\sqrt{5} & \\ & \sqrt{5}\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}}$

$= \begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}/\sqrt{2}\begin{pmatrix}90 & \\ & 10\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}/\sqrt{2}$, $\cancel{A} = \begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}/\sqrt{2}\begin{pmatrix}3\sqrt{10} & \\ & 2\sqrt{5}/\sqrt{10}\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}/\sqrt{2}$

$\cancel{= \frac{1}{2}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}3\sqrt{10} & -3\sqrt{10} \\ 2\sqrt{5} & -2\sqrt{5}\end{pmatrix}}$ $\cancel{\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}45 & 45 \\ 5 & -5\end{pmatrix}}$ $= \cancel{\frac{1}{2}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}3\sqrt{10} & 3\sqrt{10} \\ \sqrt{10} & -\sqrt{10}\end{pmatrix}} = \begin{matrix}2\sqrt{10} & \sqrt{10} \\ \sqrt{10} & \end{matrix}$

$AA^T = \begin{pmatrix}9 & 12 \\ 12 & 41\end{pmatrix}$, $(9-\lambda)(41-\lambda) - 144 = \lambda^2 - 50\lambda + 225 = (\lambda-45)(\lambda-5)$, $9 + 12q = 45$, $q_1 = (1,3)$,

$q_2 = (1, -1/3)$, $A = ?\ \cancel{\begin{pmatrix}1/\sqrt{10} & 2/\sqrt{5} \\ 3/\sqrt{10} & -2/3\sqrt{5}\end{pmatrix}\begin{pmatrix}45 & \\ & 5\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}} = \cancel{\begin{pmatrix}1/\sqrt{10} & 2/\sqrt{5} \\ 3/\sqrt{10} & -2/3\sqrt{5}\end{pmatrix}\begin{pmatrix}45 & 45 \\ 5 & -5\end{pmatrix}/\sqrt{2}} = ($

$A = ?\begin{pmatrix}1/\sqrt{10} & 3/\sqrt{10} \\ 3/\sqrt{10} & -1/\sqrt{10}\end{pmatrix}\begin{pmatrix}\sqrt{45} & \\ & \sqrt{5}\end{pmatrix}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}/\sqrt{2} = \frac{1}{\sqrt{5}\cdot 2}\begin{pmatrix}1 & 3 \\ 3 & -1\end{pmatrix}\begin{pmatrix}\sqrt{45} & \sqrt{45} \\ \sqrt{5} & -\sqrt{5}\end{pmatrix} = \frac{1}{2\sqrt{5}}\begin{pmatrix}6\sqrt{5} & 0 \\ 8\sqrt{5} & 10\sqrt{5}\end{pmatrix}$

$\cancel{\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}45 & \\ & 5\end{pmatrix}\begin{pmatrix}1 & 3 \\ 3 & -1\end{pmatrix} = \begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}45 & 90 \\ 15 & -5\end{pmatrix}} = \begin{pmatrix}3 & 0 \\ 4 & 5\end{pmatrix}$ 6.28 //

Strary

$$\begin{matrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & & & \\ & -1 & 1 & & \\ & & -1 & 1 & \\ & -1 & & & 1 \\ -1 & & 1 & & \end{matrix}$$

$b = \begin{pmatrix} -x_1+x_2 \\ -x_2+x_3 \\ -x_3+x_4 \\ -x_1+x_4 \end{pmatrix}$, $b = x_1A_1 + x_2A_2 + x_3A_3 + x_4(-A_1-A_2-A_3)$
$= (x_1-x_4)A_1 + (x_2-x_4)A_2 + (x_3-x_4)A_3$,

$N(A_1^T) = (1,1,1,-1)$ 1.6.2 // $N(A^T) = \{(1,1,1,1,-1,-1), (1,1,-1,-1,1,-1)\}$ 1.6.3 // $R(A) = 3+4$. $(m \vee n) - p - |m-n| = R(A)$, $\dim\{x : Ax = b\}$

$= \dim N(A) = n - R(A)$, $\dim\{y : A^Ty = b\} = \dim N(A^T) = n - R(A)$ 1.6.4 //

$E \le 2V, \ldots$, $y^TAx = 0$ #6.7 // $(A^{-1})^T = \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix}^T$, $(A^T)^{-1} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$ //

$R(A) = (x_1, x_2, 0)$, $N(A) = (x_1, 0, 0, x_4)$, $R(A^T) = (0, x_2, x_3, 0)$, $N(A^T) = (0, 0, x_3)$ //

A: orthogonal, inv, perm, B: sym, proj, Jordan. $\lambda(A) = 1$, $\lambda(B) = 1$ // $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} A_1 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$,

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} = I$, $A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} A_1 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$,

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} = I$, $A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$. $PA_2 = \begin{matrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{matrix}$,

$$\left.\begin{matrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{matrix}\right| \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}, \quad \left.\begin{matrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{matrix}\right| \begin{matrix} 1 & - & & & \\ -1 & 1 & - & & \\ 1 & -1 & 1 & - & \\ 0 & 0 & 0 & 1 & 0 \end{matrix}, \quad \left.\begin{matrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{matrix}\right| \begin{matrix} 1 & - & & & \\ -1 & 1 & - & & \\ 1 & -1 & 1 & - & \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 1 \end{matrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ & & 0 & \end{pmatrix} = PA_2 \quad 6.11 // \quad \left.\begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{matrix}\right| \begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix},$$

$$\left.\begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{matrix}\right| \begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}, \quad L = \begin{matrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{matrix}, \quad A = LU, \quad U^{-1} = \begin{matrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{matrix}, \quad A^{-1} = \begin{matrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{matrix}, \quad x = \begin{matrix} 4 \\ 3 \\ 2 \\ 1 \end{matrix} //$$

$x_1+x_2+x_3 = 0$, $x_2+x_3+x_4 = 0$, $(0,1,-1,0)$, $(-2,1,1,-2)$. $x_2 - x_3 = 0$, $-2x_1+x_2+x_3-2x_4$

$= 0$ // $0 \ne y^Tb = y^TAx = x^T(A^Ty) = 0$ // $A = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $A = (1\ 0)$, $A = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $A = 1$, $r = n = m - 1$,

$r = m = n - 1$, $r = m - 1 = n - 1$, $r = n = m$ // $x - 1 + 2(y-1) + 3(z-1) = 0$, $x + 2y + 3z = 6$ // T, T,

T ($\dim N(A^T) = \dim N(B^T) = m - 3$, $N(A^T + B^T) \supset N(A^T) \cap N(B^T)$, $\dim(N(A^T) \cap N(B^T)) \ge$

$m - 6$, $R(A^T + B^T) \le 6$), T, $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$, $\begin{matrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{matrix}$, F 6.19 // $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & \\ & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} \cdot \frac{1}{2}$

$= \frac{1}{2} \begin{pmatrix} 6 & 0 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{matrix} 3 & 0 \\ -\frac{1}{2} & 4 \end{matrix}$ // $A = \begin{matrix} 1 & -1 \\ 1 & -1 \end{matrix}$, ~~AAA~~, $\det \cancel{\begin{pmatrix} & -1 \\ 1 & -2 \end{pmatrix}} = \det \begin{pmatrix} 1-\lambda & -1 \\ 1 & -1-\lambda \end{pmatrix} = \lambda^2 - 1 + 1$, $\lambda = 0$,