$\hat\theta = \sum_{i,j} \varphi_{ij}/M^2 = \sum_i \varphi_{i\cdot}/M^2 = \sum_j \varphi_{\cdot j}/M^2$.

$V(x_i) = \sum_j \varphi_{ij}/M$, $V(y_j) = \sum_i \varphi_{ij}/M$ $=$ $V(x_i) = \varphi_{i\cdot}/M$, $V(y_j) = \varphi_{\cdot j}/M$

$S_x = \sum_i (V(x_i) - \hat\theta)^2$, $S_y = \sum_j (V(y_j) - \hat\theta)^2$

$S_{xy} = \sum_k (V(x_k) - \hat\theta)(V(y_k) - \hat\theta)$

$\hat\sigma^2 = S_x/M^2 + S_y/M^2 + 2S_{xy}/M^2$ $\quad [M(M-1)]^{-1} \sum_j (V(x_j) + V(y_j) - 2\hat\theta)^2$

$\theta_{-i} = \sum_{j\neq i, k\neq i} \varphi_{jk}/(M-1)^2$, $\hat\theta_j = M\hat\theta - (M-1)\theta_{-j}$, $\hat\sigma^2 = \frac{1}{M(M-1)}\sum_j (\hat\theta_j - \bar{\hat\theta})^2$,

$\bar{\hat\theta} = M\hat\theta - \frac{M-1}{M}\sum_j \theta_{-j}$, $(M-1)^2\theta_{-i} = \sum_{jk}\varphi_{jk} - \sum_j \varphi_{ij} - \sum_j \varphi_{ji} + \varphi_{ii}$

$= M^2\hat\theta - \varphi_{i\cdot} - \varphi_{\cdot i} + \varphi_{ii}$ $\quad \sum_j \theta_{-j} = \frac{M^3}{(M-1)^2}\hat\theta - \frac{2M^2\hat\theta}{(M-1)^2} - \frac{\mathrm{diag}\,\varphi}{(M-1)^2}$,

$\bar{\hat\theta} = \hat\theta\left(M - \frac{M^2}{M-1} + \frac{2M}{M-1}\right) - \frac{\mathrm{diag}\,\varphi}{M(M-1)}$, $1 - \frac{M-2}{M-1} = \frac{1}{M-1}$,

$\bar{\hat\theta} = \frac{M}{M-1}\left(\hat\theta - \mathrm{diag}\,\varphi/M\right)$, $\hat\theta_j - \bar{\hat\theta} = M\hat\theta - (M-1)\theta_{-j} - \frac{M}{M-1}\hat\theta + \frac{\mathrm{diag}\,\varphi}{M(M-1)}$

$= \frac{\hat\theta M(M-2)}{M-1} + \frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{1}{M-1}\left(M^2\hat\theta - \varphi_{j\cdot} - \varphi_{\cdot j} + \varphi_{jj}\right)$

$= -\frac{2M}{M-1}\hat\theta + \frac{\mathrm{tr}\,\varphi}{M(M-1)} + \frac{\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}}{M-1}$, $\sum_j \varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj} = 2M^2\hat\theta - \mathrm{tr}\,\varphi$,

$M\left(\frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{2M}{M-1}\hat\theta\right)^2 + 2\left(\frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{2M}{M-1}\hat\theta\right)\frac{2M^2\hat\theta - \mathrm{tr}\,\varphi}{M-1} + \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2}$

$= \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2} - M\left(\frac{\mathrm{tr}\,\varphi - 2M^2\hat\theta}{M(M-1)}\right)^2 = M(M-1)\hat\sigma_{jk}^2$ $\quad \sum_j\left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat\theta\right)^2$

$= M^{-2}\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj} + \varphi_{jj} - 2M\hat\theta)^2$ $\quad \sum(\varphi_{jj} - 2M\hat\theta)^2 = \mathrm{tr}\,\varphi - 4M\hat\theta\,\mathrm{tr}\,\varphi$

$+ 4M^3\hat\theta^2$, $\quad \sum(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})(\varphi_{jj} - 2M\hat\theta) =$

$-2M\hat\theta(2M^2\hat\theta - \mathrm{tr}\,\varphi) + \sum\varphi_{j\cdot}\varphi_{jj} + \sum\varphi_{\cdot j}\varphi_{jj} - \mathrm{tr}\,\varphi, - \mathrm{tr}\,\varphi - 4M^3\hat\theta^2 + 2\sum\varphi_{jj}($

$\varphi_{j\cdot} + \varphi_{\cdot j}) + \sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2$ $\quad \frac{\sum_j(\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{\ldots} - 4M\hat\theta^2$

$$M(M-1)\hat\sigma^2_{osv} = \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat\theta\right)^2 = M^{-2}\left(\sum_j \varphi_{j\cdot}^2 + \sum_j \varphi_{\cdot j}^2 + 2\sum \varphi_{\cdot j}\varphi_{j\cdot}\right)$$

$$-4\hat\theta/M \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j}) + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1}$$

$$\cancel{4M\hat\theta^2/M} - 4\hat\theta/M \cdot 2M^2\hat\theta + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\dots)\mathbb{1} - 4M\hat\theta^2 =$$

$$M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4/M\,\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} \qquad \frac{1}{(M-1)^2} - \frac{1}{M^2} = \frac{2M-1}{(M(M-1))^2}$$

$$0 \doteq ? \quad \frac{2M-1}{(M(M-1))^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \frac{2}{(M-1)^2}\mathbb{1}^T(\varphi + \varphi^T - I/2$$

$$\frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T + \operatorname{diag}\varphi\,\mathbb{1}^T/2M)\cdot\operatorname{diag}\varphi$$

$$\frac{M-1/2}{M^2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - \mathbb{1}^T(\varphi + \varphi^T - \frac{I}{2} - \frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T +$$

$$\operatorname{diag}\varphi\,\mathbb{1}^T/2M)\operatorname{diag}\varphi \overset{①}{=} \frac{1}{2}(M-1)^3 M(\sigma^2_{jk} - \sigma^2_{osv})$$

$$(\varphi = \varphi^T) \quad \mathbb{1}^T\varphi(I - \frac{1}{M}\mathbb{1}\mathbb{1}^T)\varphi\mathbb{1} = \sum s_j^2 - \frac{1}{M}(\sum s_j)^2 = (M-1)\operatorname{var}(\mathbb{1}^T\varphi) = \operatorname{var}(\mathbb{1}^T\varphi^T)$$

$$\leq \frac{M}{2}\left((M - M/2)^2 + (M/2)^2\right) = \frac{M^3}{4}$$

$$\mathbb{1}^T(\varphi^T\varphi - \varphi\mathbb{1}\mathbb{1}^T\varphi/M)\mathbb{1} = \mathbb{1}^T(\varphi^T - \varphi\mathbb{1}\mathbb{1}^T/M)\varphi\mathbb{1}, \quad \mathbb{1}^T\varphi\cdot\mathrm{Proj}_{\mathbb{1}^\perp}\varphi\mathbb{1}$$

$$= \mathbb{1}^T\varphi(\vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec c}), \quad (\vec c, \vec r) - (\sum \varphi_{ij})^2/M,$$

$$(\vec c, \vec r) = \sum_j (\sum_i \varphi_{ij})(\sum_i \varphi_{ji}) = \sum_j \sum_{i,k} \varphi_{ij}\varphi_{jk} = \sum_{ik}\sum_j \varphi_{ij}\varphi_{jk} - \sum_{ik}\sum_{jl} \varphi_{ij}\varphi_{lk}/M$$

$$= \sum_{ikj}(\varphi_{ij}\varphi_{jk} - \sum_l \varphi_{ij}\varphi_{lk}/M) = \sum_{ikj}\varphi_{ij}(\varphi_{jk} - \varphi_{\cdot k})$$

$$M(M-1)\sigma_{o3o}^2 = M^{-2}\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - 2M\hat\theta)^2 \qquad \varphi_{\cdot j} = 1 - \varphi_{j\cdot}, \quad \varphi_{\cdot j} = \sum_i \varphi_{ij}$$

$$= \sum_i (1 - \varphi_{ji}) = M - \varphi_{j\cdot}, \quad M^{-2}\sum_j (M - 2M\hat\theta)^2 = M - 4M\hat\theta + 4M\hat\theta^2 = M(1-2\hat\theta)^2_j$$

$$\sigma_{o3o}^2 = \frac{(1-2\hat\theta)^2}{M-1}$$

$$M(M-1)\sigma_{jk}^2 = \frac{M^4}{(M-1)^2} - M\left(\frac{-2M^2\hat\theta}{M(M-1)}\right)^2 = \frac{M^4}{(M-1)^2} - \frac{4M^3\hat\theta^2}{(M-1)^2} = \frac{M^4}{(M-1)^2}(1-4\hat\theta^2),$$

$$\sigma_{jk}^2 = \frac{M^2}{(M-1)^3}(1-4\hat\theta^2)$$

$$\varphi_{\cdot j} = \sum_i \mathbb{P}(X_i < Y_j) = M - \sum_i \mathbb{P}(X_i > Y_j)$$

$$M^{-2}\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - \mathrm{tr}\,\varphi/M^2 - 4M\hat\theta^2 + \frac{2}{M^2}\sum \varphi_{jj}(\varphi_{j\cdot} + \varphi_{\cdot j}) = ?$$

$$(M-1)^{-2}\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - M^{-1}\left(\frac{\mathrm{tr}\,\varphi - 2M^2\hat\theta}{(M-1)}\right)^2$$

$$\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} + \varphi_{jj})^2, \quad \sum_j \varphi_{j\cdot}^2 = \sum_j \left(\sum_i \varphi_{ji}\right)^2 = \sum_j \Big(\sum_i \varphi_{ji} + 2\sum_{i<k}$$

$$\varphi_{ji}\varphi_{jk}\Big) = M^2\hat\theta + 2\sum_{i<k}(\varphi_{\cdot i}, \varphi_{\cdot k}), \quad \sum_j \varphi_{j\cdot}^2 = \mathbb{1}^T\varphi^T\varphi\mathbb{1}, \quad \sum_j \varphi_{\cdot j}^2 = \mathbb{1}^T\varphi\varphi^T\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{\cdot j} = \sum_j \left(\sum_i \varphi_{ji}\right)\left(\sum_k \varphi_{kj}\right) = \sum_j \sum_{i,k} \varphi_{ji}\varphi_{kj} = \sum_{i,k}\sum_j \varphi_{ji}\varphi_{kj} = \mathbb{1}^T\varphi\varphi\mathbb{1},$$

$$\sum_j \varphi_{j\cdot}\varphi_{jj} = \sum_i\sum_j \varphi_{ji}\varphi_{jj} = \mathbb{1}^T\varphi\,\mathrm{diag}\,\varphi, \quad \sum_j \varphi_{\cdot j}\varphi_{jj} = (\mathrm{diag}\,\varphi)^T\varphi\mathbb{1},$$

$$\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} + \mathrm{tr}\,\varphi - 2\mathbb{1}^T(\varphi + \varphi^T).$$

$$\mathrm{diag}\,\varphi = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T)\,\mathrm{diag}\,\varphi,$$

$$\mathrm{tr}\,\varphi - 2M^2\hat\theta = \mathrm{tr}\,\varphi - 2\mathbb{1}^T\varphi\mathbb{1}, \quad (M-1)^{-2}\Big\{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2 - \frac{(\mathrm{tr}\,\varphi)^2}{M} -$$

$$4M^3\hat\theta^2 + 4M\hat\theta\,\mathrm{tr}\,\varphi\Big\}, \quad \hat\theta^2 = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\varphi\mathbb{1}/M^4, \quad \hat\theta\,\mathrm{tr}\,\varphi = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\,\mathrm{diag}\,\varphi,$$

$$(M-1)^{-2}\Big\{\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4M^{-1}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - 2\mathbb{1}^T\big(\varphi - I/2 + \varphi^T - 2/M\,\varphi\mathbb{1}\mathbb{1}^T$$

$$+ \mathrm{diag}\,\varphi\cdot\mathbb{1}^T/2M\big)\,\mathrm{diag}\,\varphi\Big\} = M(M-1)\hat\sigma_{jk}^2$$

$\int_0^\infty \frac{y}{\theta} e^{-y/\theta} = \cancel{\ } -ye^{-y/\theta}\Big|_0^\infty + \int e^{-y/\theta} = -\theta e^{-y/\theta}\Big|_0^\infty = \theta$, $\int \frac{y^2}{\theta} e^{-y/\theta}$

$= -y\theta e^{-y/\theta}\Big|_0^\infty + \int \theta e^{-y/\theta} = -\theta^2 e^{-y/\theta}\Big|_0^\infty = \theta^2$

$\Sigma(Y_j^2 - 2\bar{Y}Y_j + \bar{Y}^2) = \Sigma Y_j^2 - n\bar{Y}^2$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $\left(\frac{n-1}{\sigma^2}\right)^2 \operatorname{Var} S^2 = 2(n-1)$

$\operatorname{Var} S^2 = \frac{2\sigma^4}{n-1}$, $\operatorname{Var}(Y_1^2 - 2Y_1Y_2 + Y_2^2)$, $Y_1 - Y_2 \sim N(0, 2\sigma^2)$, $\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}} \sim Z$,

$1 = \frac{1}{2\sigma^2} \operatorname{Var}(Y_1 - Y_2)$, $\operatorname{Var}\left(\left(\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}}\right)^2\right) \sim \operatorname{Var} \chi^2(1) = 2$, $\operatorname{Var}(Y_1 - Y_2)^2$

$= 8\sigma^4$,

$\int_0^1 \theta y^\theta = \frac{\theta}{\theta+1} y^{\theta+1}\Big|_0^1 = \frac{\theta}{\theta+1}$

$2 \mathbb{E} Y_1^2 = 2\lambda(1+\lambda) - 2\lambda^2 = 2\lambda$, $\operatorname{Var}((Y_i - Y_j)^2)$

$\frac{\theta+1}{\theta} y^{\theta+1}\Big|_0^1 = \frac{\theta+1}{\theta}$, $\frac{\theta+1}{\theta+2} y^{\theta+2}\Big|_0 = \frac{\theta+1}{\theta+2} = \bar{X}$, $\theta(1-\bar{X}) = 2\bar{X} - 1$, $\hat{\theta} = \frac{2\bar{X}-1}{1-\bar{X}} \xrightarrow{p} \theta$

$\sigma^2 + \mu^2 = \overline{X^2}$ $\quad -y/p^2 - (n-y)\frac{1}{(1-p)^2}$ $\quad e^{-x/\theta}/\theta$, $\theta^{-n} e^{-\theta^{-1}\Sigma x}$, $-n\log\theta$

$-\Sigma x/\theta$, $-n/\theta + \Sigma x/\theta^2$, $\hat{\theta} = \bar{X}$ $\quad$, $\left(\frac{1}{2\theta+1}\right)^n \{X_{(n)} \le 2\theta+1\}$ $\quad \hat{\theta} = \frac{1}{2}(X_{(n)} - 1)$,

$\frac{1}{2\theta+1}\int_0^{2\theta+1} y^2$, $\frac{1}{3}(2\theta+1)^2 - \frac{1}{4}(2\theta+1)^2 = \frac{1}{12}(2\theta+1)^2$, $\theta^{-2n} \prod y_j e^{-\Sigma y/\theta}$, $-2n\log\theta + \Sigma y_j - \Sigma y/\theta$,

$-2n/\theta + \Sigma y/\theta^2$, $\hat{\theta} = \bar{Y}/2$,

9.36

a) $Y_1, \dots, Y_n$ iid $\sim N(\mu, \sigma^2)$, pdf $f(y) = (2\pi\sigma^2)^{-1/2} \exp\left(-\left(\frac{y-\mu}{\sigma}\right)^2/2\right)$

joint pdf $\prod_{j=1}^{n} f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j-\mu)^2\right)$

$$(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j^2 - 2\mu y_j + \mu^2)\right)$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(\frac{1}{2\sigma^2} \sum y_j^2 + \frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right)$$

$$f(y_1, \dots, y_n) = \prod f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(\frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right) \exp\left(-\frac{1}{2\sigma^2} \sum y_j^2\right)$$

$$= \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(\frac{n\mu}{\sigma^2} \bar{y} - \frac{n\mu^2}{2\sigma^2}\right)} \underbrace{\exp\left(-\frac{1}{2\sigma^2} \sum y_j^2\right)}$$

depends on $y_1, \dots, y_n$ only through $\bar{y}$

does not depend on target μ

b) $f(y_1, \dots, y_n) = \prod f(y_j) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j - \mu)^2\right)} \cdot 1$

depends on $y_1, \dots, y_n$ only through $\sum (y_j - \mu)^2$

does not depend on target σ^2

c) $f(y_1, \dots, y_n) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum y_j^2 + \frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right)} \cdot 1$

depends on $y_1, \dots, y_n$ only through $(\sum y_j^2, \sum y_j)$

does not depend on (μ, σ^2)

9.39

$Y_1, \dots, Y_n \sim \mathrm{Poi}(\lambda)$, pmf is $f_Y(j) = e^{-\lambda} \frac{\lambda^j}{j!}$ for $j = 0, 1, \dots$

also $\sum_{j=1}^n Y_j \sim \mathrm{Poi}(n\lambda)$, pmf is then $f_{\Sigma Y}(j) = e^{-n\lambda} \frac{(n\lambda)^j}{j!}$.

Then $\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \mid \Sigma Y_j = \Sigma y_j)$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \text{ and } \Sigma Y_j = \Sigma y_j)}{\mathbb{P}(\Sigma Y_j = \Sigma y_j)}$$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n)}{\mathbb{P}(\Sigma Y_j = \Sigma y_j)} = \frac{\prod_{j=1}^n f_Y(y_j)}{f_{\Sigma Y}(\Sigma y_j)}$$

$$= \prod_{j=1}^n \left(e^{-\lambda} \frac{\lambda^{y_j}}{y_j!}\right) \Big/ e^{-n\lambda} \frac{(n\lambda)^{\Sigma y_j}}{(\Sigma y_j)!} = e^{-n\lambda} \frac{\lambda^{\Sigma y_j}}{\prod y_j!} \Big/ e^{-n\lambda} \frac{(n\lambda)^{\Sigma y_j}}{(\Sigma y_j)!}$$

$$= \frac{(\Sigma y_j)!}{\prod y_j!} \quad \text{which doesn't depend on } \lambda$$

9.40 $Y_1, \dots, Y_n \sim \mathrm{Rayleigh}(\theta)$, pdf is $f_Y(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta}$ for $y > 0$,

so $f(y_1, \dots, y_n) = \prod_{j=1}^n f_Y(y_j) = \prod_{j=1}^n \left(\frac{2y_j}{\theta}\right) e^{-y_j^2/\theta} = \left(\frac{2}{\theta}\right)^n \left(\prod y_j\right) e^{-\frac{\Sigma y_j^2}{\theta}}$

$$= \underbrace{\left(\frac{2}{\theta}\right)^n e^{-\Sigma y_j^2/\theta}}_{\substack{\text{depends on} \\ y_1, \dots, y_n \text{ only} \\ \text{through } \Sigma y_j^2}} \cdot \underbrace{\left(\prod y_j\right)}_{\substack{\text{doesn't} \\ \text{depend on } \theta}}$$

9.41

$Y_1, \dots, Y_n \sim \text{Weibull}(\alpha)$, pdf is $f_Y(y) = \frac{1}{\alpha} m y^{m-1} e^{-y^m/\alpha}$ for $y>0$

so $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\frac{1}{\alpha} m y_j^{m-1} e^{-y_j^m/\alpha}\right) = \left(\frac{m}{\alpha}\right)^n \left(\prod y_j\right)^{m-1} e^{-\sum y_j^m/\alpha}$

$$= \underbrace{\left(\frac{m}{\alpha}\right)^n e^{-\sum y_j^m/\alpha}}_{\substack{\text{dependence on} \\ \text{sample thru } \sum y_j^m}} \cdot \underbrace{\left(\prod y_j\right)^{m-1}}_{\text{no } \alpha \text{ dependence}}$$

9.42 — HW

9.43

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha y_j^{\alpha-1}/\theta^\alpha\right) = \underbrace{\left(\frac{\alpha}{\theta^\alpha}\right)^n \left(\prod y_j\right)^{\alpha-1}}_{\substack{\text{depends on} \\ \text{sample thru } \prod y_j}} \cdot \underbrace{1}_{\substack{\text{no } \alpha \\ \text{dependence}}}$$

9.44

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha \beta^\alpha y_j^{-(\alpha+1)}\right) = \underbrace{(\alpha\beta^\alpha)^n \left(\prod y_j\right)^{-(\alpha+1)}}_{\text{ditto}} \cdot 1$$

9.45 $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(a(\theta) b(y_j) e^{-c(\theta) d(y_j)}\right)$

$$= \underbrace{a(\theta)^n e^{-c(\theta) \sum d(y_j)}}_{\substack{\text{depends on} \\ \text{sample thru } \sum d(y_j)}} \cdot \underbrace{\prod b(y_j)}_{\text{no } \theta \text{ dependence}}$$

9.46 pdf is $f_Y(y) = \frac{1}{\beta} e^{-y/\beta}$ which has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \beta$, $c(\theta) = 1/\beta$, $a(\theta) = 1/\beta$, $b(y) = 1$.

9.47 pdf $\alpha y^{\alpha-1}/\theta^{\alpha} = \theta^{-\alpha} \alpha \exp\left((\alpha-1)\log y\right)$ has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$, $c(\theta) =$

$\alpha - 1$, $a(\theta) = \theta^{-\alpha} \cdot \alpha$. So by 9.45 $\sum_j d(y_j) = \sum \log y_j$

is sufficient for α. No contradiction since $\sum \log y_j$

$= \log\left(\prod y_j\right)$ and "log" is one-to-one on $\mathbb{R}^+$

9.48 pdf $\alpha \beta^{\alpha} y^{-(\alpha+1)} = \alpha \beta^{\alpha} \exp\left(-(\alpha+1)\log y\right)$ has form

$a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$,

$c(\theta) = \alpha + 1$, $a(\theta) = \alpha \beta^{\alpha}$, $b(y) = 1$. Suff. stat. is

then $\sum d(y_j) = \sum \log y_j = \log\left(\prod y_j\right)$.

$f'(i) \le \frac{1! \cdot 1}{1} = 1$, $f(z) = \frac{1}{1+z-i}$, $|1+z-i| \ge 1$, $Re^{i\theta} + R^{-1}e^{-i\theta} = (R+\frac{1}{R})\cos\theta$ $+i(R-\frac{1}{R})\sin\theta$, $R^2+\frac{1}{R^2} + 2(\cos^2-\sin^2) = R^2+\frac{1}{R^2}+2\cos 2\theta$, $z-\frac{1}{z}$, $R^2+\frac{1}{R^2}-2\cos 2\theta$, $R+\frac{1}{R}+2\cos\frac{\theta}{2}$; $z^{1/4}+z^{-1/4} = \frac{z^{1/2}+1}{z^{1/4}} = z^{-1}(z^{5/4}+z^{3/4})$, $z^a+z^{-a} = z^{-a}(z^{2a}+1)$, $\frac{1}{(z-a)(z-b)}$, $\frac{1}{(z-a)^2(z-b)} + \frac{1}{(z-a)(z-b)^2}\Big|_{z=i} = (i-a)^{-2}(i-b)^{-1}+(i-a)^{-1}(i-b)^{-2}$ $= \frac{2i-a-b}{(i-a)^2(i-b)^2}$, $\frac{2i-(x_1-iy+x_2-i/y)}{(i-x_1+iy)^2(i-x_2+i/y)^2} = \frac{-(x_1+x_2)+i(2+y+1/y)}{(-x_1+i(1+y))^2(-x_2+i(1+1/y))^2}$, $((x_1+x_2)^2+16)^{1/2} \ge ? (x_1^2+4)(x_2^2+4)$; $2+y+1/y \ge ? (x_1^2+(1+y)^2)(x_2^2+(1+1/y)^2) \ge (2+y+1/y)^2 \ldots$

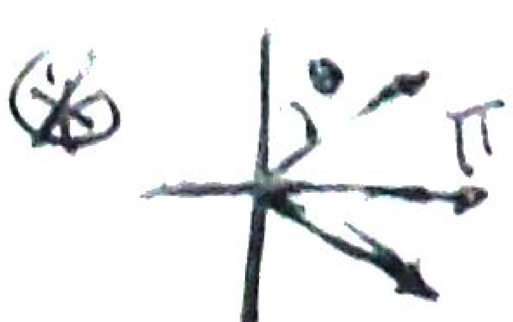
$f(x)=0 \Rightarrow |f(x)| \le \min_{y\in\bar D}|f(y)|$, $|f(x)| \ne 0$, $|f(x)| \le \inf_{y\in D}|f(y)| \Rightarrow f(y)\ne 0$, $y\in D$, $|\frac{1}{f}(x)| \ge \sup_{y\in \bar D/2}|\frac{1}{f}(y)|$, $\frac{1}{f} = \text{const.}$ #3// $f(x)\ne 0$, $x\in D \Rightarrow \max_{\bar D}|f(x)|$ $= \max_{\partial D}|f(x)| = c = \min_{\bar D}|f(x)|$, $|f(x)| = c$, $x\in \bar D$, $u^2(x,y)+v^2(x,y) = c$, uu_x+vv_x $= uu_y+vv_y = 0$, $\frac{v_x}{u_x} = \frac{v_y}{u_y}$, $-\frac{u_y}{u_x} = \frac{u_x}{u_y}$, $u_x^2 = -u_y^2$, $u_x = v_y = 0$, $u_y = v_x = 0$, $f(x) = \text{const}$, —*— #4a//

$m,n > n_0 \Rightarrow \varepsilon > \max_{z\in\partial D}|f_n(z)-f_m(z)| > \max_{z\in D}|\cdots|$ #5// $N(x) := \max_{\text{Re } z = x}|fh_\varepsilon|$, $|y| > \frac{B}{\eta\varepsilon} \Rightarrow |fh_\varepsilon| \le \frac{B}{\varepsilon}\cdot\frac{\eta\varepsilon}{B} = \eta$, $R := \{(x,y): a\le x\le b, \frac{-B}{\eta\varepsilon} \le y \le \frac{B}{\eta\varepsilon}\}$, $x_0: \{x<x_0\}\cap R \Rightarrow |f(z)| < \eta$, $|f(z)h_\varepsilon| < \eta$, $x<x_0 \Rightarrow N(x) < \eta$, $N(x) \xrightarrow[x\to a^+]{} 0$, $a<x<\xi<b \Rightarrow N(\xi) \le N(x)^{\frac{b-\xi}{b-x}} N(b)^{\frac{\xi-x}{b-x}} \xrightarrow[x\to a^+]{} 0$, $N(x)\equiv 0$, $h_\varepsilon(z)\ne 0$, $z\in\Omega$, $f\equiv 0$ #7// $M(r)^{\log b/a} \le ? M(a)^{\log b/r} M(b)^{\log r/a}$, $M(a) = M(b)$ $= 1 \Rightarrow M(r)\le 1$, $\sup_{\text{Re } z = x}\log|f(z)| = \log\sup_{\text{Re } z = x}|f(z)|$, $(\frac{b}{a})^{\log M(r)} \le ? (\frac{b}{r})^{\log M(a)} (\frac{r}{a})^{\log M(b)}$ $(f(z) = z)$ $r^{\log b/a} \le ? a^{\log b/r} b^{\log r/a}$, $\log b \log r - \log a \log r \le ? \log a \log b - \log a \log r + \log b \log r - \log b \log a$,

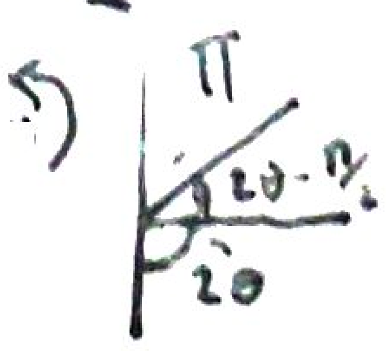
$|z|^\alpha = \exp(\alpha \log|z|)$, $e^{\beta z} \pm e^{-\beta z} = e^{\beta x}e^{i\beta y} \pm e^{-\beta x}e^{-i\beta y} = (e^{\beta x} \pm e^{-\beta x})\cos\beta y$
$+ i(e^{\beta x} \mp e^{-\beta x})\sin\beta y$, $|\cdots|^2 = e^{2\beta x} + e^{-2\beta x} \pm 2\cos\beta y \mp 2\sin\beta y = 2\cosh 2\beta x$
$\pm 2\cos(\beta y + \frac{\pi}{4}) \neq 0$, $x > x_0 \Rightarrow |e^{\beta z} + e^{-\beta z}| > \delta > 0$, $\beta y + \frac{\pi}{4} = \frac{(2k+1)\pi}{2}$, $y =$
$\beta^{-1}\frac{\pi}{2}(2k + \frac{1}{2})$, $k \in \mathbb{Z}$, $Y := \{\beta^{-1}\frac{\pi}{2}(2k+\frac{1}{2}) : k \in \mathbb{Z}\}$, $e^{\beta z} \pm e^{-\beta z} =$
$\operatorname{Im} z \in Y$, $e^{\beta z} + e^{-\beta z} : 2\cosh 2\beta x$, $|e^{\beta i z} + e^{-\beta i z}|^2 = 2\cosh 2\beta y +$
$2\cos(\beta x + \frac{\pi}{4})$ $|e^z + e^{iz}| = |e^x e^{iy} + e^{-y}e^{ix}|$, $|e^x e^{iy} + e^{-x}e^{-iy} + e^{-y}e^{ix} + e^{y}e^{-ix}|$
$\neq |e^{|x|}$, $z + iz = x + y + i(y - x)$ $|e^{z - iz}| = e^{x+y}$, $e^{z-iz} + e^{iz-z} = e^{x+y}e^{i(y-x)} + e^{-x-y}e^{i(x-y)}$
$= (e^{x+y} + e^{-x-y})\cos(x-y) + i(\cdots)$, $z^{1/n} \in ? \ H(\mathbb{C}\setminus 0)$, $z = Re^{i\theta} \mapsto R^{1/n}e^{i\theta/n} \in ?$
$H(\mathbb{C}\setminus 0)$, $n : 1 - \frac{1}{n} > \alpha$, $z^{1-1/n} := R^{1-1/n}e^{i(1-1/n)\theta}$, $\operatorname{Re} z^{1-1/n} =$
$R^{1-1/n}\cos((1-\frac{1}{n})\theta) > R^{1-1/n}\cos((1-\frac{1}{n})\frac{\pi}{2}) =: R^{1-1/n}\delta > 0$ $(z \in \Pi)$ $|\exp(-\varepsilon z^{1-1/n})|$
$= \exp(-\varepsilon \operatorname{Re} z^{1-1/n}) < \exp(-\varepsilon R^{1-1/n}\delta) < 1$ $(z \in \Pi)$, $h_\varepsilon(z) := e^{-\varepsilon z^{1-1/n}}$,
$z \in \Pi$, $fh_\varepsilon \in H(\Pi)$, $|fh_\varepsilon| \leq 1$ $(\operatorname{Re} z = 0)$, $|fh_\varepsilon| < A\exp(|z|^\alpha - \varepsilon \operatorname{Re} z^{1-1/n})$
$\xrightarrow[|z| = R \to \infty]{} 0$, $R_0 : |z| > R_0 \Rightarrow |fh_\varepsilon| < 1$, $\max\limits_{z \in \Pi \cap \{|z| \leq R_0\}} |fh_\varepsilon| = \max\limits_{z \in \partial(\cdots)} |fh_\varepsilon|$
≤ 1, $|fh_\varepsilon| \leq 1$ $(z \in \Pi, \varepsilon > 0)$, $|f| \leq 1$. $(\alpha = 1)$ $f(z) := e^z$,
$|f(z)| = e^{\operatorname{Re} z} \leq e^{|z|}$, $|f(x)| = e^x \xrightarrow[x \to \infty]{} \infty$.

$(\frac{\pi}{2} + \theta)\alpha < \frac{\pi}{2}$, $\alpha < \dfrac{\frac{\pi}{2}}{\frac{\pi}{2} + \theta}$

$|e^{ny}| \leq 1$, $|f(iy)e^{ny}| \leq 1$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| = |e^{nr\cos\alpha} f(re^{i\alpha})| = \exp(nr\cos\alpha$
$+ \log|f(re^{i\alpha})|) = \exp(r(n\cos\alpha + r^{-1}\log|f(re^{i\alpha})|))$, $n\cos\alpha + r^{-1}\log|f(re^{i\alpha})| \to -\infty$
$A := \max\limits_r \exp(r(n\cos\alpha + r^{-1}\log|f(re^{i\alpha})|))$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| \leq A$.



$\alpha < \dfrac{\frac{\pi}{2}}{\frac{\pi}{2} - \theta}$, $\alpha < \beta < \dfrac{\frac{\pi}{2}}{\frac{\pi}{2} - \theta}$, $\operatorname{Re} z^\beta = r^\beta\cos(\beta \arg z) \geq r^\beta \overbrace{\cos(\beta(\tfrac{\pi}{2} - \theta))}^{>0}$;



$z \mapsto (e^{-i\theta}z)^\beta$, $\operatorname{Re}(e^{-i\theta}z)^\beta > 0 \Leftarrow z \in \Pi$ $h_\varepsilon(z) := \exp(-\varepsilon(e^{-i\theta}z)^\beta) < 1$,
$|fh_\varepsilon| < A\exp(|z|^\alpha - \varepsilon r^\beta\cos(\beta(\frac{\pi}{2} - \theta))) \xrightarrow[|z| = r \to \infty]{} 0$. $|e^{nz}| < Ae^{|z|^\alpha}$

$(\Omega \neq \mathbb{C})$ $\Gamma = \emptyset$, M ? $(x_0 \notin \Omega)$ $\sup_{z \in \Omega} |f(z)| = \sup_{z \in \Omega - x_0} |f(z + x_0)|$, $0 \notin \Omega - x_0$ $(0 \notin \Omega)$ $r := \inf_{z \in \Omega} |z|$, $r \neq 0$, $\sup_{z \in \Omega} |f(z)| = \sup_{z \in r\Omega} |f(z/r)|$,

$r\Omega \cap U \neq \emptyset$.

$\left|\frac{f(z_0)^n}{z_0}\right| \leq \max_{\Gamma \cap V} \left|\frac{f(z)^n}{z}\right| \vee \max_{\Gamma \cap V} \left|\frac{f(z)^n}{z}\right| \leq M^n \vee \frac{B^n}{v}$

$\max(M^n, \frac{B^n}{v}) \xrightarrow[v \to \infty]{} M^n$, $|f(z_0)|^n \leq |z_0| M^n$, $n \in \mathbb{Z}$,

$|f(z_0)| \leq M$ #11 //

$\operatorname{diam} E_n < \infty \Rightarrow \|f\|_{E_n} < \|f\|_{\partial E_n} = n$, $E_n \neq \emptyset$; $\|f\|_{E_n} < n+1$

$\Rightarrow |f|_{E_n} \leq |f|_{\partial E_n} = n$, $E_n = \emptyset$; $\gamma: \gamma(1 - 1/n) \in E_n$, $|\gamma(1 - 1/n) - \gamma(1 - \frac{1}{n+1})| > 1$ #12 //

$e^x \xrightarrow[x \to \infty]{} \infty$, $e^x \xrightarrow[x \to -\infty]{} 0$, $\operatorname{Re} \gamma \to \infty \Rightarrow e^\gamma \to \infty$, $\operatorname{Re} \gamma \to -\infty$

$\Rightarrow e^\gamma \to 0$, $\operatorname{Re} \gamma \not\to \infty \Rightarrow \operatorname{Im} \gamma \to 0$, $e^\gamma \not\to L$. $e^{iz} - e^{-iz} = e^{-y+ix} - e^{y-ix}$

$= (e^y - e^{-y})\cos x + i(e^y + e^{-y})\sin x$, $|e^{iz} - e^{-iz}|^2 = e^{2y} + e^{-2y} - 2 \xrightarrow[|y| \to \infty]{} \infty$

$(|y| \not\to \infty)$ $x \to \infty$, $e^{iz} - e^{-iz} \not\to L \in \mathbb{C} \cup \infty$ #13 // $g(z) := f(1/z)$, Thrm 10.21,

$(c \in \mathbb{C} : \tilde{g}(0) := c, \tilde{g}(z) = g(z), z \neq 0, \tilde{g} \in H(\mathbb{C}))$ $z_n \to \infty$, $g(\frac{1}{z_n}) \to c$,

$f(z_n) \to c$, $f = \text{const}$; $(g(z) = \sum_{j=-m}^{\infty} c_j z^j)$ $z_n \to 0 \Rightarrow c \in \mathbb{C}$. $g(z_n) - \sum_{j=-m}^{-1} c_j z_n^j$

$\to c$, $f(w_n) - \sum_{j=1}^{m} c_{-j} w_n^j \xrightarrow[w_n \to \infty]{} c$, $f(w) - \sum_{j=1}^{m} c_{-j} w^j = \text{const}$, $f(w) = \sum_{j=0}^{m} \tilde{c}_j w^j$, $\operatorname{Im} f = \mathbb{C}$; $(\operatorname{cl}(g(D(0;r))) = \mathbb{C})$ $c \in \mathbb{C}$, $z_n : z_n \to 0$, $g(z_n) \to c$,

$f(1/z_n) \to c$ #14 //

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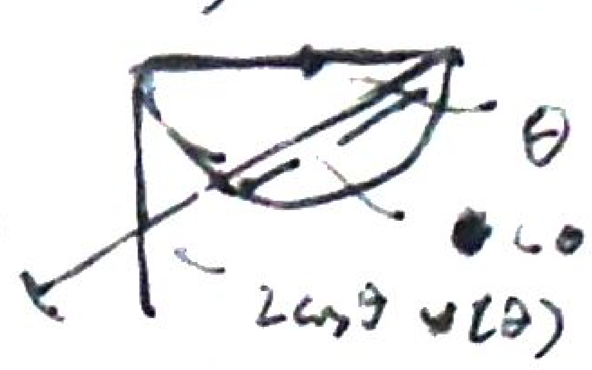
$|A| = \infty$, $\lim_{r \to 0^+} |D(\infty, r) \cap A| = 0$, $\infty \in \operatorname{cl} A \Rightarrow |A| < \infty$, $f(z) - \sum_{j=1}^{|A|} R_j$

$\in H(\mathbb{C})$ #1 //

$\varphi : D(\frac{1}{2}, \frac{1}{2}) \to V$, φ, $\varphi^{-1} \in C'$,



$\cos\theta$, $\varphi : 1 + \alpha v(\theta)$ $2\cos\theta\, v(\theta)$

$\mapsto 1 + (\cos\theta + \frac{3}{2}) v(\theta)$, $0 < \alpha < 2\cos\theta$, $\bar{v}(\theta) := -1 - \frac{1}{\cos\theta}$, $v(\theta) := \bar{v}(\theta)/|\bar{v}(\theta)|$, $\pi/2 < \theta < 3\pi/2$.

$f(z) := \frac{1}{z - \frac{1}{2}}$, $\inf_{U} |f| = \frac{2}{3}$, $\sup_{\Omega} |P - f| < ? \frac{1}{3}$, $|(z - \frac{1}{2})P - 1| < \frac{1}{2}$, $\sup_{\operatorname{cl}(\Omega)} |(z - \frac{1}{2})P - 1| \leq \frac{1}{2}$,

$\sup_{D(\frac{1}{2}, \frac{1}{2})} |(z - \frac{1}{2})P - 1| \leq \sup_{\partial D(\frac{1}{2}, \frac{1}{2})} |\cdots| = \frac{1}{2}$, $\left((z - \frac{1}{2})P - 1\right)\Big|_{z = \frac{1}{2}} = 1$ #2 //

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1] $.95 = \mathbb{P}(L < \mu < U) = \mathbb{P}(X-L > X-\mu > X-U)$, $X-\mu \sim N(0,1)$,

$X - L = z_{.025}$, $L = X - z_{.025}$, $X - U = -z_{.025}$, $U = X + z_{.025}$ //

2] $.56 \pm z_{.005}\sqrt{\frac{.56 \cdot .44}{1003}} = .56 \pm z_{.005}\frac{4}{1000}\sqrt{154}$

3] a) $\mathbb{E}X - \mu = 0$ b) $1 - 2\Phi(-1) = 2\Phi(1) - 1$ c) $\mathbb{P}(|X_1 - \mu| > \varepsilon) \geq$

$\mathbb{P}(|X_1 - \mu| > 1) = 2\Phi(1) - 1 \not\to 0$

4] a) $f_\mu(x_1, \dots, x_n) = \prod_{j=1}^{n} f_\mu(x_j) = (2\pi)^{-n/2}\exp\left(-\frac{1}{2}\sum_{j=1}^{n}(x_j - \mu)^2\right) =$

$(2\pi)^{-n/2}\exp\left(-\frac{1}{2}\sum x_j^2 + \mu\sum x_j + \mu^2/2\right) = \underbrace{(2\pi)^{-n/2}\exp(\mu n\bar{X} + \mu^2/2)}_{\text{depends on data through } \bar{X}}\ \underbrace{\exp\left(-\frac{1}{2}\sum x_j^2\right)}_{\text{doesn't depend on } \mu}$

b) $\mathbb{E}(\bar{Y}^2) = \operatorname{Var}\bar{Y} + (\mathbb{E}(\bar{Y}))^2 = 1/n + \mu^2$, $\mathbb{E}(\bar{Y}^2) - 1/n = \mu^2$

c) LLN: $\bar{Y} \xrightarrow[p]{} \mu$, so $\bar{Y}^2 \xrightarrow[p]{} \mu^2$ since $x \mapsto x^2$ is continuous.

5] $1 - \alpha = \mathbb{P}(L < \sigma^2 < U) = \mathbb{P}\left(\frac{(n-1)S^2}{L} > \frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)S^2}{U}\right)$,

$\frac{(n-1)S^2}{L} := \chi^2_{n-1, \frac{1}{2}(1-\alpha)}$, $L = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{1}{2}(1-\alpha)}}$, $U = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}$.

6] a) $(-1, \infty)$ b) $\mathbb{E}Y = \int_0^1 y(\theta+1)y^\theta\,dy = \frac{\theta+1}{\theta+2} := \bar{Y}$, $\hat{\theta}(1-\bar{Y}) = 2\bar{Y} - 1$,

$\hat{\theta}_{MOM} = \frac{2\bar{Y}-1}{1-\bar{Y}}$ c) $(\theta+1)^n\left(\prod y_j\right)^\theta$, $n\log(\theta+1) + \theta\log\sum Y_j$, $\frac{n}{\theta+1} + \log\sum Y_j$

$= 0$, $\hat{\theta}_{MLE} = -\frac{n}{\log\sum Y_j} - 1$ d) $\hat{\theta}^2_{MLE}$ e)

7.84] $\theta^{-2n}\prod y_j e^{-\sum y_j/\theta}$, $-2n\log\theta - \sum y_j/\theta$, $-2n/\theta + \sum y_j/\theta^2 = 0$, $\hat{\theta} = \sum y_j/2n$

$\mathbb{E}Y = \theta^{-2}\int y^2 e^{-y/\theta} = \theta^{-2}\left\{-\theta e^{-y/\theta}y^2\Big|_0^\infty + 2\theta\int y e^{-y/\theta}\right\} = 2\theta$

$(1-p)^{\sum y_j - n}$

$\frac{2^n \prod y_j}{\theta^n} e^{-\sum y_j^2/\theta}$, $-n \log\theta - \sum y_j^2/\theta$, $-\frac{n}{\theta} + \sum y_j^2/\theta^2$, $\hat\theta = \frac{1}{n}\sum y_j^2$,

$\int y^2 e^{-y^2/\theta} = y\left(-\frac{\theta}{2}e^{-y^2/\theta}\right)\Big|_0 + \int \frac{\theta}{2} y e^{-y^2/\theta} = \left(\frac{\theta}{2}\right)^2$

$p^n (1-p)^{\sum y_j - n}$, $n \log p + (\sum y_j - n)\log(1-p)$, $\frac{n}{p} - \frac{\sum y_j - n}{1-p} = 0$

$0 = n(1-p) - p(\sum y_j - n) = n - p(\sum y_j - n + 1)$, $p = \frac{n}{\sum y_j - n + 1}$

$\prod_{j=1}^{n} (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}(x_j - \mu)^2\right) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum (x_j - \mu)^2\right)$,

$\frac{L_a(\mu_{a10})}{L(\mu_5)} = \exp\left(\frac{1}{2\sigma^2}\sum^2 \{(x_j - \mu_5)^2 - (x_j - \mu_{10})^2\}\right) = \exp\left(\frac{1}{2\sigma^2}\{-2x_j\mu_5\right.$

$\left.+2x_j\mu_{10} + \mu_5^2 - \mu_{10}^2\}\right) < k$, $2x_j(\mu_0 - \mu_a) + \mu_a^2 - \mu_0^2 < 2\sigma^2 \log k$,

$\exp\left(\frac{1}{2\sigma^2}\{2n\bar{x}(\mu_0 - \mu_a) + n(\mu_a^2 - \mu_0^2)\}\right) < k$, $\bar{x} < \frac{1}{2n\cdot 5}\{2\sigma^2 \log k -$

$n(\mu_a^2 - \mu_0^2)\}$, $P_0(\bar{X} < k) = \alpha = P\left(z < \frac{k - \mu_0}{\sqrt{\sigma^2/n}}\right)$, $k = \mu_0 + z_{1-\alpha}\frac{S}{\sqrt{n}}$

$= 10 - t_\alpha \frac{5}{\sqrt{n}}$, $(2\theta^3)^{-n}\left(\prod y_j^2\right) e^{-\theta^{-1}\sum y_j}$ $\frac{L(\theta_0)}{L(\theta_a)} = \left(\frac{\theta_a}{\theta_0}\right)^{3n} e^{\sum y_j\left(1/\theta_0 - 1/\theta_a\right)} < k$,

$-(\sum y_j)\left(1/\theta_0 - 1/\theta_a\right) < k$, $\sum y_j > k$, $y_j \sim \Gamma(3, \theta)$, $\sum y_j \sim \Gamma($

$P(Y_1 + Y_2 < c) = \int_0^1 F_2(c - y_1) = \int_0^1 (c - y_1) = c - \frac{1}{2}$, $P(Y_1 + Y_2 < 1 + c) = c + \int_c^1 F(1 + c - y)$

$= c + \frac{1}{2}\,\cdots\, 1 - c^2 - \frac{1}{2} + \frac{1}{2}c^2 = c + \frac{1}{2} - \frac{1}{2}c^2$, $f = 1 - c$, $\frac{1}{2}(\theta + 1)^2$,

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$a^3 = n+3$, $b^3 = n^2+3n+3$, $b^3 - a^3 = \overset{= n(n+2)}{n^2+2n}$, $b^3 + a^3 = n^2 + 4n + 6 = (n+2)^2 + 2$

$b^3 = (n+3)(n+1) - n = (n+2)(n+1) + 1$, $b^3 = a^3(n+1) - n$, $b^3 = n^2 + 2n + a^3$,

$n^2 + 2n = b^3 - a^3 = (b-a)(b^2 + ab + a^2) = n(n+2)$,

$(2|n)$ $2 \nmid a$ $2 \nmid b$ $(2 \nmid n)$ $2|a$, $2 \nmid b$, $\frac{n+2}{n} + \frac{2}{n(n+2)}$,

$b^3 = a^6 - 6a^3 + 9 + 3a^3 - 6 = a^6 - 3a^3 + 3 = (a^3-3)(a^3-1) + a^3 = (a^3-2)^2$

$+ a^3 - 1$, $b^3 - a^3 = (a^3 - 3)(a^3 - 1) = (a^3-3)(a-1)(a^2+a+1)$

$2 \nmid a \Rightarrow 2 \nmid b$, $2|a \Rightarrow 2 \nmid b$, $2|(n^2+3n) = n(n+3)$, $b^3 - 3 = a^3(a^3-3)$,

$b^3 = (n+2)(n+1) + 1$, $(b-1)(b^2+b+1) = (n+1)(n+2)$, $a^3 = m$, $b^3 = m^2 - 6m + 9 + 3m - 9 + 3$

$= m^2 - 3m + 3$ $a^3 = m+1$, $b^3 = m^2 - 4m + 4 + 3m - 6 + 3 = m^2 - m + 1$, $n^2 + 6n + 9$,

$c^3 - b^3 = 3n + 6 = 3(n+2)$, $(n+3)^4 = n^4 + 4n^3 \cdot 3 + 6n^2 \cdot 9 + 4 \cdot 27n + 81$, $n^4 + 9n^2 + 9$

$+ (3n^3 + 3n^2 + 9n)2 = n^4 + 6n^3 + 15n^2 + 18n + 9$, $n^3 - 6n^2 + 12n + 9$, $(n+1)^3 =$

$n^3 + 3n^2 + 3n + 1$ $(n+2)^3 = n^3 + 6n^2 + 12n + 8$ ...

$A^p = I$, $B^q = 0$, $A^{p-k} = A^{-k}$, $A^k = A^{-(p-k)}$, $(q=2, p=1)$ $(I+B)^2$

$= I + 2B$, $3B + I$ $A^{p} = I$, $(A+I)^{-1} = \sum (-A)^j$

$5^{5^{n+2}} + 5^{5^{n+1}} - 5^{5^{n+1}} - 5^{5^n} = 5^{5^{n+2}} - 5^{5^n} = 5^{5^n}(5^{5^n \cdot 24} - 1)$, $(n=0)$ $25 \cdot 625 + 6$

$= 15631 = 7 \cdot 2233 = 49 \cdot 319 = 7^2 \cdot 11 \cdot 29$ $5^{5^n \cdot 2^3 \cdot 3} - 1$ $(n=0)$ $5^5 + 6 = 3131$

$= 31 \cdot 101$ $5^2 - 1 = 24$, $124 = 31 \cdot 4 = 5^3 - 1$, $5^{5^n \cdot 2^3 \cdot 3} - 1 = (5^3-1) \sum_{j=0}^{5^n \cdot 8 - 1} 5^j$,

$31 | (5^{5^{n+2}} - 5^{5^n})$ // $S = \sum_{j=0}^{n-1} (-1)^j \binom{n}{j} 5^{n-j-1}$, $5S = \sum_{j=0}^{n} \binom{n}{j} (-1)^j 5^{n-j} - (-1)^n$,

$5S - 1 = 6^n$, $S = \frac{1}{5}(6^n - 1)$, $(n=5)$ $36 \cdot 216 + 1 = 7777 = 7 \cdot 1111 = 7 \cdot 11 \cdot 101$

$5S_k - 1 = 4^n$, $(n=5)$ $4^n + 1 = 1025 = 25 \cdot 41$, $S_5 = 5 \cdot 41$, S_{2k+3}

$4^{2k+1}+1 \leq ? \; 5p \overset{(3)}{\geq} 2p$

#103

$(a^2 b + b^2 c + c^2 a)(bc^2 + ca^2 + ab^2) \geq (3abc)^2$ // $\sum_{j=1}^{n} a_j^4 \; (\sum a_j)(\sum \frac{a_j^4}{n})$

$\geq (\sum \frac{a_j^{5/2}}{\sqrt{n}})^2$, $(\sum a_j^{5/2})^2 \geq ? \; n^2$ $\sum a_j^4 \geq ? \sum a_j$, $n = \sum a_j \leq n^{1/2}(\sum a_j^2)^{1/2}$

#104

$n \leq \sum a_j^2 \leq n^{1/2}(\sum a_j^4)^{1/2}$, $\sum a_j^4 \geq n$ // $\leq \sum a_j$ / $(\sum f_j)(\sum f_j x_j^2) \geq (\sum f_j x_j)^2$,

#106

$(\sum f_j)(\sum f_j x^2) \geq (\sum \sqrt{f_j} \cdot \sqrt{f_j}\, x)^2$ // $\sum_{j=1}^{n} k_j = 5n-4$, $1 = \sum \frac{1}{k_j}$, $1 \leq \sum k_j^2 \sum k_j^{-4}$

$1 \leq \quad k^{-3}$, $5n-4 \leq (\sum \frac{1}{k})^2 (\sum k_j^3)^2 = (\sum k_j^3)^2$, $\sum \frac{1}{k_j^3} \geq \frac{1}{5n-4}$, $\frac{1}{k} \geq \frac{1}{5n-4}$,

$a = 176$

$1 \leq 5n^2 - 4n - 8$, $16 + 20c$, $n^2 \leq \sum k_j^2 \sum k_j^{-2}$, $n^2 = (\sum k^{1/2} k^{-1/2})^2 \leq \sum k \sum \frac{1}{k} =$

$5n-4$, $(n-1)(n-4) \leq 0$, $1 \leq n \leq 4$, $(n-1)\; k_1 = k_1$ $(n-2)\; k_1 = k_2 = 2$ $(n=3)\; k_1 = k_2 =$

$k_3 = 3$, $\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{2}$, $2(k_1 + k_2) = k_1 k_2$, $(n=4)\; k_1 = k_2 = k_3 = k_4 = \frac{1}{k_1} = \frac{1}{k_2} = \frac{1}{k_3} = \frac{1}{k_4}$,

#107

$k_j = 4$ // $\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b} \geq ? \; \frac{1}{\cos(a-b)} = \frac{1}{\cos a \cos b + \sin a \sin b}$, $1 \leq \left(\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b}\right)$

#111

$(\cos a \cos b + \sin a \sin b) \geq$ ~~$\sin^2 a + \cos^2 b$~~ $\sin^2 a + \sin^2 b$ //

$(P(x) = (x-r_1)(x-\bar{r}_1)(x-r_2)(x-\bar{r}_2)(x-r_3)$, $35 = -|r_1 r_2|^2 r_3$, -10

$= \bar{r}_1 |r_2|^2 r_3 + r_1 |r_2|^2 r_3 + |r_1|^2 \bar{r}_2 r_3 + |r_1|^2 r_2 r_3 + |r_1 r_2|^2 = 2\,\mathrm{Re}\, r_1 \cdot |r_2|^2 r_3$

$+ 2\,\mathrm{Re}\, r_2 \, |r_1|^2 r_3 + |r_1 r_2|^2$, $\frac{2}{7} = \frac{2\,\mathrm{Re}\, r_1}{|r_1|^2} + \frac{2\,\mathrm{Re}\, r_2}{|r_2|^2} + \frac{1}{r_3}$, $\frac{1}{7} = \frac{\mathrm{Re}\, r_1}{|r_1|^2} + \frac{\mathrm{Re}\, r_2}{|r_2|^2} + \frac{1}{2r_3}$

$\frac{2}{7} = \sum \frac{1}{r_j}$ ....

$(n^{n+1}(n-1)^2)^{1/2} \leq \frac{1}{2}(n^{n+1} + n^2 - 2n + 1)$, $\leq \frac{1}{2}(n^{n+1}(n-1) + n - 1)$, $n^n - 1 = (n-1)\sum_{j=0}^{n-1} n^j$

#122

$\geq (n-1)\, n \left(\prod_{j=0}^{n-1} n^j\right)^{1/n} = (n-1)\, n^{1+\frac{n-1}{2}} = (n-1)\, n^{\frac{n+1}{2}}$ // $(1 + (r_1 \cdots r_n)^{1/n}) \leq ? \; ((1+r_1)\cdots(1+r_n))^{1/n}$

$\log(1 + (\prod r_j)^{1/n}) \leq ? \; \frac{1}{n}\sum \log(1+r_j) \geq (\prod \log(1+r_j))^{1/n}$, $\log(1 + (\prod r_j)^{1/n}) \leq \log(1+\bar{r})$

$= \log(\frac{1}{n}\sum(1+r_j)) \geq \frac{1}{n}\sum \log(1+r_j)$ $\log(1+r_1)\log(1+r_2) \geq ? \; (\log(1+\sqrt{r_1 r_2}))^2$,

$\frac{1}{n}\sum \log\log(1+r_j) \geq ? \; \log\log(1+(\prod r_j)^{1/n})$ $1+\sqrt{r_1 r_2} \leq ? \; ((1+r_1)(1+r_2))^{1/2}$ $\sum (\;)(\prod \;)^{j}$

Wooldridge

$\hat{y}_2 = z\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy^T)$, $\hat{v}_2 = y_2 - z\Pi_z(y_2)$, $\ddot{x} = (z_1, y_2) - \hat{v}_2\Pi_{\hat{v}_2}(z_1, y_2)$; $\hat{v}_2\Pi_{\hat{v}_2}(z_1,y_2)$

$= \hat{v}_2\,\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2(z_1,y_2)^T)$, $\Pi_{\ddot{x}}(y) = \mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\mathbb{E}(\ddot{x}y^T)$, $\mathbb{E}(\ddot{x}\ddot{x}^T)$

$= \mathbb{E}(xx^T) - \mathbb{E}(xx^T\Pi_{\hat{v}_2}\hat{v}_2^T) - \mathbb{E}(\hat{v}_2\Pi_{\hat{v}_2}xx^T) + \mathbb{E}(\hat{v}_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}\hat{v}_2^T) =$

$\mathbb{E}(xx^T) - \mathbb{E}(xx^T\Pi_{\hat{v}_2}y_2^T) + \mathbb{E}(xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T) - \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T) + \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T)$

$+ \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T) - \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T) - \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T)$

$+ \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T)$; $\Pi_{\ddot{x}}y = ?\,\Pi_{\hat{x}}y$, $\Pi_{\hat{x}}y = \mathbb{E}(\hat{x}\hat{x}^T)^{-1}\mathbb{E}(\hat{x}y)$,

$\hat{x} = \Pi_z x = \bar{\Pi}_z x$, $\Pi_{\hat{x}}y$ $\mathbb{E}$ [illegible] $(\bar{\Pi}_z\mathbb{E}(xx^T)\bar{\Pi}_z)^{-1}\bar{\Pi}_z\mathbb{E}(xy)$,

$P_{\hat{x}}y = \mathbb{E}(y\hat{x}^T)\mathbb{E}(\hat{x}\hat{x}^T)^{-1}\hat{x}$, $\hat{x} = P_z x = \mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z$, $\hat{y} = \mathbb{E}(yz)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T)$

$\{\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T)\}\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z = \mathbb{E}(P_z y\cdot x^T)\cdot\mathbb{E}(P_z x\cdot x^T)^{-1}$

$\cdot P_z x$, $P_{\ddot{x}}y = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\ddot{x}$, $\ddot{x} = x - P_x\hat{v}_2$ $= \bar{P}_{\hat{v}_2}x$, $\hat{v}_2 = \bar{P}_z y_2$,

$\bar{P}_{\hat{v}_2}x = x - \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$ ~~$= x - \mathbb{E}(x\,\ldots\,y_2)^T)$~~ $\hat{v}_2 = y_2 - P_{y_2}z = y_2 - \mathbb{E}(y_2$ [illegible]

$P_x\hat{v}_2 = \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$

$\mathbb{E}(zz^T)^{-1}z$, $\mathbb{E}(y\hat{v}_2^T P_x^T) = \mathbb{E}(y(y_2^T - z^T\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T))\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2 x^T)$, $x = \begin{pmatrix} z_1 \\ y_2\end{pmatrix}$

$P(x|\hat{v}_2) = \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$, $\mathbb{E}\left(\begin{pmatrix} z_1 \\ y_2\end{pmatrix}(y_2^T - \mathbb{E}(y_2 z^T)\,z^T\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T))\right) = \begin{pmatrix} 0 \\ \mathbb{E}y_2^2 - \mathbb{E}y_2P(y_2|z)\end{pmatrix}$

$\mathbb{E}(y_2^2) - \mathbb{E}(y_2P(y_2|z)^T) = \mathbb{E}y_2^2 - \mathbb{E}(y_2z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2) = \mathbb{E}y_2^2 - \mathbb{E}(y_2z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)$

$\mathbb{E}(y_2z^T) = \mathbb{E}(y_2^2) - \mathbb{E}(P(y_2|z)^2) = \mathbb{E}\hat{v}_2^2$, ~~$P(x|\hat{v}_2) = \hat{v}_2$~~ $\mathbb{E}(x\hat{v}_2) = \begin{pmatrix} 0 \\ \mathbb{E}\hat{v}_2^2\end{pmatrix}$, $P(x|\hat{v}_2) = \begin{pmatrix} 0 \\ \hat{v}_2\end{pmatrix}$

$\ddot{x} = x - P(x|\hat{v}_2) = \begin{pmatrix} z_1 \\ y_2 - \hat{v}_2\end{pmatrix}$, $(\hat{\delta}_1, \hat{\alpha}_1) = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}$ ~~$\mathbb{E}(y\ddot{x}^T)$~~ $= (\mathbb{E}yz_1^T,$

$\mathbb{E}yy_2 - \mathbb{E}y\hat{v}_2)\begin{pmatrix} \mathbb{E}z_1z_1^T & \mathbb{E}z_1y_2 \\ \mathbb{E}y_2z_1^T & \mathbb{E}(y_2-\hat{v}_2)^2\end{pmatrix}^{-1} = ?\ \mathbb{E}(P(y|z)\cdot x^T)\cdot\mathbb{E}(P(x|z)\cdot x^T)^{-1}$

$\mathbb{E}(P(y|z)\cdot x^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(z\cdot(z_1^T, y_2^T)) = \mathbb{E}(yz^T)[\underset{|z|\times|z_1|}{I} ; \mathbb{E}(zz^T)$

$\mathbb{E}(zy_2^T)] = \mathbb{E}(yz_1^T) ; \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T) = (\mathbb{E}yz_1^T, \mathbb{E}(yP(y_2|z)^T)) = (\mathbb{E}yz_1^T, \mathbb{E}(y(y_2-\hat{v}_2))$

$\mathbb{E}(P(x|z)\cdot x^T) = \mathbb{E}(P(x|z)P(x|z)^T) = \mathbb{E}\left(\begin{pmatrix} z_1 \\ y_2-\hat{v}_2\end{pmatrix}^{\otimes 2}\right)$ #5.1 // [illegible] $\hat{x} = P(x|z)$

$= \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}z$, $\hat{\beta}_1 = \mathbb{E}(y\hat{x})\mathbb{E}(\hat{x}^2)^{-1} = \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}\mathbb{E}(yz)\mathbb{E}(z^2)\mathbb{E}(xz)^{-2}\mathbb{E}(z^2)$ [illegible]

$\beta_1 = \mathbb{E}(yz)/\mathbb{E}(xz)$, $\hat\beta_1 = \sum yz/\sum xz = (\sum xz)^{-1}(\beta_0 \sum z + \beta_1 \sum xz + \sum zu) = \beta_0 \bar z/\overline{xz}$

$+\beta_1 + \sum xu/\sum xz$, $\cancel{\sqrt{N}(\hat\beta_1-\beta_1) = \beta_0\sqrt N \bar z/\overline{xz} + \sqrt N\, \overline{xu}/\overline{xz} \rightsquigarrow \frac{\beta_0}{\sigma^2_{xz}} N(0,\ldots)}$ $\sqrt N(\hat\beta_1-\beta_1)$

$= \frac{\sqrt N\, \overline{zu}}{\overline{zx}} \rightsquigarrow \sigma_{zx}^{-1} N(0, \mathrm{Var}\, zu) = N(0, \sigma_z^2\sigma_u^2/\sigma_{zx}^2) = N(0, \rho_{zx}^{-2}\sigma_u^2/\sigma_x^2)$ #5.10 //

$\tilde y_2 = P(y_2|z_2) = \mathbb{E}(y_2 z_2^T)\mathbb{E}(z_2z_2^T)^{-1}z_2$, $(\tilde\delta_1^T, \tilde\alpha_1) = \mathbb{E}(y_1(z_1^T,\tilde y_2))\mathbb{E}\left(\binom{z_1}{\tilde y_2}(z_1^T,\tilde y_2)\right)^{-1}$

$= \mathbb{E}\left\{(\delta_1^T,\alpha_1)\binom{z_1}{y_2}(z_1^T,\tilde y_2) + u_1(z_1^T,\tilde y_2)\right\}\mathbb{E}\begin{pmatrix} z_1z_1^T & z_1\tilde y_2\\ z_1^T\tilde y_2 & \tilde y_2^2\end{pmatrix}^{-1} = (\delta_1^T,\alpha_1)\mathbb{E}\left(\binom{z_1}{y_2}(z_1^T,\tilde y_2)\right)$

$\mathbb{E}\left(\binom{z_1}{\tilde y_2}^{\otimes 2}\right)^{-1}$, $\mathbb{E}(\tilde y_2^2) = \mathbb{E}(y_2\tilde y_2)$, $\mathbb{E}(y_2z_1) \ne \mathbb{E}(\tilde y_2 z_1)$

#5.11 //

$\begin{pmatrix}\mathbb{E}x_{-k}x_{-k}^T & \mathbb{E}x_{-k}z^T\\ \mathbb{E}zx_{-k}^T & \mathbb{E}zz^T\end{pmatrix}$, $\begin{pmatrix}\mathbb{E}x_{-k}x^T\\ \mathbb{E}zx^T\end{pmatrix} = \begin{pmatrix}\mathbb{E}x_{-k}x_{-k}^T & \mathbb{E}x_{-k}x_k\\ \mathbb{E}zx_{-k}^T & \mathbb{E}zx_k\end{pmatrix}$, $(\delta,\theta) = \mathbb{E}(x_k(x_{-k}^T,z^T))\mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}$

$\theta=0 \Rightarrow$ $\mathbb{E}((x_kx_{-k}^T, x_kz^T)) \perp \cancel{\begin{pmatrix}\mathbb{E}x_{-k}z^T\\ \mathbb{E}zz^T\end{pmatrix}}$ $\mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[, k:(k+M)]$

$(\mathbb{E}x_{-k}x_{-k}^T, \mathbb{E}x_{-k}z^T) \perp \mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[,k:(k+M)]$, $\mathrm{rank}\begin{pmatrix}\mathbb{E}x_{-k}x^T\\ \mathbb{E}zx^T\end{pmatrix} = K-1$.

$\begin{pmatrix}\mathbb{E}x_{-k}x_k\\ \mathbb{E}zx_k\end{pmatrix} \in \mathrm{sp}\begin{pmatrix}\mathbb{E}x_{-k}x_{-k}^T\\ \mathbb{E}zx_{-k}^T\end{pmatrix} \Rightarrow \begin{pmatrix}\mathbb{E}x_{-k}x_k\\ \mathbb{E}zx_k\end{pmatrix} \perp \mathbb{E}^{-1}\binom{x_{-k}}{z}^{\otimes 2}[,k:(k+M)]$, $\theta=0$ #5.12 //

$\beta_1 = \mathrm{Cov}(y,z)/\mathrm{Cov}(x,z) = (\mathbb{E}(yz) - \mathbb{E}y\,\mathbb{E}z)/(\mathbb{E}(xz)-\mathbb{E}x\,\mathbb{E}z) =$

$(\sum y\{z=1\} - \sum y\cdot\sum z/n)/(\sum x\{z=1\} - \sum x\cdot \sum z/n)$, $\mathbb{E}yz = \beta_0\mathbb{E}z + \beta_1\mathbb{E}xz$,

$\mathbb{E}\left(\binom{1}{x}(1\ z)\right)\mathbb{E}^{-1}\begin{pmatrix}1&z\\z&z^2\end{pmatrix} = \begin{pmatrix}1 & \mathbb{E}z\\ \mathbb{E}x & \mathbb{E}xz\end{pmatrix}\begin{pmatrix}1&\mathbb{E}z\\ \mathbb{E}z & \mathbb{E}z^2\end{pmatrix}^{-1} = (\mathrm{Var}\,z)^{-1}\begin{pmatrix}1&\mathbb{E}z\\ \mathbb{E}x&\mathbb{E}xz\end{pmatrix}\begin{pmatrix}\mathbb{E}z^2 & -\mathbb{E}z\\ -\mathbb{E}z & 1\end{pmatrix}$,

$\mathbb{E}\left(\binom{1}{z}(1\ x)\right)^{-1}\mathbb{E}\binom{y}{yz} = \mathbb{E}\begin{pmatrix}1&x\\z&xz\end{pmatrix}^{-1}\mathbb{E}\binom{y}{yz} = (\mathrm{Cov}(x,z))^{-1}\mathbb{E}\begin{pmatrix}xz & -x\\ -z & 1\end{pmatrix}\mathbb{E}\binom{y}{yz}$,

$\hat\beta_1 = \frac{\mathbb{E}yz - \mathbb{E}y\,\mathbb{E}z}{\mathbb{E}xz-\mathbb{E}x\,\mathbb{E}z}$, $\frac1n(\sum y\sum z) = \frac1n(\sum y\{z=1\} + \sum_{j\ne k} y_j z_k)$, $\sum y_j(\{z_j=1\} - \bar z)$,

$\sum y_j(\{z_j=1\}-\{z_j=0\}) = 2\sum y_j\{z_j=1\} - \sum y_j$, $\sum y\{z=1\} - \bar y\sum\{z=1\}$, $\sum y\{z=1\}$

$-\bar z\sum y(\{z=1\}+\{z=0\}) = (1-\bar z)\sum y\{z=1\} - \bar z\sum y\{z=0\} = n\bar z(1-\bar z)\bar y_1 - \bar z n(1-\bar z)\bar y_0$ #5.13 //

$L(y|z) = L(\sum_{j=1}^{K} \beta_j x_j + u|z) = \sum_{j=1}^{K-1}\beta_jx_j + \beta_K L(x_K|z)$, $L(x_K|z) = \sum^{K-1}\delta_jx_j + \sum\theta_jz_j$,

$\theta\ne0$ $\mathrm{rank}\,\mathbb{E}(zx^T) = K$ #5.14 //

vaVaart Ch 6

$\frac{dP_n}{dQ_n} = \exp\left(\frac{1}{2}(x-\mu_n)^2 - \frac{1}{2}x^2\right) = \exp\left(-x\mu_n + \frac{\mu_n^2}{2}\right)$, $P_{Q_n}\left(\frac{dP_n}{dQ_n} < c\right) = P_{Q_n}\left(X > -\frac{1}{\mu_n}\left(\log c - \frac{\mu_n^2}{2}\right)\right) = P\left(Z > -\frac{\log c}{\mu_n} + \frac{\mu_n}{2}\right)$, $\frac{\partial}{\partial c} = \varphi\left(-\frac{\log c}{\mu_n} - \frac{\mu_n}{2}\right)/c\mu_n$, $P_{Q_n}\left(\frac{dP_n}{dQ_n} > 0\right)$

$\xrightarrow[n_k\to\infty]{} P_{Q_\infty}\left(\exp\left(-x\mu_\infty + \frac{\mu_\infty^2}{2}\right) > 0\right) = 1 \Leftarrow -X\mu_\infty + \frac{\mu_\infty^2}{2} > -\infty$ wp $1-Q_\infty$,

$X\mu_\infty + \frac{\mu_\infty^2}{2} \underset{Q_\infty}{\sim} Z\mu_\infty - \frac{\mu_\infty^2}{2} > 0$ wp 1 $\Leftarrow \mu_n \in [-K, K]$. $E_{Q_n}\frac{dP_n}{dQ_n}$

$= \int \exp\left(-x\mu_n + \frac{\mu_n^2}{2}\right)(2\pi)^{-1/2}\exp\left(-\frac{1}{2}(x-\mu_n)^2\right)dx = (2\pi)^{-1/2}\int \exp\left(-\frac{x^2}{2}\right) = 1$ #1

$\frac{dP_n}{dQ_n} = (2\pi/n)^{-1}\exp\left(\frac{1}{2n}\left(-(x-\theta_n)^2 - x^2\right)\right) = \frac{n}{2\pi}\exp\left(-(\theta_n^2 - 2x\theta_n)/2n\right)$, $P_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right)$

$(\theta_n = O(n^{-1/2}))$

$= P_{Q_n}\left((\theta_n^2 - 2x\theta_n)/2n < \log\left(\delta\frac{2\pi}{n}\right)\right) = P\left(N\left(-\frac{\theta_n^2}{2n}, \frac{\theta_n^2}{n^2}\right) < \log\left(\delta\frac{2\pi}{n}\right)\right)$

$= \Phi\left(\left\{\log\left(\delta\frac{2\pi}{n}\right) + \frac{\theta_n^2}{2n}\right\}/\frac{\theta_n}{n}\right) = \cancel{\Phi\left(O(\sqrt{n})\left(\log\left(\delta\frac{2\pi}{n}\right) + O(1)\right)\right)} \to 0$

$= \Phi\left(O(n^{3/2})\log\left(\delta\frac{2\pi}{n}\right)\right)$ $\frac{dP_n}{dQ_n} = (\ )^{-1}\exp\left(\frac{n}{2}(\theta_n^2 - 2x\theta_n)\right)$, $P_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right)$

$= P\left(N\left(-\theta_n^2\frac{n}{2}, n\theta_n^2\right) < \log(\delta)\right) = \Phi\left(\left\{\log(\delta) + \theta_n^2\frac{n}{2}\right\}/\sqrt{n}\theta_n\right)$

$= \Phi\left(O(1)\left\{\log(\delta) + O(1)\right\}\right)$ $\frac{\log\left(\frac{2\pi}{n}\delta\right)}{\sqrt{n}} \to \frac{-\infty}{\infty}$, $\cancel{\left(\frac{n}{2\pi\delta}\frac{\sqrt{n}\delta}{n^2}\right)/\left(\frac{1}{2\sqrt{n}}\right)} \to 0$,

$\xrightarrow{\delta\to 0} 0$, $\frac{dP_n}{dQ_n} \overset{Q_n}{\rightsquigarrow} V \Rightarrow P(V=0) = 0$, $P_{P_n}\left(\frac{dQ_n}{dP_n} < \delta\right) =$

$P\left(N\left(-\frac{\theta_n^2}{2}n, n\theta_n^2\right) < \log\delta\right) \xrightarrow{\delta\to 0, n\to\infty} 0$, $\frac{dQ_n}{dP_n} \overset{P_n}{\rightsquigarrow} W \Rightarrow P(W=0) = 0$, $P_n \lhd\rhd Q_n$,

$\theta_n \neq O(n^{-1/2})$, $\limsup \theta_n n^{1/2} = \infty$; $P_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right) = \Phi\left(\frac{\log\delta}{\sqrt{n}\theta_n} + \frac{\theta_n\sqrt{n}}{2}\right)$

$\xrightarrow[n_k]{} 1 \Leftarrow \theta_{n_k} n_k^{1/2} \xrightarrow[k\to\infty]{} \infty$ #2 $\frac{dP_n}{dQ_n} = \frac{n}{n+1}1\{[0,1]\} \overset{Q_n}{\rightsquigarrow} V$, $P_{Q_n}(V < \delta)$

$\lim_{k\to\infty} P_{Q_{n_k}}\left(\frac{dP_{n_k}}{dQ_{n_k}} < \delta\right) = \lim_{k\to\infty} 1\left\{\frac{n_k}{n_k+1} < \delta\right\} = 0 \Leftarrow \delta < \frac{1}{2}$, $Q_n \lhd P_n$. $\frac{dQ_n}{dP_n} = \frac{n+1}{n}1\{[0,1]\}$,

$P_{P_n}\left(\frac{dQ_n}{dP_n} < \delta\right) = P_{P_n}\left(1\{0 \le X \le 1\} < \frac{\delta n}{n+1}\right) \overset{(\delta<1)}{=} P_{P_n}(X \notin [0,1]) = \frac{1}{n+1} \to 0$, $P_n \lhd Q_n$ #3

$P_n(A) \to 0 \Rightarrow Q_n(A) = P_n(A) + Q_n(A) - P_n(A) \le P_n(A) + \|P_n - Q_n\|_{TV} \to 0$ #4

$P_n(0) = 1-\varepsilon$, $P_n(1) = \varepsilon$, $Q_n(0) = \varepsilon$, $Q_n(1) = 1-\varepsilon$, $\|P_n - Q_n\| = 1-2\varepsilon$, $P_n(A) \to 0 \Rightarrow A \cap \{0,1\}$

Andreescu

$a+b+c=2 \geq 3(abc)^{1/3}$, $(1-a)(1-b)(1-c) = 1-a-b-c+ab+bc+ac-abc =$

$-1+\alpha$, $0 \leq ? (1-a)(1-b)(1-c)$ $a+b>c$, $1-a-b<1-c$, $(1-a)(1-b)(1-c)$

$= \alpha - 1 \leq (\frac{1}{3}\cancel{(a+b+c))^3 = \frac{8}{27}, \alpha \leq \frac{35}{27}}$ $(\frac{1}{3}(3-a-b-c))^3 = \frac{1}{27}$, $\alpha \leq \frac{28}{27}$.

$\alpha \geq ab+bc+ac-(\frac{1}{3}(a+b+c))^3 = ab+bc+ac-8/27$, $(1-a)(1-b)(a+b-1)$

$=(b-1)(a-1)(b+a-1)$, $0 \leq ? (1-a)(1-b)(1-c)$, $a>1 \Rightarrow b+c=2-a<1$,

$a>b+c$, $a,b,c<1$ #124 // $365^{-25}\prod(365-j) \leq ? \frac{1}{2}$, $(\frac{1}{25}\sum(365-j))^{25} \leq ?$

$365^{25}/2$, $365-\frac{25\cdot 26}{50} \leq ? \; 365/2^{1/25}$, $\cancel{352/365}$ $2^{1/25} \leq ? \frac{365}{352} = \frac{13}{11\cdot 32}+1$

$\geq 1+1/32 \geq 1.02$, $\frac{13}{352} \geq \frac{13}{390} = \frac{1}{30}$, $(1.03)^{25} \geq ! \; 2$, $1.03^2 = 1.0609$, 1.12, 1.25,

16 1.56, $9: 1.2075$, 156 124 1243 312 156 ... #125/ ($k=3$) 1, 5 = 1 + (2-1) + k, 12 = 5+1+2k, 13+

$3\cdot 4 = 22$, $a_{n-1} \underset{(k)}{\equiv} n-1$, $a_n - a_{n-1} \underset{(k)}{\equiv} 1$, $a_n - a_{n-1} = k(n-1)+1$, $a_n = \sum_{m=1}^{n} k(m-1)+1$

$= n + \frac{n(n-1)}{2}k$ #697 // $f(0)+2f(f(0)) = 5$, $f(f(0)) = \frac{1}{2}(5-f(0)) = f(\cancel{\ldots})$, $5-f(0) \neq 0$,

$2 \mid (5-f(0))$, $f(0) \in \{1,3,5\}$ $(f(0)=1)$ $f(1) = 2 = \cancel{\ldots}$, $f(2) = f(f(n))$

$= \frac{1}{2}(3n+5-f(n))$, $f(2) = f^2(1) = 3$, $f(3) = f^2(2)$ $f(n+1) = f(f(n)) = \frac{1}{2}(3n+5-(n+1))$

$= n+2$, $f(n) = n+1$ $(f(0)=3)$ $f^2(\ldots$ $(3)) \notin \mathbb{N}$

$(f(0)=5)$ $f(5)=0$, $f(1)$ #701 // $0 = f(n)+3f(0)f(n)$

$f(0) = -1/3$, $f(n) = 0$, $=0$, $f(n^2) =$

$2f(n)+3f(n)^2 = (3f(n)$ $f(2n) = f(2)+f(n)+3f(2)f(n)$,

$\frac{f(mn)}{f(m)} = 1 + \frac{f(n)}{f(m)} + \ldots \geq 0 \Rightarrow f(n) \geq 1$, $f(6) \geq 3+6 = 9$ $f(4)$

$= 2f(2)+3f(2)^2$, $f(8)$ $f(4)+f(2)+3f(4)f(2) = 3f(2)+3f(2)^2+6f(2)^2+9f(2)^3$

$= 3f_2+9f_2^2+9f_2^3$, $f(nf^{-1}(1)) = 1+4f(n)$

rdUmrt chb cnit

$P(A_n) \to 0$, $Q(A_n) \not\to 0 \Rightarrow \{A_{n_k}\}_k$, $\delta > 0$: $Q(A_{n_k}) > \delta$, $\{A_{n_{k_j}}\}_j$, $P(A_{n_{k_j}}) < 2^{-j}$, $B_m := \bigcup_{j=m}^{\infty} A_{n_{k_j}}$, $P(B_m) \leq 2^{-m+1} \xrightarrow[m\to\infty]{} 0$, $B_m \downarrow B_\infty$, $0 = \lim P(B_m) = P(\lim B_m) = Q(\lim B_m) = \lim Q(B_m)$, $Q(B_m) > \delta$, $\leftarrow$ $\#$ // $\mathbb{P}_{P}\left(\frac{dQ}{dP} > 0\right) = \mathbb{P}_{P}(dQ > 0) = 1 - \mathbb{P}_P(dQ = 0) = 1$ $\#$ //

Andreescu

$(mx+ny)^2+10(jx+ky)^2=(m^2+10j^2)x^2+(2mn+20jk)xy+(n^2+10k^2)y^2$, $mn=-10jk$

$m^2+10j^2 \equiv 1$, $n^2+10k^2 \equiv 10$, $n^2+10k^2=10(m^2+10j^2)$, $2mn+20jk=0$, $mn=-10jk$

$n^2+10 \overset{10}{?}\ 10m^2+100$, $j=k=1$, $m=-1, n=10$, $m^2+10j^2=u^2$, $n^2+10k^2=10u^2$, $n^2u^2-10j^2n^2=$

$100j^2k^2=n^2(u^2-10j^2)$, $4,9,16,25,36,49,64,81,100,121,144,169,\ldots$ $1,4,9,5,6$, $3z^2=\_\{3,2,7,5,8$

$x^2+10y^2=\_\{1,4,9,5,6\}$, $n=\{5\}$, $5|x, 5|y, 5|z$ #706 / $(2x+2y)(2x-2y)=t^2-t^2$

$=(z+t)(z-t)$, $4|(z^2-t^2)$, $x^2-y^2=\frac{1}{4}(z^2-t^2)$, $24y^2=5z^2-t^2$, $\begin{pmatrix}x^2\\y^2\end{pmatrix}=-\frac{1}{24}\begin{pmatrix}1&-5\\-5&1\end{pmatrix}\begin{pmatrix}z^2\\t^2\end{pmatrix}$

$24x^2=5t^2-z^2$, $24y^2=5z^2-t^2$; $5|(z-x)(z+x)$, $(z=x+5k, t=y+5j)$

$5y^2=10kx+25k^2$, $y^2=2kx+5k^2$, $x^2=2jy+5j^2$, $(z-x)(z+x)=5y^2$

$=y_1y_2$, $z=\frac{1}{2}(y_1+y_2)$, $x=\frac{1}{2}(y_2-y_1)$, $(t-y)(t+y)=5x^2=x_1x_2$, $t=\frac{1}{2}(x_1+x_2)$

$y=\frac{1}{2}(x_2-x_1)$, $4y^2=x_1^2+x_2^2-10x^2=\frac{4}{5}(z^2-x^2)$, $-\frac{46}{5}x^2+x_1^2+x_2^2=z^2$, $5|x$,

$5|z$, $5|t$, $25\left(\frac{x}{5}\right)^2+5y^2=25\left(\frac{z}{5}\right)^2$, $5x_{(1)}^2+y^2=5z_{(1)}^2$

$x^2+5y_{(1)}^2=t_{(1)}^2$, $5|?y$, $25|(z^2-x^2)$, $5|y$, $\left(\frac{x}{5}\right)^2+5\left(\frac{y}{5}\right)^2=\left(\frac{t}{5}\right)^2$,

$5\left(\frac{x}{5}\right)^2+\left(\frac{y}{5}\right)^2=\left(\frac{t}{5}\right)^2$ #707 // $p|x \Rightarrow p|(x-y)(x+y)$, $p|y$, $x=y$ #718 // $x_j^2, x_{j+1}^2, x_{j+2}^2, \ldots$,

$x_{j+1}^2-x_j^2=d$, $x_{j+1}-x_j=\dfrac{d}{x_j+x_{j+1}}\to 0$ #719 // $n^3-4n^2+1+n^2-1+4=n^3+n^2+3+(1-2n)\cdot$

$(1+2n)$, $(2n-1)|(n^3+n^2+3)$, $2n^3-n^3-n^2-2n^2+4=n^2(2n-1)-n^3-2n^2+4$, $(2n-1)|$

(n^3+2n^2-4), $(2n-1)|(n^2-7)$; $n^3-2n^2-n^2-n+n+4=-n(2n-1)+n^3-n^2-n+4$, $(2n-1)|$

(n^3-n^2-n+4), $(2n-1)|(2n^2+n-1)=(2n-1)(n+1)$, $n^2=\_\{1,4,9,6,5,0\}$, $n^2-7=\_\{4,7,2,9,8,3\}$

$(2n-1)|(4n^2-1)$, $(2n-1)|(n^3-17)$, $(2n-1)|(5n^2-4)$, $(2n-1)|(-27)$,

$2n-1\in\{1,3,9\}$ #724 // $1!\,2!\ldots100!=1^{100}2^{99}3^{98}\ldots 99^2\cdot100^1=$

$x^2(2\cdot4\cdot6\cdots100)=x^2\,50!\,2^{50}=y^2\,50!$ #725 // , $a_j=m+j$, $m|a_k \Rightarrow$

$(a_j,a_k)=m$, $(a_j,a_{2j})=j \Rightarrow j|m$, $(a_j,a_m)=j$, $a_j=b_j\,j$, $(j,k)=(b_j\cdot j, b_k\cdot k)\Rightarrow(b_j,b_k)=1$

$b_{j_0}\neq1$?, $(a_{j_0}, a_{b_{j_0}})=(j_0 b_{j_0}, b_{b_{j_0}} b_{j_0})=(j_0, b_{j_0})$, $b_{j_0}|j_0$

IV

$\frac{1}{2}\frac{\partial T}{\partial \gamma} \pm \left(\frac{1}{2}\frac{\partial T}{\partial \gamma} - \frac{\partial D}{\partial \gamma}\right)\frac{1}{2}(T^2/4 - D)^{-1/2} = 0$, $T = p^2(\gamma^2/\delta_0 + (1-\gamma)^2/\delta_1) + (1-p)^2(\gamma^2/\delta_1 + (1-\gamma)^2/\delta_0)$,

$\frac{\partial T}{\partial \gamma} = 2p^2(\gamma/\delta_0 - (1-\gamma)/\delta_1) + 2(1-p)^2(\gamma/\delta_1 - (1-\gamma)/\delta_0)$, $\frac{1}{4}(p^4 + (1-p)^4)\cdot(2p-1)^2 \ldots$

$T = p^2(\gamma/\delta_0 + (1-\gamma)/\delta_1) + (1-p)^2(\gamma/\delta_1 + (1-\gamma)/\delta_0)$, $\frac{\partial T}{\partial \gamma} = p^2(1/\delta_0 - 1/\delta_1) + (1-p)^2(1/\delta_1 - 1/\delta_0)$

$= (\frac{1}{\delta_0} - \frac{1}{\delta_1})(2p-1)$, $T^2 = p^4\left(\frac{\gamma^2}{\delta_0^2} + \frac{(1-\gamma)^2}{\delta_1^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right) + (1-p)^4\left(\frac{\gamma^2}{\delta_1^2} + \frac{(1-\gamma)^2}{\delta_0^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}\right)$

$+ 2p^2(1-p)^2\left(\frac{\gamma^2}{\delta_0\delta_1} + \frac{(1-\gamma)^2}{\delta_0\delta_1} + \gamma_0\gamma_1(1/\delta_0^2 + 1/\delta_1^2)\right)$ $(\delta_0 = \delta_1)$ $T = p^2\delta^{-2}(\gamma_0^2 + \gamma_1^2)(2p^2 - 2p + 1)$, $D = (2p-1)^2\delta^{-4}(\gamma_0\gamma_1)^2$, $(w_0\gamma_0 + w_1\gamma_1)/\delta_0 + (w_0\gamma_1 + w_1\gamma_0)/\delta_1$

$= ((w_0 - w_1)\gamma_0 + w_1)/\delta_0 + ((w_1 - w_0)\gamma_0 + w_0)/\delta_1$

$\mathbb{P}(A_{t+1}=a \mid L_t = l, U_t = u) = \sum_b \overset{p_B^b(1-p_B)^{1-b}}{\mathbb{P}}(A_{t+1}=a \mid L_t = l, U_t = u, B_{t+1} = b) = \overset{?p_B}{\mathbb{P}}(A^L_{t+1} = a \mid L_t = l) + \overset{(1-p_B)}{\mathbb{P}}(A^U_{t+1} = a \mid U_t = u) = \frac{1}{2} p_B p_{LA}^{a=l}(1-p_{LA})^{a\neq l} + (1-p_B)p_{UA}^{a=u}(1-p_{UA})^{a\neq u}$, $\mathbb{P}(L_{t+1} = l, U_{t+1} = u \mid A_t = a)$

(32aa) $= p_B\,\mathbb{P}(L_{t+1} = l, U_{t+1} = u \mid A^L_t = a, B_t = 1) + (1-p_B)\,\mathbb{P}(L_{t+1} = l, U_{t+1} = u \mid A^U_t = a, B_t = 0) = \frac{1}{2}p_B p_{AL}^{a=l}(1-p_{AL})^{a\neq l} + \frac{1}{2}(1-p_B)p_{AU}^{a=u}(1-p_{AU})^{a\neq u}$, $\mathbb{P}(L_{t+1} = l \mid A_t = a)$

$= p_B\,\mathbb{P}(L_{t+1} = l \mid A^L_t = a) + (1-p_B)\,\mathbb{P}(L_{t+1} = l) = p_B p_{AL}^{a=l}(1-p_{AL})^{a\neq l} + \frac{1}{2}(1-p_B)$,

$\mathbb{P}(A_t = a \mid L_t = l) = p_B p_{LA}^{a=l}(1-p_{LA})^{a\neq l} + (1-p_B)/2$

$P(L_t = l \mid L_{t-1} = l) = \sum_a \mathbb{P}(L_t = l \mid A_{t-1} = a)\,\mathbb{P}(A_{t-1} = a \mid L_{t-1} = l)$

$\delta_j^{-2} \mapsto \delta_j$

$T = p(1/\delta_0^2 + 1/\delta_1^2)$, $\Delta = p^2/\delta_0^2\delta_1^2 - (1-p)^2/\delta_0^2\delta_1^2 = (2p-1)/\delta_0^2\delta_1^2$, $T^2/4 - \Delta =$

$\frac{1}{4}(p^2(\delta_0^2 + \delta_1^2) + 2p^2\delta_0\delta_1) - (2p-1)\delta_0\delta_1 = \frac{p^2}{4}(\delta_0^2 + \delta_1^2) + \delta_0\delta_1(p^2/2 - 2p + 1)$

$= \frac{p^2}{4}(\delta_0 - \delta_1)^2 + \delta_0\delta_1(p^2 - 2p + 1) = \frac{p^2}{4}(\delta_0 - \delta_1)^2 + \delta_0\delta_1(p-1)^2$ $(\delta_0 = \delta_1)$

$\cancel{p(1/\delta_0^2 + 1/\delta_1^2)} \pm p/\delta_0^2 \pm (1-p)/\delta_0^2 = \frac{1}{\delta_0^2}, \frac{(2p-1)}{\delta_0^2}$

tz $1/\delta_0 + 1/\delta_1 = s$, $\quad T = p^2((\gamma_0 - \gamma_1)/\delta_0 + \gamma_1 s) + (1-p)^2((\gamma_1 - \gamma_0)/\delta_0 + \gamma_0 s)$

$= (\gamma_0 - \gamma_1)\delta_0^{-1}(2p-1) + s(p^2\gamma_1 + (1-p)^2\gamma_0)$ =

Andreescu

$n^a-1=(n^{(a,b)}-1)(1+n+\dots+n^{a'-1})$, $a':=\frac{a}{(a,b)}$, $n^{(a,b)}-1 \mid (n^a-1, n^b-1)$. & $d \mid (n^a-1, n^b-1)$,

$d \mid (n^a-n^b) = (n^{(a,b)a'}-n^{(a,b)b'}) = (n^{a'}-n^{b'})(n^{a'((a,b)-1)}+n^{a'((a,b)-2)}n^{b'}+\dots+n^{b'((a,b)-1)})$

$=(n^{a'}-n^{b'})(n^{a-a'}+n^{a-2a'+b'}+n^{a-3a'+2b'}+\dots+n^{b-b'})$, $n^a-1=(n^g-1)\sum_{j=0}^{a'-1} n^{jg}$

$n^b-1=(n^g-1)\sum_{j=0}^{b'-1} n^{jg}$, $(b>a)$ & $n^b-1-(n^a-1)=(n^g-1)\sum_{j=a'}^{b'-1} n^j$, $\sum_{b'}^{b'+a'-1}$

$(a',b')=1$, $a'x+b'y=1$, $\frac{1+n+n^2}{1+n+n^2+n^3+n^4}$, $\frac{1+n+n^2}{1+n}$, $1=(\sum^{5-1}-n^2\sum^{3-1})(-n)+\sum^{3-1}=-n\sum^{5-1}+$

$(n^3+1)\sum^{3-1}$, $S_{a'-1}:=\sum_{j=0}^{a'-1} n^j$, $\alpha S_{a'-1}+\beta S_{b'-1}=?\,1$, $\alpha,\beta\in\mathbb{Z}$, $(a'x-b'y=1)$

$(\sum_{j=0}^{x} n^{ja'})S_{a'-1}=(\sum_{j=0}^{x} n^{ja'})\sum_{j=0}^{a'-1} n^j=\sum_{j=0}^{a'x} n^j$, $(\sum_{j=0}^{y} n^{jb'})S_{b'-1}=\sum_{j=0}^{b'y} n^j=\sum_{j=0}^{a'x-1} n^j$, $\sum_{j=0}^{x}$

$(\sum_{j=0}^{x} n^{ja'})S_{a'-1}-n(\sum_{j=0}^{j} n^{jb'})S_{b'-1}=1$ #727 // $g=(a,b)$, $a=ga'$, $b=gb'$, $(a',b')=1$,

$(2\nmid a', 2\nmid b')$, $2^a+1=(2^g+1)\sum_{j=0}^{a'-1} 2^{jg}$, $2^b+1=(2^g+1)\sum_{j=0} 2^{jg}$ $(\#727)\Rightarrow \alpha,\beta\in\mathbb{Z}$,

$\alpha\sum_{j=0}^{a'} 2^j+\beta\sum_{j=0}^{b'} 2^j=1$, $2^g+1=\alpha(2^a+1)+\beta(2^b+1)$, $(2^a+1,2^b+1)\mid(2^g+1)$

$(2\nmid a', b'=2^k b'', 2\nmid b'')$ $2^a+1=(2^g+1)\sum_{j=0}^{a'-1}(-2^g)^j$, $2^b+1=(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{g2^k})^j$

$\alpha,\beta\in\mathbb{N}?:\alpha(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j=(2^g+1)\{\beta\sum_{j=0}^{a'-1}(-2^g)^j-1\}$, $a^{2^{k+1}}$ 2^k.

$=\sum_{j=0}^{k-1} 2^j+1=\sum_{j=1}^{k-1} 2^j+2$, $(\alpha:=(2^g+1))?\beta:(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j=\beta\sum_{j=0}^{a'-1}(-2^g)^j-1$,

$LHS=\sum_{j=0}^{b''-1}(-1)^j 2^{2^k g(j+1)}+\sum_{j=0}^{b''-1}(-1)^j(2^{2^k g})^j=-\sum_{j=1}^{b''}(-1)^j 2^{2^k gj}+\sum_{j=0}^{b''-1}(-1)^j(2^{2^k g})^j$

$=1-(-2^{2^k g})^{b''}=?\ \beta\sum_{j=0}^{a'-1}(-2^g)^j-1$, $(-2^{2^k g})^{b''}=?\ \beta\sum_{j=0}^{a'-1}(-2^g)^j$,

$?\alpha',\beta:\{-2^b-1\}\alpha'=\beta\sum_{j=0}^{a'-1}(-2^g)^j-1=\beta\frac{2^a+1}{2^g+1}-1$, $\alpha'(2^b+1)(2^g+1)=?$

$\beta(2^a+1)-1$, $\alpha'(2^{b+g}+2^g(1+2^{b-g})+1)=?\ \beta 2^a+\beta-1$, $2^g\{\alpha'(1+2^{b-g}+2^g)$

$-\beta 2^{a-g}\}=?\ \beta-1-\alpha'$,

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$L(\mu) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\Sigma(x_j-\mu)^2\right)$, $\frac{L(\mu_0)}{L(\mu_a)} = \exp\left(\frac{1}{50}\{\Sigma[(x_j-\mu_a)^2 - (x_j-\mu_0)^2]\}\right)$

$= \exp\left(\frac{1}{50}\Sigma(2x_j(\mu_0-\mu_a) + \mu_a^2 - \mu_0^2)\right) < k$, $2(\mu_0-\mu_a)n\bar{X} + n(\mu_a^2-\mu_0^2) < k$

$\bar{x} < k$, $\alpha = \mathbb{P}_{\mu=10}(\bar{X} < k) = \mathbb{P}\left(Z < \frac{k-10}{5/\sqrt{n}}\right)$, $k = 10 - \frac{5}{\sqrt{n}} z_{.025}$, $\mathbb{P}_{\mu=5}(\bar{X} <$

$10 - \frac{5}{\sqrt{n}} z_\alpha) = \mathbb{P}\left(Z < \frac{5 - \frac{5}{\sqrt{n}} z_\alpha}{5/\sqrt{n}}\right) = \mathbb{P}(Z < \sqrt{n} - z_\alpha)$, $\frac{+(.018)}{.01} > \chi^2_{.05}$

$\left(\frac{\sigma_a}{\sigma_0}\right)^n \exp\left(\Sigma(x_j-\mu)^2\left(\frac{1}{2\sigma_a^2} - \frac{1}{2\sigma_0^2}\right)\right) < k$, $\Sigma(x_j-\mu)^2\left(\frac{1}{2\sigma_a^2} - \frac{1}{2\sigma_0^2}\right) < k$

$\Sigma(x_j-\mu)^2 > k$, $\chi^2(n) > k = \chi^2_\alpha$, $\theta^{2N_1}(2\theta(1-\theta))^{N_2}(1-\theta)^{2N_3}$

$= 2^{N_2}\theta^{2N_1+N_2}(1-\theta)^{N_2+2N_3}$, $L(\theta) = \left(\frac{\theta_0}{\theta_a}\right)^{2N_1+N_2}\left(\frac{1-\theta_0}{1-\theta_a}\right)^{N_2+2N_3} < k$ (>1, <1)

treatment

first continue to [illegible]

$(2N_1+N_2)\log\left(\frac{\theta_0}{\theta_a}\right) + (N_2+2N_3)\log(\ ) < k$

$2N_1 + N_2 > k$

stick with DAGs not linear a's

alternate slide

at top of slide

$L(\lambda) = e^{-n\lambda}\frac{\lambda^{n\bar{x}}}{\prod x_j!}$, $\frac{L(\lambda_0)}{L(\lambda_a)}$

$= e^{n(\lambda_a-\lambda_0)}\left(\frac{\lambda_0}{\lambda_a}\right)^{n\bar{x}} < k$, $\bar{x}\log(\ ) < k$, $\bar{x} > k$,

$\alpha = \mathbb{P}(\Sigma x_j > k)$, $\binom{124}{76}p^{76}(1-p)^{48}\binom{147}{53}p^{53}(1-p)^{94}\cdots = (\ )p^{\Sigma}(1-p)^{\Sigma}$, $(2\pi)^{-3n/2}(\sigma_1^2\sigma_2^2$

$\sigma_3^2)^{-n/2}\exp\left(-\frac{1}{2\sigma_1^2}\sum^{n_1}(x_j-\mu_1)^2 - \frac{1}{2\sigma_2^2}\sum^{n_2}(y_j-\mu_2)^2 - \frac{1}{2\sigma_3^2}\sum^{n_3}(w_j-\mu_3)^2\right)$, $L(\theta_0) = (2\pi)^{-3n/2}\sigma^{-3}$

$\exp\left(-\frac{1}{2\sigma^2}\left(\sum^{n_1} + \sum^{n_2} + \sum^{n_3}\right)\right)$, $\log = -\frac{3}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\left(\sum^{n_1} + \sum^{n_2} + \sum^{n_3}\right)$,

$\frac{\partial\log}{\partial\sigma^2} = -\frac{3}{2\sigma^2} + \frac{1}{2\sigma^4}(\Sigma\cdots) = 0$, $\sigma^{-2} = 3/(\Sigma\cdots)$, $\hat{\sigma}^2 = \frac{1}{3}(\Sigma + \cdots)$, $\frac{\partial\log}{\partial\mu_1} = \frac{1}{\sigma^2}\Sigma(x_j-\mu_1) = 0$

$\mu_1 = \bar{x}$, $L(\theta) =$

$\frac{1}{\|u\|^2}(u,b) = u/2$, $\lambda(A) = 1$, $A^2 = uu^Tuu^T = A$ // $uu^T = \begin{pmatrix} 4/9 & 4/9 & 2/9 \\ 4/9 & 4/9 & 2/9 \\ 2/9 & 2/9 & 1/9 \end{pmatrix}$, $P = \begin{pmatrix} 5/9 & -4/9 & -2/9 \\ -4/9 & 5/9 & -2/9 \\ -2/9 & -2/9 & 8/9 \end{pmatrix}$

$Pb = (5/9, -4/9, -2/9)$, $\lambda(P) = \{0, 1, 1\}$, $P^2 = I - 2uu^T + uu^Tuu^T = P$ //

$H = \begin{pmatrix} 4/9 & 4/9 & 5/9 \\ 4/9 & 4/9 & 5/9 \\ 5/9 & 5/9 & 7/9 \end{pmatrix}$, $\lambda(H) = \{1, 1, -1\}$, $H^2 = I - 4uu^T + 4uu^Tuu^T = I$ // $R(A) = c\cdot(1,1)$,

$N(A) = (1,1,2)^\perp$, $R(A^T) = \{(2,1,1)\}$, $N(A^T) = (1,1)^\perp = c\cdot(1,-1)$ //

$A^TA = wv^Tvw^T = (v^Tv)ww^T$, $A^TAw = \|v\|^2\|w\|^2 w$ // $I = BB^{-1} = I - (c+1)vw^T$

$+ cvw^Tvw^T = I + (-c-1+cw^Tv)vw^T$, $c(w^Tv-1) = 1$, $c = (w^Tv-1)^{-1}$ //

$BB^{-1} = I - cvw^TA^{-1} - vw^TA^{-1} + cvw^TA^{-1}vw^TA^{-1} = I + (-c-1)vw^TA^{-1} +$

$cw^TA^{-1}v\,vw^TA^{-1} = I + vw^TA^{-1}(-c-1+cw^TA^{-1}v)$, $c = (w^TA^{-1}v-1)$.

$v = w = (1,0)$, $c = \bar{a}_{11} - 1$, $c\begin{pmatrix} \bar{a}_{11} \\ \bar{a}_{21} \end{pmatrix}(\bar{a}_{11}\ \bar{a}_{12})$ 6.85 //

$\lambda(A)=0$, $\{1,-1\}$, $\begin{pmatrix}1&-1\\1&-1\end{pmatrix}=\begin{pmatrix}1&0\\-1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}&\\&\end{pmatrix}=\begin{pmatrix}0&1\\0&-1\end{pmatrix}\begin{pmatrix}&\\1&1\end{pmatrix}$, $\begin{pmatrix}1&1\end{pmatrix}$, $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}\frac{1}{ad-bc}$

$=\begin{pmatrix}0&a\\0&c\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}\frac{1}{ad-bc}=\begin{pmatrix}-ac&a^2\\-c^2&ac\end{pmatrix}\cdot\frac{1}{\Delta}$, $\cancel{aa=a^2=c^2}$ $a^2=ac$, $a=c$, $\cancel{ac}$ -1 1 1 d

$d-b=-1$, $M=\begin{pmatrix}1&b\\1&b-1\end{pmatrix}$, $\begin{pmatrix}1&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}\cdot-1=\begin{pmatrix}0&-1\\0&-1\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}=\begin{matrix}-1&-1\\-1&-1\end{matrix}$

$\begin{pmatrix}-a&a\\-a&a\end{pmatrix}\cdot\frac{1}{d-b}$, $a=c=-1$, $d=1$, $b=0$, $\begin{pmatrix}-1&0\\-1&1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\1&-1\end{pmatrix}\cdot-1=\begin{pmatrix}0&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&-1\end{pmatrix}=A$ 6.21 //

$A^T=(S^T)^{-1}\Lambda S^T$, $(S^T)^{-1}$ 6.22 // $A^T(1,1,1)=0$, $N(A^T)=\{(1,1,1)\}$, $R(A^T)=\{(1,0,-1),$

$(1,-1,0)\}$ 6.25 // $\begin{pmatrix}3&0\\0&5\end{pmatrix}=\begin{pmatrix}1&0\\-\frac{4}{3}&1\end{pmatrix}A$, $A=\begin{pmatrix}1&0\\4/3&1\end{pmatrix}\begin{pmatrix}3&0\\0&5\end{pmatrix}$, $\begin{pmatrix}3&0\\0&5\end{pmatrix}=A\begin{pmatrix}1&0\\-4/5&1\end{pmatrix}$,

$\begin{pmatrix}3&-15/4\\4&0\end{pmatrix}=A\begin{pmatrix}1&-5/4\\0&1\end{pmatrix}$, $\begin{pmatrix}0&1\\1&0\end{pmatrix}A=\begin{pmatrix}4&5\\3&0\end{pmatrix}$, $\begin{pmatrix}1&0\\&\end{pmatrix}$, $\begin{pmatrix}r_1q_{11}&r_2q_1+r_3q_2\\r_1q_{21}&\end{pmatrix}=A$, $q_1\propto\begin{pmatrix}3\\4\end{pmatrix}$,

$q_2\propto\begin{pmatrix}3\\4\end{pmatrix}^\perp\propto\begin{pmatrix}1\\-3/4\end{pmatrix}$, $q_1=\begin{pmatrix}3\\4\end{pmatrix}/5$, $q_2=\begin{pmatrix}1\\-3/4\end{pmatrix}/\frac{5}{4}=\begin{pmatrix}4/5\\-3/5\end{pmatrix}$, $r_1=5$, $r_2\begin{pmatrix}3/5\\4/5\end{pmatrix}+r_3\begin{pmatrix}4/5\\-3/5\end{pmatrix}$

$=\begin{pmatrix}0\\5\end{pmatrix}$, $r_2=-\frac{4}{3}r_3$, $-\frac{16}{3}-\frac{3}{5}\cdot 9r_3=5$, $r_3=\frac{-25}{19}$, $r_2=\frac{100}{57}$, $A=\begin{pmatrix}3/5&4/5\\4/5&-3/5\end{pmatrix}\begin{pmatrix}5&\\0&-25\end{pmatrix}$

$A^TA=\begin{pmatrix}25&20\\20&25\end{pmatrix}=\begin{pmatrix}b_1&b_2\\b_2&b_3\end{pmatrix}^2=\begin{pmatrix}b_1^2+b_2^2&b_1b_2+b_2b_3\\ \cdot&b_2^2+b_3^2\end{pmatrix}$, $b_1^2=b_3^2$, $b_1=b_3$, $b_1^2+b_2^2=25$,

$2b_1b_2=20$, $b_1^2+\frac{100}{b_1^2}=25$, $b_1^4-25b_1^2+100=0$, $b_1^2=\frac{1}{2}(25\pm15)=20,5$, $B=\begin{pmatrix}&\\&\end{pmatrix}$

$Q=AB^{-1}=\begin{pmatrix}3&0\\4&5\end{pmatrix}\begin{pmatrix}\sqrt5&-2\sqrt5\\ \cdot&\sqrt5\end{pmatrix}/-15=\begin{pmatrix}-1/\sqrt5&2/\sqrt5\\2/\sqrt5&1/\sqrt5\end{pmatrix}$ // $(3-\lambda)(5-\lambda)=\cancel{\lambda^2-8\lambda+15}$, $(1,-2)$, $4+5v_2=3v_2$

$(0,1)$, $A=\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}3&\\&5\end{pmatrix}\begin{pmatrix}1&0\\2&1\end{pmatrix}$, $\begin{pmatrix}0&1\\1&-2\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&-2\\0&1\end{pmatrix}$, $S^{-1}=\begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$,

$(25-\lambda)^2-400=\lambda^2-50\lambda+225$, $\frac{1}{2}(50\pm40)=45,5$, $25+20q=45$, $q=1$, $q_1=(1,1)$

$q_2=(1,-1)$, $A^TA=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ $\cancel{A=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt5&\\&\sqrt5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\begin{pmatrix}1&1\\1&-1\end{pmatrix}}$

$=\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2\begin{pmatrix}90&\\&10\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2$, $\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2\begin{pmatrix}3\sqrt{10}&\\&2\sqrt5/\sqrt{10}\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2$

$\cancel{=\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt{10}&-3\sqrt{10}\\2\sqrt5&-2\sqrt5\end{pmatrix}}$ $\cancel{\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&45\\5&-5\end{pmatrix}}$ $=\cancel{\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt{10}&3\sqrt{10}\\\sqrt{10}&-\sqrt{10}\end{pmatrix}}=\frac{2\sqrt{10}}{\sqrt{10}}\ \sqrt{10}$

$AA^T=\begin{pmatrix}9&12\\12&41\end{pmatrix}$, $(9-\lambda)(41-\lambda)-144=\lambda^2-50\lambda+225=(\lambda-45)(\lambda-5)$, $9+12q=45$, $q_1=(1,3)$,

$q_2=(1,-1/3)$, $A=?$ $\cancel{\begin{pmatrix}1/\sqrt{10}&2/\sqrt5\\3/\sqrt{10}&-2/3\sqrt5\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}1/\sqrt2&1/\sqrt2\\1/\sqrt2&-1/\sqrt2\end{pmatrix}=\begin{pmatrix}1/\sqrt{10}&2/\sqrt5\\3/\sqrt{10}&-2/3\sqrt5\end{pmatrix}\begin{pmatrix}45&45\\5&-5\end{pmatrix}/\sqrt2}=\big($

$A=?\begin{pmatrix}1/\sqrt{10}&3/\sqrt{10}\\3/\sqrt{10}&-1/\sqrt{10}\end{pmatrix}\begin{pmatrix}\sqrt{45}&\\&\sqrt5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2=\frac{1}{\sqrt5\cdot2}\begin{pmatrix}1&3\\3&-1\end{pmatrix}\begin{pmatrix}\sqrt{45}&\sqrt{45}\\\sqrt5&-\sqrt5\end{pmatrix}=\frac{1}{2\sqrt5}\begin{pmatrix}6\sqrt5&0\\8\sqrt5&10\sqrt5\end{pmatrix}$

$\cancel{\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}1&3\\3&-1\end{pmatrix}=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&90\\15&-5\end{pmatrix}}=\begin{pmatrix}3&0\\4&5\end{pmatrix}$ 6.28 //

$$\hat{\Theta} = \sum_{i<j} \varphi_{ij} \Big/ \binom{M}{2} = \sum_{i,j} \varphi_{ij} \Big/ M^2 .$$

$$V(x_i) = \sum_j \varphi_{ij}/M, \quad V(y_j) = \sum_i \varphi_{ij}/M = \quad V(x_i) = \varphi_{i\cdot}/M, \quad V(y_i) = \varphi$$

$$S_x = \sum_i (V(x_i) - \hat{\theta})^2, \quad S_y = \sum_j (V(y_j) - \hat{\theta})^2$$

$$S_{xy} = \sum_k (V(x_k) - \hat{\theta})(V(y_k) - \hat{\theta})$$

$$\hat{\sigma}^2 = S_x/M^2 + S_y/M^2 + 2S_{xy}/M^2 = [M(M-1)]^{-1} \sum_j (V(x_j) + V(y_j) - 2\hat{\theta})^2$$

$$\theta_{-i} = \sum_{j \neq i, k \neq i} \varphi_{jk} \Big/ (M-1)^2, \quad \hat{\theta}_j = M\hat{\theta} - (M-1)\theta_{-j}, \quad \hat{\sigma}^2 = (M(M-1))^{-1} \sum_j (\hat{\theta}_j - \bar{\hat{\theta}})^2,$$

$$\bar{\hat{\theta}} = M\hat{\theta} - \frac{M-1}{M} \sum_j \theta_{-j}, \quad (M-1)^2 \theta_{-i} = \sum_{jk} \varphi_{jk} - \sum_j \varphi_{ij} - \sum_i \varphi_{ij} + \varphi_{ii}$$

$$= M^2 \hat{\theta} - \varphi_{i\cdot} - \varphi_{\cdot i} + \varphi_{ii}, \quad \sum_j \theta_{-j} = \frac{M^3}{(M-1)^2}\hat{\theta} - \frac{2M^2\hat{\theta}}{(M-1)^2} + \frac{\mathrm{diag}\,\varphi}{(M-1)^2},$$

$$\bar{\hat{\theta}} = \hat{\theta}\left(M - \frac{M^2}{M-1} + \frac{2M}{M-1}\right) - \frac{\mathrm{diag}\,\varphi}{M(M-1)}, \quad 1 - \frac{M-2}{M-1} = \frac{1}{M-1},$$

$$\bar{\hat{\theta}} = \frac{1}{M-1}\left(\overset{M}{\hat{\theta}} - \mathrm{diag}\,\varphi/M\right), \quad \hat{\theta}_j - \bar{\hat{\theta}} = M\hat{\theta} - (M-1)\theta_{-j} - \frac{M}{M-1}\hat{\theta} + \frac{\mathrm{diag}\,\varphi}{M(M-1)}$$

$$= \frac{\hat{\theta} M(M-2)}{M-1} + \frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{1}{M-1}\left(M^2\hat{\theta} - \varphi_{j\cdot} - \varphi_{\cdot j} + \varphi_{jj}\right)$$

$$= -\frac{2M}{M-1}\hat{\theta} + \frac{\mathrm{tr}\,\varphi}{M(M-1)} + \frac{\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}}{M-1}, \quad \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj}) = 2M^2\hat{\theta} - \mathrm{tr}\,\varphi,$$

$$M\left(\frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)^2 + 2\left(\frac{\mathrm{tr}\,\varphi}{M(M-1)} - \frac{2M}{M-1}\hat{\theta}\right)\frac{2M^2\hat{\theta} - \mathrm{tr}\,\varphi}{M-1} + \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})}{(M-1)^2}$$

$$= \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{(M-1)^2} - M\left(\frac{\mathrm{tr}\,\varphi - 2M^2\hat{\theta}}{M(M-1)}\right)^2 = M(M-1)\hat{\sigma}^2_{jk}, \quad \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat{\theta}\right)^2$$

$$= M^{-2} \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj} + \varphi_{jj} - 2M\hat{\theta})^2, \quad \sum (\varphi_{jj} - 2M\hat{\theta})^2 = \mathrm{tr}\,\varphi - 4M\hat{\theta}\,\mathrm{tr}\,\varphi$$

$$+ 4M^3\hat{\theta}^2, \quad \sum (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})(\varphi_{jj} - 2M\hat{\theta}) =$$

$$-2M\hat{\theta}(2M^2\hat{\theta} - \mathrm{tr}\,\varphi) + \sum \varphi_{j\cdot}\varphi_{jj} + \sum \varphi_{\cdot j}\varphi_{jj} - \mathrm{tr}\,\varphi, - \mathrm{tr}\,\varphi - 4M^3\hat{\theta}^2 + 2\sum \varphi_{jj}($$

$$\varphi_{j\cdot} + \varphi_{\cdot j}) + \sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2, \quad \frac{\sum_j (\varphi_{j\cdot} + \varphi_{\cdot j} - \varphi_{jj})^2}{M^2} - \frac{\mathrm{tr}\,\varphi}{} - 4M\hat{\theta}^2$$

$$M(M-1)\hat\sigma^2_{osv} = \sum_j \left(\frac{\varphi_{j\cdot}}{M} + \frac{\varphi_{\cdot j}}{M} - 2\hat\theta\right)^2 = M^{-2}\Big(\sum_j \varphi_{j\cdot}^2 + \sum_j \varphi_{\cdot j}^2 + 2\sum \varphi_{\cdot j}\varphi_{j\cdot}\Big)$$

$$- 4\hat\theta/M \sum_j(\varphi_{j\cdot}+\varphi_{\cdot j}) + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1}$$

$$\cancel{\hat\theta^2/M^2}\ \cancel{M} - 4\hat\theta/M \cdot 2M^2\hat\theta + 4M\hat\theta^2 = M^{-2}\mathbb{1}^T(\dots)\mathbb{1} - 4M\hat\theta^2 =$$

$$M^{-2}\mathbb{1}^T(\varphi^T\varphi+\varphi\varphi^T+2\varphi\varphi - 4/M\,\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} \qquad \frac{1}{(M-1)^2}-\frac{1}{M^2} = \frac{2M-1}{(M(M-1))^2}$$

$$0 \stackrel{?}{=} \frac{2M-1}{(M(M-1))^2}\mathbb{1}^T\Big(\varphi^T\varphi+\varphi\varphi^T+2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi\Big)\mathbb{1} - \frac{2}{(M-1)^2}\mathbb{1}^T\Big(\varphi+\varphi^T - I/2$$

$$- \frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T + \operatorname{diag}\varphi\,\mathbb{1}^T/2M\Big)\cdot\operatorname{diag}\varphi$$

$$\frac{M-1/2}{M^2}\mathbb{1}^T\Big(\varphi^T\varphi+\varphi\varphi^T+2\varphi\varphi - \frac{4}{M}\varphi\mathbb{1}\mathbb{1}^T\varphi\Big)\mathbb{1} - \mathbb{1}^T\Big(\varphi+\varphi^T - \frac{I}{2} - \frac{2}{M}\varphi\mathbb{1}\mathbb{1}^T +$$

$$\operatorname{diag}\varphi\,\mathbb{1}^T/2M\Big)\operatorname{diag}\varphi \stackrel{(1)}{=} \frac12 (M-1)^3 M(\sigma^2_{jk} - \sigma^2_{osv})$$

$$(\varphi=\varphi^T)\quad \mathbb{1}^T\varphi\Big(I - \frac1M\mathbb{1}\mathbb{1}^T\Big)\varphi\mathbb{1} = \sum s_j^2 - \frac1M\Big(\sum s_j\Big)^2 = (M-1)\operatorname{var}(\mathbb{1}^T\varphi) = \operatorname{var}(\mathbb{1}^T\varphi^T)$$

$$\le \frac{M}{2}\big((M-M/2)^2 + (M/2)^2\big) = \frac{M^3}{4}$$

$$\mathbb{1}^T(\varphi^T\varphi - \varphi\mathbb{1}\mathbb{1}^T\varphi/M)\mathbb{1} = \mathbb{1}^T(\varphi^T - \varphi\mathbb{1}\mathbb{1}^T/M)\varphi\mathbb{1}, \quad \mathbb{1}^T\varphi\cdot \mathrm{Proj}_{\mathbb{1}^\perp}\varphi\mathbb{1}$$

$$= \mathbb{1}^T\varphi(\vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec r}) = (\vec c, \vec r - \bar{\vec c}), \quad (\vec c,\vec r) - \Big(\sum\varphi_{ij}\Big)^2/M,$$

$$(\vec c,\vec r) = \sum_j\Big(\sum_i\varphi_{ij}\Big)\Big(\sum_i\varphi_{ji}\Big) = \sum_j\sum_{i,k}\varphi_{ij}\varphi_{jk}, \quad \sum_{ik}\sum_j\varphi_{ij}\varphi_{jk} - \sum_{ik}\sum_{jl}\varphi_{ij}\varphi_{lk}/M$$

$$= \sum_{ikj}\Big(\varphi_{ij}\varphi_{jk} - \sum_l\varphi_{ij}\varphi_{lk}/M\Big) = \sum_{ikj}\varphi_{ij}(\varphi_{jk} - \varphi_{\cdot k})$$

$M(M-1)\sigma^2_{03c} = M^{-2}\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}-2M\hat\theta)^2$, $\varphi_{\cdot j} = 1-\varphi_{j\cdot}$, $\varphi_{\cdot j} = \sum_i \varphi_{ij}$

$= \sum_i (1-\varphi_{ji}) = M - \varphi_{j\cdot}$, $M^{-2}\sum_j (M-2M\hat\theta)^2 = M - 4M\hat\theta + 4M\hat\theta^2 = M(1-2\hat\theta)^2$,

$\sigma^2_{03c} = \frac{(1-2\hat\theta)^2}{M-1}$

$M(M-1)\sigma^2_{jk} = \frac{M^4}{(M-1)^2} - M\left(\frac{-2M^3\hat\theta^2}{M(M-1)}\right)^2 = \frac{M^3}{(M-1)^2} - \frac{4M^3\hat\theta^2}{(M-1)^2} = \frac{M^3}{(M-1)^2}(1-4\hat\theta^2)$,

$\sigma^2_{jk} = \frac{M^2}{(M-1)^3}(1-4\hat\theta^2)$

$\sum \varphi_{\cdot j} = \sum_i \mathbb{P}(X_i < Y_j) = M - \sum_i \mathbb{P}(X_i > Y_j)$

$M^{-2}\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}-\varphi_{jj})^2 - \mathrm{tr}\,\varphi/M^2 - 4M\hat\theta^2 + \frac{2}{M^2}\sum \varphi_{jj}(\varphi_{j\cdot}+\varphi_{\cdot j}) = ?$

$(M-1)^{-2}\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}-\varphi_{jj})^2 - M^{-1}\left(\frac{\mathrm{tr}\,\varphi - 2M^2\hat\theta}{M(M-1)}\right)^2$

$\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}+\varphi_{jj})^2$, $\sum_j \varphi_{j\cdot}^2 = \sum_j \left(\sum_i \varphi_{ji}\right)^2 = \sum_j \left(\sum_i \varphi_{ji} + 2\sum_{i<k} \varphi_{ji}\varphi_{jk}\right) = M^2\hat\theta + 2\sum_{i<k}(\varphi_{\cdot i}, \varphi_{\cdot k})$, $\sum_j \varphi_{j\cdot}^2 = \mathbb{1}^T\varphi^T\varphi\mathbb{1}$, $\sum_j \varphi_{\cdot j}^2 = \mathbb{1}^T\varphi\varphi^T\mathbb{1}$,

$\sum_j \varphi_{j\cdot}\varphi_{\cdot j} = \sum_j \left(\sum_i \varphi_{ji}\right)\left(\sum_k \varphi_{kj}\right) = \sum_j \sum_{i,k} \varphi_{ji}\varphi_{kj} = \sum_{i,k}\sum_j \varphi_{ji}\varphi_{kj} = \mathbb{1}^T\varphi\varphi\mathbb{1}$,

$\sum_j \varphi_{j\cdot}\varphi_{jj} = \sum_i\sum_j \varphi_{ji}\varphi_{jj} = \mathbb{1}^T\varphi\,\mathrm{diag}\,\varphi$, $\sum_j \varphi_{\cdot j}\varphi_{jj} = (\mathrm{diag}\,\varphi)^T\varphi\mathbb{1}$,

$\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}-\varphi_{jj})^2 = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} + \mathrm{tr}\,\varphi - 2\mathbb{1}^T(\varphi+\varphi^T)\cdot$

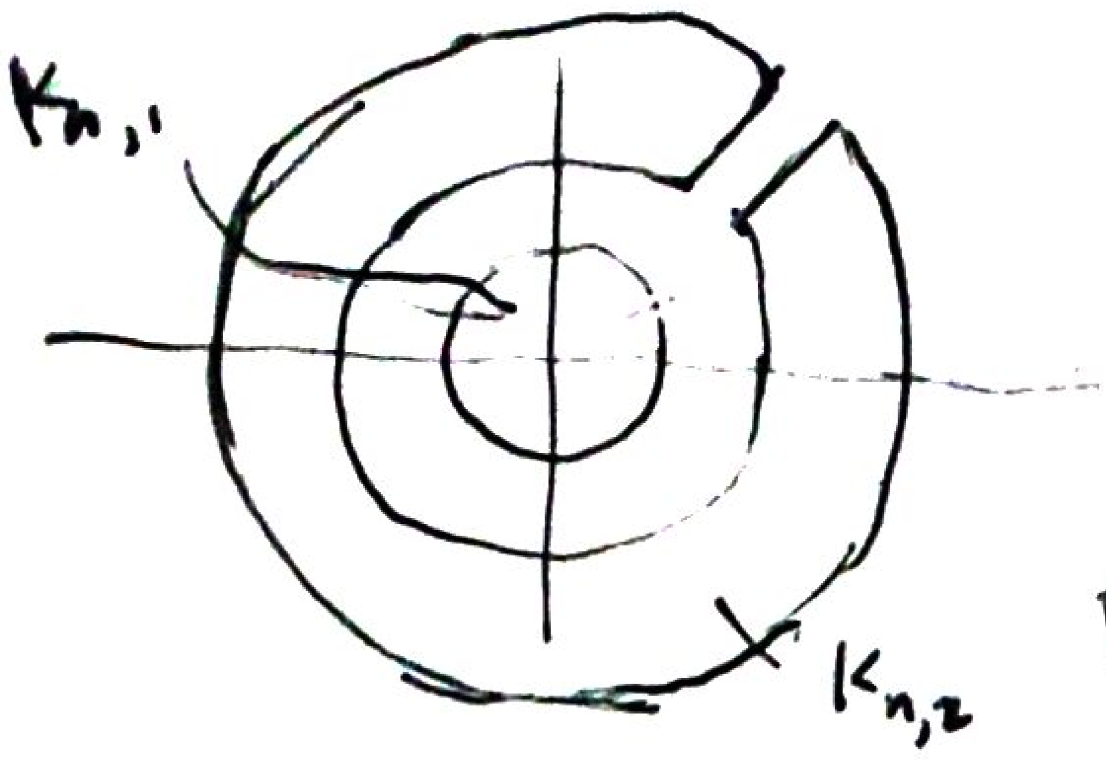
$\mathrm{diag}\,\varphi = \mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T)\,\mathrm{diag}\,\varphi$,

$\mathrm{tr}\,\varphi - 2M^2\hat\theta = \mathrm{tr}\,\varphi - 2\mathbb{1}^T\varphi\mathbb{1}$, then $(M-1)^{-2}\left\{\sum_j (\varphi_{j\cdot}+\varphi_{\cdot j}-\varphi_{jj})^2 - (\mathrm{tr}\,\varphi)^2/M - 4M^3\hat\theta^2 + 4M\hat\theta\,\mathrm{tr}\,\varphi\right\}$, $\hat\theta^2 = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\varphi\mathbb{1}/M^4$, $\hat\theta\,\mathrm{tr}\,\varphi = \mathbb{1}^T\varphi\mathbb{1}\mathbb{1}^T\mathrm{diag}\,\varphi$,

$(M-1)^{-2}\Big\{\mathbb{1}^T(\varphi^T\varphi + \varphi\varphi^T + 2\varphi\varphi - 4M^{-1}\varphi\mathbb{1}\mathbb{1}^T\varphi)\mathbb{1} - 2\mathbb{1}^T(\varphi - I/2 + \varphi^T - 2/M\,\varphi\mathbb{1}\mathbb{1}^T$

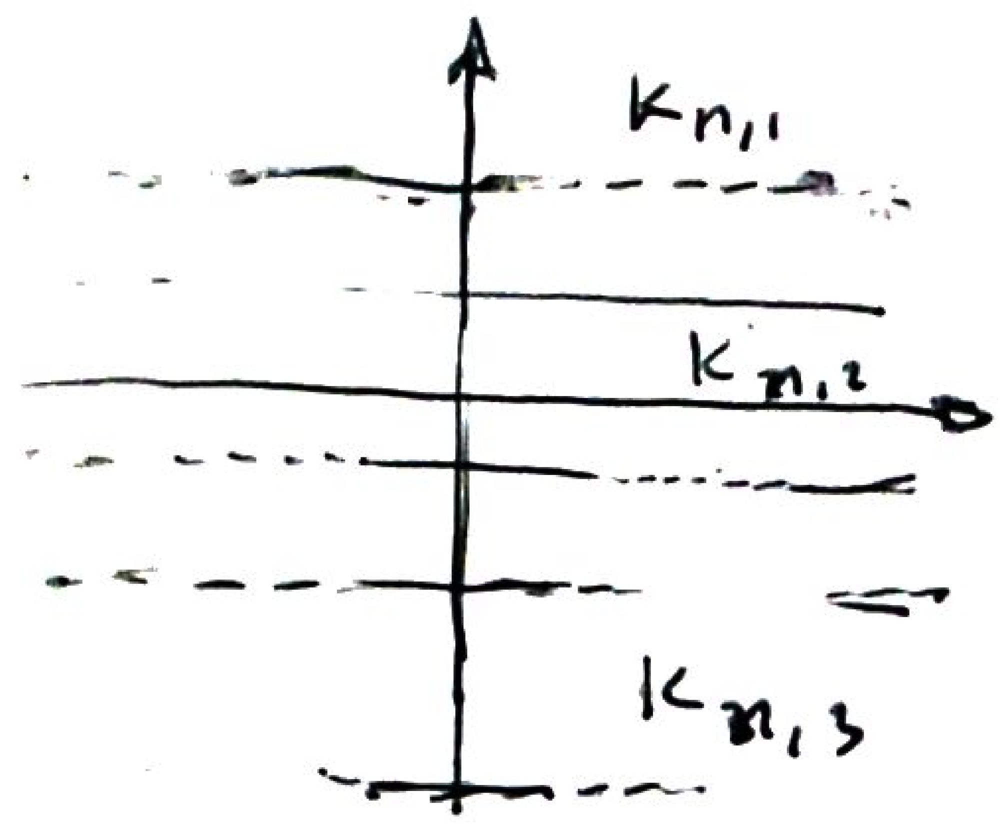
$+ \mathrm{diag}\,\varphi\cdot\mathbb{1}^T/2M)\,\mathrm{diag}\,\varphi\Big\} = M(M-1)\hat\sigma^2_{jk}$

Rudin ch 13, cont



$K_n := \underbrace{\bar D(0,\tfrac1n)}_{K_{n,1}} \cup \underbrace{\{re^{i\theta} : \tfrac1n + \tfrac1{n^2} \le r \le n, \ \theta \notin \tfrac{1}{2\pi}(\sum_{j=1}^{n-1} \tfrac1{j^2}, \sum_{j=1}^{n} \tfrac1{j^2})\}}_{K_{n,2}}$, $P_n(z) : z \in K_n \Rightarrow |P_n(z) - f_n(z)| < \tfrac1n$, $f_n(z) := \begin{cases} 0, & z \in K_{n,1} \\ 1, & z \in K_{n,2} \end{cases}$,

$P_n(z) \xrightarrow[n\to\infty]{} \begin{cases} 1, & z = 0 \\ 0, & z \ne 0 \end{cases}$, $\tilde P_n(z) := P_n(z)/P_n(0)$ #3 //



$K_n := \{|\mathrm{Im}\, z| \le \tfrac1n\} \cup \{\tfrac1n + \tfrac1{n^2} \le \mathrm{Im}\, z \le n\} \cup \{-\tfrac1n - \tfrac1{n^2} \ge \mathrm{Im}\, z \ge -n\} =: K_{n,1} \cup K_{n,2} \cup K_{n,3}$, $f_n(z) := \begin{cases} 1, & z \in K_{n,1} \\ 0, & z \in K_{n,2} \\ -1, & z \in K_{n,3} \end{cases}$,

$P_n(z) : |f - P_n|_{K_n} < \tfrac1n$ #4 // $a \in \partial\Omega \Rightarrow a \notin K_n$ #7 //

$A_n := A \cap \{n < |z| \le n+1\}$, $|A_n| < \infty$, $Q_n := \sum_{a \in A_n} P_a(z)$,

$Q_n \in H(\bar D(0,n))$, $\sum_{j=0}^{\infty} a_j z^j : |\sum a_j z^j - Q_n|_{\bar D(0,n)} \to 0$, $m : |\sum_{j=1}^{m} a_j z^j - Q_n(z)|_{\bar D(0,n)} < 2^{-n}$, $\tilde Q_n(z) := \sum_{j=1}^{m} a_j z^j$, $f(z) := Q_1 + \sum_{j=2}^{\infty} (Q_j(z) - \tilde Q_j(z))$,

$(z \in \bar D(0,n))$, $|\sum_{j=n}^{\infty} (Q_j(z) - \tilde Q_j(z))|_{\bar D(0,n)} \le 2^{-n+1}$,

$\sum_{j=n}^{\infty} (Q_j(z) - \tilde Q_j(z)) \in H(\bar D(0,n))$, $f(z) - \sum_{j=1}^{n-1} Q_j(z) \in H(\bar D(0,n))$ #8 //

$n_k \xrightarrow[k\to\infty]{} \infty$, $z^{2n_k} \to 0$, $k_0 : k > k_0 \Rightarrow z^{2n_{k-1}} < 5^{-1}$, $5^k z^{2n_{k_0-1}} < 1$, $\sum_{k_0}^{\infty} 5^k z^{n_k}$

$< \sum (5 z^{n_{k_0-1}})^k < \infty$. $|z| = 1 - \tfrac1{n_m}$, $\cancel{|h(z)| \le \sum_1^{\infty} 5^k (\tfrac{n_m-1}{n_m})^{n_k}}$, $(z = \tfrac{n_m-1}{n_m})$

$\sum_1^{\infty} 5^k (\tfrac{n_m-1}{n_m})^{n_k} = 5^m \sum_1^{\infty} 5^{k-m} (\tfrac{n_m-1}{n_m})^{2kn_{k-1}}$, $\lim_{m\to\infty} \sum_k 5^{k-m} (\tfrac{n_m-1}{n_m})^{2kn_{k-1}} > 0$?

$(1 - \tfrac1{n_m})^{n_m} \to e^{-1}$, $(|z| = 1 - \tfrac1{n_m})$ $\sum_{k=1}^{m-1} 5^{k-m} (\tfrac{n_m-1}{n_m})^{n_k - n_m} < \sum_{k=1}^{m-1} 5^{k-m} (\tfrac{n_m}{n_m-1})^{n_m}$

$= (e + o_m(1)) \, 5^{-m} \tfrac{1-5^m}{1-4} \le \tfrac13 (e + o_m(1))$, $\sum_{k=m+1}^{\infty} 5^{k-m} (\tfrac{n_m-1}{n_m})^{n_k - n_m}$

$< \sum (5 \tfrac{n_m-1}{n_m})^{k-m} (\tfrac{n_m-1}{n_m})^{n_{k-1} - kn_m}$ $\sum_{k=1}^{\infty} 5^k (1 - \tfrac1{n_m})^{n_{k+m} - n_m} = \sum_{k=1}^{\infty} 5^k (\tfrac1e + o(1))^{\frac{n_{k+m}}{n_m}}$, $\tfrac{n_{k+m}}{n_m} \to$

$= o(1)$, $|\sum_{k=1}^{\infty} 5^k z^{n_k}| \ge 5^m |z|^m - \sum_{j \ne m} 5^j |z|^j \ge 5^m \cdot c$ //

3.16

$\int_0^\infty \frac{y}{\theta} e^{-y/\theta} = -ye^{-y/\theta}\Big|_0^\infty + \int e^{-y/\theta} = -\theta e^{-y/\theta}\Big|_0^\infty = \theta$, $\int \frac{y^2}{\theta} e^{-y/\theta}$

$= -y\theta e^{-y/\theta}\Big|_0^\infty + \int \theta e^{-y/\theta} = -\theta^2 e^{-y/\theta}\Big|_0^\infty = \theta^2$

$\sum(Y_j^2 - 2\bar{Y}Y_j + \bar{Y}^2) = \sum Y_j^2 - n\bar{Y}^2$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $\left(\frac{n}{\sigma^2}\right)^2 \operatorname{Var} S^2 = 2(n-1)$

$\operatorname{Var} S^2 = \frac{2\sigma^4}{n-1}$, $\operatorname{Var}(Y_1^2 - 2Y_1Y_2 + Y_2^2)$, $Y_1 - Y_2 \sim N(0, 2\sigma^2)$, $\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}} \sim Z$,

$\cancel{1 = \frac{1}{2\sigma^2}\operatorname{Var}(Y_1 - Y_2)}$, $\operatorname{Var}\left(\left(\frac{Y_1 - Y_2}{\sqrt{2\sigma^2}}\right)^2\right) \sim \operatorname{Var} \chi^2(1) = 2$, $\operatorname{Var}(Y_1 - Y_2)^2$

$= 8\sigma^4$,

$\int_0^1 \theta y^\theta = \frac{\theta}{\theta+1} y^{\theta+1}\Big|_0^1 = \frac{\theta}{\theta+1}$

$2EY_1^2 = 2\lambda(1+\lambda) - 2\lambda^2 = 2\lambda$, $\operatorname{Var}((Y_i - Y_j)^2)$

$\frac{\theta+1}{\theta} y^{\theta+1}\Big|_0^1 = \frac{\theta+1}{\theta}$, $\frac{\theta+1}{\theta+2} y^{\theta+2}\Big| = \frac{\theta+1}{\theta+2} = \bar{X}$, $\theta(1-\bar{X}) = 2\bar{X} - 1$, $\hat{\theta} = \frac{2\bar{X}-1}{1-\bar{X}} \xrightarrow{p} \theta$

$\sigma^2 + \mu^2 = \overline{X^2}$ $\quad -\frac{y}{p^2} - (n-y)\frac{1}{(1-p)^2}$ $\quad e^{-x/\theta}/\theta$, $\theta^{-n} e^{-\theta^{-1}\sum X}$, $-n\log\theta$

$-\sum X/\theta$, $-n/\theta + \sum X/\theta^2$, $\hat{\theta} = \bar{X}$, $\left(\frac{1}{2\theta+1}\right)^n \{X_{(n)} \le 2\theta+1\}$, $\hat{\theta} = \frac{1}{2}(X_{(n)} - 1)$,

$\frac{1}{2\theta+1}\int_0^{2\theta+1} y^2$, $\frac{1}{3}(2\theta+1)^2 - \frac{1}{4}(2\theta+1)^2 = \frac{1}{12}(2\theta+1)^2$, $\theta^{-2n}\prod y_j e^{-\sum y/\theta}$, $-2n\log\theta + \sum y_j - \sum y/\theta$,

$-2n/\theta + \sum y/\theta^2$, $\hat{\theta} = \bar{Y}/2$,

9.38

a) $Y_1, \dots, Y_n$ iid $\sim N(\mu, \sigma^2)$, pdf $f(y) = (2\pi\sigma^2)^{-1/2} \exp\left(-\left(\frac{y-\mu}{\sigma}\right)^2 / 2\right)$

joint pdf $\prod_{j=1}^{n} f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j - \mu)^2\right)$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j^2 - 2\mu y_j + \mu^2)\right)$$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum y_j^2 + \frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right)$$

$$f(y_1, \dots, y_n) = \prod f(y_j) = (2\pi\sigma^2)^{-n/2} \exp\left(\frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right) \exp\left(-\frac{1}{2\sigma^2} \sum y_j^2\right)$$

$$= \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(\frac{n\mu}{\sigma^2} \bar{y} - \frac{n\mu^2}{2\sigma^2}\right)}_{\text{depends on } y_1, \dots, y_n \text{ only through } \bar{y}} \underbrace{\exp\left(-\frac{1}{2\sigma^2} \sum y_j^2\right)}_{\text{does not depend on target } \mu}$$

b) $$f(y_1, \dots, y_n) = \prod f(y_j) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} (y_j - \mu)^2\right)}_{\text{depends on } y_1, \dots, y_n \text{ only through } \sum (y_j - \mu)^2} \cdot \underbrace{1}_{\text{does not depend on target } \sigma^2}$$

c) $$f(y_1, \dots, y_n) = \underbrace{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2} \sum y_j^2 + \frac{\mu}{\sigma^2} \sum y_j - \frac{n\mu^2}{2\sigma^2}\right)}_{\text{depends on } y_1, \dots, y_n \text{ only through } (\sum y_j^2, \sum y_j)} \cdot \underbrace{1}_{\text{does not depend on } (\mu, \sigma^2)}$$

9.39

$Y_1, \dots, Y_n \sim \text{Poi}(\lambda)$, pmf is $f_Y(j) = e^{-\lambda} \frac{\lambda^j}{j!}$ for $j = 0, 1, \dots$

also $\sum_{j=1}^{n} Y_j \sim \text{Poi}(n\lambda)$, pmf is then $f_{\Sigma Y}(j) = e^{-n\lambda} \frac{(n\lambda)^j}{j!}$.

Then $\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \mid \sum Y_j = \sum y_j)$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n \text{ and } \sum Y_j = \sum y_j)}{\mathbb{P}(\sum Y_j = \sum y_j)}$$

$$= \frac{\mathbb{P}(Y_1 = y_1, \dots, Y_n = y_n)}{\mathbb{P}(\sum Y_j = \sum y_j)} = \frac{\prod_{j=1}^{n} f_Y(y_j)}{f_{\Sigma Y}(\sum y_j)}$$

$$= \prod_{j=1}^{n} \left(e^{-\lambda} \frac{\lambda^{y_j}}{y_j!} \right) \Big/ e^{-n\lambda} \frac{(n\lambda)^{\sum y_j}}{(\sum y_j)!} = e^{-n\lambda} \frac{\lambda^{\sum y_j}}{\prod y_j!} \Big/ e^{-n\lambda} \frac{(n\lambda)^{\sum y_j}}{(\sum y_j)!}$$

$$= \frac{(\sum y_j)!}{\prod y_j!} \quad \text{which doesn't depend on } \lambda$$

9.40 $Y_1, \dots, Y_n \sim \text{Rayleigh}(\theta)$, pdf is $f_Y(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta}$ for $y > 0$,

so $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\frac{2y_j}{\theta}\right) e^{-y_j^2/\theta} = \left(\frac{2}{\theta}\right)^n \left(\prod y_j\right) e^{-\frac{\sum y_j^2}{\theta}}$

$$= \underbrace{\left(\frac{2}{\theta}\right)^n e^{-\sum y_j^2/\theta}}_{\substack{\text{depends on} \\ y_1, \dots, y_n \text{ only} \\ \text{through } \sum y_j^2}} \cdot \underbrace{\left(\prod y_j\right)}_{\substack{\text{doesn't} \\ \text{depend on } \theta}}$$

9.41

$Y_1, \dots, Y_n \sim \text{Weibull}(\alpha)$, pdf is $f_Y(y) = \frac{1}{\alpha} m y^{m-1} e^{-y^m/\alpha}$ for $y > 0$

so $f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\frac{1}{\alpha} m \, y_j^{m-1} e^{-y_j^m/\alpha} \right) = \left(\frac{m}{\alpha} \right)^n \left(\prod y_j \right)^{m-1} e^{-\sum y_j^m/\alpha}$

$$= \underbrace{\left(\frac{m}{\alpha} \right)^n e^{-\sum y_j^m/\alpha}}_{\substack{\text{dependence on} \\ \text{sample thru } \sum y_j^m}} \cdot \underbrace{\left(\prod y_j \right)^{m-1}}_{\text{no } \alpha \text{ dependence}}$$

9.42 — HW

9.43

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha y_j^{\alpha-1} / \theta^\alpha \right) = \underbrace{\left(\frac{\alpha}{\theta^\alpha} \right)^n \left(\prod y_j \right)^{\alpha-1}}_{\substack{\text{depends on} \\ \text{sample thru } \prod y_j}} \cdot \underbrace{1}_{\substack{\text{no } \alpha \\ \text{dependence}}}$$

9.44

$$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(\alpha \beta^\alpha y_j^{-(\alpha+1)} \right) = \underbrace{(\alpha \beta^\alpha)^n \left(\prod y_j \right)^{-(\alpha+1)}}_{\text{ditto}} \cdot 1$$

9.45

$f(y_1, \dots, y_n) = \prod_{j=1}^{n} f_Y(y_j) = \prod_{j=1}^{n} \left(a(\theta) b(y_j) e^{-c(\theta) d(y_j)} \right)$

$$= \underbrace{a(\theta)^n e^{-c(\theta) \sum d(y_j)}}_{\substack{\text{depends on} \\ \text{sample thru } \sum d(y_j)}} \cdot \underbrace{\prod b(y_j)}_{\text{no } \theta \text{ dependence}}$$

9.46 pdf is $f_Y(y) = \frac{1}{\beta} e^{-y/\beta}$ which has form $a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \beta$, $c(\theta) = 1/\beta$, $a(\theta) = 1/\beta$, $b(y) = 1$.

9.47 pdf $\alpha y^{\alpha-1}/\theta^{\alpha} = \theta^{-\alpha} \alpha \exp((\alpha-1)\log y)$ has form $a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$, $c(\theta) = \alpha - 1$, $a(\theta) = \theta^{-\alpha} \cdot \alpha$. So by 9.45 $\sum_j d(y_j) = \sum \log y_j$ is sufficient for α. No contradiction since $\sum \log y_j = \log(\prod y_j)$ and "log" is one-to-one on $\mathbb{R}^+$

9.48 pdf $\alpha \beta^{\alpha} y^{-(\alpha+1)} = \alpha \beta^{\alpha} \exp(-(\alpha+1)\log y)$ has form $a(\theta) b(y) e^{-c(\theta) d(y)}$ by taking $\theta = \alpha$, $d(y) = \log y$, $c(\theta) = \alpha + 1$, $a(\theta) = \alpha \beta^{\alpha}$, $b(y) = 1$. Suff. stat. is then $\sum d(y_j) = \sum \log y_j = \log(\prod y_j)$.

$f'(i) \le \frac{1! \cdot 1}{1} = 1$, $f(z) = \frac{1}{1+z-i}$, $|1+z-i| \ge 1$, $Re^{i\theta} + R^{-1}e^{-i\theta} = (R+\frac{1}{R})\cos\theta$ $+ i(R-\frac{1}{R})\sin\theta$, $R^2 + \frac{1}{R^2} + 2(\cos^2 - \sin^2) = R^2 + \frac{1}{R^2} + 2\cos 2\theta$, $z - \frac{1}{z}$, $R^2 + \frac{1}{R^2} - 2\cos 2\theta$, $R + \frac{1}{R} + 2\cos\frac{\theta}{2}$; $z^{1/4} + z^{-1/4} = \frac{z^{1/2}+1}{z^{1/4}} = z^{1}(z^{5/4} + z^{3/4})$, $z^{a} + z^{-a} =$ $z^{-a}(z^{2a}+1)$, $\frac{1}{(z-a)(z-b)}$, $\frac{1}{(z-a)^2(z-b)} + \frac{1}{(z-a)(z-b)^2}\Big|_{z=i} = (i-a)^{-2}(i-b)^{-1} + (i-a)^{-1}(i-b)^{-2}$ $= \frac{2i-a-b}{(i-a)^2(i-b)^2}$, $\frac{2i-(x_1-iy+x_2-i/y)}{(i-x_1+iy)^2(i-x_2+i/y)^2} = \frac{-(x_1+x_2)+i(2+y+1/y)}{(-x_1+i(1+y))^2(-x_2+i(1+1/y))^2}$; $((x_1+x_2)^2+16)^{1/2} \overset{?}{\le} (x_1^2+4)(x_2^2+4)$; $2+y+1/y \le ?\ (x_1^2+(1+y)^2)(x_2^2+(1+1/y)^2) \ge$ $(2+y+1/y)^2 \ldots$

$f(x)=0 \Rightarrow |f(x)| \le \min_{y\in\bar D}|f(y)|$, $|f(x)| \ne 0$, $|f(x)| \le \inf_{y\in D}|f(y)| \Rightarrow f(y) \ne 0$, $y \in D$, $|\frac{1}{f}(x)| \ge \sup_{y\in \overline{D/2}} |\frac{1}{f}(y)|$, $\frac{1}{f} = \text{const.}$ #3// $f(x)\ne 0$, $x\in D \Rightarrow \max_{\bar D}|f(x)|$ $= \max_{\partial D}|f(x)| = c = \min_{\bar D}|f(x)|$, $|f(x)| = c$, $x \in \bar D$, $u^2(x,y)+v^2(x,y) = c$, $uu_x + vv_x$ $= uu_y + vv_y = 0$, $\frac{v_x}{u_x} = \frac{v_y}{u_y}$, $-\frac{u_y}{u_x} = \frac{u_x}{u_y}$, $u_x^2 = -u_y^2$, $u_x = v_y = 0$, $u_y = v_x = 0$, $f(x) = \text{const}$, —×— #4a//

$m,n > n_0 \Rightarrow \varepsilon > \max_{z\in\partial D}|f_n(z) - f_m(z)| > \max_{z\in D}|\cdots|$ #5// $N(x) := \max_{\mathrm{Re}\, z = x}|fh_\varepsilon|$, $|y| > \frac{B}{\eta\varepsilon} \Rightarrow |fh_\varepsilon| \le \frac{B}{\varepsilon}\cdot\frac{\eta\varepsilon}{B} = \eta$, $R := \{(x,y): a \le x \le b,\ -\frac{B}{\eta\varepsilon} \le y \le \frac{B}{\eta\varepsilon}\}$, $x_0: \{x < x_0\} \cap R \Rightarrow |f(z)| < \eta$, $|f(z)h_\varepsilon| < \eta$, $x < x_0 \Rightarrow N(x) \le \eta$, $N(x) \underset{x\to a^+}{\to} 0$, $a < x < \xi < b \Rightarrow N(\xi) \le N(x)^{\frac{b-\xi}{b-x}} N(b)^{\frac{\xi-x}{b-x}} \underset{x\to a^+}{\to} 0$, $N(x) \equiv 0$, $h_\varepsilon(z) \ne 0$, $z \in \Omega$, $f \equiv 0$ #7// $M(r)^{\log b/a} \le ?\ M(a)^{\log b/r} M(b)^{\log r/a}$, $M(a) = M(b)$ $= 1 \Rightarrow M(r) \le 1$, $\sup_{\mathrm{Re}\, z = x}\log|f(z)| = \log\sup_{\mathrm{Re}\, z=x}|f(z)|$, $(\frac{b}{a})^{\log M(r)} \le ?\ (\frac{b}{r})^{\log M(a)} (\frac{r}{a})^{\log}$ $(f(z) = z)$ $r^{\log b/a} \le ?\ a^{\log b/r} b^{\log r/a}$; $\log b \log r - \log a \log r \le ?\ \log a \log b - \log a \log r$ $+ \log b \log r - \log b \log a$;

$|z|^\alpha = \exp(\alpha\log|z|)$, $e^{\beta z}\pm e^{-\beta z} = e^{\beta x}e^{i\beta y}\pm e^{-\beta x}e^{-i\beta y} = (e^{\beta x}\pm e^{-\beta x})\cos\beta y$

$+ i(e^{\beta x}\mp e^{-\beta x})\sin\beta y$, $|\dots|^2 = e^{2\beta x}+e^{-2\beta x} \pm 2\cos\beta y \mp 2\sin\beta y = 2\cosh 2\beta x$

$\pm 2\cos(\beta y + \frac{\pi}{4}) \ge 0$, $x > x_0 \Rightarrow |e^{\beta z}+e^{-\beta z}| > \delta > 0$, $\beta y + \frac{\pi}{4} = \frac{(2k+1)\pi}{2}$, $y =$

$\beta^{-1}\frac{\pi}{2}(2k+\frac12)$, $k\in\mathbb{Z}$, $Y := \{\beta^{-1}\frac{\pi}{2}(2k+\frac12) : k\in\mathbb{Z}\}$,

$\operatorname{Im} z \in Y \Rightarrow e^{\beta z}+e^{-\beta z} = 2\cosh 2\beta x$, $|e^{\beta i z}+e^{-\beta i z}|^2 = 2\cosh 2\beta y +$

$2\cos(\beta x + \frac{\pi}{4})$, $|e^z + e^{iz}| = |e^x e^{iy} + e^{-y}e^{ix}|$, $|e^x e^{iy} + e^{-x}e^{-iy} + e^{-y}e^{ix} + e^{y}e^{-ix}|$

$\ge |e^{|x|}$, $z - iz = x+y+i(y-x)$, $|e^{z-iz}| = e^{x+y}$, $e^{z-iz}+e^{iz-z} = e^{x+y}e^{i(y-x)} + e^{-x-y}e^{i(x-\dots}$

$= (e^{x+y}+e^{-x-y})\cos(x-y) + i(\dots), \dots$ $z^{1/n} \in ?\ H(\mathbb{C}\setminus 0)$, $\overset{Re^{i\theta}}{z}\mapsto R^{1/n}e^{i\theta/n} \in ?$

$H(\mathbb{C}\setminus 0)$, $n : 1-\frac1n > \alpha$, $z^{1-1/n} := R^{1-1/n}e^{i(1-1/n)\theta}$, $\operatorname{Re} z^{1-1/n} =$

$R^{1-1/n}\cos((1-\frac1n)\theta) \overset{(z\in\Pi)}{>} R^{1-1/n}\cos((1-\frac1n)\frac{\pi}{2}) =: R^{1-1/n}\delta > 0$, $|\exp(-\varepsilon z^{1-1/n})|$

$= \exp(-\varepsilon \operatorname{Re} z^{1-1/n}) < \exp(-\varepsilon R^{1-1/n}\delta) < 1$ $(z\in\Pi)$, $h_\varepsilon(z) := e^{-\varepsilon z^{1-1/n}}$,

$z\in\Pi$, $fh_\varepsilon \in H(\Pi)$, $|fh_\varepsilon| \overset{(\operatorname{Re} z = 0)}{<} 1$, $|fh_\varepsilon| < A\exp(|z|^\alpha - \varepsilon\operatorname{Re} z^{1-1/n})$

$\xrightarrow[|z|=R\to\infty]{} 0$, $R_0 : |z| > R_0 \Rightarrow |fh_\varepsilon| < 1$, $\max\limits_{z\in\Pi\cap\{|z|> R_0\}} |fh_\varepsilon| = \max\limits_{z\in\partial(\dots)} |fh_\varepsilon|$

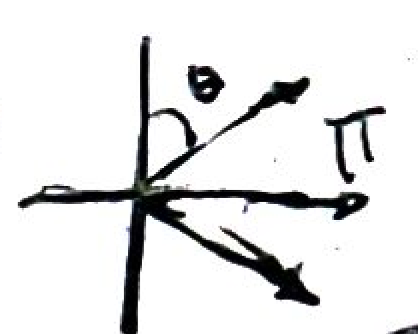
≤ 1, $|fh_\varepsilon| \le 1$ $(z\in\Pi,\ \varepsilon>0)$, $|f|\le 1$. $(\alpha = 1)$ $f(z) := e^z$,

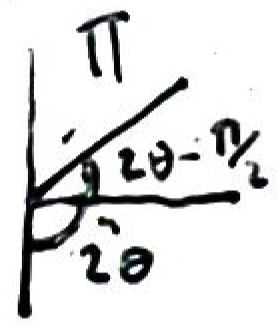
$|f(z)| = e^{\operatorname{Re} z} \le e^{|z|}$, $|f(x)| = e^x \xrightarrow[x\to\infty]{} \infty$.  $(\frac{\pi}{2}+\theta)\alpha < \frac{\pi}{2}$, $\alpha < \frac{\pi/2}{\pi/2+\theta}$ (*)

$|e^{niy}| \le 1$, $|f(iy)e^{niy}| \le 1$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| = |e^{nr\cos\alpha} f(re^{i\alpha})| = \exp(nr\cos\alpha$

$+ \log|f(re^{i\alpha})|) = \exp(r(n\cos\alpha + r^{-1}\log|f(re^{i\alpha})|))$, $n\cos\alpha + r^{-1}\log|f(re^{i\alpha})| \to -\infty$

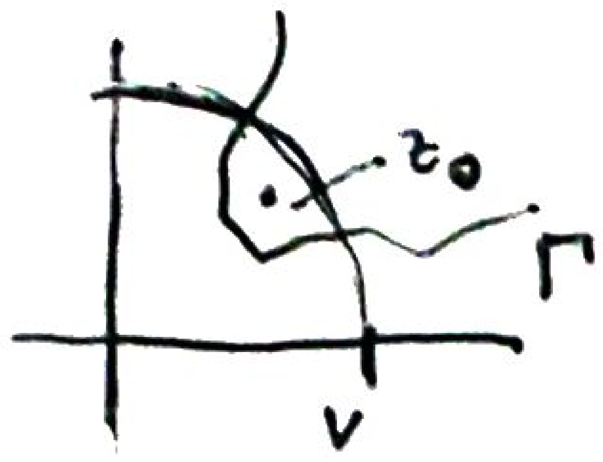
$1/A := \max\limits_r \exp(r(n\cos\alpha + r^{-1}\log|f(re^{i\alpha})|))$, $|f(re^{i\alpha})e^{nre^{i\alpha}}| \le A$.

(*)  $\alpha < \frac{\pi/2}{\pi/2-\theta}$, $\alpha<\beta<\frac{\pi/2}{\pi/2-\theta}$, $\operatorname{Re} z^\beta = r^\beta\cos(\beta\arg z) \ge r^\beta \overbrace{\cos(\beta(\frac{\pi}{2}-\theta))}^{>0}$;

 $z\mapsto (e^{-i\theta}z)^\beta$, $\operatorname{Re}(e^{-i\theta}z)^\beta > 0 \Leftarrow z\in\Pi$, $h_\varepsilon(z) := \exp(-\varepsilon(e^{-i\theta}z)^\beta) < 1$,

$|fh_\varepsilon| < A\exp(|z|^\alpha - \varepsilon r^\beta\cos(\beta(\frac{\pi}{2}-\theta))) \xrightarrow[|z|=r\to\infty]{} 0$. $|e^{nz}| < Ae^{|z|^\alpha}$

$(\Omega=\mathbb{C})$ $\Gamma=\emptyset$, M ? $(x_0 \notin \Omega)$ $\sup_{z\in\Omega}|f(z)| = \sup_{z\in\Omega - x_0}|f(z+x_0)|$, $0\notin \Omega - x_0$ $(0\notin\Omega)$ $r := \inf_{z\in\Omega}|z|$, $r\neq 0$, $\sup_{z\in\Omega}|f(z)| = \sup_{z\in r\Omega}|f(z/r)|$, $r\Omega \cap U \neq \emptyset$.



$\left|\frac{f(z_0)^n}{z_0}\right| \le \max_{\Gamma\cap V}\left|\frac{f(z)^n}{z}\right| \vee \max_{r=v}\left|\frac{f(z)^n}{z}\right| \le M^n \vee \frac{B^n}{v}$, $\max(M^n, B^n/v) \xrightarrow[v\to\infty]{} M^n$, $|f(z_0)|^n \le |z_0| M^n$, $n\in\mathbb{Z}$, $|f(z_0)|\le M$ #11 //

$\operatorname{diam} E_n < \infty \Rightarrow \|f\|_{E_n} < \|f\|_{\partial E_n} = n$, $E_n = \emptyset$; $\|f\|_{E_n} < n+1 \Rightarrow |f|_{E_n} \le |f|_{\partial E_n} = n$, $E_n = \emptyset$;

$\gamma : \gamma(1-1/n) \in E_n$, $|\gamma(1-1/n) - \gamma(1-\frac{1}{n+1})| > 1$ #12 //

$e^x \xrightarrow[x\to\infty]{} \infty$, $e^{-x}\xrightarrow[x\to\infty]{} 0$, $\operatorname{Re}\gamma\to\infty \Rightarrow e^{\gamma}\to\infty$, $\operatorname{Re}\gamma\to-\infty \Rightarrow e^\gamma \to 0$, $\operatorname{Re}\gamma \not\to \infty \Rightarrow \operatorname{Im}\gamma\to 0$, $e^\gamma \not\to L$. $e^{iz} - e^{-iz} = e^{-y+ix} - e^{y-ix}$ $= (e^y - e^{-y})\cos x + i(e^y+e^{-y})\sin x$, $|e^{iz}-e^{-iz}|^2 = e^{2y}+e^{-2y} - 2 \xrightarrow[|y|\to\infty]{} \infty$ $(|y|\not\to\infty)$ $x\to\infty$, $e^{iz}-e^{-iz}\not\to L \in \mathbb{C}\cup\infty$ #13 //

$g(z) := f(1/z)$, Thrm 10.21, $(c\in\mathbb{C} : \tilde g(0) := c, \tilde g(z) = g(z), z\neq 0, \tilde g \in H(\mathbb{C}))$ $z_n\to\infty$, $g(\frac{1}{z_n})\to c$, $f(z_n)\to c$, $f = \text{const}$; $(g(z) = \sum_{j=-m}^{\infty} c_j z^j)$ $z_n\to 0 \Rightarrow c\in\mathbb{C} : g(z_n) - \sum_{j=-m}^{1} c_j z_n^j \to c$, $f(w_n) - \sum_{j=1}^{m} c_{-j} w_n^j \xrightarrow[w_n\to\infty]{} c$, $f(w_n) - \sum_{j=1}^{m} c_{-j}w_n^j = \text{const.}$, $f(w_n) = \sum_{j=0}^{m}\tilde c_j w^j$, $\operatorname{Im} f = \mathbb{C}$; $(\mathrm{cl}(g(D(0;r))) = \mathbb{C})$ $c\in\mathbb{C}$, $z_n : z_n\to 0$, $g(z_n)\to c$, $f(1/z_n)\to c$ #14 //

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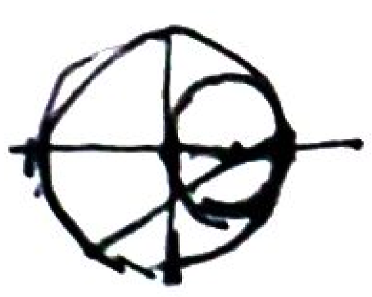
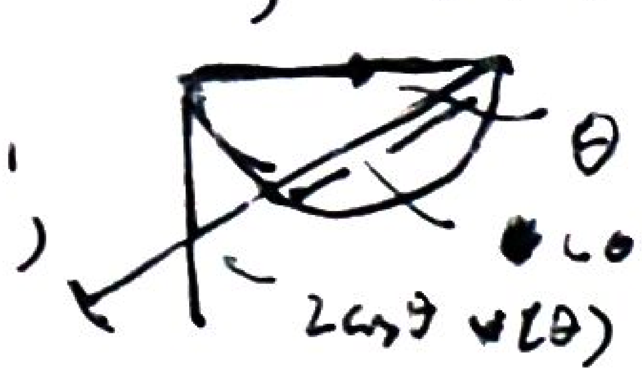
$|A| = \infty$, $\lim_{r\to 0^+}|D(\infty, r)\cap A| > 0$, $\infty \in \mathrm{cl} A \Rightarrow |A| < \infty$, $f(z) - \sum_{j=1}^{|A|} R_j \in H(\mathbb{C})$. #1 //



$\varphi : D(\frac12,\frac12)\to V$, φ, $\varphi^{-1}\in C^1$,



$\Phi : 1+\alpha v(\theta) \mapsto 1 + (\cos\theta + \frac{\alpha}{2})v(\theta)$, $0<\alpha<2\cos\theta$, $\bar v(\theta) := -1 - \frac{1}{\cos\theta}$, $v(\theta) := \bar v(\theta)/|\bar v(\theta)|$, $\pi/2 < \theta < 3\pi/2$.

$f(z) := \frac{1}{z-\frac12}$, $\inf_V |f| = \frac23$, $\sup_\Omega |P-f| < ?\ \frac13$, $|(z-\frac12)P - 1| < \frac12$, $\sup_{\mathrm{cl}(\Omega)}|(z-\frac12)P-1| \le \frac12$, $\sup_{D(\frac12,\frac12)}|(z-\frac12)P-1| \le \sup_{\partial D(\frac12,\frac12)}|\cdots| = \frac12$, $\left.((z-\frac12)P-1)\right|_{z=\frac12} = 1$ #2 //

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1] $.95 = \mathbb{P}(L < \mu < U) = \mathbb{P}(X-L > X-\mu > X-U)$, $X-\mu \sim N(0,1)$,

$X - L = z_{.025}$, $L = X - z_{.025}$, $X - U = -z_{.025}$, $U = X + z_{.025}$ //

2] $.56 \pm z_{.005}\sqrt{\frac{.56 \cdot .44}{100^3}} = .56 \pm z_{.005}\frac{4}{1000}\sqrt{154}$

3] a) $\mathbb{E}X - \mu = 0$ b) $1 - 2\Phi(-1) = 2\Phi(1) - 1$ c) $\mathbb{P}(|X_1 - \mu| > \epsilon) \geq$.

$\mathbb{P}(|X_j - \mu| > 1) = 2\Phi(1) - 1 \not\to 0$

4] a) $f_\mu(x_1,\dots,x_n) = \prod_{j=1}^n f_\mu(x_j) = (2\pi)^{-n/2}\exp\left(-\frac{1}{2}\sum_{j=1}^n (x_j-\mu)^2\right) =$

$(2\pi)^{-n/2}\exp\left(-\frac{1}{2}\sum x_j^2 + \mu\sum x_j + \mu^2/2\right) = (2\pi)^{-n/2}\underbrace{\exp(\mu n\bar{X} + \mu^2/2)}_{\text{depends on data through } \bar{X}}\underbrace{\exp\left(-\frac{1}{2}\sum x_j^2\right)}_{\text{doesn't depend on } \mu}$

b) $\mathbb{E}(\bar{Y}^2) = \operatorname{Var}\bar{Y} + (\mathbb{E}(\bar{Y}))^2 = 1/n + \mu^2$, $\mathbb{E}(\bar{Y}^2) - 1/n = \mu^2$

c) LLN: $\bar{Y} \xrightarrow[p]{} \mu$, so $\bar{Y}^2 \xrightarrow[p]{} \mu^2$ since $x \mapsto x^2$ is continuous.

5] $1-\alpha = \mathbb{P}(L < \sigma^2 < U) = \mathbb{P}\left(\frac{(n-1)S^2}{L} > \frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)S^2}{U}\right)$,

$\frac{(n-1)S^2}{L} := \chi^2_{n-1,\frac{1}{2}(1-\alpha)}$, $L = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{1}{2}(1-\alpha)}}$, $U = \frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}$.

6] a) $(-1,\infty)$ b) $\mathbb{E}Y = \int_0^1 y(\theta+1)y^\theta\,dy = \frac{\theta+1}{\theta+2} := \bar{Y}$, $\hat\theta(1-\bar{Y}) = 2\bar{Y} - 1$,

$\hat\theta_{MOM} = \frac{2\bar{Y} - 1}{1-\bar{Y}}$ c) $(\theta+1)^n\left(\prod y_j\right)^\theta$, $n\log(\theta+1) + \theta\log\sum Y_j$, $\frac{n}{\theta+1} + \log\sum Y_j$

$=0$, $\hat\theta_{MLE} = -\frac{n}{\log\sum Y_j} - 1$ d) $\hat\theta^2_{MLE}$ e)

9.84] $\theta^{-2n}\prod y_j\, e^{-\sum y_j/\theta}$, $-2n\log\theta - \sum y_j/\theta$, $-2n/\theta + \sum y_j/\theta^2 = 0$, $\hat\theta = \sum y_j/2$

$\mathbb{E}Y = \theta^{-2}\int y^2 e^{-y/\theta} = \theta^{-2}\{-\theta e^{-y/\theta}y^2|_0^\infty + 2\theta\int y e^{-y/\theta}\} = 2\theta$

$(1-p)^{\sum y_j - n}$

$\frac{2^n \prod y_j}{\theta^n} e^{-\sum y_j^2/\theta}$, $-n\log\theta - \sum y_j^2/\theta$, $-n/\theta + \sum y_j^2/\theta^2$, $\hat\theta = \frac{1}{n}\sum y_j^2$,

$\int y^2 e^{-y^2/\theta} = y\left(-\frac{\theta}{2}e^{-y^2/\theta}\right)\Big|_0^\infty + \int \frac{\theta}{2} y e^{-y^2/\theta} = \left(\frac{\theta}{2}\right)^2$

$p^n(1-p)^{\sum y_j - n}$, $n\log p + (\sum y_j - n)\log(1-p)$, $n/p - \frac{\sum y_j - n}{1-p} = 0$

$0 = n(1-p) - p(\sum y_j - n) = n - p(\sum y_j - n + 1)$, $p = \frac{n}{\sum y_j - n + 1}$

$\prod_{j=1}^{n} (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}(x_j - \mu)^2\right) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum (x_j-\mu)^2\right)$,

$\frac{L(\mu_{10})}{L(\mu_5)} = \exp\left(\frac{1}{2\sigma^2}\sum\{(x_j-\mu_5)^2 - (x_j - \mu_{10})^2\}\right) = \exp\left(\frac{1}{2\sigma^2}\{-2x_j\mu_5\right.$

$\left.+2x_j\mu_{10} + \mu_5^2 - \mu_{10}^2\}\right) < k$, $2x_j(\mu_0 - \mu_a) + \mu_a^2 - \mu_0^2 < 2\sigma^2\log k$,

$\exp\left(\frac{1}{2\sigma^2}\{2n\bar{x}(\mu_0-\mu_a) + n(\mu_a^2 - \mu_0^2)\}\right) < k$, $\bar{x} < \frac{1}{2n\cdot 5}\{2\sigma^2\log k -$

$n(\mu_a^2 - \mu_0^2)\}$, $P_0(\bar{X} < k) = \alpha = P\left(Z < \frac{k-\mu_0}{\sqrt{\sigma^2/n}}\right)$, $k = \mu_0 + z_{1-\alpha}\frac{5}{\sqrt{n}}$

$= 10 - z_\alpha \frac{5}{\sqrt n}$, $(2\theta^3)^{-n}\left(\prod y_j^2\right)e^{-\theta^{-1}\sum y_j}$, $\frac{L(\theta_0)}{L(\theta_a)} = \left(\frac{\theta_a}{\theta_0}\right)^{3n} e^{-\sum y_j(1/\theta_0 - 1/\theta_a)} < k$,

$-(\sum y_j)(1/\theta_0 - 1/\theta_a) < k$, $\sum y_j > k$, $y_j \sim \Gamma(3,\theta)$, $\sum y_j \sim \Gamma($

$P(Y_1 + Y_2 < c) = \int_0^1 F_2(c - y_1) = \int_0^1 (c - y_1) = c - 1/2$, $P(Y_1 + Y_2 < 1+c) = c + \int_c^1 F(1+c-y)$

$= c + 1 - c^2 - \frac{1}{2} + \frac{1}{2}c^2 = c + \frac{1}{2} - \frac{1}{2}c^2$, $F = 1-c$, $\frac{1}{2}(\theta+1)^2$,

Andreescu

$a^3 = n+3$, $b^3 = n^2+3n+3$, $b^3 - a^3 = n^2+2n = n(n+2)$, $b^3+a^3 = n^2+4n+6 = (n+2)^2+2$

$b^3 = (n+3)(n+1) - n = (n+2)(n+1)+1$, $b^3 = a^3(n+1) - n$, $b^3 = n^2+2n+a^3$,

$n^2+2n = b^3 - a^3 = (b-a)(b^2+ab+a^2) = n(n+2)$,

$(2|n)\ 2\nmid a$, $2\nmid b$ $(2\nmid n)\ 2|a$, $2\nmid b$, $\frac{n+2}{n} + \frac{2}{n(n+2)}$,

$b^3 = a^6 - 6a^3 + 9 + 3a^3 - 6 = a^6 - 3a^3 + 3 = (a^3-3)(a^3-1) + a^3 = (a^3-2)^2$

$+a^3-1$, $b^3 - a^3 = (a^3-3)(a^3-1) = (a^3-3)(a-1)(a^2+a+1)$

$2\nmid n \Rightarrow 2\nmid b$, $2|n \Rightarrow 2\nmid b$, $2|(n^2+3n) = n(n+3)$, $b^3 - 3 = a^3(a^3-3)$,

$b^3 = (n+2)(n+1)+1$, $(b-1)(b^2+b+1) = (n+1)(n+2)$, $a^3 = m$, $b^3 = m^2 - 6m + 9 + 3m - 9 + 3$

$= m^2 - 3m + 3$, $a^3 = m+1$, $b^3 = m^2 - 4m + 4 + 3m - 6 + 3 = m^2 - m + 1$, n^2+6n+9,

$c^3 - b^3 = 3n+6 = 3(n+2)$, $(n+3)^4 = n^4 + 4n^3 \cdot 3 + 6n^2 \cdot 9 + 4 \cdot 27n + 81$, $n^4 + 9n^2 + 9$

$+ (3n^3 + 3n^2 + 9n)2 = n^4 + 6n^3 + 15n^2 + 18n + 9$, $n^3 - 6n^2 + 12n + 9$, $(n+1)^3 =$

$n^3 + 3n^2 + 3n + 1$, $(n+2)^3 = n^3 + 6n^2 + 12n + 8 \ldots$

$A^p = I$, $B^q = 0$, $A^{p-k} = A^{-k}$, $A^k = A^{-(p-k)}$, $(q=2, p=1)\ (I+B)^2$

$= I + 2B$, $3B + I$, $A^{tp} = I$, $(A+I)^{-1} = \sum (-A)^j$

$5^{5^{n+2}} + 5^{5^{n+1}} - 5^{5^{n+1}} - 5^{5^n} = 5^{5^{n+2}} - 5^{5^n} = 5^{5^n}(5^{5^n \cdot 24} - 1)$, $(n=0)\ 25 \cdot 625 + 6$

$= 15631 = 7 \cdot 2233 = 49 \cdot 319 = 7^2 \cdot 11 \cdot 29$, $5^{5^n \cdot 2^3 \cdot 3} - 1$ $(n=0)\ 5^5 + 6 = 3131$

$= 31 \cdot 101$, $5^2 - 1 = 24$, $124 = 31 \cdot 4 = 5^3 - 1$, $5^{5^n \cdot 2^3 \cdot 3} - 1 = (5^3 - 1)\sum_{j=0}^{5^n \cdot 8 - 1} 5^j$,

$31 | (5^{5^{n+2}} - 5^{5^n})$ #44 // $S = \sum_{j=0}^{n-1} (-1)^j \binom{n}{j} 5^{n-j-1}$, $5S = \sum_{j=0}^{n} \binom{n}{j}(-1)^j 5^{n-j} - (-1)^n$,

$5S - 1 = 6^n$, $S_n = \frac{1}{5}(6^n - 1)$, $(n=5)\ 36 \cdot 216 + 1 = 7777 = 7 \cdot 1111 = 7 \cdot 11 \cdot 101$

$5S_k - 1 = 4^n$, $(n=5)\ 4^n + 1 = 1025 = 25 \cdot 41$, $S_5 = 5 \cdot 41$, S_{2k+3}

216
36
96

$4^{2k+1}+1 \equiv ?\ 5p \overset{(3)}{\equiv} 2p$

$(a^2 b + b^2 c + c^2 a)(bc^2 + ca^2 + ab^2) \geq (3abc)^2$ #103 // $\sum_{j=1}^{n} a_j^4 = (\sum a_j)(\sum \frac{a_j^4}{n})$

$\geq (\sum \frac{a_j^{5/2}}{\sqrt{n}})^2$, $(\sum a_j^{5/2})^2 \geq ?\ n^2$, $\sum a_j^4 \geq ?\ \sum a_j$, $n = \sum a_j \leq \sqrt{n}(\sum a_j^2)^{1/2}$,

$n \leq \sum a_j^2 \leq n^{1/2}(\sum a_j^4)^{1/2}$, $\sum a_j^4 \geq n$ // #104 $\leq \sum a_j^2$ / $(\sum f_j)(\sum f_j x_j^2) \geq (\sum f_j x_j)^2$,

$(\sum f_j)(\sum f_j x^2) \geq (\sum \sqrt{f_j}\cdot\sqrt{f_j}\, x)^2$ #106 // $\sum^{n} k_j = 5n-4$, $1 = \sum \frac{1}{k_j}$, $1 \leq \sum k_j^2 \sum k_j^{-4}$

$1 \leq$ k^{-3}, $5n-4 \leq (\sum \frac{1}{k})^2(\sum k_j^3)^2 = (\sum k_j^3)^2$, $\sum^{n} \frac{1}{k_j^3} \geq \frac{1}{5n-4}$, $\frac{1}{?} \geq \frac{1}{5n-4}$,

$\Delta = 176$

$0 \leq 5n^2 - 4n - 8$, $16+20c$, $n^2 \leq \sum k_j^2 \sum k_j^{-2}$, $n^2 = (\sum k^{1/2} k^{-1/2})^2 \leq \sum k \sum \frac{1}{k} =$

$5n-4$, $(n-1)(n-4) \leq 0$, $1 \leq n \leq 4$, $(n=1)\ k_1 = 1$ $(n=2)\ k_1 = k_2 = 2$ $(n=3)\ k_1 = k_2 =$

$k_3 = 3$, $\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{2}$, $\cancel{2(k_1+k_2) = k_1 k_2}$, $(n=4)\ k_1 = k_2 = k_3 = k_4 = \frac{1}{k_1} = \frac{1}{k_2} = \frac{1}{k_3} = \frac{1}{k_4}$,

$k_j = 4$ #107 // $\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b} \geq ?\ \frac{1}{\cos(a-b)} = \frac{1}{\cos a\cos b + \sin a \sin b}$, $1 \leq (\frac{\sin^3 a}{\sin b} + \frac{\cos^3 a}{\cos b})$

$(\cos a \cos b + \sin a \sin b) \geq \sin^2 a + \sin^2 b$ #111 //

$(P(x) = (x-r_1)(x-\bar{r}_1)(x-r_2)(x-\bar{r}_2)(x-r_3)$, $35 = -|r_1 r_2|^2 r_3$, -10

$= \bar{r}_1 |r_2|^2 r_3 + r_1 |r_2|^2 r_3 + |r_1|^2 \bar{r}_2 r_3 + |r_1|^2 r_2 r_3 + |r_1 r_2|^2 = 2\operatorname{Re} r_1 \cdot |r_2|^2 r_3$

$+ 2\operatorname{Re} r_2 |r_1|^2 r_3 + |r_1 r_2|^2$, $\frac{2}{7} = \frac{2\operatorname{Re} r_1}{|r_1|^2} + \frac{2\operatorname{Re} r_2}{|r_2|^2} + \frac{1}{r_3}$, $\frac{1}{7} = \frac{\operatorname{Re} r_1}{|r_1|^2} + \frac{\operatorname{Re} r_2}{|r_2|^2} + \frac{1}{2r_3}$

$\frac{2}{7} = \sum \frac{1}{r_j}$ ....

$(n^{n+1}(n-1)^2)^{1/2} \leq \frac{1}{2}(n^{n+1} + n^2 - 2n + 1)$, $\leq \frac{1}{2}(n^{n+1}(n-1) + n - 1) = \frac{?}{2} n^n$, $n^n - 1 = (n-1)\sum_{j=0}^{n-1} n^j$

$\geq (n-1)\, n (\prod_{j=0}^{n-1} n^j)^{1/n} = (n-1) n^{1+\frac{n-1}{2}} = (n-1) n^{\frac{n+1}{2}}$ #122 // $1 + (r_1 \cdots r_n)^{1/n} \leq ?\ ((1+r_1)\cdots(1+r_n))^{1/n}$

$\log(1 + (\prod r_j)^{1/n}) \leq ?\ \frac{1}{n}\sum \log(1+r_j) \geq (\prod \log(1+r_j))^{1/n}$, $\log(1+(\prod r_j)^{1/n}) \leq \log(1+\bar{r})$

$= \log(\frac{1}{n}\sum(1+r_j)) \geq \frac{1}{n}\sum \log(1+r_j)$ $\log(1+r_1)\log(1+r_2) \geq ?\ (\log(1+\sqrt{r_1 r_2}))^2$,

$\frac{1}{n}\sum \log\log(1+r_j) \geq ?\ \log\log(1+(\prod r_j)^{1/n})$, $1+\sqrt{r_1 r_2} \leq ?\ ((1+r_1)(1+r_2))^{1/2}$ $\sum^{n} \binom{n}{j} (\prod \ldots)^{j}$

Wooldridge

$\hat{y}_2 = z\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy^T)$, $\hat{v}_2 = y_2 - z\Pi_z(y_2)$, $\ddot{x} = \overbrace{(z_1, y_2) - \hat{v}_2^T \Pi_{\hat{v}_2}(z_1, y_2)}^{\text{"}x\text{"}}$, $\Pi_{\hat{v}_2}(z_1, y_2)$

$= \hat{v}_2 \mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2(z_1, y_2)^T)$, $\Pi_{\ddot{x}}(y) = \mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\mathbb{E}(\ddot{x}y^T)$, $\mathbb{E}(\ddot{x}\ddot{x}^T)$

$= \mathbb{E}(xx^T) - \mathbb{E}(xx^T\Pi_{\hat{v}_2}\hat{v}_2^T) - \mathbb{E}(\hat{v}_2\Pi_{\hat{v}_2}xx^T) + \mathbb{E}(\hat{v}_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}\hat{v}_2^T) =$

$\mathbb{E}(xx^T) - \mathbb{E}(xx^T\Pi_{\hat{v}_2}y_2^T) + \mathbb{E}(xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T) - \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T) + \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T)$

$+ \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T) - \mathbb{E}(y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T) - \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T)$

$+ \mathbb{E}(z\Pi_z y_2\Pi_{\hat{v}_2}xx^T\Pi_{\hat{v}_2}y_2^T\Pi_z z^T)$; $\Pi_{\ddot{x}}y = ?\ \Pi_{\hat{x}}y$, $\Pi_{\hat{x}}y = \mathbb{E}(\cancel{\hat{x}\hat{x}^T})^{-1}\mathbb{E}(\hat{x}y)$,

$\hat{x} = \Pi_z x = \Pi_z x$, $\Pi_{\hat{x}}y = \mathbb{E}\,\Pi_z(\Pi_z\mathbb{E}(xx^T)\Pi_z)^{-1}\Pi_z\mathbb{E}(xy)$,

$P_{\hat{x}}y = \mathbb{E}(y\hat{x}^T)\mathbb{E}(\hat{x}\hat{x}^T)^{-1}\hat{x}$, $\hat{x} = P_z x = \mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z$, $P_z y = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T)$

$\{\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T)\}\mathbb{E}(xz^T)\mathbb{E}(zz^T)^{-1}z = \mathbb{E}(P_z y\cdot x^T)\cdot\mathbb{E}(P_z x\cdot x^T)^{-1}$

$\cdot P_z x$, $P_{\ddot{x}}y = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}\ddot{x}$, $\ddot{x} = x - P_x\hat{v}_2$, $\hat{v}_2 = \bar{P}_z y_2$,

$\bar{P}_{\hat{v}_2}x = x - \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$ $\cancel{= x - \mathbb{E}(x\,\mathbb{E}(y_2)^T)}$ $\hat{v}_2 = y_2 - P_{y_2}z = y_2 - \mathbb{E}(y_2 z^T)$

$P_x\hat{v}_2 = \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$

$\mathbb{E}(zz^T)^{-1}z$, $\mathbb{E}(y\hat{v}_2^T P_x^T) = \mathbb{E}(y(y_2^T - z^T\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T)))\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\mathbb{E}(\hat{v}_2 x^T)$, $x = \begin{pmatrix} z_1 \\ y_2 \end{pmatrix}$

$P(x|\hat{v}_2) = \mathbb{E}(x\hat{v}_2^T)\mathbb{E}(\hat{v}_2\hat{v}_2^T)^{-1}\hat{v}_2$; $\mathbb{E}(\begin{pmatrix} z_1 \\ y_2 \end{pmatrix}(y_2^T - z^T\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2^T))) = \begin{pmatrix} 0 \\ \mathbb{E}y_2^2 - \mathbb{E}y_2 P \end{pmatrix}$

$\mathbb{E}(y_2^2) - \mathbb{E}(y_2 P(y_2|z)^T) = \mathbb{E}y_2^2 - \mathbb{E}(y_2 z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2) = \mathbb{E}y_2^2 - \mathbb{E}(y_2 z^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zz^T)\mathbb{E}(zz^T)$

$\mathbb{E}y_2^2 = \mathbb{E}(y_2^2) - \mathbb{E}(P(y_2|z)^2) = \mathbb{E}\hat{v}_2^2$, $\cancel{P(\hat{v}_2 \cdot \hat{v}_2)}$ $\mathbb{E}(x\hat{v}_2) = \begin{pmatrix} \\ \mathbb{E}\hat{v}_2^2 \end{pmatrix}$, $P(x|\hat{v}_2) = \begin{pmatrix} 0 \\ \hat{v}_2 \end{pmatrix}$

$\ddot{x} = x - P(x|\hat{v}_2) = \begin{pmatrix} z_1 \\ y_2 - \hat{v}_2 \end{pmatrix}$, $(\hat{\delta}_1, \hat{\alpha}_1) = \mathbb{E}(y\ddot{x}^T)\mathbb{E}(\ddot{x}\ddot{x}^T)^{-1}$ $\cancel{\mathbb{E}(y\ddot{x}^T)}$ $= (\mathbb{E}yz_1^T,$

$\mathbb{E}yy_2 - \mathbb{E}y\hat{v}_2)\begin{pmatrix} \mathbb{E}z_1z_1^T & \mathbb{E}z_1y_2 \\ \mathbb{E}y_2z_1^T & \mathbb{E}(y_2 - \hat{v}_2)^2 \end{pmatrix}^{-1} = ?\ \mathbb{E}(P(y|z)\cdot x^T)\cdot\mathbb{E}(P(x|z)\cdot x^T)^{-1}$

$\mathbb{E}(P(y|z)\cdot x^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zx^T) = \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(z\cdot(z_1^T, y_2^T)) = \mathbb{E}(yz^T)[\underset{|z|\times|z_1|}{I}; \mathbb{E}(zz^T)$

$\mathbb{E}(zy_2^T)] = \mathbb{E}(yz_1^T) : \mathbb{E}(yz^T)\mathbb{E}(zz^T)^{-1}\mathbb{E}(zy_2) = (\mathbb{E}yz_1^T, \mathbb{E}(yP(y_2|z)^T)) = (\mathbb{E}yz_1^T, \mathbb{E}(y(y_2 - \hat{v}_2)))$

$\mathbb{E}(P(x|z)\cdot x^T) = \mathbb{E}(P(x|z)P(x|z)^T) = \mathbb{E}(\begin{pmatrix} z_1 \\ y_2 - \hat{v}_2 \end{pmatrix}^{\otimes 2})$ #5.1 // $\hat{\beta}_1 = P(y|\hat{x})$, $\hat{x} = P(x|z)$

$= \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}z$, $\hat{\beta}_1 = \mathbb{E}(y\hat{x})\mathbb{E}(\hat{x}^2)^{-1} = \mathbb{E}(xz)\mathbb{E}(z^2)^{-1}\mathbb{E}(yz)\mathbb{E}(z^2)\mathbb{E}(xz)\mathbb{E}(z^2)^{-1}(x^2)$

$\beta_1 = \mathbb{E}(yz)/\mathbb{E}(xz)$, $\hat\beta_1 = \sum yz/\sum xz = (\sum xz)^{-1}(\beta_0 \sum z + \beta_1 \sum xz + \sum zu) = \cancel{\beta_0 \bar z/\overline{xz}}$

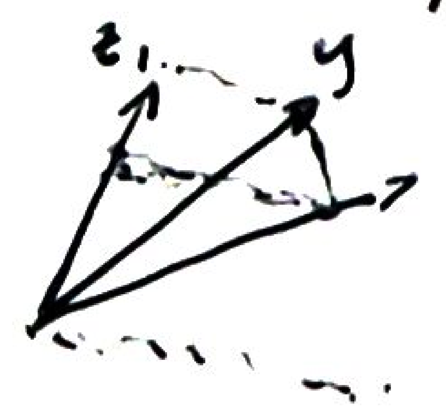
$+\beta_1 + \sum xu/\sum xz$, $\cancel{\sqrt N(\hat\beta_1-\beta_1) = \beta_0\sqrt N \bar z/\overline{xz} + \sqrt N\, \overline{xu}/\overline{xz} \rightsquigarrow \frac{\beta_0}{\sigma^2_{xz}} N(0,\sigma^2)}$ $\sqrt N(\hat\beta_1-\beta_1)$

$= \frac{\sqrt N\, \overline{zu}}{\overline{zx}} \rightsquigarrow \sigma_{zx}^{-1} N(0, \operatorname{Var} zu) = N(0, \sigma_z^2\sigma_u^2/\sigma_{zx}^2) = N(0, \rho_{zx}^{-2}\frac{\sigma_u^2}{\sigma_x^2})$ #5.10 //

$\tilde y_2 = P(y_2|z_2) = \mathbb{E}(y_2 z_2^T)\mathbb{E}(z_2z_2^T)^{-1} z_2$, $(\tilde\delta_1^T,\tilde\alpha_1) = \mathbb{E}(y_1(z_1^T,\tilde y_2))\mathbb{E}\left(\binom{z_1}{\tilde y_2}(z_1^T,\tilde y_2)\right)^{-1}$

$=\mathbb{E}\left\{(\delta_1^T,\alpha_1)\binom{z_1}{y_2}(z_1^T,\tilde y_2) + u_1(z_1^T,\tilde y_2)\right\}\mathbb{E}\begin{pmatrix} z_1z_1^T & z_1\tilde y_2\\ z_1^T\tilde y_2 & \hat y_2^2\end{pmatrix}^{-1} = (\delta_1^T,\alpha_1)\mathbb{E}\left(\binom{z_1}{y_2}(z_1^T,\tilde y_2)\right)$

$\cdot\mathbb{E}\left(\binom{z_1}{\tilde y_2}^{\otimes 2}\right)^{-1}$, $\mathbb{E}(\hat y_2^2) = \mathbb{E}(y_2\tilde y_2)$, $\mathbb{E}(y_2z_1)\ne\mathbb{E}(\tilde y_2 z_1)$



#5.11 //

$\begin{pmatrix}\mathbb{E}X_{-k}X_{-k}^T & \mathbb{E}X_{-k}z^T\\ \mathbb{E}zX_{-k}^T & \mathbb{E}zz^T\end{pmatrix}$, $\begin{pmatrix}\mathbb{E}X_{-k}X^T\\ \mathbb{E}zX^T\end{pmatrix} = \begin{pmatrix}\mathbb{E}X_{-k}X_{-k}^T & \mathbb{E}X_{-k}x_k\\ \mathbb{E}zX_{-k}^T & \mathbb{E}zx_k\end{pmatrix}$, $(\delta,\theta) = \mathbb{E}(x_k(X_{-k}^T,z^T))\mathbb{E}\left(\binom{X_{-k}}{z}^{\otimes 2}\right)^{-1}$

$\theta = 0 \Rightarrow$ $\mathbb{E}((x_kX_{-k}^T, x_kz^T)) \perp \cancel{\begin{pmatrix}\mathbb{E}X_{-k}z^T\\ \mathbb{E}zz^T\end{pmatrix}}$ $\mathbb{E}^{-1}\binom{X_{-k}}{z}^{\otimes 2}[,k:(k+M)]$;

$(\mathbb{E}X_{-k}X_{-k}^T, \mathbb{E}X_{-k}z^T) \perp \mathbb{E}^{-1}\binom{X_{-k}}{z}^{\otimes 2}[,k:(k+M)]$, $\operatorname{rank}\begin{pmatrix}\mathbb{E}X_{-k}X^T\\ \mathbb{E}zX^T\end{pmatrix} = K-1$.

$\begin{pmatrix}\mathbb{E}X_{-k}x_k\\ \mathbb{E}zx_k\end{pmatrix}\in \operatorname{sp}\begin{pmatrix}\mathbb{E}X_{-k}X_{-k}^T\\ \mathbb{E}zX_{-k}^T\end{pmatrix} \Rightarrow \begin{pmatrix}\mathbb{E}X_{-k}x_k\\ \mathbb{E}zx_k\end{pmatrix}\perp \mathbb{E}^{-1}\binom{X_{-k}}{z}^{\otimes 2}[,k:(k+M)]$, $\theta=0$ #5.12 //

$\beta_1 = \operatorname{Cov}(y,z)/\operatorname{Cov}(x,z) = (\mathbb{E}(yz) - \mathbb{E}y\,\mathbb{E}z)/(\mathbb{E}(xz)-\mathbb{E}x\,\mathbb{E}z) \doteq$

$(\sum y\{z=1\} - \sum y\cdot\sum z/n)/(\sum x\{z=1\} - \sum x\cdot \sum z/n)$, $\mathbb{E}yz = \beta_0\mathbb{E}z + \beta_1\mathbb{E}xz$,

$\mathbb{E}\left(\binom{1}{x}(1\ z)\right)\mathbb{E}\begin{pmatrix}1 & z\\ z & z^2\end{pmatrix} = \begin{pmatrix}1 & \mathbb{E}z\\ \mathbb{E}x & \mathbb{E}xz\end{pmatrix}\begin{pmatrix}1&\mathbb{E}z\\ \mathbb{E}z & \mathbb{E}z^2\end{pmatrix}^{-1} = (\operatorname{Var} z)^{-1}\begin{pmatrix}1&\mathbb{E}z\\ \mathbb{E}x & \mathbb{E}xz\end{pmatrix}\begin{pmatrix}\mathbb{E}z^2 & -\mathbb{E}z\\ -\mathbb{E}z & 1\end{pmatrix}$,

$\mathbb{E}\left(\binom{1}{z}(1\ x)\right)^{-1}\mathbb{E}\binom{y}{yz} = \mathbb{E}\begin{pmatrix}1&x\\ z&xz\end{pmatrix}^{-1}\mathbb{E}\binom{y}{yz} = (\operatorname{Cov}(x,z))^{-1}\mathbb{E}\begin{pmatrix}xz & -x\\ -z & 1\end{pmatrix}\mathbb{E}\binom{y}{yz}$,

$\hat\beta_1 = \frac{\mathbb{E}yz - \mathbb{E}y\,\mathbb{E}z}{\mathbb{E}xz - \mathbb{E}x\,\mathbb{E}z}$, $\frac1n(\sum y\sum z) = \frac1n(\sum y\{z=1\} + \sum_{j\ne k} y_j z_k)$, $\sum y_j(\{z_j=1\} - \bar z)$,

$\sum y_j(\{z_j=1\} - \{z_j=0\}) = 2\sum y_j\{z_j=1\} - \sum y_j$, $\sum y\{z=1\} - \bar y\sum\{z=1\}$, $\sum y\{z=1\}$

$-\bar z\sum y(\{z=1\}+\{z=0\}) = (1-\bar z)\sum y\{z=1\} - \bar z\sum y\{z=0\} = n\bar z(1-\bar z)\bar y_1 - \bar z n(1-\bar z)\bar y_0$ #5.13 //

$L(y|z) = L(\sum_{j=1}^K \beta_jx_j + u|z) = \sum_{j=1}^{K-1}\beta_jx_j + \beta_K L(x_K|z)$. $L(x_K|z) = \sum^{K-1}\delta_jx_j + \sum\theta_jz_j$,

$\theta\ne0$, $\dim|\mathbb{E}(zx^T)| = K$ #5.14 //

vdVaart Ch 6

$\frac{dP_n}{dQ_n} = \exp\left(\frac{1}{2}(x-\mu_n)^2 - \frac{1}{2}x^2\right) = \exp\left(-x\mu_n + \frac{\mu_n^2}{2}\right)$, $\mathbb{P}_{Q_n}\left(\frac{dP_n}{dQ_n} \overset{<c}{\cancel{>c}}\right) = \mathbb{P}_{Q_n}\left(X > -\frac{1}{\mu_n}(\log c - \frac{\mu_n^2}{2})\right) = \mathbb{P}\left(Z > -\frac{\log c}{\mu_n} + \frac{\mu_n}{2}\right)$, $\frac{\partial}{\partial c} = \varphi\left(-\frac{\log c}{\mu_n} - \frac{\mu_n}{2}\right)/c\mu_n$, $\mathbb{P}_{Q_n}\left(\frac{dP_n}{dQ_n} > 0\right)$

$\xrightarrow[n_k\to\infty]{} \mathbb{P}_{Q_\infty}\left(\exp(-x\mu_\infty + \frac{\mu_\infty^2}{2}) > 0\right) = 1 \Leftarrow -X\mu_\infty + \frac{\mu_\infty^2}{2} > -\infty$ wp $1 - Q_\infty$,

$X\mu_\infty + \frac{\mu_\infty^2}{2} \underset{Q_\infty}{\sim} Z\mu_\infty - \frac{\mu_\infty^2}{2} > 0$ wp 1 $\Leftarrow \mu_n \in [-K, K]$. $\mathbb{E}_{Q_n}\frac{dP_n}{dQ_n}$

$= \int \exp\left(-x\mu_n + \frac{\mu_n^2}{2}\right)(2\pi)^{-1/2}\exp\left(-\frac{1}{2}(x-\mu_n)^2\right)dx = (2\pi)^{-1/2}\int \exp\left(-\frac{x^2}{2}\right) = 1$ #1 //

$\frac{dP_n}{dQ_n} = (2\pi/n)^{-1}\exp\left(\frac{1}{2n}((x-\theta_n)^2 - x^2)\right) = \sqrt{\frac{n}{2\pi}}\exp\left((\theta_n^2 - 2x\theta_n)/2n\right)$, $\mathbb{P}_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right)$

$(\theta_n = O(n^{-1/2}))$

$= \mathbb{P}_{Q_n}\left((\theta_n^2 - 2x\theta_n)/2n < \log\left(\delta\sqrt{\frac{2\pi}{n}}\right)\right) = \mathbb{P}\left(N\left(-\frac{\theta_n^2}{2n}, \frac{\theta_n^2}{n^2}\right) < \log\left(\delta\sqrt{\frac{2\pi}{n}}\right)\right)$

$= \Phi\left(\left\{\log\left(\delta\sqrt{\frac{2\pi}{n}}\right) + \frac{\theta_n^2}{2n}\right\}/\frac{\theta_n}{n}\right) = \cancel{\Phi\left(O(\sqrt{n})\left(\log(\delta\sqrt{2\pi/n}) + O(1)\right)\right) \to 0}$

$= \Phi\left(O(n^{3/2})\log\left(\delta\sqrt{\frac{2\pi}{n}}\right)\right)$ $\frac{dP_n}{dQ_n} = (\sqrt{2\pi})^{-1}\exp\left(\frac{n}{2}(\theta_n^2 - 2x\theta_n)\right)$, $\mathbb{P}_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right)$

$= \mathbb{P}\left(N\left(-\theta_n^2\frac{n}{2}, n\theta_n^2\right) < \log(\sqrt{2\pi}\delta)\right) = \Phi\left(\left\{\log(\sqrt{2\pi}\delta) + \theta_n^2\frac{n}{2}\right\}/\sqrt{n}\theta_n\right)$

$= \Phi\left(O(1)\left\{\log(\sqrt{2\pi}\delta) + O(1)\right\}\right)$ $\frac{\log(\frac{2\pi}{n}\delta)}{\sqrt{n}}$ $\cancel{\to \frac{-\infty}{\infty}, \left(\frac{n}{\sqrt{2\pi}\delta}\frac{\sqrt{2\pi}\delta}{n^2}\right)/\left(\frac{1}{2\sqrt{n}}\right)} \to 0$,

$\xrightarrow{\delta\to 0} 0$, $\frac{dP_n}{dQ_n} \overset{Q_n}{\rightsquigarrow} V \Rightarrow \mathbb{P}(V=0) = 0$, $\mathbb{P}_{P_n}\left(\frac{dQ_n}{dP_n} < \delta\right) =$

$\mathbb{P}\left(N\left(-\frac{\theta_n^2}{2}n, n\theta_n^2\right) < \log\delta\right) \xrightarrow{\delta\to 0, n\to\infty} 0$, $\frac{dQ_n}{dP_n} \overset{P_n}{\rightsquigarrow} W \Rightarrow \mathbb{P}(W=0) = 0$, $P_n \lhd\rhd Q_n$,

$\theta_n \neq O(n^{-1/2})$, $\cancel{\theta_n n^{1/2} \to \infty}$ $\limsup \theta_n n^{1/2} = \infty$, $\mathbb{P}_{Q_n}\left(\frac{dP_n}{dQ_n} < \delta\right) = \Phi\left(\frac{\log\delta}{\sqrt{n}\theta_n} + \frac{\theta_n\sqrt{n}}{2}\right)$

$\xrightarrow[n_k]{} 1 \Leftarrow \theta_{n_k} n_k^{1/2} \xrightarrow[k\to\infty]{} \infty$ #2 // $\frac{dP_n}{dQ_n} = \frac{n}{n+1}\{[0,1]\} \overset{Q_n}{\rightsquigarrow} V$, $\mathbb{P}_{Q_n}(V < \delta)$

$= \lim_{k\to\infty} \mathbb{P}_{Q_{n_k}}\left(\frac{dP_{n_k}}{dQ_{n_k}} < \delta\right) = \lim_{n_k\to\infty}\left\{\frac{n_k}{n_k+1} < \delta\right\} = 0 \Leftarrow \delta < \frac{1}{2}$, $Q_n \lhd P_n$. $\frac{dQ_n}{dP_n} = \frac{n+1}{n}\{[0,1]\}$,

$\mathbb{P}_{P_n}\left(\frac{dQ_n}{dP_n} < \delta\right) = \mathbb{P}_{P_n}\left(\{0 \le X \le 1\} < \frac{\delta n}{n+1}\right) \overset{(\delta<1)}{=} \mathbb{P}_{P_n}(X \notin [0,1]) = \frac{1}{n+1} \to 0$, $P_n \lhd Q_n$ #3 //

$P_n(A) \to 0 \Rightarrow Q_n(A) = P_n(A) + Q_n(A) - P_n(A) \le P_n(A) + \|P_n - Q_n\|_{TV} \to 0$ #4 //

$P_n\{0\} = 1-\varepsilon$, $P_n(1) = \varepsilon$, $Q_n(0) = \varepsilon$, $Q_n(1) = 1-\varepsilon$, $\|P_n - Q_n\| = 1 - 2\varepsilon$, $P_n(A) \to 0 \Rightarrow A \cap \{0,1\}$

Andreescu

$a+b+c=2 \geq 3(abc)^{1/3}$, $(1-a)(1-b)(1-c)=1-a-b-c+ab+bc+ac-abc=$

$-1+\alpha$, $0\leq ? (1-a)(1-b)(1-c)$ $a+b>c$, $1-a-b<1-c$, $(1-a)(1-b)(1-c)$

$=\alpha-1\leq (\frac{1}{3}\cancel{(a+b+c)})^3 \cancel{= \frac{8}{27}}, \cancel{\alpha\leq\frac{35}{27}}$ $(\frac{1}{3}(3-a-b-c))^3=\frac{1}{27}$, $\alpha\leq\frac{28}{27}$.

$\alpha \geq ab+bc+ac-(\frac{1}{3}(a+b+c))^3 = ab+bc+ac-8/27$, $(1-a)(1-b)(a+b-1)$

$=(b-1)(a-1)(b+a-1)$, $0\leq ?(1-a)(1-b)(1-c)$, $a>1 \Rightarrow b+c=2-a<1$,

$a>b+c$, $a,b,c<1$ #124 // $365^{-25}\prod (365-j)\leq ? \frac{1}{2}$, $(\frac{1}{25}\Sigma(365-j))^{25} < ?$

$365^{25}/2$, $365-\frac{25\cdot 26}{50}\leq ? 365/2^{1/25}$, $\cancel{352/365}$ $2^{1/25}\leq ?\frac{365}{352}$ $\frac{13}{11\cdot 32}+1$

$\geq 1+1/32 \approx 1.02$, $\frac{13}{352}\geq\frac{13}{390}=\frac{1}{30}$, $(1.03)^{25}\geq ! 2$, $1.03^2=1.0609$, 1.12^4, 1.25^4,

1.56^{16}, $9: 1.2085$, $156 \cdot 124$ ... #125/ $(k=3)$ $1, 5 = 1+(2-1)+k$, $12=5+1+2k$, $13+$

$3\cdot k=22$, $a_{n-1}\underset{(k)}{\equiv} n-1$, $a_n-a_{n-1}\underset{(k)}{\equiv} 1$, $a_n-a_{n-1}=k(n-1)+1$, $a_n=\sum_{m=1}^{n}k(m-1)+1$

$=n+\frac{n(n-1)}{2}k$ #697 // $f(0)+2f(f(0))=5$, $f(f(n))=\frac{1}{2}(5-f(0))=f(\cancel{\ldots})$, $5-f(0)\neq 0$,

$2|(5-f(0))$, $f(0)\in\{1,3,5\}$ $(f(0)=1)$ $f(1)=2=\cancel{f(\frac{1}{2}(5-f(1)))}$, $f(2)=f(f(n))$

$=\frac{1}{2}(3n+5-f(n))$, $f(2)=f^2(1)=3$, $f(3)=f^2(2)$ ... $f(n+1)=f(f(n))=\frac{1}{2}(3n+5-(n+1))$

$=n+2$, $f(n)=n+1$ $(f(0)=3)$ $f^2($ ... $(3))\notin\mathbb{N}$

$(f(0)=5)$ $f(5)=0$, $f(0)$ ... #701 // $0=f(n)+3f(0)f(n)$

$f(0)\neq -1/3$, $f(n)=0$, ... $=0$, $f(n^2)=$

$2f(n)+3f(n)^2=(3f(n)$ ... $f(2n)=f(2)+f(n)+3f(2)f(n)$,

$\frac{f(mn)}{f(m)}=1+\frac{f(n)}{f(m)}+$ ... $>0 \Rightarrow f(n)\geq 1$, $f(6)\geq 3+6=9$ $f(4)$

$=2f(2)+3f(2)^2$, $f(8)$ $f(4)+f(2)+3f(4)f(2)=3f(2)+3f(2)^2+9f(2)^2+9f(2)^3$

$=3f_2+9f_2^2+9f_2^3$, $f(nf^{-1}(1))=1+4f(n)$

causal

$p(1-p) = \frac{1}{4} - (p-\frac{1}{2})^2$, $1+4\frac{p_2(1-p_2)}{p_1(1-p_1)}$, $1+4\frac{1-4(p_2-\frac{1}{2})^2}{\frac{1}{4}-(p_1-\frac{1}{2})^2}(p_1-\frac{1}{2})^2$, $1+4$ 

$1+\frac{1-4(p_2-\frac{1}{2})^2}{\frac{1}{4\dots}-1}$, $1+\frac{p_2(1-p_2)}{p_1(1-p_1)} - 4p_2(1-p_2) = 4(p_2-\frac{1}{2})^2 + \frac{p_2(1-p_2)}{p_1(1-p_1)}$

$T = p(\delta_0+\delta_1)$, $\Delta = p^2\delta_0\delta_1 - (1-p)^2\delta_0\delta_1 = \delta_0\delta_1(2p-1)$, $\frac{T^2}{4}-\Delta = \frac{p^2}{4}(\delta_0^2+\delta_1^2+2\delta_0\delta_1)$

$-\delta_0\delta_1(2p-1)$, $\frac{T^2}{4}-\Delta = (1-p)^2\delta_0\delta_1 + \frac{p^2}{4}(\delta_0^2+\delta_1^2+2\delta_0\delta_1) - p^2\delta_0\delta_1 = (1-p)^2\delta_0\delta_1$

$+\frac{p^2}{4}(\delta_0-\delta_1)^2 \overset{(\delta_0\pm\delta_1)}{=} (\delta_0-\delta_1)^2\left(\frac{p^2}{4} + (1-p)^2\frac{\delta_0\delta_1}{\delta_0^2-2\delta_0\delta_1+\delta_1^2}\right) = (\delta_0-\delta_1)^2\Big(\frac{p^2}{4}+(1-p)^2$

$\left(\frac{\delta_0}{\delta_1}+\frac{\delta_1}{\delta_0}-2\right)^{-1}\Big)$, $\frac{T^2}{4}-\Delta = \frac{p^2}{4}(\delta_0+\delta_1)^2\left(1-\frac{4}{p^2}(2p-1)\frac{1}{\delta_0/\delta_1+\delta_1/\delta_0+2}\right)$, $y=x+\delta$, $1-\frac{x}{x+\delta}$

$<?\ \frac{x+\delta}{x}-1 = \frac{\delta}{x}$, $1<?\ \frac{x^2+x\delta+\delta x}{x(x+\delta)} = 1+\frac{\delta}{x+\delta}$, $\inf_{\delta_0,\delta_1} \delta_0/\delta_1+\delta_1/\delta_0 = 2$, $\frac{T^2}{4}-\Delta \le \frac{p^2}{4}(\delta_0+\delta_1)^2$

$(1-\frac{2}{p}+\frac{1}{4}) \gtrsim \frac{T}{4}\cdot\frac{p^2}{4}(\delta_0+\delta_1)^2$ $\quad \frac{T^2}{4}-\Delta \le \frac{p^2}{4}(\delta_0+\delta_1)^2(1-\frac{2}{p}+\frac{1}{p^2})$

$\frac{p^2}{4}(\delta_0-\delta_1)^2+(1-p)^2\delta_0\delta_1$, $\frac{T^2}{4}-\Delta \le \left(\frac{p^2}{4}(\delta_0-\delta_1)^2 \vee \delta_0\delta_1\right)\cdot 2$, $\sqrt{\frac{T^2}{4}-\Delta} \le \frac{p}{2}|\delta_0-\delta_1|$

$+(1-p)\sqrt{\delta_0\delta_1}$, $\frac{p}{2}(\delta_0+\delta_1)+(1-p)\sqrt{\delta_0\delta_1} = \frac{p}{2}(\sqrt{\delta_0}-\sqrt{\delta_1})^2+\sqrt{\delta_0\delta_1}$, $|\delta_0-\delta_1| = \overset{\ge 1}{|\sqrt{\delta_0}+\sqrt{\delta_1}|}|\sqrt{\delta_0}-\sqrt{\delta_1}|$

$\delta_0,\delta_1\ge 1 \Rightarrow |\sqrt{\delta_0}-\sqrt{\delta_1}| \le |\delta_0-\delta_1|$, $\cancel{\lambda \le \frac{p}{2}(\delta_0+\delta_1)+}$ $\lambda \le \frac{p}{2}(\sqrt{\delta_0}-\sqrt{\delta_1})^2+\sqrt{\delta_0\delta_1}+\frac{p}{2}|\delta_0-\delta_1|$

$= \sqrt{\delta_0\delta_1} + \frac{p}{2}\left(|\sqrt{\delta_0}-\sqrt{\delta_1}||\sqrt{\delta_0}+\sqrt{\delta_1}| + (\sqrt{\delta_0}-\sqrt{\delta_1})^2\right) \le \sqrt{\delta_0\delta_1} + |\delta_0-\delta_1|\cdot\frac{p}{2}$

vdVaart ch6 cont

$P(A_n) \to 0$, $Q(A_n) \not\to 0 \Rightarrow \{A_{n_k}\}_k$, $\delta > 0$: $Q(A_{n_k}) > \delta$, $\{A_{n_{k_j}}\}_j$, $P(A_{n_{k_j}}) < 2^{-j}$, $B_m := \bigcup_{j=m}^{\infty} A_{n_{k_j}}$, $P(B_m) \leq 2^{-m+1} \xrightarrow[m\to\infty]{} 0$, $B_m \downarrow B_\infty$, $0 = \lim P(B_m)$

$= P(\lim B_m) = Q(\lim B_m) = \lim Q(B_m)$, $Q(B_m) > \delta$, $\mathbb{P}_{P_\theta}\left(\frac{dQ_\theta}{dP_\theta} > 0\right)$

$= \mathbb{P}_{P_\theta}(dQ > 0) = 1 - \mathbb{P}_P(dQ = 0) = 1$

Andreescu

$(mx+ny)^2+10(jx+ky)^2=(m^2+10j^2)x^2+(2mn+20jk)xy+(n^2+10k^2)y^2$, $mn=-10jk$

$m^2+10j^2 \propto 1$, $n^2+10k^2 \propto 10$, $n^2+10k^2=10(m^2+10j^2)$, $2mn+20jk=0$, $mn=-10jk$

$n^2+10?\ 10m^2+100$, $j=k=1$, $m=-1, n=10$, $m^2+10j^2=u^2$, $n^2+10k^2=10u^2$, $n^2u^2-10j^2n^2=$

$100j^2k^2=n^2(u^2-10j^2)$, $4,9,16,25,36,49,64,81,100,121,144,169,\dots$ $1,4,9,5,6$, $3z^2=\_\{3,2,7,5,8$

$x^2+10y^2=\_\{1,4,9,5,6\}$, $n=\{5\}$, $5|x, 5|y, 5|z$ #706 // $(2x+2y)(2x-2y)=z^2-t^2$

$=(z+t)(z-t)$, $4|(z^2-t^2)$, $x^2-y^2=\frac{1}{4}(z^2-t^2)$, $24y^2=5z^2-t^2$, $\begin{pmatrix}x^2\\y^2\end{pmatrix}=-\frac{1}{24}\begin{pmatrix}1&-5\\-5&1\end{pmatrix}\begin{pmatrix}z^2\\t^2\end{pmatrix}$

$24x^2=5t^2-z^2$, $24y^2=5z^2-t^2$, $5|(z-x)(z+x)$, $(z=x+5k, t=y+5j)$

$5y^2=10kx+25k^2$, $y^2=2kx+5k^2$, $x^2=2jy+5j^2$, $(z-x)(z+x)=5y^2$

$=y_1y_2$, $z=\frac{1}{2}(y_1+y_2)$, $x=\frac{1}{2}(y_2-y_1)$; $(t-y)(t+y)=5x^2=x_1x_2$, $t=\frac{1}{2}(x_1+x_2)$

$y=\frac{1}{2}(x_2-x_1)$, $4y^2=x_1^2+x_2^2-10x^2=\frac{4}{5}(z^2-x^2)$, $-\frac{46}{5}x^2+x_1^2+x_2^2=z^2$, $5|x$,

$5|z$, $5|t$, $25(\frac{x}{5})^2+5y^2=25(\frac{z}{5})^2$, $5x_{(1)}^2+y^2=5z_{(1)}^2$,

$x^2+5y_{(1)}^2=t_{(1)}^2$, $5|?y$, $25|(z^2-x^2)$, $5|y$, $(\frac{x}{5})^2+5(\frac{y}{5})^2=(\frac{z}{5})^2$,

$5(\frac{x}{5})^2+(\frac{y}{5})^2=(\frac{t}{5})^2$ #707 // $p|x \Rightarrow p|(x-y)(x+y)$, $p|y$, $x=y$ #708 // $x_j^2, x_{j+1}^2, x_{j+2}^2,\dots$,

$x_{j+1}^2-x_j^2=d$, $x_{j+1}-x_j=\dfrac{d}{x_j+x_{j+1}}\to 0$ #709 // $n^3-4n^2+1+n^2-1+4=n^3+n^2+3+(1-2n)-$

$(1+2n)$, $(2n-1)|(n^3+n^2+3)$, $2n^3-n^3-n^2-2n^2+4=n^2(2n-1)-n^3-2n^2+4$, $(2n-1)|$

(n^3+2n^2-4), $(2n-1)|(n^2-7)$, $n^3-2n^2-n^2-n+n+4=-n(2n-1)+n^3-n^2-n+4$, $(2n-1)|$

(n^3-n^2-n+4), $(2n-1)|(2n^2+n-1)=(2n-1)(n+1)$, $n^2=\_\{1,4,9,6,5,0\}$, $n^2-7=\_\{4,7,2,9,8,3\}$

$(2n-1)|(4n^2-1)$, $(2n-1)|(n^3-17)$, $(2n-1)|(5n^2-4)$, $(2n-1)|(-27)$,

$2n-1\in\{1,3,9\}$ #724 // $1!\,2!\cdots100!=1^{100}2^{99}3^{98}\cdots 99^2\cdot100^1=$

$x^2(2\cdot4\cdot6\cdots100)=x^2\,50!\,2^{50}=y^2\,50!$ #725 // , $a_j=m+j$, $m|a_k \Rightarrow$

$(a_j,a_k)=m$, $(a_j,a_{2j})=j \Rightarrow j|m$, $(a_j,\overset{m}{a_m})=j$, $a_j=b_j\cdot j$, $(j,k)=(b_j\cdot j, b_k\cdot k)\Rightarrow(b_j,b_k)=$

$b_{j_0}\neq1?$, $(a_{j_0},a_{b_{j_0}})=(j_0 b_{j_0}, b_{b_{j_0}} b_{j_0})=(j_0,b_{j_0})$, $b_{j_0}|j_0$, IV

$\frac{1}{2}\frac{\partial T}{\partial\gamma} \pm \left(\frac12\frac{\partial T}{\partial\gamma} - \frac{\partial D}{\partial\gamma}\right)\frac12 (T^2/4 - D)^{-1/2} = 0$, $T = p^2(\gamma^2/\delta_0 + (1-\gamma)^2/\delta_1) + (1-p)^2(\gamma^2/\delta_1 + (1-\gamma)^2/\delta_0)$,

$\frac{\partial T}{\partial \gamma} = 2p^2(\gamma/\delta_0 - (1-\gamma)/\delta_1) + 2(1-p)^2(\gamma/\delta_1 - (1-\gamma)/\delta_0)$, $\frac14(p^4+(1-p)^4)\cdot(2p-1)^2 \ldots$

$T = p^2(\gamma/\delta_0 + (1-\gamma)/\delta_1) + (1-p)^2(\gamma/\delta_1 + (1-\gamma)/\delta_0)$, $\frac{\partial T}{\partial\gamma} = p^2(1/\delta_0 - 1/\delta_1) + (1-p)^2(1/\delta_1 - 1/\delta_0)$

$= (\frac{1}{\delta_0} - \frac{1}{\delta_1})(2p-1)$, $T^2 = p^4(\frac{\gamma^2}{\delta_0^2} + \frac{(1-\gamma)^2}{\delta_1^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1}) + (1-p)^4(\frac{\gamma^2}{\delta_1^2} + \frac{(1-\gamma)^2}{\delta_0^2} + 2\frac{\gamma_0\gamma_1}{\delta_0\delta_1})$

$+2p^2(1-p)^2(\frac{\gamma^2}{\delta_0\delta_1} + \frac{(1-\gamma)^2}{\delta_0\delta_1} + \gamma_0\gamma_1(1/\delta_0^2 + 1/\delta_1^2))$ $(\delta_0=\delta_1)$ $T = p^2\delta^{-2}(\gamma_0^2+\gamma_1^2)(2p^2$

$-2p+1)$, $D = (2p-1)^2\delta^{-4}(\gamma_0\gamma_1)^2$, $(w_0\gamma_0 + w_1\gamma_1)/\delta_0 + (w_0\gamma_1 + w_1\gamma_0)/\delta_1$

$= ((w_0-w_1)\gamma_0 + w_1)/\delta_0 + ((w_1-w_0)\gamma_0 + w_0)/\delta_1$

$\mathbb{P}(A_{t}=a \mid L_t=l, U_t=u) = \sum_b p_B^b(1-p_B)^{1-b}\,\mathbb{P}(A_t=a\mid L_t=l, U_t=u, B_{t+1}=b) = p_B\,\mathbb{P}(A^L_t = a \mid L_t = l) + (1-p_B)\,\mathbb{P}(A^U_{t} = a \mid$

$U_t=u) = \frac12 p_B p_{LA}^{a=l}(1-p_{LA})^{a\neq l} + (1-p_B) p_{UA}^{a=u}(1-p_{UA})^{a\neq u}$, $\mathbb{P}(L_{t+1}=l, U_{t+1}=u \mid$

$A_t=a)$ $= p_B\,\mathbb{P}(L_{t+1}=l, U_{t+1}=u\mid A^L_t=a, B_t=1) + (1-p_B)\,\mathbb{P}(L_{t+1}=l, U_{t+1}=u \mid A^U_t = a,$

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$B_t=0) = \frac12 p_B p_{AL}^{a=l}(1-p_{AL})^{a\neq l} + \frac12(1-p_B)p_{AU}^{a=u}(1-p_{AU})^{a\neq u}$, $\mathbb{P}(L_{t+1}=l\mid A_t=a)$

$= p_B\,\mathbb{P}(L_{t+1}=l\mid A^L_t=a) + (1-p_B)\,\mathbb{P}(L_{t+1}=l) = p_B p_{AL}^{a=l}(1-p_{AL})^{a\neq l} + \frac12(1-p_B)$,

$\mathbb{P}(A_t=a\mid L_t=l) = p_B p_{LA}^{a=l}(1-p_{LA})^{a\neq l} + (1-p_B)/2$

$P(L_t=l\mid L_{t-1}=l) = \sum_a \mathbb{P}(L_t=l\mid A_{t-1}=a)\,\mathbb{P}(A_{t-1}=a\mid L_{t-1}=l)$

$\delta_j^{-2} \mapsto \delta_j$

$T = p(1/\delta_0^2 + 1/\delta_1^2)$, $\Delta = p^2/\delta_0^2\delta_1^2 - (1-p)^2/\delta_0^2\delta_1^2 = (2p-1)/\delta_0^2\delta_1^2$, $T^2/4 - \Delta =$

$\frac14(p^2(\delta_0^2+\delta_1^2) + 2\delta_0\delta_1) - (2p-1)\delta_0\delta_1 = \frac{p^2}{4}(\delta_0^2+\delta_1^2) + \delta_0\delta_1(p^2/2 - 2p+1)$

$= \frac{p^2}{4}(\delta_0-\delta_1)^2 + \delta_0\delta_1(p^2-2p+1) = \frac{p^2}{4}(\delta_0-\delta_1)^2 + \delta_0\delta_1(p-1)^2$ $(\delta_0=\delta_1)$

$\frac12 p\cancel{(1/\delta_0^2 + 1/\delta_1^2)} \pm p/\delta_0^2 \pm (1-p)/\delta_0^2 = \frac{1}{\delta_0^2}, (2p-1)\frac{1}{\delta_0^2}$

th $\frac{1}{\delta_0} + \frac{1}{\delta_1} = s$, $T = p^2((\gamma_0 - \gamma_1)/\delta_0 + \gamma_1 s) + (1-p)^2((\gamma_1 - \gamma_0)/\delta_0 + \gamma_0 s)$

$= (\gamma_0 - \gamma_1)\delta_0^{-1}(2p-1) + s(p^2\gamma_1 + (1-p)^2\gamma_0)$ =

Andreescu

$n^a-1=(n^{(a,b)}-1)(1+n+\dots+n^{a'-1})$, $a' := \frac{a}{(a,b)}$, $n^{(a,b)}-1 \mid (n^a-1, n^b-1)$. & $d \mid (n^a-1, n^b-1)$,

$d \mid (n^a-n^b) = (n^{(a,b)a'} - n^{(a,b)b'}) = (n^{a'}-n^{b'})(n^{a'((a,b)-1)} + n^{a'((a,b)-2)}n^{b'} + \dots + n^{b'((a,b)-1)})$

$=(n^{a'}-n^{b'})(n^{a-a'}+n^{a-2a'+b'}+n^{a-3a'+2b'}+\dots+n^{b-b'})$, $n^a-1=(n^g-1)\sum_{j=0}^{a'-1} n^{jg}$,

$n^b-1=(n^g-1)\sum_{j=0}^{b'-1} n^{jg}$, $(b>a)$ & $n^b-1-(n^a-1) = (n^g-1)\sum_{j=a'}^{b'-1} n^j$, $\sum_{b'}^{b'+a'-1}$

$(a',b')=1$, $a'x+b'y=1$, $\frac{1+n+n^2}{1+n+n^2+n^3+n^4}$, $\frac{1+n+n^2}{1+n}$, $1 = (\sum^{5-1} - n^2\sum^{3-1})(-n)+\sum^{3-1} = -n\sum^{5-1} +$

$(n^2+1)\sum^{3-1}$, $S_{a'-1} := \sum_{j=0}^{a'-1} n^j$, $\alpha S_{a'-1}+\beta S_{b'-1} =? 1$, $\alpha,\beta\in\mathbb{Z}$, $(a'x-b'y=1)$.

$(\sum_{j=0}^{x} n^{ja'}) S_{a'-1} = (\sum_{j=0}^x n^{ja'})\sum_{j=0}^{a'-1} n^j = \sum_{j=0}^{a'x} n^j$, $(\sum_{j=0}^{y} n^{jb'}) S_{b'-1} = \sum_{j=0}^{b'y} n^j = \sum_{j=0}^{a'x-1} n^j$,

$(\sum_{j=0}^x n^{ja'})S_{a'-1} - n(\sum_{j=0}^{y} n^{jb'})S_{b'-1} = 1$ #727 // $g=(a,b)$, $a=ga'$, $b=gb'$, $(a',b')=1$,

$(2\nmid a', 2\nmid b')$, $2^a+1 = (2^g+1)\sum_{j=0}^{a'-1} 2^{jg}$, $2^b+1=(2^g+1)\sum_{j=0}^{b'-1} 2^{jg}$ (#727) => $\alpha,\beta\in\mathbb{Z}$,

$\alpha\sum_{j=0}^{a'} 2^j + \beta\sum_{j=0}^{b'} 2^j = 1$, $2^g+1=\alpha(2^a+1)+\beta(2^b+1)$, $(2^a+1,2^b+1)\mid(2^g+1)$

$(2\nmid a', b'=2^k b'', 2\nmid b'')$ $2^a+1=(2^g+1)\sum_{j=0}^{a'-1}(-2^g)^j$, $2^b+1=(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{g2^k})^j$

$\alpha,\beta\in\mathbb{N}?: \alpha(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j = (2^g+1)\{\beta\sum_{j=0}^{a'-1}(-2^g)^j - 1\}$, a^{2^k-1} 2^k.

$=\sum_{j=0}^{k-1} 2^j + 1 = \sum_{j=1}^{k-1} 2^j + 2$, $(\alpha := (2^g+1))$? β: $(2^{2^k g}+1)\sum_{j=0}^{b''-1}(-2^{2^k g})^j = \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1$,

$\sum_{j=0}^{b''-1}(-1)^j 2^{2^k g(j+1)} + \sum_{j=0}^{b''-1}(-1)^j(2^{2^k g})^j = -\sum_{j=1}^{b''}(-1)^j 2^{2^k g j} + \sum_{j=0}^{b''-1}(-1)^j (2^{2^k g})^j$

$=1-(-2^{2^k g})^{b''} =? \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1$, $(-2^{2^k g})^{b''} =? \beta\sum_{j=0}^{a'-1}(-2^g)^j$,

$?\alpha',\beta: \{-2^b - 1\}\alpha' = \beta\sum_{j=0}^{a'-1}(-2^g)^j - 1 = \beta\frac{2^a+1}{2^g+1} - 1$, $\alpha'(2^b+1)(2^g+1) = ?$

$\beta(2^a+1)-1$, $\alpha'(2^{b+g}+2^g(1+2^{b-g})+1) =? \beta 2^a+\beta-1$, $2^g\{\alpha'(1+2^{b-g}+2^g)$

$-\beta 2^{a-g}\} =? \beta-1-\alpha'$,

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$L(\mu) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum(x_j-\mu)^2\right)$, $\frac{L(\mu_0)}{L(\mu_a)} = \exp\left(\frac{1}{50}\{\sum[(x_j-\mu_a)^2 - (x_j-\mu_0)^2]\}\right)$

$= \exp\left(\frac{1}{50}\sum(2x_j(\mu_0-\mu_a) + \mu_a^2 - \mu_0^2)\right) < k$, $2(\mu_0-\mu_a)n\bar{X} + n(\mu_a^2-\mu_0^2) < k$

$\bar{X} < k$; $\alpha = \mathbb{P}_{\mu=10}(\bar{X} < k) = \mathbb{P}\left(Z < \frac{k-10}{5/\sqrt{n}}\right)$, $k = 10 - \frac{5}{\sqrt{n}} z_{.025}$, $\mathbb{P}_{\mu=5}(\bar{X} <$

$10 - \frac{5}{\sqrt{n}} z_\alpha) = \mathbb{P}\left(Z < \frac{5 - 5/\sqrt{n}\, z_\alpha}{5/\sqrt{n}}\right) = \mathbb{P}(Z < \sqrt{n} - z_\alpha)$, $\frac{t(.018)}{.01} > \chi^2_{.05}$

$\left(\frac{\sigma_a}{\sigma_0}\right)^n \exp\left(\sum(x_j-\mu)^2\left(\frac{1}{2\sigma_a^2} - \frac{1}{2\sigma_0^2}\right)\right) < k$, $\sum(x_j-\mu)^2\left(\frac{1}{2\sigma_a^2} - \frac{1}{2\sigma_0^2}\right) < k$

$\sum(x_j-\mu)^2 > k$, $\chi^2(n) > k = \chi^2_\alpha$, $\theta^{2N_1}(2\theta(1-\theta))^{N_2}(1-\theta)^{2N_3}$

$\cancel{L(\theta)=\frac{\theta_0}{\theta_a}} = 2^{N_2}\theta^{2N_1+N_2}(1-\theta)^{N_2+2N_3}$, $L(\theta) = \left(\frac{\theta_0}{\theta_a}\right)^{2N_1+N_2}\left(\frac{1-\theta_0}{1-\theta_a}\right)^{N_2+2N_3} < k$

treatment

$(2N_1+N_2)\log\left(\frac{\theta_0}{\theta_a}\right) + (N_2+2N_3)\log(\ ) < k$

first continue to [illegible]

$2N_1 + N_2 > k$

stick with DAGs not linear a's

$L(\lambda) = e^{-n\lambda}\frac{\lambda^{n\bar{x}}}{\prod x_j!}$, $\frac{L(\lambda_0)}{L(\lambda_a)}$

alternate slide

[illegible] at top of slide

$= e^{n(\lambda_a-\lambda_0)}\left(\frac{\lambda_0}{\lambda_a}\right)^{n\bar{x}} < k$, $\bar{x}\log(\ ) < k$, $\bar{x} > k$,

$\alpha = \mathbb{P}(\sum x_j > k)$, $\binom{124}{76}p^{76}(1-p)^{48}\binom{147}{53}p^{53}(1-p)^{94}\cdots = (\ )p^2(1-p)^2$, $(2\pi)^{-3n/2}(\sigma_1^2\sigma_2^2$

$\sigma_3^2)^{-1/2}\exp\left(-\frac{1}{2\sigma_1^2}\sum^1(x_j-\mu_1)^2 - \frac{1}{2\sigma_2^2}\sum^2(y_j-\mu_2)^2 - \frac{1}{2\sigma_3^2}\sum^3(w_j-\mu_3)^2\right)$, $L(\theta_0) = (2\pi)^{-3n/2}\sigma^{-3}$

$\exp\left(-\frac{1}{2\sigma^2}\left(\hat{\sum}^1 + \hat{\sum}^2 + \hat{\sum}^3\right)\right)$, $\log = -\frac{3}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\left(\hat{\sum}^1 + \hat{\sum}^2 + \hat{\sum}^3\right)$,

$\frac{\partial \log}{\partial\sigma^2} = -\frac{3}{2\sigma^2} + \frac{1}{2\sigma^4}(\sum\cdots) = 0$, $\sigma^{-2} = 3/(\sum\cdots)$, $\hat{\sigma}^2 = \frac{1}{3}(\sum + \cdots)$, $\frac{\partial\log}{\partial\mu_1} = \frac{1}{\sigma^2}\sum(x_j-\mu_1) = 0$,

$\mu_1 = \bar{x}$, $L(\theta) =$

$\frac{1}{\|u\|^2}(u,b) = u/2$, $\lambda(A) = 1$, $A^2 = uu^Tuu^T = A$ // $uu^T = \begin{pmatrix} 4/9 & 4/9 & 2/9 \\ 4/9 & 4/9 & 2/9 \\ 2/9 & 2/9 & 1/9 \end{pmatrix}$, $P = \begin{pmatrix} 5/9 & -4/9 & -2/9 \\ -4/9 & 5/9 & -2/9 \\ -2/9 & -2/9 & 8/9 \end{pmatrix}$

$Pb = (5/9, -4/9, -2/9)$, $\lambda(P) = \{0, 1, 1\}$, $P^2 = I - 2uu^T + uu^Tuu^T = P$ //

$H = \begin{matrix} 1/9 & 1/9 & 5/9 \\ 1/9 & 1/9 & 5/9 \\ 5/9 & 5/9 & 7/9 \end{matrix}$ $\lambda(H) = \{1, 1, -1\}$, $H^2 = I - 4uu^T + 4uu^Tuu^T = I$ // $R(A) = c\cdot(1,1)$,

$N(A) = (1,1,2)^\perp$, $R(A^T) = \{(2,1,1)\}$, $N(A^T) = (1,1)^\perp = c\cdot(1,-1)$ //

$A^TA = wv^Tvw^T = (v^Tv)ww^T$, $A^TAw = \|v\|^2\|w\|^2 w$ // $I = BB^{-1} = I - (c+1)vw^T$

$+ cvw^Tvw^T = I + (-c-1+cw^Tv)vw^T$, $c(w^Tv - 1) = 1$, $c = (w^Tv-1)^{-1}$ //

$BB^{-1} = I - cvw^TA^{-1} - vw^TA^{-1} + cvw^TA^{-1}vw^TA^{-1} = I + (-c-1)vw^TA^{-1} +$

$cw^TA^{-1}vvw^TA^{-1} = I + vw^TA^{-1}(-c-1+cw^TA^{-1}v)$, $c = (w^TA^{-1}v - 1)$.

~~ww~~ $v = w = (1,0)$, $c = \bar{a}_{11} - 1$, $c\begin{pmatrix} \bar{a}_{11} \\ \bar{a}_{21} \end{pmatrix}(\bar{a}_{11} \;\; \bar{a}_{12})$ 6.85 //

$\lambda(A)=0$, $\{1,-1\}$, $\begin{pmatrix}1&-1\\1&-1\end{pmatrix}=\begin{pmatrix}1&0\\-1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}&\\&\end{pmatrix}=\begin{pmatrix}0&1\\0&-1\end{pmatrix}\begin{pmatrix}&\\1&1\end{pmatrix}$, $\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}\frac{1}{ad-bc}$

$=\begin{pmatrix}0&a\\0&c\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}\frac{1}{ad-bc}=\begin{pmatrix}-ac&a^2\\-c^2&ac\end{pmatrix}\cdot\frac{1}{\Delta}$, $a^2=ac$, $a=c$,

$d-b=-1$, $M=\begin{pmatrix}1&b\\1&b-1\end{pmatrix}$, $\begin{pmatrix}1&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}\cdot-1=\begin{pmatrix}0&-1\\0&-1\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}=\begin{matrix}-1&-1\\-1&-1\end{matrix}$

$\begin{pmatrix}-a&a\\-a&a\end{pmatrix}\cdot\frac{1}{d-b}$, $\begin{matrix}a=c=-1\\d=1,\ b=0\end{matrix}$, $\begin{pmatrix}-1&0\\-1&1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1&0\\1&-1\end{pmatrix}\cdot-1=\begin{pmatrix}0&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\1&-1\end{pmatrix}=A$ 6.21 //

$A^T=(S^T)^{-1}\Lambda S^T$, $(S^T)^{-1}$ 6.22 // $A^T(1,1,1)=0$, $N(A^T)=\{(1,1,1)\}$, $R(A^T)=\{(1,0,-1),$

$(1,-1,0)\}$ 6.25 // $\begin{pmatrix}3&0\\0&5\end{pmatrix}=\begin{pmatrix}1&0\\-\frac{4}{3}&1\end{pmatrix}A$, $A=\begin{pmatrix}1&0\\\frac{4}{3}&1\end{pmatrix}\begin{pmatrix}3&0\\0&5\end{pmatrix}$, $\begin{pmatrix}3&0\\0&5\end{pmatrix}=A\begin{pmatrix}1&0\\-\frac{4}{5}&1\end{pmatrix}$,

$\begin{pmatrix}3&-\frac{15}{4}\\4&0\end{pmatrix}=A\begin{pmatrix}1&-\frac{5}{4}\\0&1\end{pmatrix}$, $\begin{pmatrix}0&1\\1&0\end{pmatrix}A=\begin{pmatrix}4&5\\3&0\end{pmatrix}$, $\begin{pmatrix}r_1q_{11}&r_2q_1+r_3q_2\\r_1q_{21}&\end{pmatrix}=A$, $q_1\propto\begin{pmatrix}3\\4\end{pmatrix}$,

$q_2\propto\begin{pmatrix}3\\4\end{pmatrix}^\perp\propto\begin{pmatrix}1\\-\frac{3}{4}\end{pmatrix}$, $q_1=\begin{pmatrix}3\\4\end{pmatrix}/5$, $q_2=\begin{pmatrix}1\\-\frac{3}{4}\end{pmatrix}/\frac{5}{4}=\begin{pmatrix}4/5\\-3/5\end{pmatrix}$, $r_1=5$, $r_2\begin{pmatrix}3/5\\4/5\end{pmatrix}+r_3\begin{pmatrix}4/5\\-3/5\end{pmatrix}$

$=\begin{pmatrix}0\\5\end{pmatrix}$, $r_2=-\frac{4}{3}r_3$, $-\frac{16}{5}\cdot-\frac{3}{5}\cdot 9r_3=5$, $r_3=\frac{-25}{17}$, $r_2=\frac{100}{57}$, $A=\begin{pmatrix}3/5&4/5\\4/5&-3/5\end{pmatrix}\begin{pmatrix}5&\\0&-25\end{pmatrix}$

$A^TA=\begin{pmatrix}25&20\\20&25\end{pmatrix}=\begin{pmatrix}b_1&b_2\\b_2&b_3\end{pmatrix}^2=\begin{pmatrix}b_1^2+b_2^2&b_1b_2+b_2b_3\\ &b_2^2+b_3^2\end{pmatrix}$, $b_1^2=b_3^2$, $b_1=b_3$, $b_1^2+b_2^2=25$,

$2b_1b_2=20$, $b_1^2+\frac{100}{b_1^2}=25$, $b_1^4-25b_1^2+100=0$, $b_1^2=\frac{1}{2}(25\pm 15)=20,5$, $B=($

$Q=AB^{-1}=\begin{pmatrix}3&0\\4&5\end{pmatrix}\begin{pmatrix}\sqrt5&-2\sqrt5\\ &\sqrt5\end{pmatrix}/_{15}=\begin{pmatrix}\frac{1}{\sqrt5}&\frac{2}{\sqrt5}\\\frac{2}{\sqrt5}&\frac{1}{\sqrt5}\end{pmatrix}$ // $(3-\lambda)(5-\lambda)=$, $4+5v_2=3v_2$ $(1,-2)$,

$(0,1)$, $A=\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}3&\\&5\end{pmatrix}\begin{pmatrix}1&0\\2&1\end{pmatrix}$, $\begin{pmatrix}0&1\\1&-2\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&-2\\0&1\end{pmatrix}$, $S^{-1}=\begin{pmatrix}1&2\\0&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$,

$(25-\lambda)^2-400=\lambda^2-50\lambda+225$, $\frac{1}{2}(50\pm 40)=45,5$, $25+20g=45$, $g=1$, $g_1=(1,1)$

$g_2=(1,-1)$, $A^TA=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, ~~$A=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt5&\\&\sqrt5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$~~

$=\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2\begin{pmatrix}90&\\&10\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2$, $\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2\begin{pmatrix}3\sqrt{10}&\\&\frac{2\sqrt5}{\sqrt{10}}\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

~~$=\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt{10}&-3\sqrt{10}\\2\sqrt5&-2\sqrt5\end{pmatrix}$~~ ~~$\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&45\\5&-5\end{pmatrix}$~~ $=$ ~~$\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}3\sqrt{10}&3\sqrt{10}\\\sqrt{10}&-\sqrt{10}\end{pmatrix}$~~ $=\begin{matrix}2\sqrt{10}&\sqrt{10}\\\sqrt{10}&\end{matrix}$

$AA^T=\begin{pmatrix}9&12\\12&41\end{pmatrix}$, $(9-\lambda)(41-\lambda)-144=\lambda^2-50\lambda+225=(\lambda-45)(\lambda-5)$, $9+12g=45$, $g_1=(1,3)$,

$g_2=(1,-\frac{1}{3})$, $A=?$ ~~$\begin{pmatrix}\frac{1}{\sqrt{10}}&\frac{2}{\sqrt5}\\\frac{3}{\sqrt{10}}&-\frac{2}{3\sqrt5}\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{10}}&\frac{2}{\sqrt5}\\\frac{3}{\sqrt{10}}&-\frac{2}{3\sqrt5}\end{pmatrix}\begin{pmatrix}45&45\\5&-5\end{pmatrix}/\sqrt2$~~ $=($

$A=?\begin{pmatrix}\frac{1}{\sqrt{10}}&\frac{3}{\sqrt{10}}\\\frac{3}{\sqrt{10}}&-\frac{1}{\sqrt{10}}\end{pmatrix}\begin{pmatrix}\sqrt{45}&\\&\sqrt5\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}/\sqrt2=\frac{1}{\sqrt5\cdot2}\begin{pmatrix}1&3\\3&-1\end{pmatrix}\begin{pmatrix}\sqrt{45}&\sqrt{45}\\\sqrt5&-\sqrt5\end{pmatrix}=\frac{1}{2\sqrt5}\begin{pmatrix}6\sqrt5&0\\8\sqrt5&10\sqrt5\end{pmatrix}$

~~$\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&\\&5\end{pmatrix}\begin{pmatrix}1&3\\3&-1\end{pmatrix}=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}45&90\\15&-5\end{pmatrix}$~~ $=\begin{pmatrix}3&0\\4&5\end{pmatrix}$ 6.28 //