# Group project 2 – CS340/MATH321 – Geometrical modelling and numerical analysis

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## Instructions

This group project will contribute 20% towards the final marks. The deadline to submit the project is October 25, 2019 18:30 hours. The project must be submitted via **Github**. You should include both the tex and pdf files, and any code files you may have. Please note that this is a **group** project. Groups for this project are mentioned in the "group.txt" file.

### Github Classroom instructions

Please use this link to join the GitHub Classroom

- One of the persons in the group should open that link and select the email address from the list
- On the next page you will be asked to create your team. Your team name must by named 'f19-cs340-math321-assignment2 – ⟨useridOfMember1⟩ – ⟨useridOfMember2⟩ – ⟨useridOfMember3⟩ – ⟨useridOfMember4⟩ ',
- Then the remaining members of the group must use the Github link and select their email address from the page to join the team which was created earlier.

### Marking scheme

The project will be marked out of 100.

- 10% marks will be for the presentation.
- 10% for following good coding practises
- 80% for the correctness and understanding of the topic. Both questions carry equal marks
- Each student will grade the remaining students in their group as a ratio highlighting how much effort others have put in. For example, if A, B, C, and D are in one group, then A will grade B, C, and D. If A thinks that B, C, and D have put in equal effort then A should write "B:C:D = 1:1:1". On the other hand, if A thinks B's contribution to the project is twice as much as C and D, then A should write " B:C:D = 1:0.5:0.5". If a group has three students A, B, and C, then A will grade the efforts of B and C. For example, A thinks that C has put in a bit more effort than B. In that case A may write "B:C = 0.91:1". All students in a group will **individually** submit the contribution of others in a group via LMS. <u>Please make sure your contributions are normalised by the highest contribution</u>.

The parts you are responsible for must be written by you in LaTeX. Just typing in LaTeX itself is not a contribution.

In addition to writing the ratios, please ensure that you specifically identify which parts of the question **other** group members were responsible for.

50% of the marks will come from the individual contributions and the remaining 50% will be the group effort. For example, a group has members A, B, and C. The group gets 90 points because Question 2 was wrong and it member A's responsibility. Then the group members will get the following score, assuming each member contributed equally i.e. ratios of 1:1:1.

– $A = \frac{23.333}{33.333} \times 50 + \frac{90}{100} \times 50 \approx 80$

– $B = \frac{33.333}{33.333} \times 50 + \frac{90}{100} \times 50 \approx 95$

– $C = \frac{33.333}{33.333} \times 50 + \frac{90}{100} \times 50 \approx 95$

If A and B had jointly attempted question 2, then the scores would have been 80, 80, and 95 respectively.

A good project would contain, but not limited to, the following:

- Document typed in LaTeX. The basic template for LaTeX and the associated makefile is available to download from the course website on LMS. You are welcome to use your own LaTeX template
- A document free of typing errors
- Using figures/diagrams/set diagram, where possible, to explain your answers. As the cliché goes – a picture is worth a thousand words
- Concise and thorough answers. A long report doesn't necessarily mean a good report
- List of references
- Figures should be properly labelled
- Please make sure you read your submission from the reader's perspective
- Useful comments in the code.
- Code free of magic numbers
- Good unit tests in test_meshlib.py
- No dangling print statements
- Keeping the repository neat and tidy. For example, there should be just one code/tex file i.e. we don't want to see XXXv1.tex, XXXv2.tex, XXXv2Final.tex.
- Regular code commits

## Late submission policy

Late submissions, unless approved beforehand, will be penalised according to the following.

| # hours past the deadline | Percentage penalty |
|---|---|
| < 1 hour | 5% |
| 1-2 hours | 15% |
| 2-3 hours | 30% |
| 3-4 hours | 45% |
| >4 hours | 0% (not accepted) |

### 0.1 Plagiarism and collusion

There is a zero tolerance policy towards plagiarism and/or collusion. If a student(s) is found to have plagiarised and/or colluded, or the work submitted is not their own, (s)he will be given an **F in this course**. Furthermore, they will be reported to the academic code of conduct committee which would affect your academic standing in the university. If you are unsure whether you are plagiarising, please ask.

Please note that even if you understand everything, copying someone else's work is still plagiarism.

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In the event that something is not clear from the question, you are strongly encouraged to use the discussion forum on LMS. **Individual enquiries to instructor and Basem will not be entertained**

## Questions

1. **(50 points)** The following question asks you to explore and implement the equation

$$\vec{p}(s,t) = \sum N_i(s,t)\vec{p}_i$$

in two- and three-dimensions

   a. **(3 points)** Draw all the linear and quadratic Bézier basis functions over a unit interval and a unit square

   b. **(3 points)** Draw a linear, quadratic, cubic, quartic b-splines basis functions in 1D over a uniform knot vector

   c. **(3 points)** Draw quadratic, cubic, and quartic Bézier curves by arbitrary defining the appropriate control points

   d. **(6 points)** The curve generated using Bézier basis functions do not usually pass through the control points. Derive new quadratic and cubic polynomial basis functions $N_i(t)$ which when linearly combined with the control points $\vec{p}_i$ will generate a curve which passes through the control points. Draw them and comment on potential advantages and disadvantages of these basis functions. You can define the control points arbitrarily.

   e. **(6 points)** Now imagine you have a cubic Bézier curves $\vec{P}(t)$ and $\vec{Q}(t)$, derive the conditions on the control points such that the curves $\vec{P}(t)$ join the curve $\vec{Q}(t)$ with $C^1$ and with $C^2$ continuity. What is the continuity of the cubic Bézier curve $\vec{P}(t)$ by itself.

   f. **(9 points)** For part e), draw a figure which demonstrates that the curve $\vec{P}(t)$ joins the curve $\vec{Q}(t)$ with $C^1$ continuity. Draw a similar figure(s) which demonstrates that the curves join with $C^2$ continuity. The figure must be generated by some algorithm/function.

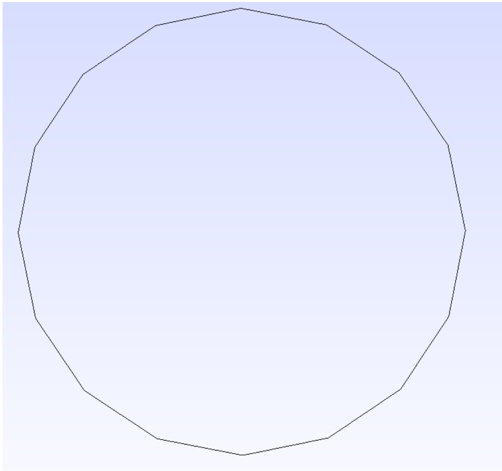

Figure 1: Line mesh of the circle

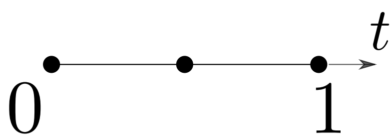


Figure 2: Parametrisation of quadratic line element

g. **(9 points)** Repeat parts c-d for a single patch with tensor product structure

h. **(2 points)** Consider two arbitrary tensor product quadratic Bézier patches $\vec{P}(s,t)$ and $\vec{Q}(s,t)$, describe (qualitatively or mathematically) the conditions for $C^1$ continuity along the edge at which they meet?

i. **(9 points)** Define arbitrary control points for patches $\vec{P}(s,t)$ and $\vec{Q}(s,t)$ and generate a figure which demonstrates whether the patches connects with $C^1$ continuity. The figure must be generated by some algorithm/function.

2. **(10 points)** This question asks you to generate a curve defined by the control points using the quadratic Bézier basis functions. You are given a circle.txt file. It has 16 vertices and 8 faces (line elements). Figure 1 shows this mesh. Describe succinctly what you have done

   Each face (element) of the mesh comprises of three vertices. For example circle.txt file shows that the second face comprises of vertices {2, 3, 4}. Using the parametrisation show in figure 2, generate the resulting geometry using the quadratic Bézier basis functions.

3. **(20 points)** You are now asked to generalise your algorithm in question 2 from curves to surfaces. Given the mesh for the one-eighth of a sphere in sphere.txt. The screenshot of the mesh is shown in Figure 3

   The 'sphere.txt' file has 224 vertices and 60 quadrilateral faces/elements. Each quadrilateral face/element comprises of 9 points. The quadrilateral in the sphere.txt numbers the vertices as shown in figure 4. Generate the surface represented by the mesh using the quadratic Bézier basis functions

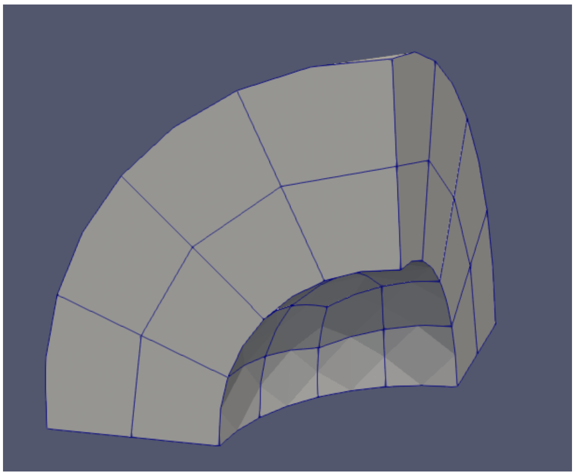

Figure 3: Mesh for the one-eighth of a sphere

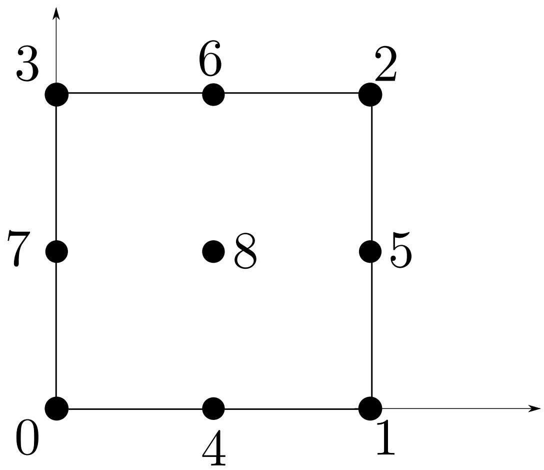


Figure 4: Mesh for the one-eighth of a sphere