

Machine Learning, Winter 2024  
Practice Assignment 1  
Solution

**Exercise 1-1** Definitions\*

Discuss the following questions:

- a) What is *empirical* data?
- b) What is the formal definition of *machine learning*? What does each of **E**, **T** and **P** stand for?

**Exercise 1-2** Machine Learning\*

Consider the formal definition of *machine learning*

For each of the following tasks (**T**), suggest a suitable choice of each of **E** and **P**:

- a) Predicting the price of a stock in the stock market
- b) Playing chess
- c) Face recognition

**Exercise 1-3** Vectors- Algebra

Solve the following equations using the given vector

$$w = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

- $\|w\|$
- $\frac{-w}{\|w\|} *$
- $\|w\|^2$

$$a = \begin{bmatrix} 1 \\ 8 \\ 4 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

- $a \cdot b$  (solve using 2 different equations)

**Exercise 1-4**      Matrices - Algebra

Solve the following equations using the given matrices

$$a = \begin{bmatrix} 7 & 1 & 2 \\ 6 & 4 & 3 \\ 5 & 1 & 9 \end{bmatrix} \quad b = \begin{bmatrix} 2 & 1 & 6 \\ 5 & 8 & 9 \\ 6 & 4 & 3 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \quad t = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad n = 3$$

- $a + b$
- $a - b$
- $a + nb$
- $a + nbt$
- $(a + b)^T$
- $(a + b)^T x$
- $(a + b)^T x + c^*$

$$z = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$$

- $(z^T z)^{-1}$

This is an additional resource to know how to perform the inverse of a 4x4 matrix  
<https://semath.info/src/inverse-cofactor-ex4.html>

**Exercise 1-5**      Matrices - Calculus\*

Find the derivative of  $\mathbf{F}$  with respect to  $x$

$$F = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1 x_2 x_3 \end{bmatrix}$$

**Exercise 1-6**      Vectors - Calculus

Find the derivative of  $C\mathbf{x}$  with respect to  $\mathbf{x}$

$$C = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**Exercise 1-7**      Matrices - Calculus\*

Find the derivative of the matrix product  $\mathbf{M}\mathbf{x}$  with respect to  $\mathbf{x}$ .

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**Exercise 1-8**      Matrices - Calculus\*

Find the derivative of  $\mathbf{y}^T \mathbf{B} \mathbf{y}$  with respect to  $\mathbf{y}$ .

$$B = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

**Exercise 1-9**      Matrices - Calculus

Given the matrix function  $\mathbf{f}(\mathbf{x}) = \mathbf{x}^T \mathbf{G} \mathbf{x} + 2\mathbf{h}^T \mathbf{x} + c$ , find the derivative with respect to  $\mathbf{x}$ .

$$G = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}, \quad h = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad c = 5, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**Exercise 1-10**      Matrices - Calculus\*

Find the derivative of  $\mathbf{A}x$  and  $x^T \mathbf{A}x$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**Exercise 1-11**      Matrices - Calculus

Find the derivative of  $\mathbf{x}^T \mathbf{C} \mathbf{x}$  with respect to  $\mathbf{x}$ .

$$C = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$