Eclipsing binaries 000 and 017

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Introduction

In this project, the goal was to find physical parameters of two pairs of spectroscopic binary stars. The period and mass of the stars are examples of such parameters. The analysis was done separately for each pair, however, the same methods were used. The Kepler light curves and radial velocity data was given for the binary pair 000 and 017. The Kepler light curves data consisted of the raw, corrected and detrended flux as well as their corresponding errors. It also included the time measurement (Barycentric Julian Date). Furthermore, the radial velocity data gave us the observation date, the measured radial velocities for the two stars, their uncertainties and the corresponding phase.

Note that for the rest of this report, the detrended flux will be referred to as the flux of the star as it is what is mainly used for the rest of the analysis.

Determining the transit period

Figure 1 and 2 show us the flux vs time for pair 017 and 000, respectively. This was plotted in order to estimate the period of the stars. By looking at only two transits, it was estimated that the period should be around 0.7 days for both pairs. In order to find what the true period is, we set a period range and plotted it against the standard deviations of the scatter of the flux. As the plot became more and more clear to what the period range should be, we decreased the range until this scatter phase plot was produced shown in figure 3 and 4.

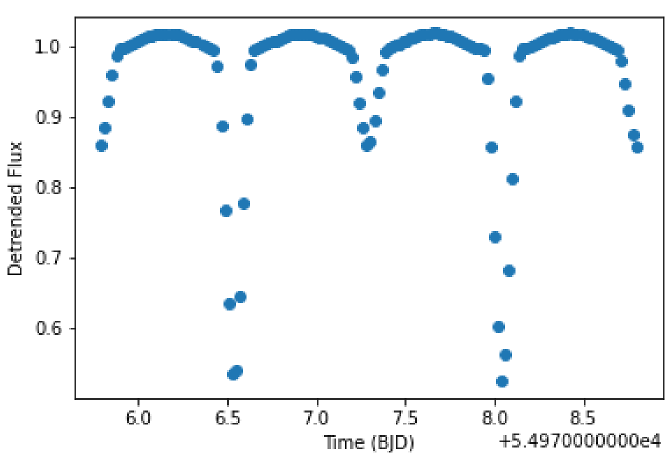

Figure 1 - Transits for binary pair 017

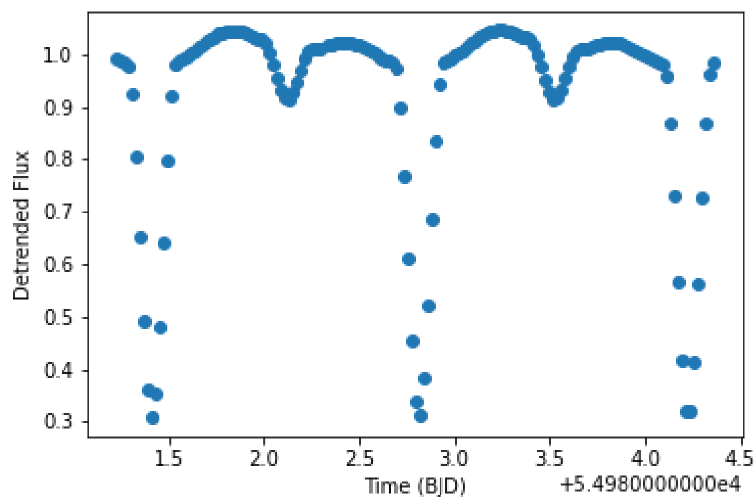

Figure 2 - Transits for binary pair 000

The initial period estimates from this plot came to be 0.75325x2 = 1.5065 for 017 (we multiply by 2 because we looked at two transits only making the value we found to be half of the period) and 0.70236x2=1.40472 for 000.

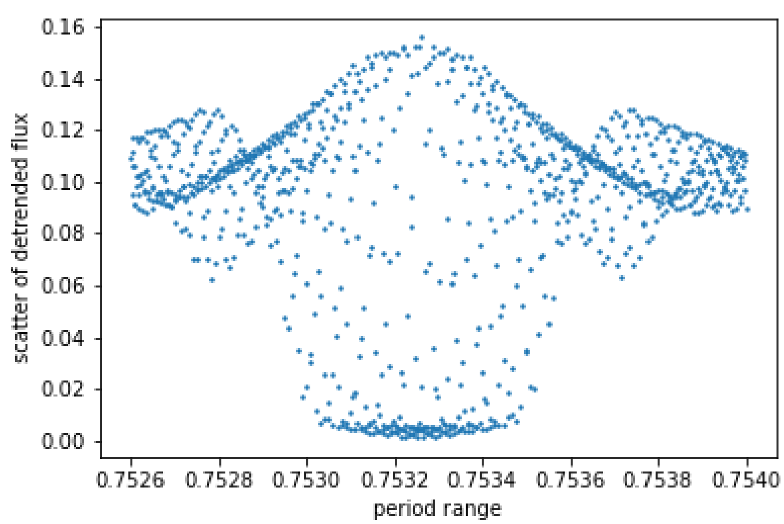

Figure 3 - scatter phase plot (017)

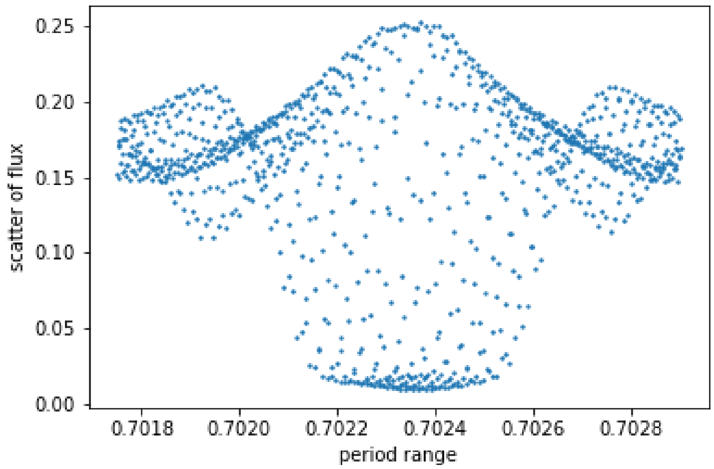

Figure 4 - scatter phase plot (000)

To find the period as accurately as possible, we calculated the phase and plotted the phase vs flux. The plots can be seen in figure 5 and 6 below. By trial and error, we found the best estimate of the period to be 1.50648 (star 017) and 1.40469 (star 000).

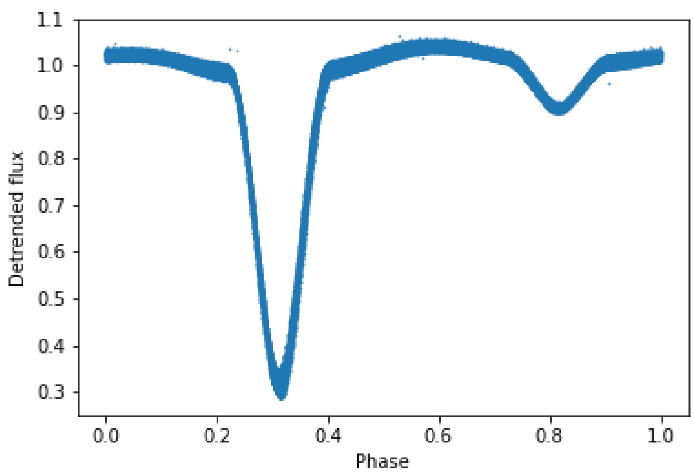

Figure 5 - Phase pot (000)

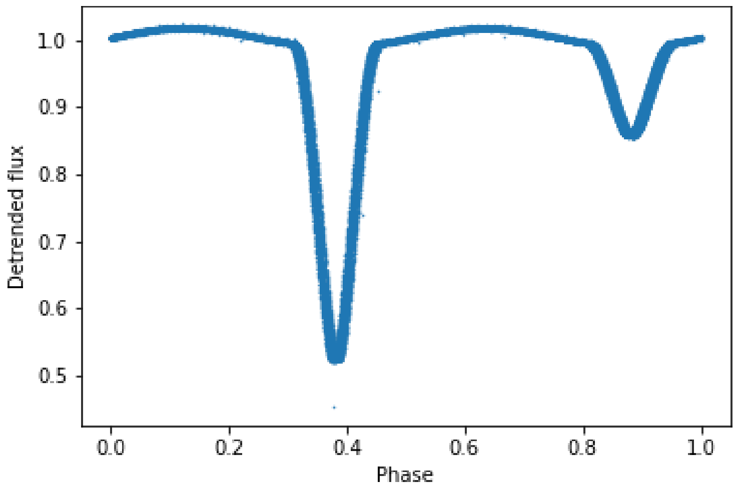

Figure 6 - Phase plot (017)

Table 1 - Period estimates for binary pairs 000 and 017

	Binary Pair 000	**Binary Pair 017**
Initial period estimate	1.40472 days	1.5065 days
Final period estimate	1.40469 days	1.50648 days
Uncertainty	0.00001	0.00001

Fitting radial velocities

The offset was taken to be 0 for pairs 017 and 000 because as we can see in figure 7 and 8, the lines, if extended, would approximately cross at 0. There is no vertical shift to be accounted for.
The estimate for the amplitude of the radial velocity curves was estimated by using curve_fit from the library scipy.optimize. This gave us an estimate of what the maximum amplitude was. The final expression for the sin wave we used to fit the data is A * sin(2pi+phi).

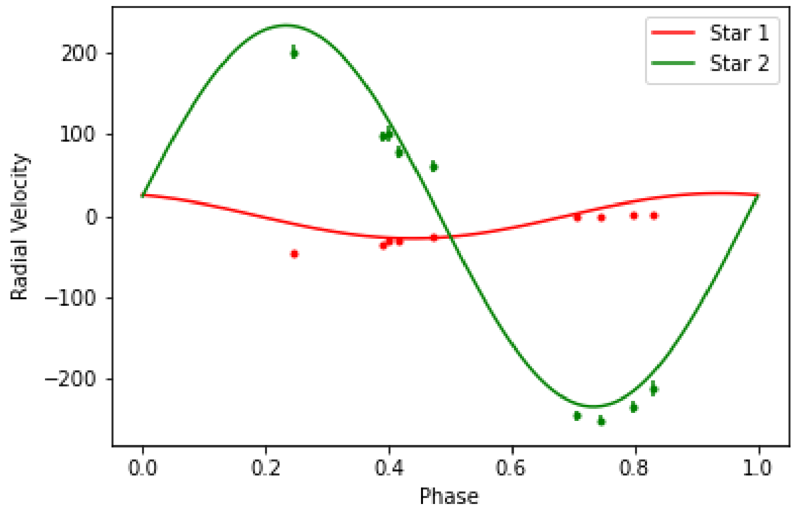

Figure 7 - sine wave model fit (017)

The reduced chi-squared was calculated using the formula for chi-squared

$$\chi^2 = \sum_i \left(\frac{\mathrm{O}_i - \mathrm{C}_i}{\mathrm{E}_i}\right)^2$$

divided by N-2 where N is the size of the dataset. We subtract by 2 instead of 1 because we have two parameters. The observed values come from our data and the fitted values (C) are the ones we found using the sin function.

For star 1, $\chi^2_{red} = 0.222$. Since this value is very close to 1 we can say that the sin curve model fit the data quite well. For star 2 however, $\chi^2_{red} = 6.9$. Since this value is larger than 1, the sought for value when calculating reduced chi-squared, it could mean that we overestimated the errors, as well as the model not being a great fit for our data for star 2.

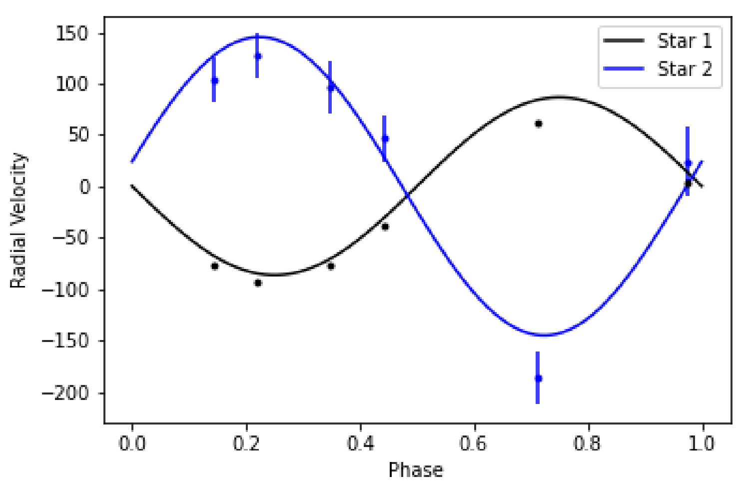

Figure 8 - sine wave model fit (000)

The reduced chi-squared values for star 1 and star 2 for pair 000 were calculated to be 15.45 and 15.75, respectively. Both these values are much larger than 1, indicating an underestimation in our uncertainties. This also tells us that the sin curve model fits the binary pair 017 better than 000 since the reduced chi-squared values for 017 were much lower.

Table 2 - Binary pair 017

	Star 1	**Star 2**
Amplitude of radial velocity curve	27.66	233.71
Amplitude uncertainty	0.01	0.01
Offset	0	0
Reduced chi-squared	0.222	6.9

Table 3 - Binary pair 000

	Star 1	**Star 2**
Amplitude of radial velocity curve	86.51	145.17
Amplitude uncertainty	0.01	0.01
Offset	0	0
Reduced chi-squared	15.45	15.75

Measuring the transit timing and depths

Here we try and find an estimate the timings for the transit ingress and egress. We first do this for the primary transit. The first red dashed line (starting from the left) represents the first contact, the second represents the beginning of minimum light and the third marks the end of minimum light.

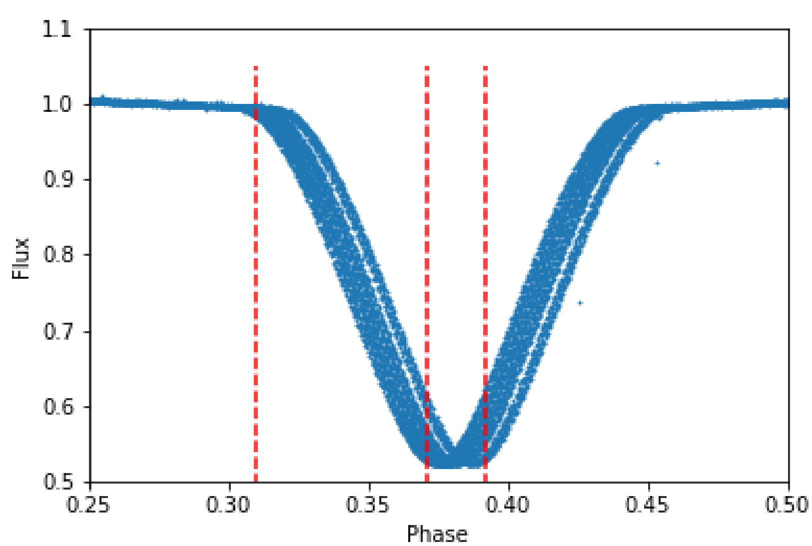

Figure 9 – primary transit highlighting first contact and beginning and end of minimum (017)

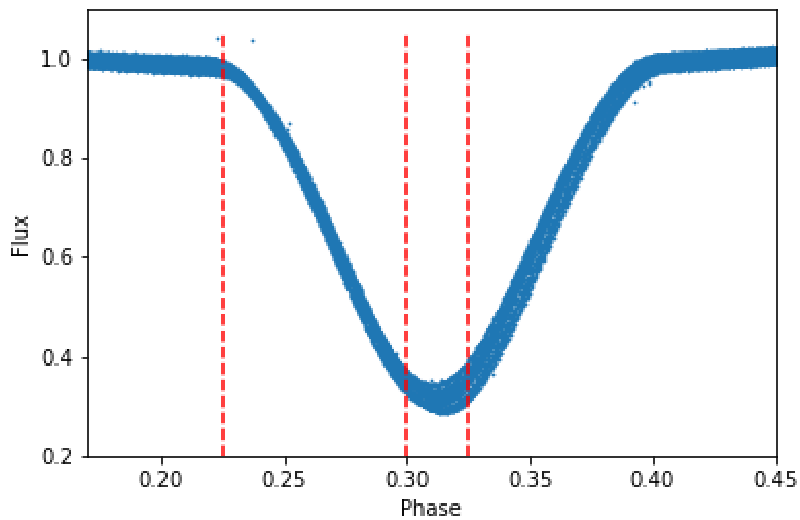

Figure 10 – primary transit highlighting first contact and beginning and end of minimum (000)

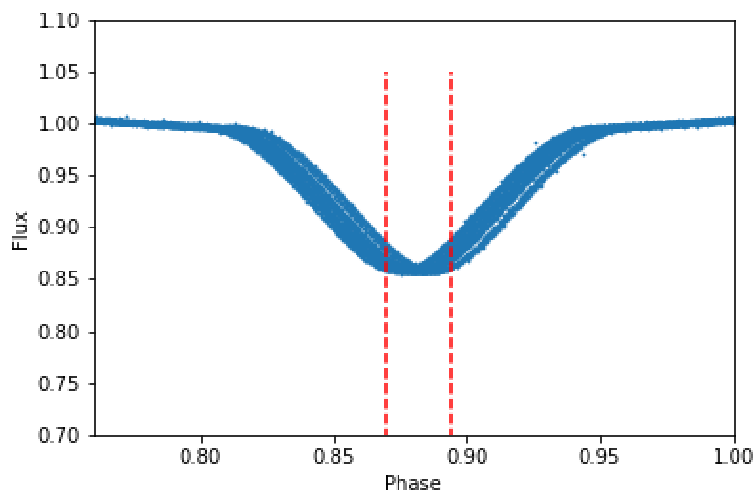

Figure 11 - Secondary transit (017)

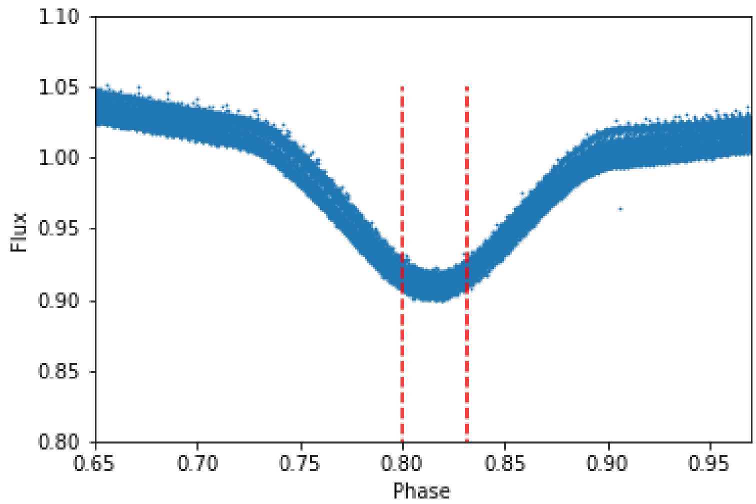

Figure 12 - Secondary transit (000)

These values used are given in the tables below where ta represents first contact, tb is the beginning of the minimum and tc is the end of the minimum.

	Primary transit	**Secondary Transit**
ta	0.31 ± 0.005	-
tb	0.371 ± 0.001	0.87 ± 0.001
tc	0.392 ± 0.001	0.894 ± 0.001
median	28239 ± 16623.5	28567 ± 16648.5

Figure 13 - Binary pair 017

	Primary transit	**Secondary transit**
Ta	0.225 ± 0.005	-
Tb	0.30 ± 0.01	0.80 ± 0.01
Tc	0.325 ± 0.001	0.832 ± 0.001
Median	28635 ± 16412.3	28669 ± 16476.7

Figure 14 - Binary pair 000

If the bottom of the light curves is flat, this means that a total eclipse happens. The light curve has the shape it does due to there being one dimmer and one brighter star. When the bright star is eclipsed we get the deeper minimum and vice versa.

Orbital and stellar parameter calculations

Finally, from the analysis above, we have everything we need to calculate a few parameters for the stars that are important to us. First we find the mass ratio of the stars. We use the formula

$$m_2/m_1 = v_{1r}/v_{2r}$$

Where $v_{1r} and\ v_{2r}$ are the radial velocities of stars 1 and 2, respectively.

	Binary 017	**Binary 000**

Mass ratio	0.118	0.596

We then use this to find the total mass of the system.

$$m_{total} = (v_{1r} + v_{2r})^3 P / 2\pi G$$

	Binary 017	**Binary 000**
Total mass (in solar masses, kg)	$3.2 \cdot 10^{-14}$	$2.096 \cdot 10^{-14}$

We know that we can use Keplers third law

$$m = a^3 / P^2$$

To find the semi major axis.

	Binary 017	**Binary 000**
Semi major axis	$4.18 \cdot 10^{-5} km$	$3.46 \cdot 10^{-5} km$

To find the mass of each star, we first found mass of star 2 and then subtracted it from the total mass to find mass of star 1.

$$m_2^3 = (m_1 + m_2)^2 v_{1r}^3 P / 2\pi G$$

	Binary 017	**Binary 000**
M1	$8.54 \cdot 10^{25} kg$	$1.96 \cdot 10^{26}\ kg$
M2	$8.54 \cdot 10^{25} kg$	$1.96 \cdot 10^{26}\ kg$

When using jupyter to calculate these masses, it was found that they actually differed by 0.00000001 · 10^26kg for binary pair 000 and by 0.000000006 · 10^25kg for binary pair 017. In both cases, the mass of star 2 was larger.

Finally, the radius for each star was calculated using these formulas for the 'smaller' star and the 'larger' star.

$$r_{small} = (tb - ta) \cdot \frac{v_{1r} + v_{2r}}{2}$$

$$r_{large} = (tc - ta) \cdot \frac{v_{1r} + v_{2r}}{2}$$

	Binary 017	**Binary 000**
R_small	7.972 km	8.688 km
R_large	10.716 km	11.584 km

To find the temperature ratio, we can use this equation:

$$\left(\frac{T_{small}}{T_{large}}\right)^4 = \frac{(B_0 - B_p)}{B_o - B_s}$$

Where B_p is the brightness of the primary minimum, B_s is the brightness of the secondary minimum and B_o is the amount of light detected from the binary when both stars are visible.

Due to the use of trial and error to find the period, the uncertainty is large and so will the uncertainty for the masses of the stars as well as the total mass. This is because we use the period in our calculation.