Lab 4 Report

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Q1a

Pseudocode

- define constants (c,k,m) + set the number of samples to a power of 2, from t = 0 to t = 150
- create function that computes the equation of motions for the relativistic particle on a spring when given initial position + velocity and other constants
  - use the Euler-Cromer method and code in Lab 1
  - calculate + return position and velocity values of the system using For loop
- determine the motion of the system in three cases: x = 1, x_c, and 10*x_c (x_c calculated during Lab 3)

Plots and written answer

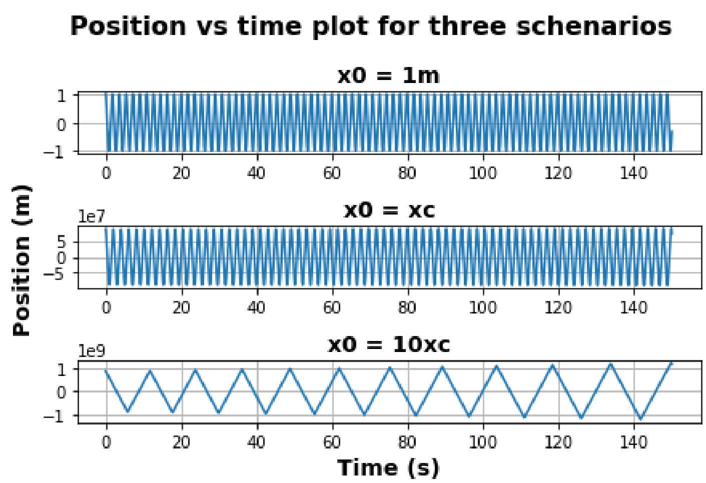


Figure 1: position plot of x = 1m (top), x = x_c (middle), and x = $10x_c$ (bottom).
Note that visually we can determine that the period of x = $10x_c$ is approximately 11-12 seconds.
*See Appendix for zoomed in plot for each scenario (easier to approximate the period).

The position vs time plot for when $x_0 = 1$, x_c, and $10x_c$ was plotted in Figure 1 for 150 seconds with a the number of time steps equal to 1048576. The reason for using these parameters is as follows: Firstly, to obtain at least ten periods when $x_0 = 10x_c$, we needed to plot for around 150 seconds to obtain at least ten oscillations. Secondly, as directed by the TA announcement for Lab 5, the number of samples was set to 1048576 so that it was a power of 2 (2^{20}).

Q1b

Pseudocode cont.

- use np.fft.fft to find fourier transform for position x(t)
- scale the amplitudes of the FFT by dividing Fourier components by the maximum value of the amplitude so that maximum value of the amplitude = 1
- plot the scaled FFT plots for $x_c = 1$, x_c, $10x_c$

Plots and written answer

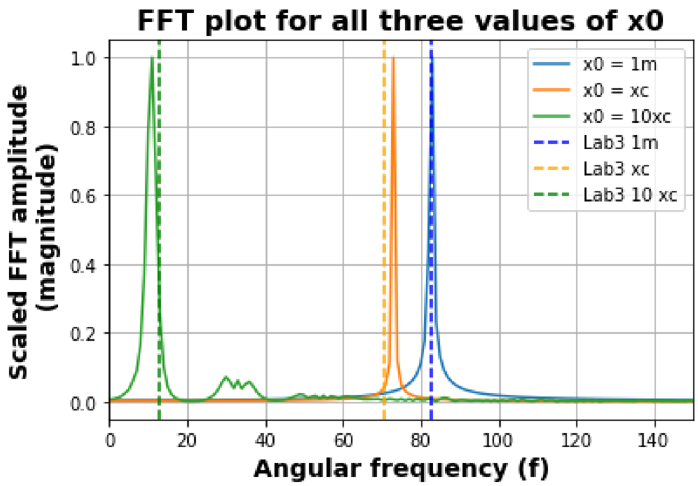


Figure 2: FFT plot for all three values of x_0 along with the calculated periods from lab 3 plotted as vertical lines as comparison (for part c)

After using FFT (fast Fourier transform) the positional values of the spring and scaling the amplitudes, Figure 2 was produced. According to Figure 2, the peaks of $x_0 = 1$m, x_c, $10x_c$ are located around angular frequencies 80, 70, and 10 respectively on the FFT plot. These values makes sense from our understanding of how the system's period change when x_0 is changed. Observing Figure 1, as the x_0 increases, the period of the system increases ie. the period of the system when $x_0 = 1$m is shorter/smaller than $x_0 = x_c$ and the period when $x_0 = x_c$ is shorter/smaller than when $x_0 = 10x_c$. As a result, it makes sense that when $x_0 = 1$m, the location of the peak corresponds to a larger frequency, as its period is smaller than the rest of the scenarios. On the other hand, the location of the peak when $x_0 = 10x_c$ in figure 2 corresponds to a lower frequency as its period is longer/larger.

Q1c

Pseudocode cont.

- use the code from lab 3 that uses Gaussian quadrature to determine the expected period of the system
  - calculate the expected period for $x_0 = 1$m, x_c, $10x_c$
  - convert the calculated period to frequency (FFT): frequency of the period = 150s/period, where 150s is how long we plotted the values for (t = 0-150s)
  - see https://faraday.physics.utoronto.ca/PVB/Harrison/FourierTransform.pdf pg 10, + see code comments for more indepth explanation as to how to convert the values
- plot the expected period (lab 3) on the FFT graph in part b using plt.axvline()

Written answer

By using Gaussian quadrature to integrate the equation Lab 3 equation (7), the periods (seconds) when $x_0 = 1$m, x_c, and $10x_c$ were calculated to be 1.8137282762355023s, 2.130549017713685s, and 11.664361478211765s respectively. By comparing the obtained periods to the position plots of Figure 1, visually, the periods returned by equation (7) is reflected in the position plots. After calculating the periods they were converted to frequency by dividing 150 (our end time) by the calculated periods. The resulting calculated frequencies from equation (7) were plotted in Figure 2. By observing the location of the FFT's peaks and the vertical lines, representing the calculated frequencies, the frequencies from the FFT and from Lab 3 equation (7) are approximately the same.

Question 2b

Pseudocode

- read in the file GraviteaTime and split into 2 channels
- find the time by creating an array then dividing by the frequency (definition of period = 1/f)
- make one plot for each channel

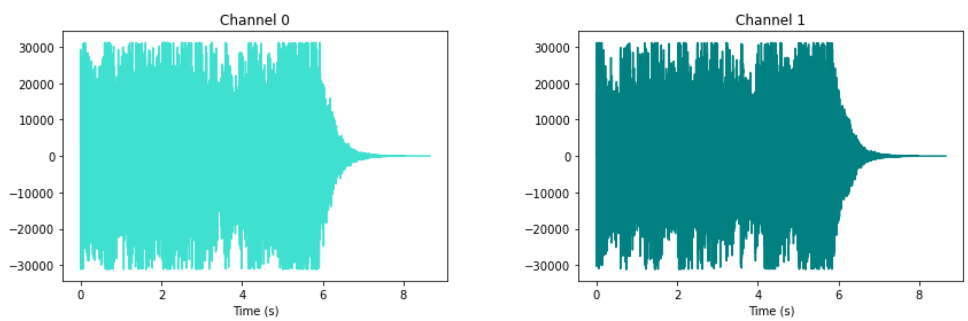
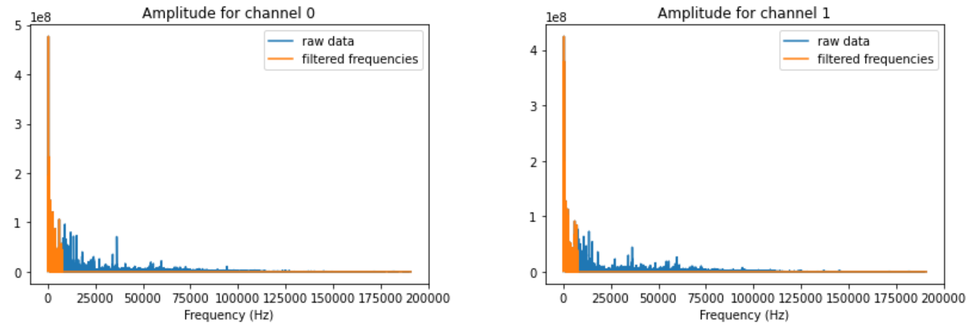


Figure 3: Plots of the data given as channel 0 and channel 1

Question 2d

Pseudocode

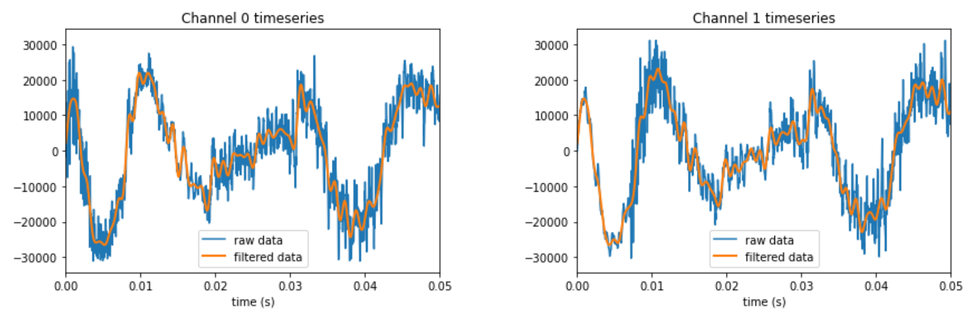
- compute the fourier transform for channel 0 and 1
- find the frequencies for channel 0 and 1 using fft.rfftfreq
- create copies of the frequency arrays in order to filter out the unwanted noise
- use a for loop to loop over all frequencies and only keep the ones lower than 880Hz and set the rest to 0
- plot the frequencies vs transformations for channel 0 and 1
- set the indices corresponding to the max frequencies to 0
- transform back to the time domain once filtering is done by fft.irfft (inverse)
- plot the amplitudes of both channels and with the filtered channels on the same plot



(a) Channel 0

(b) Channel 1

Figure 4: Amplitudes of Fourier coefficients for each channel before and after filtering



(a) Channel 0

(b) Channel 1

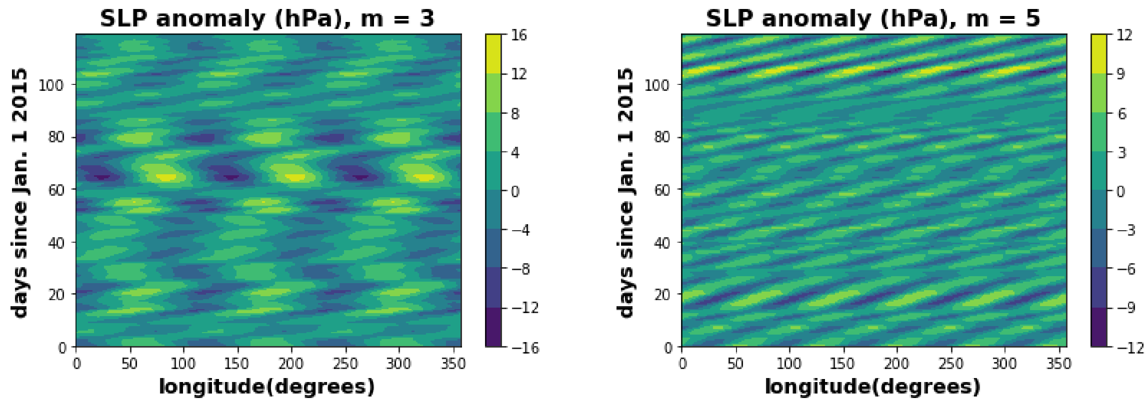
Figure 5: Time series from 0s-0.05s for both channel 1 and 2 before and after filtering

Question 3a

Pseudocode

- use the code snippet from lab handout to generate contour plot
- take the FFT of SLP using np.fft.rfft (SLPrfft)
- create a storage array shaped to the FFT of SLP (same dimensions) for storage during a loop
  - use np.zeros for the storage array - place values corresponding to the wave number into the storage array, while values corresponding to wave numbers other than the one we want = 0
- use For loop to loop through the values of the SLPrfft[i]
  - place values corresponding to the wave number (SLPrfft[i][3]) into the storage array storage[i][3]
- compute the inverse FFT, using np.fft.irrft, of the storage array
- repeat the For loop when m = 5, change j = 3 condition to j = 5 and plot the contour plots for both situations

Plots



(a) wavenumber=3

(b) wavenumber=5

Figure 6: The contour plots of when m = 3 (left) and when m = 5 (right)

Question 3b

Written answer

Some important characteristics of the contour plots for m = 3 and 5 is the alternations between blue (low pressure) and yellow-green (higher pressure). On the m = 3 contour plot, vertically, there are approximately three blue regions and three yellow-green regions. On the other hand, on the m = 5 contour plot, there are approximately five blue regions and five yellow-green regions as well, however they are more slanted than in m = 3. Evidently, the number of alternations between low and high pressure correspond to the wave number (m) of the contour plot. Another observation is that the blue and yellow-green areas of m = 5 contour plot look more stretched out diagonally than in m = 3.

Observing the contour plot for m = 3, with the assumption the wave is heading eastwards (right), the "slanting" of the area of similar pressures on the same plot represents the speed at which the waves are moving at. As mentioned before, in general, the patterns of the m = 3 plot seem to be less slanted and more vertical, indicating that the wave is traveling slower. If we were to draw imaginary lines through the low and high pressure areas in both plots, we can see that on any given day, the waves move faster eastward when m = 5 as there are more alternations (our imaginary lines are closer together) in low and high pressures than when m = 3. As a result, because m = 5 contains 5 alteration, it has more and smaller waves than m =3 and thus travels faster Eastwards. On the other hand, because m = 3 contain less alterations, and thus larger and fewer waves, it travels Eastwards at a slower rate.

Appendix

Plots

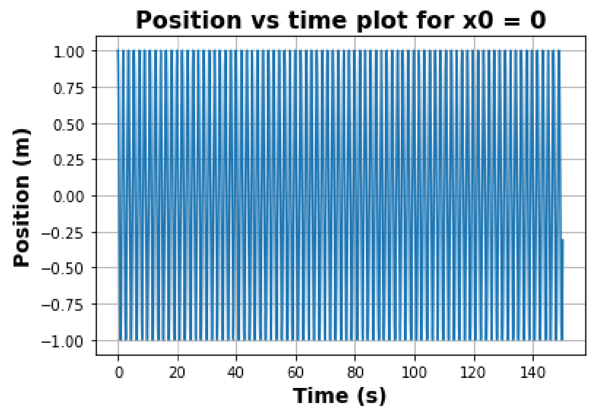


Figure 7: full sized position vs time plot for when x_0=1m

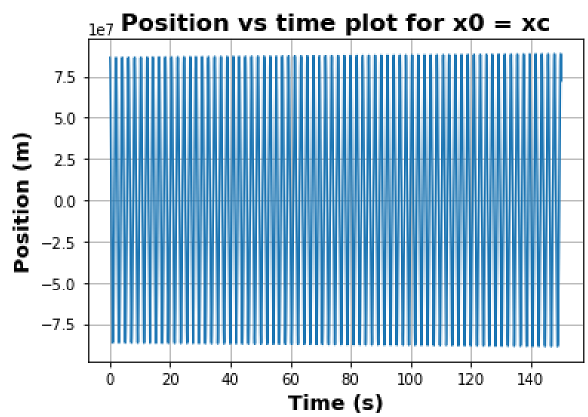


Figure 8: full sized position vs time plot for when x_0=x_c

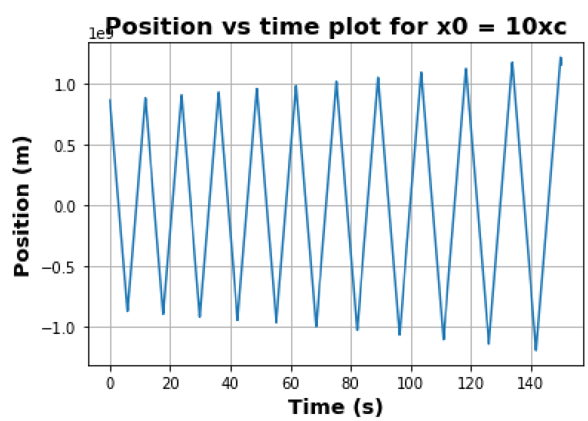


Figure 9: full sized position vs time plot for when x_0=10x_c