Department of Statistical Sciences

Time Series Analysis

STA457H1/STA2202H1 S-LEC0201

STA457H1/STA2202H S-LEC0101 & STA457H1S LEC2001

Winter 2021

Assignment 1

Due: Jan. 27th at 5.00 pm EST

Submit an RMarkdown file and the corresponding pdf report for Q1 and Q2. Please confirm the code's readability by leaving an appropriate comment on each line to get the full credit.

Q1. Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with mean zero and $\sigma_w = 1$. Simulate and plot $n = 200$ observations from each of the following two models.

(a) $x_t = s_t + w_t$, for $t = 1, \ldots, 200$, where

$$s_t = \begin{cases} 0 & \mathrm{t} = 1, \ldots, 100 \\ 10 \exp\{-\frac{(\mathrm{t}-100)}{20}\} \cos(2\pi \mathrm{t}/4), & \mathrm{t} = 101, \ldots, 200 \end{cases}$$

(b) $x_t = s_t + w_t$, for $t = 1, \ldots, 200$, where

$$s_t = \begin{cases} 0 & \mathrm{t} = 1, \ldots, 100 \\ 10 \exp\{-\frac{(\mathrm{t}-100)}{200}\} \cos(2\pi \mathrm{t}/4), & \mathrm{t} = 101, \ldots, 200 \end{cases}$$

(c) Compare the general appearance of the series (a) and (b) with the earthquake series and the explosion series shown in Fig. 1.7. In addition, plot (or sketch) and compare the signal modulators (a) $\exp\{-t/20\}$ and (b) $exp\{-t/200\}$, for $t = 1, 2, \ldots, 100$.

(d) For the two series, x_t, compute and plot the mean functions $\mu_x(t)$ for $t = 1, 2, \ldots, 200$.

Q2. For the Johnson & Johnson data, say y_t, shown in the following figure, let $x_t = \log(y_t)$. In this problem, we are going to fit a special type of structural model, $x_t = T_t + S_t + N_t$ where T_t is a trend component, S_t is a seasonal component, and N_t is noise. In our case, time t is in quarters (1960.00, 1960.25, . . .) so one unit of time is a year.

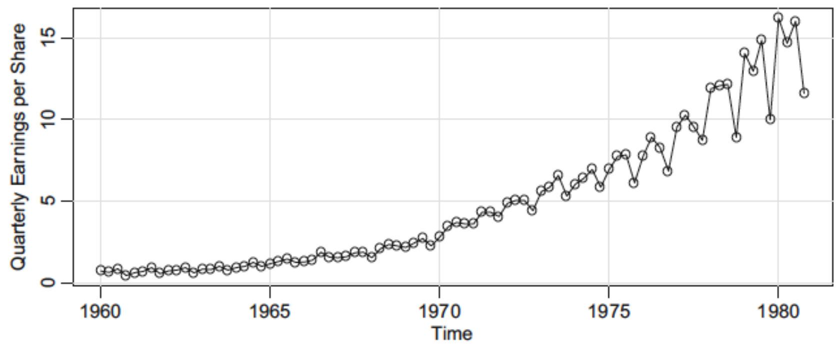

Fig. 1.1. Johnson & Johnson quarterly earnings per share, 84 quarters, 1960-I to 1980-IV

(a) Fit the regression model

$$x_t = \underbrace{\beta t}_{\text{trend}} + \underbrace{\alpha_1 Q_1(t) + \alpha_2 Q_2(t) + \alpha_3 Q_3(t) + \alpha_4 Q_4(t)}_{\text{seasonal}} + \underbrace{w_t}_{\text{noise}}$$

where $Q_i(t) = 1$ if time t corresponds to quarter $i = 1, 2, 3, 4$, and zero otherwise. The $Q_i(t)$'s are called indicator variables. We will assume for now that w_t is a Gaussian white noise sequence. Write the estimated model. Test the significance of estimated parameters and the full model.

(b) If the model is correct, what is the estimated average annual increase in the logged earnings per share?

(c) If the model is correct, does the average logged earnings rate increase or decrease from the third quarter to the fourth quarter? And, by what percentage does it increase or decrease?

(d) What happens if you include an intercept term in the model in (a)? Explain why there was a problem.

(e) Graph the data, x_t , and superimpose the fitted values, say $\hat{x}_t$, on the graph. Examine the residuals, $x_t - \hat{x}_t$, and state your conclusions. Does it appear that the model fits the data well (do the residuals look white)?

Q3. For a moving average process of the form

$$x_t = w_{t-1} + 1.2w_t + w_{t+1},$$

where w_t are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and plot the ACF as a function of h in R.

There will be a penalty for those who submit up to 30 minutes late and quizzes/tests/final submitted more than 30 minutes late will not be accepted.

Penalty	**# of minutes late**
5%	1-5
10%	6-10
20%	11-15
30%	16-20
40%	21-25
50%	$>$ 26-30
100%	$>$ 30