

Digital Modulation Techniques

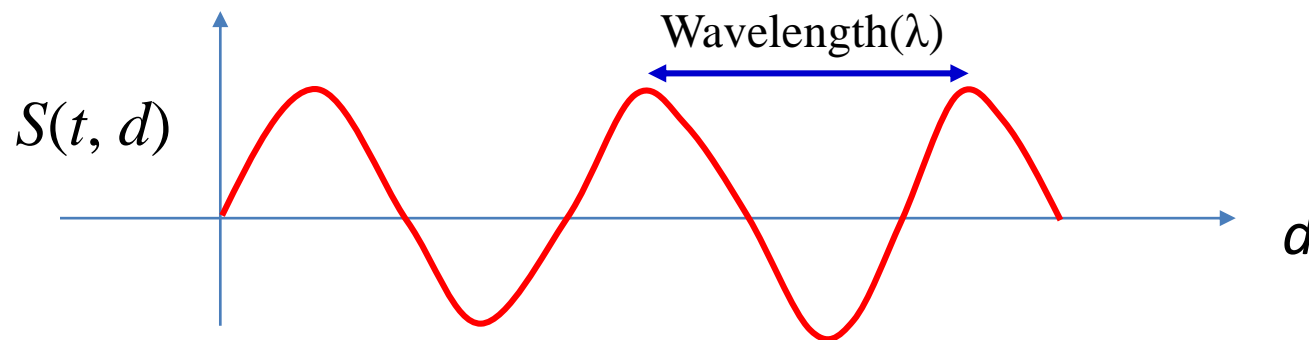
Baseband (pulses) → Passband (sinusoidal waves)

✓ The objective of digital modulation is to convert the **rectangular digital pulses** to **smooth sinusoidal wave** hence considerable reduction in bandwidth is achieved. The bandwidth reduction is essential to cope the transmitted wave with the transmission medium (channel) of lower capacity bandwidth.

✓ For example **MODEM** is connected between PC and Internet cable to convert the rectangular data pulse from the computer (infinite bandwidth) into continuous wave (called **modulation** during transmission) of lower bandwidth to cope with the allowed bandwidth of the Internet line (transmission medium). During reception (for example downloading of data) the modulated wave is converted to digital pulse train called **de-modulation**.

In wired communication the digital modulation is necessary for,

- ❖ To reduce the bandwidth of the baseband signal according to the capacity of the transmission medium and receiver circuits.
- ❖ Multiple signals can be transmitted based on FDMA.
- ❖ If the wavelength of the signal is considerable with the length of the wire, then the wire acts as an antenna and radiates most of the signal energy. The remedial measure of the situation is to increase the frequency of the signal i.e. modulation with high frequency carrier.



In wireless communications the reasons of digital modulation are,

- ❖ In wireless communication digital modulation is necessary to avoid interferences of surrounding users with choose of appropriate carrier.
- ❖ Noisy immunity is increased for a high frequency modulated wave.
- ❖ Size of an antenna depends on wavelength of a signal. After modulation (using high frequency carrier) wavelength of the modulated wave is reduced hence size of the antenna is reduced.
- ❖ A modulated wave can propagate far longer way than the baseband signal.

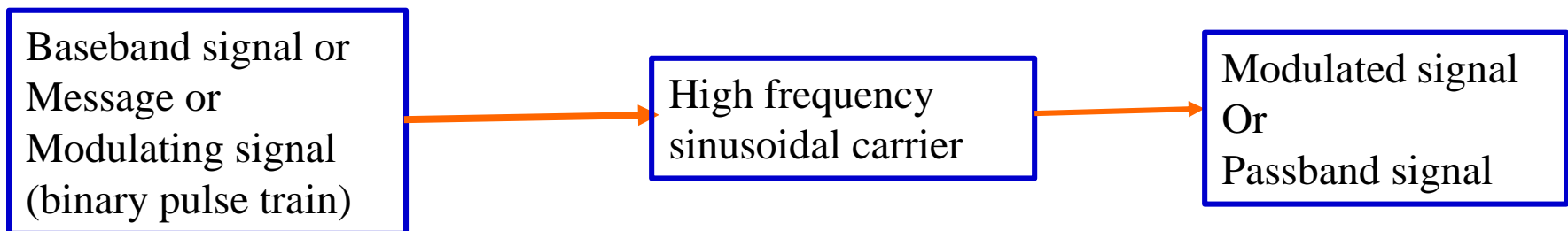
Binary Frequency Shift Keying (BFSK)

In binary FSK two sinusoidal waves of frequency f_1 and f_2 over a period of T or $[kT, (k+1)T]$ are used to represent binary logic 1 and 0 respectively. The waves are like,

$$S_1(t) = A \cos(2\pi f_1 t + \theta) \quad ; \quad kT \leq t \leq (k+1)T \quad \text{for logic 1}$$

$$S_2(t) = A \cos(2\pi f_2 t + \theta) \quad ; \quad kT \leq t \leq (k+1)T \quad \text{for logic 0}$$

; where $f_1 > f_2$ and θ is the initial phase of sinusoidal wave i.e. at the starting point symbol wave.



FSK wave can be generated using two oscillators of frequency f_1 and f_2 connected to a multiplexer like fig.1. The multiplexer is switched between the oscillators by binary input bits.

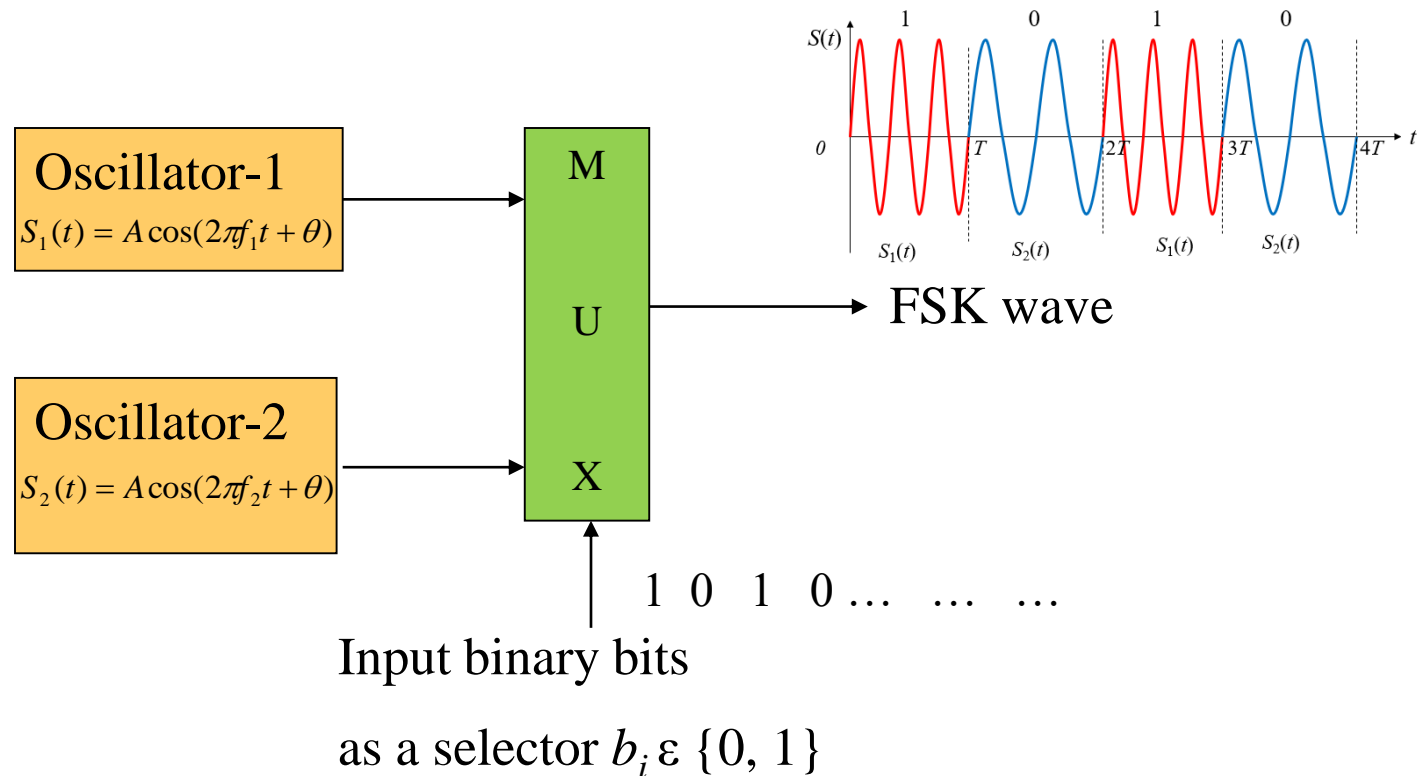


Fig. 1 Binary FSK modulator

Coherent demodulator of BFSK

The received signal is contaminated by noise is expressed as,
 $r(t)=S_i(t)+n(t)$; where $i=1, 2$ and $n(t)$ is the noise. The coherent detector of orthogonal FSK is shown in fig.4.

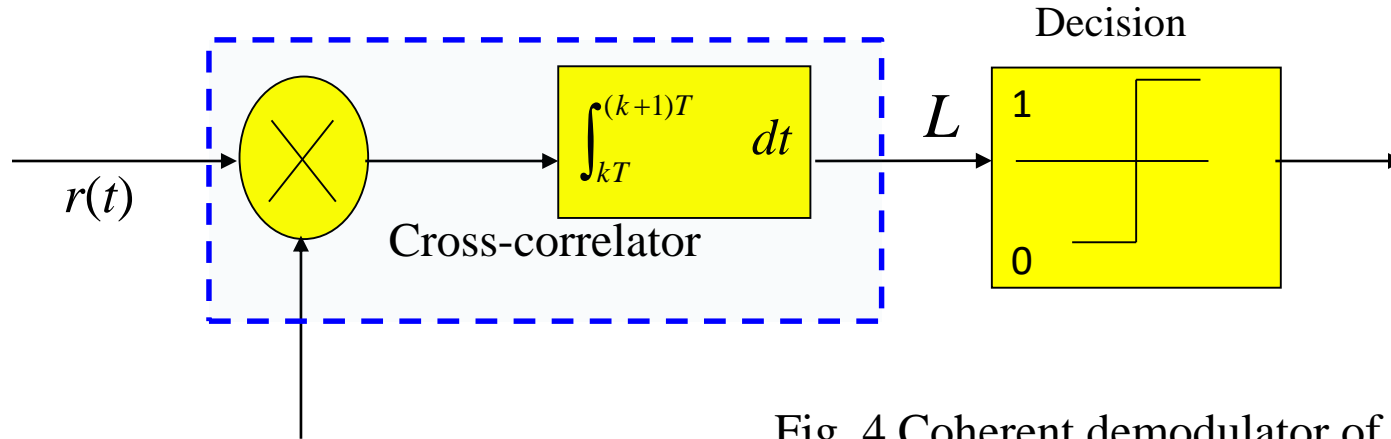


Fig. 4 Coherent demodulator of FSK

$$\cos(2\pi f_1 t) - \cos(2\pi f_2 t)$$

Let us assume logic 1 i.e. $S_1(t)$ wave is transmitted. Therefore,
 $r(t)=S_1(t)+n(t)$ then, the output of the cross-correlator L will be,

$$L = \int_{kT}^{(k+1)T} \{\cos(2\pi f_1 t) - \cos(2\pi f_2 t)\} \{A \cos(2\pi f_1 t) + n(t)\} dt = \frac{AT}{2} + n' > 0$$

$$\text{Considering, } (f_1 - f_2) = \frac{m}{T} - \frac{n}{T} = \frac{m-n}{T} = \frac{k}{T}$$

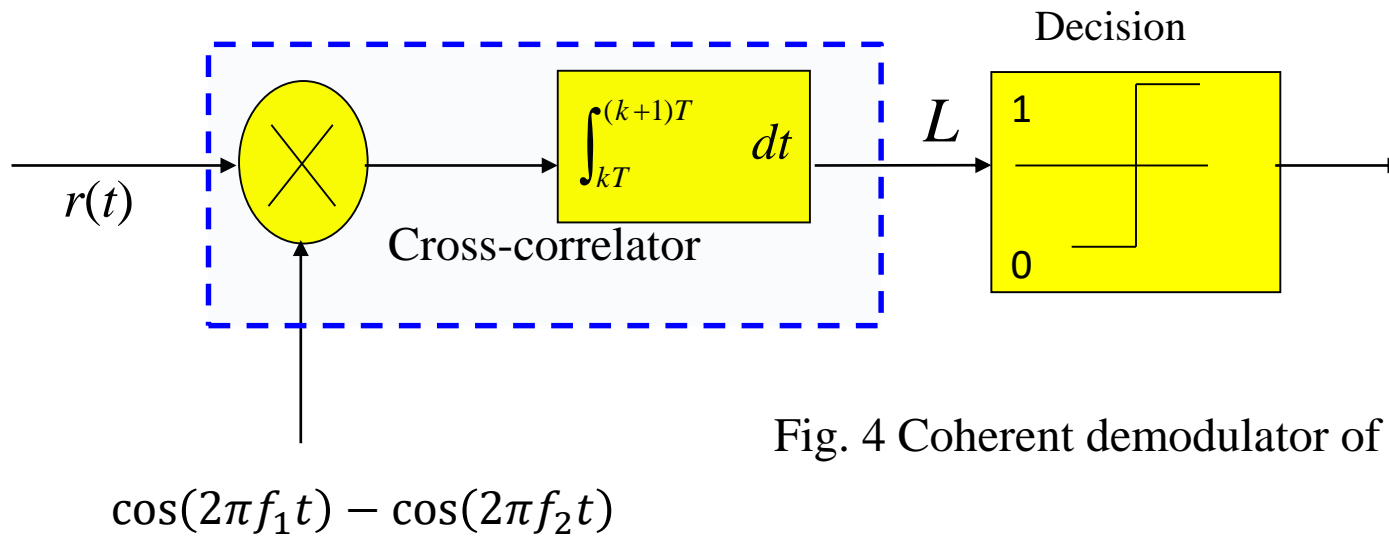


Fig. 4 Coherent demodulator of FSK

Similarly when $r(t) = S_2(t) + n(t)$ then, the output of the cross-correlator will be,

$$L = -\frac{AT}{2} + n' < 0$$

Considering, $(f_1 - f_2) = \frac{m}{T} - \frac{n}{T} = \frac{m-n}{T} = \frac{k}{T}$

Now decision can be taken based on the value of L .

Binary Phase Shift Keying (BPSK)

In BPSK binary logic 1 and 0 are presented by two sinusoidal waves of same frequency and same peak amplitude over the period $[kT, (k+1)T]$ but initial phases are 0 and π respectively. BPSK wave is expressed like,

$$S_1(t) = A \cos(2\pi f_c t + 0) = A \cos(2\pi f_c t) \quad ; \quad kT \leq t \leq (k+1)T \quad \text{for logic 1}$$

$$S_2(t) = A \cos(2\pi f_c t + \pi) = -A \cos(2\pi f_c t) \quad ; \quad kT \leq t \leq (k+1)T \quad \text{for logic 0}$$

Wave shape of BPSK is shown in fig.5.

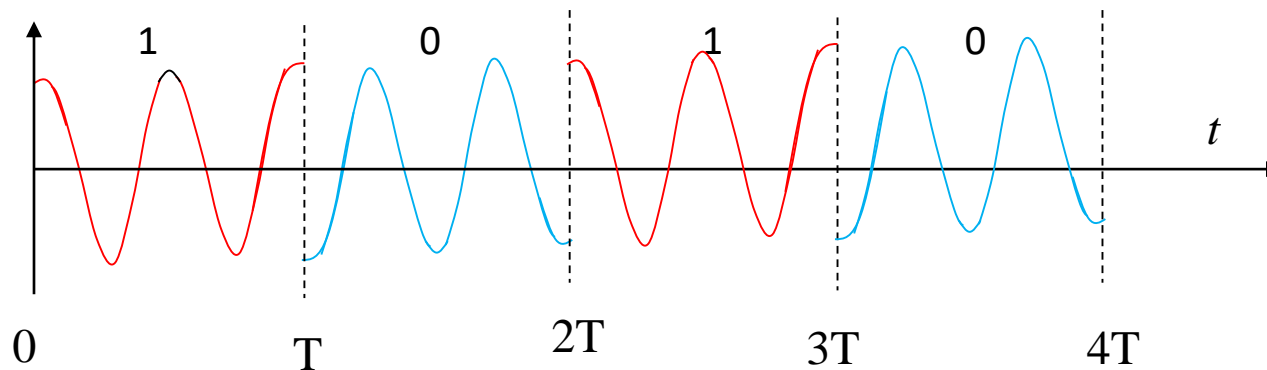
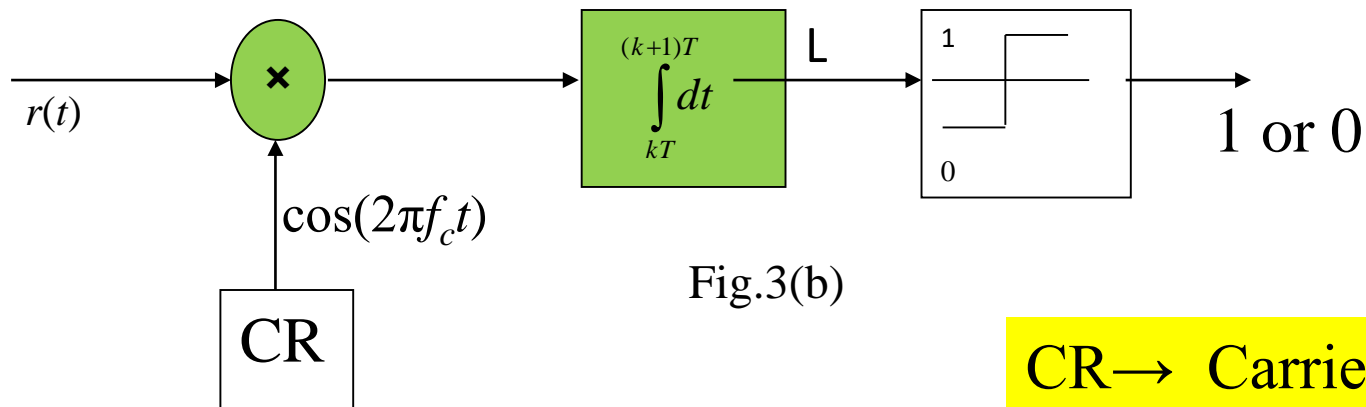
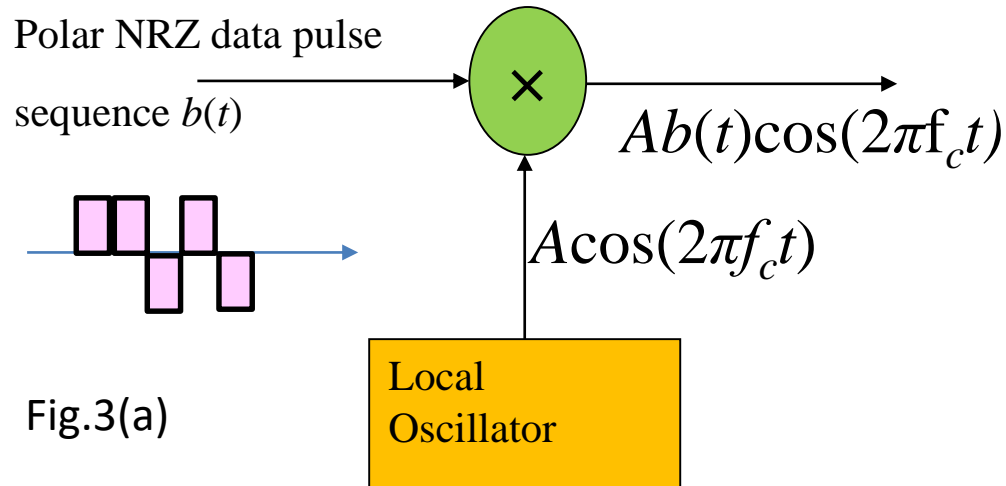


Fig. 5 BPSK waveform

Now BPSK **modulator** and its **coherent demodulator** can be implemented like fig. 3.

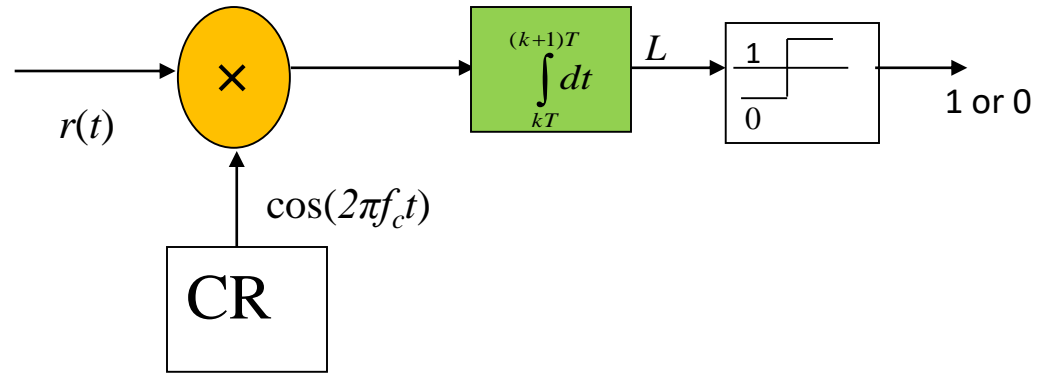


CR → Carrier Recovery

Fig.3 BPSK (a) Modulator (b) Demodulator

Here output of the correlator of demodulator assuming $S_1(t)$ is transmitted,

$$\begin{aligned}
 L &= \int_{kT}^{(k+1)T} r(t) \cos(2\pi f_c t) dt \\
 &= \int_{kT}^{(k+1)T} (S_1(t) + n(t)) \cos(2\pi f_c t) dt \\
 &= \int_{kT}^{(k+1)T} (A \cos(2\pi f_c t) + n(t)) \cos(2\pi f_c t) dt \\
 &= \int_{kT}^{(k+1)T} \{A \cos^2(2\pi f_c t) + n(t) \cos(2\pi f_c t)\} dt \\
 &= \frac{A}{2} \int_{kT}^{(k+1)T} \{1 + \cos(4\pi f_c t)\} dt + \int_{kT}^{(k+1)T} n(t) \cos(2\pi f_c t) dt \\
 &= \frac{AT}{2} + 0 + \int_{kT}^{(k+1)T} n(t) \cos(2\pi f_c t) dt = \frac{AT}{2}
 \end{aligned}$$



$$n' \approx 0$$

,Taking, $f_c = m/T$, i.e. m full cycles of carrier over the symbol period T .

If $S_2(t)$ is transmitted then, $L = -\frac{AT}{2}$

M-ARY PSK

In BPSK bit rate and modulation symbol rate are equal. In MPSK $n = \log_2 M$ bits are represented by a modulation symbol hence bit rate is n times higher than symbol rate. Therefore n times more information can be transmitted by MPSK (for the same **baud** or **symbol** rate) scheme compared to that of BPSK at the expense of bit error rate. MPSK wave is expressed as,

Throughput \rightarrow Information per unit time

$$S_i(t) = A \cos(2\pi f_c t + \theta_i)$$

$$n = \log_2 8 = \log_2 2^3 = 3 \log_2 2 \\ = 3 * 1 = 3 \text{ bits/symbol}$$

Where $0 \leq t \leq T$ $i = 1, 2, 3, \dots, M$; $\theta_i = \frac{(2i-1)\pi}{M}$

For example 8-PSK, $M = 8$ and $\theta_i = \frac{(2i-1)\pi}{8}$.

$$\text{Therefore, } \theta = \left\{ \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\}$$

The waves of 8-PSK becomes,

$$S_1(t) = A \cos\left(2\pi f_c t + \frac{\pi}{8}\right)$$

$$S_2(t) = A \cos\left(2\pi f_c t + \frac{3\pi}{8}\right)$$

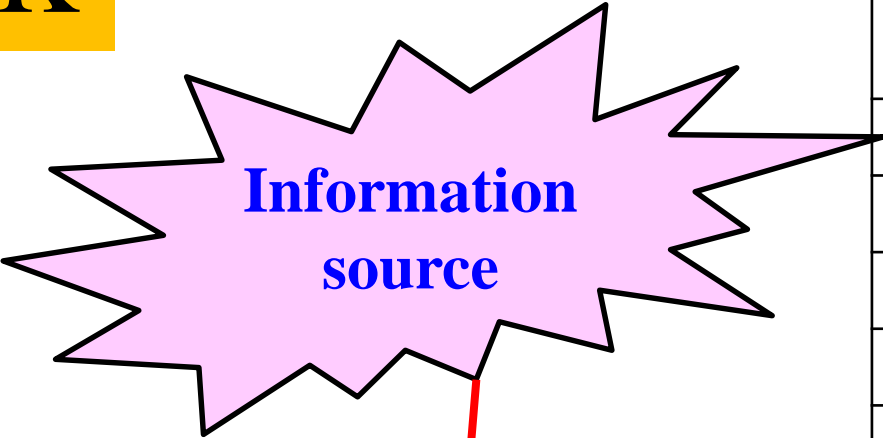
$$S_3(t) = A \cos\left(2\pi f_c t + \frac{5\pi}{8}\right)$$

...

$$S_8(t) = A \cos\left(2\pi f_c t + \frac{15\pi}{8}\right)$$

$$S_i(t) = A \cos(2\pi f_c t + \theta_i)$$
$$\theta = \left\{ \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\}$$

8-PSK

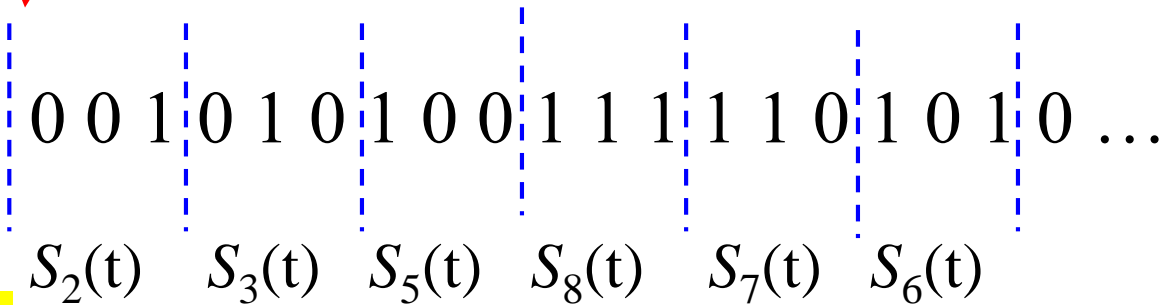


Bit Sequence	Waves	θ_i
000	$S_1(t)$	$\pi/8$
001	$S_2(t)$	$3\pi/8$
010	$S_3(t)$	$5\pi/8$
011	$S_4(t)$	$7\pi/8$
100	$S_5(t)$	$9\pi/8$
101	$S_6(t)$	$11\pi/8$
110	$S_7(t)$	$13\pi/8$
111	$S_8(t)$	$15\pi/8$

$S_1(t) = A \cos \left(2\pi f_c t + \frac{\pi}{8} \right)$

$S_2(t) = A \cos \left(2\pi f_c t + \frac{3\pi}{8} \right)$

... ..
 $S_8(t) = A \cos \left(2\pi f_c t + \frac{15\pi}{8} \right)$



$n = \log_2 8 = \log_2 2^3 = 3 \log_2 2$
 $= 3 * 1 = 3 \text{ bits/symbol}$

QPSK

Quadrature PSK is a special case of M -ary PSK (MPSK) where $M = 4$; Therefore the initial phases, $\theta_i = \frac{(2i-1)\pi}{4}$; $i = 1, 2, 3, 4$ provides $\pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$. Bit rate of QPSK scheme is lower than that of M-ARY PSK for $M > 4$ (for same baud rate) but provide better performance in context of bit error rate.

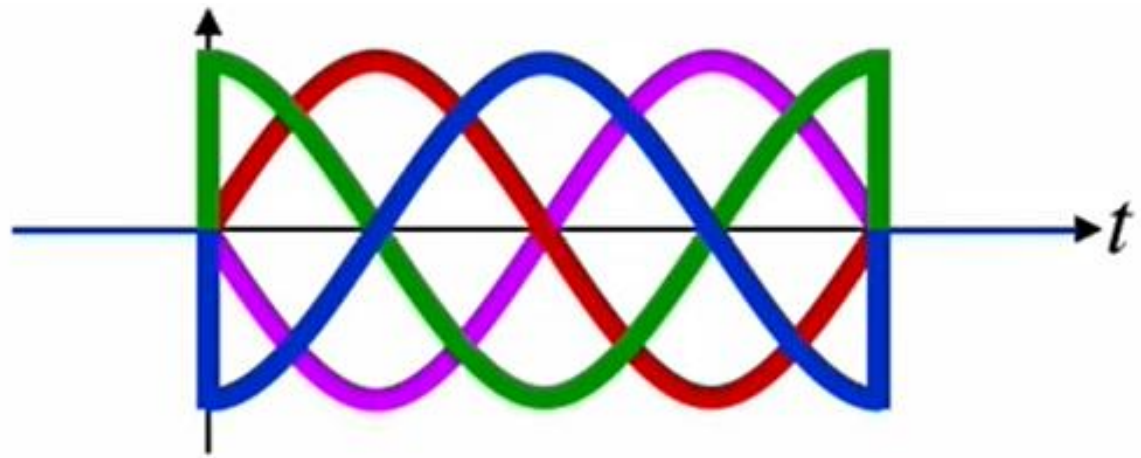
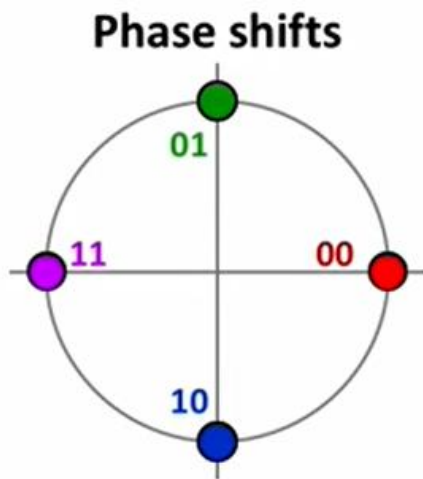
$$n = \log_2 M = \log_2 4 = 2 \text{ bits /symbol}$$

$$S_1(t) = A \sin(2 \pi f_c t + \pi/4)$$

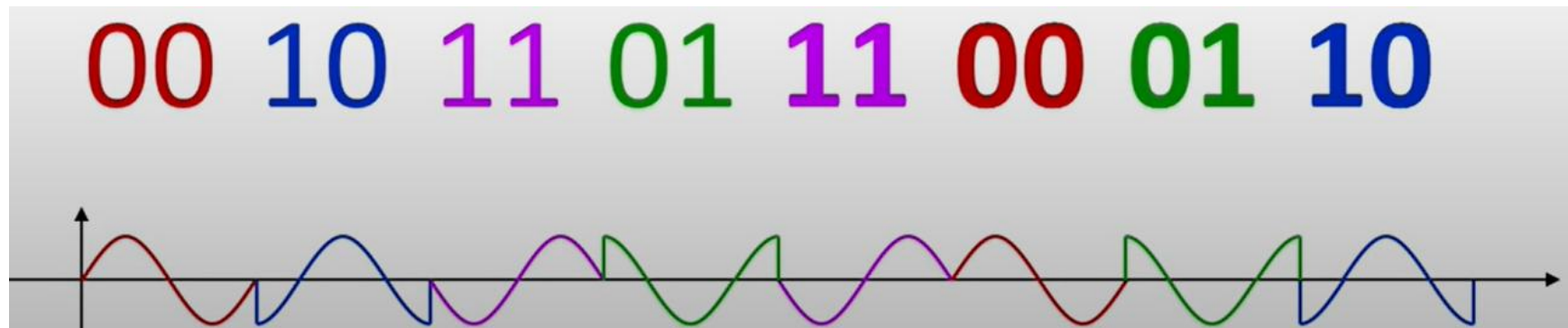
$$S_2(t) = A \sin(2 \pi f_c t + 3\pi/4)$$

$$S_3(t) = A \sin(2 \pi f_c t + 5\pi/4)$$

$$S_4(t) = A \sin(2 \pi f_c t + 7\pi/4)$$



The transmitted sequence under QPSK



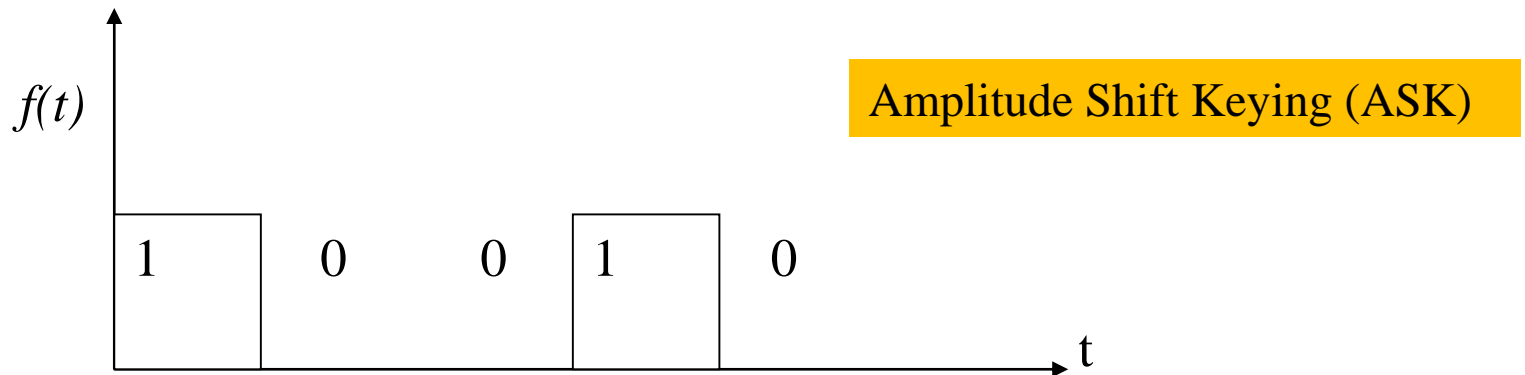
On-Off Keying (OOK)

In OOK binary logic 1 provides carrier wave over the symbol period $[0, T]$ and logic 0 turns the carrier off. The OOK signal is expressed as,

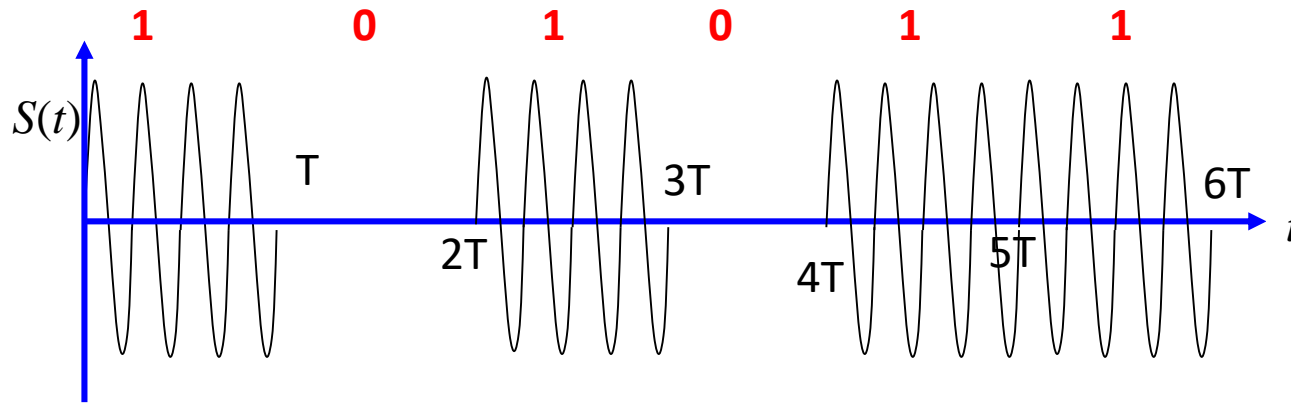
$$S_1(t) = A \cos 2\pi f_c t \quad \text{For logic 1} \quad kT \leq t \leq (k+1)T$$

$$S_2(t) = 0 \quad \text{for logic 0} \quad kT \leq t \leq (k+1)T$$

For binary sequence 1 0 0 1 0, $f(t)$ is shown in fig. below



OOK signal now can be expressed as, $S(t) = f(t).A \cos 2\pi f_c t$. The wave of OOK for 1 0 1 0 1 1 is shown in fig. below

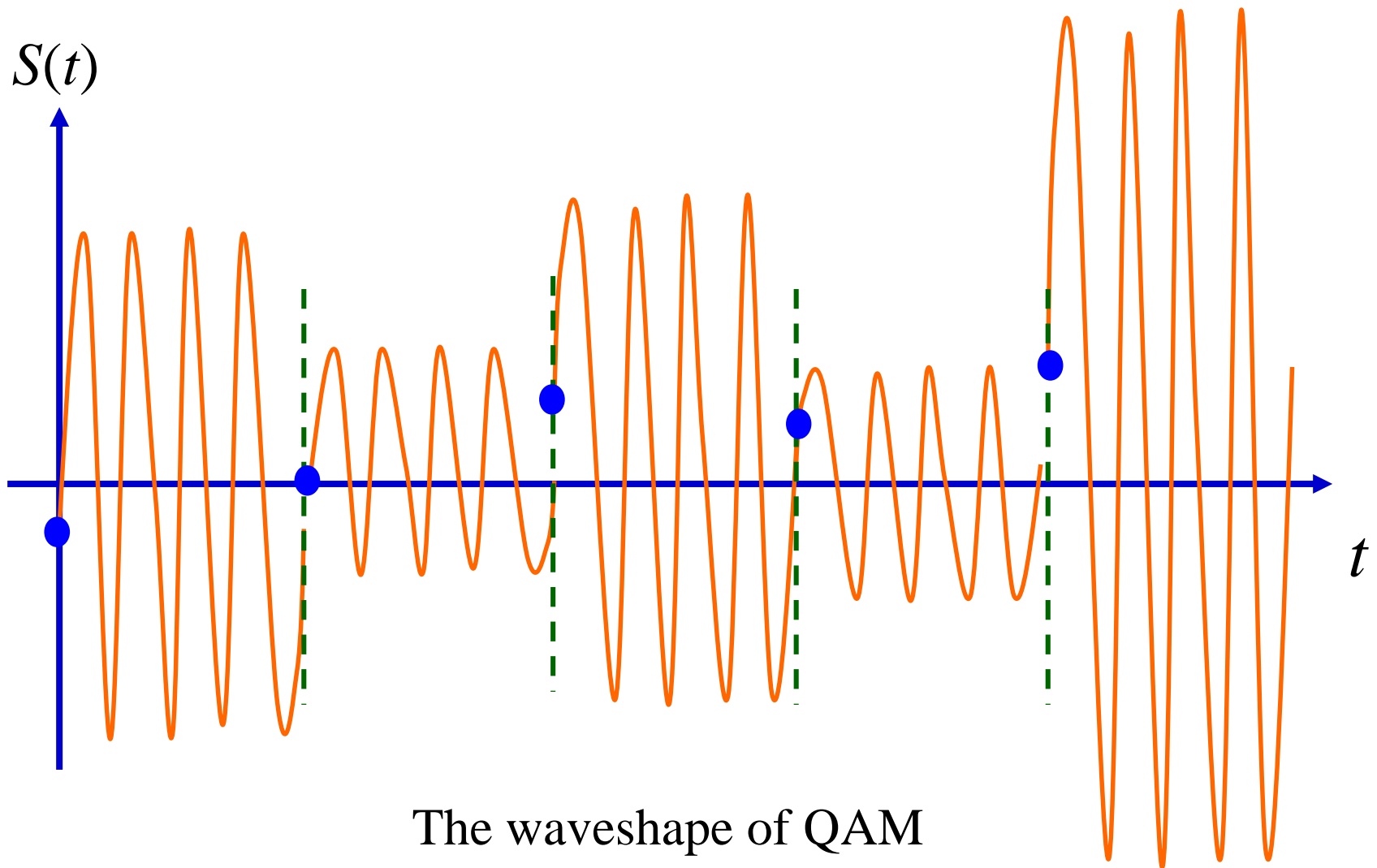


Quadrature Amplitude Modulation

In **QAM** both amplitude and phase of the carrier are modulated to enhance bandwidth efficiency. For M-QAM the i th symbol wave is expressed as,

$$\begin{aligned} S_i(t) &= A_i \cos(2\pi f_c t + \theta_i) \quad ; i = 1, 2, 3 \dots M \\ &= A_i \cos \theta_i \cos 2\pi f_c t - A_i \sin \theta_i \sin 2\pi f_c t \end{aligned}$$

Where $0 \leq t \leq T$



The waveshape of QAM