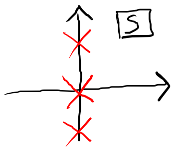
Esercizi su posizione dei poli e corrispondente andamento nel tempo

1) $F(s) = \dfrac{s^2+1}{s^3+10s} = \dfrac{s^2+1}{s(s^2+10)}$ $\quad a(s) = s(s^2+10)$

$a(s) = 0 \iff \begin{array}{l} s=0 \\ s^2+10=0 \quad s^2=-10 \quad s=\pm j\sqrt{10} \end{array}$

$p_1 = 0 \quad m_1 = 1$

$p_{2,3} = \pm j\sqrt{10} \quad m_{2,3} = 1$



$p_1 = 0 \Rightarrow$ modo di evoluzione $1(t)$ limitato non oscillante

$p_{2,3} = \pm j\sqrt{10} \Rightarrow$ modi di evoluzione $\sin(\sqrt{10}\,t)\cdot 1(t)$, $\cos(\sqrt{10}\,t)\cdot 1(t)$ limitati oscillanti

tutti poli con Re ≤ 0
AND
tutti quelli con Re $= 0$
hanno molteplicità 1

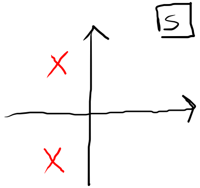
$\Rightarrow$ caso b) $f(t)$ è limitata

$\exists M : |f(t)| \leq M \quad \forall t \geq 0$

2) $F(s) = \frac{5s}{(s^2+s+1)^2}$ $\qquad a(s) = (s^2+s+1)^2$ $\qquad a(s)=0 \iff s^2+s+1=0$

$s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$p_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ $\qquad m_1 = m_2 = 2$



$\sigma_1 = -\frac{1}{2}$

$\omega_1 = \frac{\sqrt{3}}{2}$

tutti poli con $Re < 0$ $\Rightarrow$ caso e) $\qquad \lim_{t\to\infty} f(t) = 0$

modi di evoluzione

$\sin\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}t} \cdot 1(t)$, $\cos\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}t} \cdot 1(t)$ convergenti oscillanti

$t \sin\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}t} \cdot 1(t)$, $t \cos\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}t} \cdot 1(t)$ convergenti oscillanti

$t^{m_i-1} = t$

3) $F(s) = \dfrac{5\cancel{s}}{\cancel{s}(s^2+s+1)} = \dfrac{5}{s^2+s+1}$ $\quad a(s) = s^2+s+1$

$p_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ $\quad m_{1,2} = 1$

tutti poli con $Re < 0 \Rightarrow$ caso a) $\quad \lim_{t\to\infty} f(t) = 0$

modi di evoluzione $\quad e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\cdot 1(t),\ e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\cdot 1(t)$ $\quad$ convergenti oscillanti

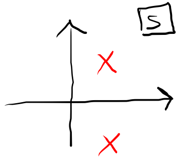
NOTA: possiamo applicare il teorema del valore finale

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s\cdot 5}{s^2+s+1} = 0$$

4) $F(s) = \frac{4s}{s^2 - s + 1}$ $\qquad Q(s) = s^2 - s + 1$ $\qquad Q(s) = 0 \iff s^2 - s + 1 = 0$

$$s = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$P_{1,2} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ $\qquad m_{1,2} = 1$



$\sigma_1 = \frac{1}{2}$

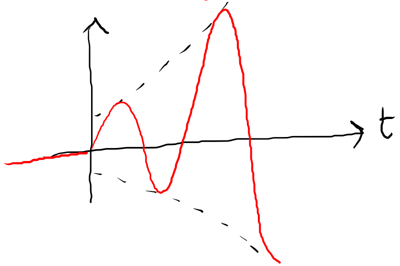
$\omega_1 = \frac{\sqrt{3}}{2}$

Esistono poli con Re > 0 $\Rightarrow$ caso c) $\qquad \lim_{t \to \infty} |f(t)| = \infty$

modi di evoluzione $\qquad e^{\sigma_1 t} \sin(\omega_1 t)\, 1(t),\; e^{\sigma_1 t} \cos(\omega_1 t)\, 1(t)$ $\qquad$ divergenti oscillanti

$$e^{\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) \cdot 1(t) \;,\; e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) \cdot 1(t)$$

$e^{\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) 1(t)$

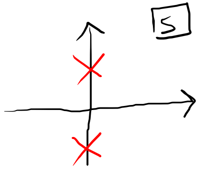


5) $F(s) = \dfrac{2s+1}{(s^2+4)^2}$ $Q(s) = (s^2+4)^2$ $Q(s)=0 \iff s^2+4=0$

$\iff s^2 = -4$

$s = \pm j2$

$p_{1,2} = \pm j2$ $m_{1,2} = 2$



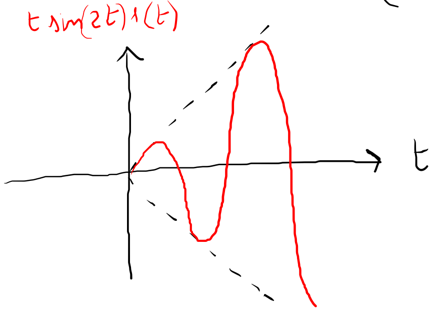
$\sigma_1 = 0$

$\omega_1 = 2$

Esistono poli con $Re = 0$ e molteplicità > 1 $\Rightarrow$ caso c) $\lim_{t \to \infty} |f(t)| = \infty$

modi di evoluzione $\sin(2t) \cdot 1(t)$, $\cos(2t) \cdot 1(t)$ limitati oscillanti

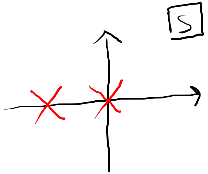
$t \sin(2t) \cdot 1(t)$, $t \cos(2t) \cdot 1(t)$ divergenti oscillanti



6) $F(s) = \dfrac{s}{s^3+2s^2} = \dfrac{\cancel{s}}{s^{\cancel{2}}(s+2)} = \dfrac{1}{s(s+2)}$ $\qquad Q(s) = s(s+2) \qquad Q(s)=0 \iff \begin{array}{l} s=0 \\ s=-2 \end{array}$

$P_1 = 0 \qquad m_1 = 1$

$P_1 = -2 \qquad m_2 = 1$



Tutti poli con Re ≤ 0
AND
i poli con Re $= 0$ hanno molteplicità 1

$\Rightarrow$ caso b) $\quad f(t)$ è limitata

Poli con Re < 0
+
1 polo in 0 con molteplicità 1

$\Rightarrow$ Vale il Teorema del valore finale

$$\lim_{t \to \infty} f(t) = \underbrace{\lim_{s \to 0} sF(s)}_{\text{residuo del polo in } 0} = \lim_{s \to 0} \frac{\overset{1}{\cancel{s}}}{\cancel{s}(s+2)} = \frac{1}{2}$$