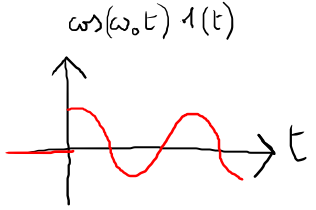
Esercizi sulla trasformata di Laplace

1) $f(t) = \cos(\omega_0 t) \cdot 1(t)$



$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$F(s) = \mathcal{L}\left\{\cos(\omega_0 t)\cdot 1(t)\right\} = \mathcal{L}\left\{\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\cdot 1(t)\right\}$$

linearità

$$= \frac{1}{2}\mathcal{L}\left\{e^{j\omega_0 t}\cdot 1(t)\right\} + \frac{1}{2}\mathcal{L}\left\{e^{-j\omega_0 t}\cdot 1(t)\right\}$$

$$\mathcal{L}\left\{e^{\lambda t}\cdot 1(t)\right\} = \frac{1}{s-\lambda}$$

$\lambda = j\omega_0$

$\lambda = -j\omega_0$

$$F(s) = \frac{1}{2}\frac{1}{s-j\omega_0} + \frac{1}{2}\frac{1}{s+j\omega_0} = \frac{1}{2}\frac{s+j\omega_0 + s-j\omega_0}{(s-j\omega_0)(s+j\omega_0)}$$

$$= \frac{1}{2}\frac{2s}{s^2-(j\omega_0)^2} = \frac{s}{s^2+\omega_0^2}$$

2) $f(t) = 2\left(1 - e^{-3t}\right) 1(t)$

$$F(s) = \mathcal{L}\left\{2\left(1 - e^{-3t}\right) 1(t)\right\} = 2\left(\mathcal{L}\left\{1(t)\right\} - \mathcal{L}\left\{e^{-3t}\, 1(t)\right\}\right)$$

$$\mathcal{L}\left\{1(t)\right\} = \frac{1}{s} \qquad \mathcal{L}\left\{e^{\lambda t}\, 1(t)\right\} = \frac{1}{s-\lambda} \qquad \lambda = -3$$

$$F(s) = 2\left(\frac{1}{s} - \frac{1}{s+3}\right)$$

3) $f(t) = \left[2\sin(3t) + 4\cos(3t)\right] 1(t)$

$$F(s) = \mathcal{L}\left\{\left[2\sin(3t) + 4\cos(3t)\right] 1(t)\right\}$$

$$= 2\,\mathcal{L}\left\{\sin(3t)\, 1(t)\right\} + 4\,\mathcal{L}\left\{\cos(3t) \cdot 1(t)\right\}$$

$$\mathcal{L}\left\{\sin(\omega_0 t) \cdot 1(t)\right\} = \frac{\omega_0}{s^2 + \omega_0^2} \qquad \mathcal{L}\left\{\cos(\omega_0 t) \cdot 1(t)\right\} = \frac{s}{s^2 + \omega_0^2}$$

$$\omega_0 = 3$$

$$F(s) = 2\,\frac{3}{s^2+9} + 4\,\frac{s}{s^2+9} = \frac{4s+6}{s^2+9}$$

4) $f(t) = e^{-2t} \sin(t) \cdot 1(t)$

$F(s) = \mathcal{L}\{ e^{-2t} \sin(t) \cdot 1(t) \}$

$\mathcal{L}\{ e^{\lambda t} g(t) \} = G(s)\Big|_{s=s-\lambda} = G(s-\lambda)$ ← Proprietà 2

$\lambda = -2 \qquad g(t) = \sin(t)$

$F(s) = \mathcal{L}\{ \sin(t) \cdot 1(t) \}\Big|_{s=s+2} = \dfrac{1}{s^2+1}\Big|_{s=s+2}$

$= \dfrac{1}{(s+2)^2+1} = \dfrac{1}{s^2+4s+4+1} = \dfrac{1}{s^2+4s+5}$