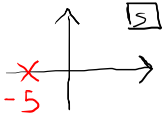
Esercizi sull'antitrasformata di Laplace

1) $$F(s) = \frac{5-s}{2s^2-50} = \frac{b(s)}{a(s)} = \frac{5-s}{2(s^2-25)} = \frac{\overset{-1}{\cancel{5-s}}}{2(s+5)\cancel{(s-5)}} = \frac{-\frac{1}{2}}{s+5}$$

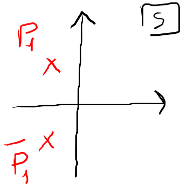


$a(s) = s+5 \qquad b(s) = -\frac{1}{2} \qquad\qquad p_1 = -5$

$$f(t) = \mathcal{L}^{-1}\left\{ \frac{-\frac{1}{2}}{s+5} \right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{ \frac{1}{s+5} \right\} = -\frac{1}{2} e^{-5t} \cdot 1(t)$$

modo di evoluzione e^{-5t} convergente non oscillante

2) $$F(s) = \frac{1}{s^2+s+1}$$



$a(s) = s^2+s+1 \qquad b(s) = 1 \qquad\qquad p_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{-3}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

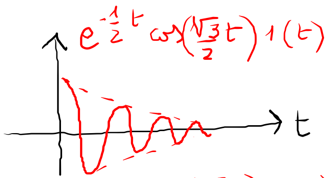
$p_2 = \bar{p}_1$

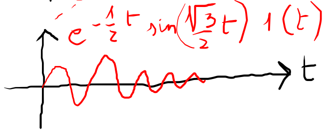
$p_1 = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = \sigma_1 + j\omega_1 \qquad \sigma_1 = -\frac{1}{2} \qquad \omega_1 = \frac{\sqrt{3}}{2}$

modi di evoluzione $\quad e^{\sigma_1 t}\cos(\omega_1 t)\cdot 1(t)\,, \quad e^{\sigma_1 t}\sin(\omega_1 t)\cdot 1(t)$

$e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\cdot 1(t)\,, \quad e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\cdot 1(t)$

convergenti oscillanti





$$F(s) = \frac{1}{s^2+s+1} \qquad P_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$F(s) = \frac{K_1}{s-P_1} + \frac{K_2}{s-P_2} = \frac{K_1}{s-P_1} + \frac{\overline{K_1}}{s-\overline{P_1}}$$

$$\boxed{K_1 = \alpha_1 + j\beta_1}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \left(K_1 e^{P_1 t} + \overline{K_1}\, e^{\overline{P_1} t}\right) 1(t)$$
$$= 2\left[\alpha_1 \cos(\omega_1 t) - \beta_1 \sin(\omega_1 t)\right] e^{\sigma_1 t}\, 1(t)$$

$$K_1 = \lim_{s \to P_1} (s-P_1)F(s) = \lim_{s \to -\frac{1}{2}+j\frac{\sqrt{3}}{2}} \frac{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}{s^2+s+1} = \lim_{s \to -\frac{1}{2}+j\frac{\sqrt{3}}{2}} \frac{\cancel{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}}{\underbrace{\cancel{\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}}_{(s-P_1)}\underbrace{\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}_{(s-\overline{P_1})}}$$

$$= \frac{1}{-\cancel{\frac{1}{2}}+j\frac{\sqrt{3}}{2}+\cancel{\frac{1}{2}}+j\frac{\sqrt{3}}{2}} = \frac{1}{j\sqrt{3}} = -\frac{j}{\sqrt{3}} = -j\frac{\sqrt{3}}{3}$$

$$\alpha_1 = 0 \qquad \beta_1 = -\frac{\sqrt{3}}{3} \qquad \sigma_1 = -\frac{1}{2} \qquad \omega_1 = \frac{\sqrt{3}}{2}$$

$$f(t) = \underbrace{-2\left(-\frac{\sqrt{3}}{3}\right)}_{2\frac{\sqrt{3}}{3}} \sin\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}t} \cdot 1(t) = \frac{2\sqrt{3}}{3} \sin\left(\frac{\sqrt{3}}{2}t\right) e^{-\frac{1}{2}} \cdot 1(t)$$

3) $F(s) = \dfrac{2}{s^3 + 4s}$

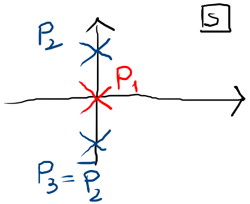
$a(s) = s^3 + 4s = s(s^2+4)$

$b(s) = 2$

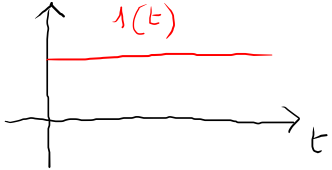
$a(s) = 0 \iff \begin{array}{l} s=0 \\ s^2+4=0 \quad s^2=-4 \quad s=\pm j2 \end{array}$

$\boxed{P_1 = 0}$

$\boxed{P_{2,3} = \pm j2}$



$P_1 = 0 \Rightarrow$ modo di evoluzione $e^{P_1 t} \cdot 1(t) = 1(t)$ limitato non oscillante

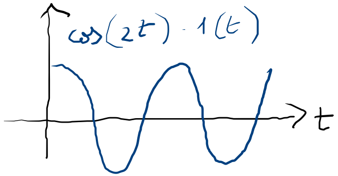


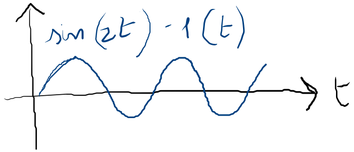
$P_{2,3} = \pm j2 \Rightarrow$ modi di evoluzione $e^{\sigma_2 t}\cos(\omega_2 t)\, 1(t)$, $e^{\sigma_2 t}\sin(\omega_2 t) \cdot 1(t)$

$\sigma_2 = 0 \quad \omega_2 = 2$

$\cos(2t) \cdot 1(t)$, $\sin(2t) \cdot 1(t)$

limitati oscillanti





$F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{s-j2} + \dfrac{\overline{K}_2}{s+j2}$

$K_2 = \alpha_2 + j\beta_2$

$f(t) = \mathcal{L}^{-1}\{F(s)\} = K_1 \cdot 1(t) + K_2 e^{j2t} + \overline{K}_2 e^{-j2t}$

$= K_1 1(t) + 2[\alpha_2 \cos(\omega_2 t) - \beta_2 \sin(\omega_2 t)] e^{\sigma_2 t} \cdot 1(t)$

$= K_1 1(t) + 2[\alpha_2 \cos(2t) - \beta_2 \sin(2t)] \cdot 1(t)$

$$F(s) = \frac{2}{s^3+4s}$$

$p_1 = 0 \qquad K_1 = \lim_{s \to p_1} (s-p_1) F(s) = \lim_{s \to 0} s \frac{2}{s^3+4s} = \lim_{s \to 0} \frac{2}{s^2+4} = \frac{1}{2}$

$p_2 = j2 \qquad K_2 = \lim_{s \to p_2} (s-p_2) F(s) = \lim_{s \to j2} (s-j2) \frac{2}{s(s^2+4)} = \lim_{s \to j2} \frac{2\cancel{(s-j2)}}{s\cancel{(s-j2)}(s+j2)}$

$$e(s) = s^3+4s = (s-p_1)(s-p_2)(s-p_3) = s(s-j2)(s+j2)$$

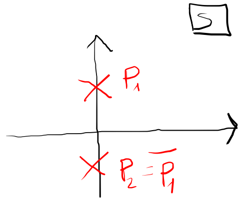
$$K_2 = \frac{2}{j2 \cdot j4} = \frac{2}{-8} = -\frac{1}{4}$$

$K_2 = \alpha_2 + j\beta_2 \qquad \alpha_2 = -\frac{1}{4} \qquad \beta_2 = 0$

$$f(t) = K_1 \, 1(t) + 2\left[\alpha_2 \cos(2t) - \beta_2 \sin(2t)\right] 1(t)$$

$$= \underbrace{\frac{1}{2} \cdot 1(t)}_{\text{limitato non oscillante}} \underbrace{- \frac{1}{2} \cos(2t) \cdot 1(t)}_{\text{limitato oscillante}}$$

4) $F(s) = \dfrac{5s+2}{s^2+1}$



$b(s) = 5s+2 \qquad a(s) = s^2+1 \qquad\qquad a(s)=0 \iff s^2=1 \iff s=\pm j$

$P_1 = j$

$P_2 = -j$

$$F(s) = \frac{5s}{s^2+1} + \frac{2}{s^2+1}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = 5\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$= 5\cos(t)\cdot 1(t) + 2\sin(t)\cdot 1(t)$$

modi di evoluzione $\cos(t)\cdot 1(t)$, $\sin(t)\cdot 1(t)$ limitati oscillanti

<u>NOTA</u>: nel caso di coppie di poli complessi coniugati $\sigma_i \pm j\omega_i$ i corrispondenti modi di evoluzione $e^{\sigma_i t}\cos(\omega_i t)1(t)$, $e^{\sigma_i t}\sin(\omega_i t)1(t)$ possono essere entrambi presenti (come nell'esercizio 4) oppure può essere presente solo uno dei due modi (come negli esercizi 2 e 3). <u>Dipende dal residuo K_i</u>