Custodial symmetry is one of those topics that nobody really explains thoroughly under the assumption that it is kind of straight forward. So let us go through it in a bit more detail than most books care to present. This article assumes some knowledge of the Standard Model and the underlying mathematics. We will skip many of the (relatively straight-forward) calculations. The emphasis is laid on the conceptual understanding.

In essence, custodial symmetry is the biggest symmetry we can find in the Higgs sector after SSB. Consider the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2,$$

where $\phi \to U\phi$ with $U \in \mathrm{SU}(2)$.
As it turns out the largest symmetry of $\mathcal{L}$ is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. We are interested in seeing how this symmetry carries through to the gauge bosons. If it does, it will generate a symmetry between gauge boson masses but if it doesn't then the masses will differ. Custodial symmetry is not an exact symmetry, we will see that it is always broken once we add hypercharge.

What does $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ mean exactly?

First, let us discuss what we mean when we say something like $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R (=: G)$. Let ϕ be a Higgs doublet, i.e. transforming in the **1/2** representation of SU(2). How does ϕ transform under G? Well, $\mathrm{SU}(2)_L$ is straight-forward, for $L \in \mathrm{SU}(2)_L$, $\phi \to L\phi$. But what about $\mathrm{SU}(2)_R$? Naively, $\phi \to \phi R^\dagger$ but this is incorrect. If ϕ is a column vector there is no such thing, mathematically. If it is a row vector, then $R^\dagger$ is just mixing the components of ϕ in a manner of SU(2), so it is identical to how $\mathrm{SU}(2)_L$ acts and we can simply write $G = \mathrm{SU}(2)_L$. So what exactly does $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ mean? To understand this let us write the transformation in index notation, $\phi_i \to L_{ij}\phi_j$. This is to say that i is the $\mathrm{SU}(2)_L$ index. If we want there to be another SU(2) which acts independently from $\mathrm{SU}(2)_L$ we have to add another index, since acting on the same index would make the groups identical. Adding another index to ϕ_i would make it a 2×2 matrix and

$$\begin{aligned}
\phi_{ij} &\underset{\mathrm{SU}(2)_L}{\to} L_{ik}\phi_{kj} \\
\phi_{ij} &\underset{\mathrm{SU}(2)_R}{\to} \phi_{ik}R^\dagger_{kj}.
\end{aligned}$$

We can also write the action of the entire group G more succinctly, $\phi_{ij} \to L_{ik}\phi_{kl}R^\dagger_{lj}$. This is known as the **bifundamental representation**. Of course, in general, this is not an irreducible representation. In fact, I have not told you how to non-trivially turn a doublet into a matrix. In general this is only possible by adding a second doublet. For example, in the Standard Model, one could add two different Higgs doublets ϕ and $\tilde{\phi}$. Note that L, the left part of the group, mixes the components of a multiplet with each other, while the right part mixes different multiplets with each other. So in this case R would mix ϕ with $\tilde{\phi}$ which might break the symmetry of your Lagrangian if not written in a way that treats the two multiplets on equal footing.

Constructing the bifundamental Higgs

There are two other ways one can generate a matrix out of a multiplet in two special cases. The first is the doublet (which is what makes the Standard Model Higgs doublet so special). To see it, note that the fundamental representation of SU(2) is pseudoreal, meaning its complex conjugate representation differs from the real representation by a similarity transformation and is therefore not an independent representation. Therefore, one can always find two doublets which transform the same way under the left group but, once a fixed basis is picked, are actually different objects. This allows us to write a matrix out of two doublets which have the same transformation properties under $\mathrm{SU}(2)_L$ but are distinct objects nonetheless, such that the action of $\mathrm{SU}(2)_R$ is non-trivial. Defining $\tilde{\phi} = \epsilon\phi^*$, we can write

$$\Phi = \frac{1}{\sqrt{2}}(\tilde{\phi}, \phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}.$$

Φ is now a 2×2 matrix with non-trivial transformation properties under both left and right. Left will mix the rows with each other while right will mix the matrix horizontally. This is why we say that the largest symmetry of $\mathcal{L}$ is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, rather than just SU(2) – the other SU(2) is hiding under the fact that SU(2) is pseudoreal.
Φ is in the (bi)fundamental representation which means that one of its basis matrices is the ordinary identity matrix $\mathbb{1}_2$. Hence we can pick the VEV in that direction,

$$\Phi_0 = \frac{v}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Φ_0 has the residual symmetry $L\Phi_0 R^\dagger$ which is only a symmetry if $L = R$. Thus the custodial symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is broken to the diagonal subgroup $\mathrm{SU}(2)_V$ which is our final custodial symmetry for the Standard Model Higgs doublet. The fact that the VEV has an SU(2) symmetry will have consequences for the gauge boson masses down the line. But before we discuss those, let us look at the other way of constructing a Higgs matrix using one representation only.
If, instead of a doublet, we work with a Higgs triplet $\phi = \phi^a T^a$, where T^a are the generators of SU(2) in some representation, then ϕ already is a matrix. The dimension depends on the exact representation chosen for the algebra but the important thing is now the vev must be picked in the direction of one of the generators since $\mathbb{1}_2$ is no longer a basis vector and does not, in fact, lie in the Lie algebra since it is not traceless. One simple choice is T^3, assuming that we work in a basis where it is the diagonal one. So,

$$\phi_0 = \frac{v}{2} T^3.$$

For example, in the triplet representation,

$$\phi_0 \to \frac{v}{2} L \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} R^\dagger.$$

The only way this is equal to ϕ_0 is if L, $R \in \mathrm{U}(1)$. This is the leftover "custodial symmetry", a much smaller symmetry than the one we got for the doublet in the bifundamental. In fact, this will yield no symmetry between the gauge boson masses and is therefore not even considered to be custodial. And given that the triplet is not a pseudoreal representation, we cannot employ the same trick with $\tilde{\phi}$. Thus we say that the triplet Higgs breaks custodial symmetry.

Gauging the custodial symmetry

Finally let us look at how all this affects the gauge boson masses. We will work with the Higgs doublet only. The task here is to see how our gauge bosons transform under the custodial symmetry. The gauge bosons arise from gauged symmetries but the custodial symmetry is a global one. To proceed, we want to determine which global custodial symmetries we can lift to local ones and identify with the Standard Model gauge symmetries. Writing $L = e^{il^a T^a}$, $R^\dagger = e^{-ir^a T^a}$, we get the covariant derivative

$$D_\mu \Phi = \partial_\mu \Phi + i\mathbf{l}\Phi - i\Phi\mathbf{r},$$

and the field now transforms as

$$\Phi \to \Phi + i\mathbf{l}\Phi - i\Phi\mathbf{r}.$$

Notice that when the symmetry breaks spontaneously, then the ground state $\Phi_0 \propto \mathbb{1}_2$ has the smaller symmetry $\mathrm{SU}(2)_V$ and $L = R$ so the transformation becomes

$$\Phi \to \Phi + i[\mathbf{l}, \Phi].$$

Now we have gauged both $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. We can now easily identify the gauged $\mathrm{SU}(2)_L$ with the one in the Standard Model. However, the Standard Model famously does not have any gauged $\mathrm{SU}(2)_R$ so what can we do? The trick is to somehow identify a part of this $\mathrm{SU}(2)_R$ with the only other gauged symmetry, $\mathrm{U}(1)_Y$.
So how does Φ transform under the hypercharge? Well Φ is made of two column vectors. The first ϕ, is the familiar Higgs doublet and it famously transforms with charge 1/2, so

$$\phi \to e^{i\frac{1}{2}B_\mu}\phi.$$

The other column, $\epsilon\phi^*$, transforms oppositely to ϕ due to the complex conjugation (and the real antisymmetric tensor ϵ doesn't act on the Abelian group), so

$$\epsilon\phi^* \to e^{-i\frac{1}{2}B_\mu}\epsilon\phi^*.$$

Thus

$$\begin{aligned}\Phi \to &\begin{pmatrix} e^{-i\frac{1}{2}B}\begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix} & e^{i\frac{1}{2}B}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\end{pmatrix}\\ &= \begin{pmatrix}\phi^{0*} & \phi^+\\ -\phi^- & \phi^0\end{pmatrix}\begin{pmatrix}e^{-i\frac{1}{2}B} & 0\\ 0 & e^{i\frac{1}{2}B}\end{pmatrix}.\end{aligned}$$

The second matrix we recognize as $e^{i\frac{1}{2}(-i\sigma^3)}$, so, formally

$$\begin{aligned}\Phi &\to \Phi e^{-i\frac{1}{2}\sigma^3}\\ &= \Phi - iYB_\mu\Phi\sigma^3.\end{aligned}$$

The covariant derivative then becomes

$$D_\mu\Phi = \partial_\mu\Phi - iYB_\mu\Phi\sigma^3.$$

Thus we have promoted the subgroup $\mathrm{U}(2)_Y \subseteq \mathrm{SU}(2)_R$ to a gauge symmetry by utilizing the fact that (only) the diagonal generator of $\mathrm{SU}(2)_R$ can be made local in accordance with the Standard Model.
We now have the full covariant derivative of the Higgs sector,

$$D_\mu\Phi = \partial_\mu\Phi + igW^a\sigma^a\Phi - ig'B\Phi\sigma^3.$$

The Lagrangian built out of this covariant derivative is invariant under a global $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ but only under a local $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ where we have identified the third direction of $\mathrm{SU}(2)_R$ with the $\mathrm{U}(1)_Y$.

So then, how do our gauge bosons respond to the custodial symmetry? Well, W^a transform in the adjoint, so $W^a\sigma^a \to UW^a\sigma^aU^\dagger$ (where $U \in \mathrm{SU}(2)_V$) gives an SU(2) triplet. However, $B\sigma^3 \to UB\sigma^3U^\dagger$ will no longer be aligned with $B\sigma^3$. In fact, if we look at the coefficients of the three basis generators, we see that W^1, W^2 are the coefficients of σ^1 and σ^2, making them a symmetric but the coefficient of σ^3 is $gW^3 - g'B$ which effectively rips W^3 out of the clean triplet structure, thus breaking the custodial symmetry. This demonstrates that the custodial symmetry we found in the Higgs sector is merely an approximate symmetry for small g'.

You can easily see the effects of the custodial symmetry and lack thereof by simply calculating the gauge boson masses. If you set $g' = 0$ and calculate the gauge boson masses for the doublet case you will see that the coefficients of $W^{1,2}$ and W^3 are equal and only the hypercharge mixes W^3 and B. Whereas, in the triplet case, W^3 will have a different coefficient even for $g' = 0$, manifestation of the lack of any natural structures to which L and R can couple non-trivially and independently.