

Série d'exercices 1

Mars 2020

Introduction

Exercice 1

Derive the matrices associated to the following differential operators and for a 3×3 image: D_x , D_y , Δ .

Exercice 2

Perform a SVD decomposition of the following matrices :

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -1 & 1 \end{pmatrix}.$$

Exercice 3

- Does the matrix $A = \begin{pmatrix} 4 & 3 \\ 8 & 6 \end{pmatrix}$ have a left inverse? A right inverse? A pseudoinverse?
If the answer to any of these questions is "yes", find the appropriate inverse.
- Prove that the left singular vectors of A are the right singular vectors of A^T .

Exercice 4

Let \mathbf{x} and \mathbf{c} are $(n \times 1)$ vectors and A an $(m \times n)$ matrix. Prove the following relations.

$f(\mathbf{x})$	$\nabla f(\mathbf{x})$
$\mathbf{c}^T \mathbf{x}$	\mathbf{c}
$A\mathbf{x}$	A
$\mathbf{x}^T A\mathbf{x}$	$\mathbf{x}^T (A^T + A)$
$\text{Tr}(XA)$	A^T
$\text{Tr}(X^T A)$	A
$\text{Tr}(X^T AX)$	$(A + A^T)X$