CS/ECE 374 Spring 2025 Heng An Cheng (hacheng2@illinois.edu)

Homework 3 Problem 1

Solution:

(a) NFA for all strings that contain 0110 or end in 101 is defined as follows:

$$M = (Q, \Sigma, \delta, s, A)$$

where:

- $Q = \{s, q_0, q_1, q_2, q_3, q_4, p_0, p_1, p_2, p_3\}$
- Alphabet: $\Sigma = \{0, 1\}$
- Transition function δ is defined as:

$$\delta(s, 0) = \delta(s, 1) = \{s\}$$

- Tracking 0110:

$$\delta(s, \epsilon) = \{q_0, p_0\}$$
$$\delta(q_0, 0) = \{q_1\}$$
$$\delta(q_1, 1) = \{q_2\}$$
$$\delta(q_2, 1) = \{q_3\}$$
$$\delta(q_3, 0) = \{q_4\}$$
$$\delta(q_4, 0) = \{q_4\}, \quad \delta(q_4, 1) = \{q_4\}$$

- Tracking 101:

$$\delta(p_0, 1) = \{p_1\}$$
$$\delta(p_1, 0) = \{p_2\}$$
$$\delta(p_2, 1) = \{p_3\}$$

- Start state: s
- Accepting states: $A = \{q_4, p_3\}$

- States q_0, q_1, q_2, q_3, q_4 tracking pattern 0110.

- States p_0, p_1, p_2, p_3 tracking pattern 101.

(b) NFA for

$$L = (111)^* + 1(01)^* + 0$$

is defined as follows:

$$M = (Q, \Sigma, \delta, s, A)$$

where:

- $Q = \{s, q_0, q_1, q_2, q_3, p_0, p_1, p_2, p_3, r_0\}$
- Alphabet: $\Sigma = \{0, 1\}$
- Transition function δ is defined as:

$$\delta(s, \epsilon) = \{q_0, p_0\}$$

- For $(111)^*$:

$$\delta(q_0, 1) = \{q_1\}$$
$$\delta(q_1, 1) = \{q_2\}$$
$$\delta(q_2, 1) = \{q_3\}$$
$$\delta(q_3, 1) = \{q_1, q_3\}$$

- For $1(01)^*$:

$$\delta(p_0, 1) = \{p_1\}$$
$$\delta(p_1, 0) = \{p_2\}$$
$$\delta(p_2, 1) = \{p_3\}$$
$$\delta(p_3, 0) = \{p_2, p_3\}$$

- For 0:

$$\delta(s, 0) = \{r_0\}$$

- Start state: s
- Accepting states: $A = \{q_0, q_3, p_1, p_3, r_0\}$

This NFA consists of:

- States q_0, q_1, q_2, q_3 tracking language $(111)^*$.

- States p_0, p_1, p_2, p_3 tracking language $1(01)^*$.

- State r_0 for language 0.

(c) NFA for

$$L = \{xbyb \mid x \in \Sigma^*,\, b \in \Sigma,\, y \in \Sigma^{372}\}$$

is defined as follows:

$$M = (Q, \Sigma, \delta, s, A)$$

where:

- $Q = \{s, q_0, q_1, \ldots, q_{373}, p_0, p_1, \ldots, p_{373}\}$
- Alphabet: $\Sigma = \{0, 1\}$

- Transition function δ is defined as:
  - For any input before the first b:

$$\delta(s,0) = \delta(s,1) = \{s, q_0, p_0\}$$

  - If $b = 0$:

$$\delta(q_i, 0) = \{q_{i+1}\}, \quad \delta(q_i, 1) = \{q_{i+1}\} \quad \text{for } 0 \leq i \leq 371$$

$$\delta(q_{372}, 0) = \{q_{373}\}$$

  - If $b = 1$:

$$\delta(p_i, 0) = \{p_{i+1}\}, \quad \delta(p_i, 1) = \{p_{i+1}\} \quad \text{for } 0 \leq i \leq 371$$

$$\delta(p_{372}, 1) = \{p_{373}\}$$

- Start state: s
- Accepting states: $A = \{q_{373}, p_{373}\}$

This NFA ensures that:

1. The automaton starts in s, which loops on any input to allow reading x.
2. It nondeterministically picks either q_0 (if $b = 0$) or p_0 (if $b = 1$).
3. It processes exactly 372 symbols in y.
4. The last character must match b to reach an accepting state.

(d) NFA for all strings over $\Sigma = \{a, b, \ldots, z\} \cup \{0, 1, \ldots, 9\}$ where every instance of the substring $cs173$ occurs before any instance of the substring $cs374$ is defined as follows:

$$M = (Q, \Sigma, \delta, s, A)$$

where:

- $Q = \{s, q_0, q_1, q_2, q_3, q_4, p_0, p_1, p_2, p_3, p_4\}$
- Alphabet: $\Sigma = \{a, b, \ldots, z, 0, 1, \ldots, 9\}$
- Transition function δ is defined as follows:
  - Detecting $cs374$:

$$\delta(s, \Sigma - \{c\}) = \{s\}, \quad \delta(s, c) = \{q_0\}$$
$$\delta(q_0, \Sigma - \{s\}) = \{s\}, \quad \delta(q_0, s) = \{q_1\}$$
$$\delta(q_1, \Sigma - \{3\}) = \{s\}, \quad \delta(q_1, 3) = \{q_2\}$$
$$\delta(q_2, \Sigma - \{7\}) = \{s\}, \quad \delta(q_2, 7) = \{q_3\}$$
$$\delta(q_3, \Sigma - \{4\}) = \{s\}, \quad \delta(q_3, 4) = \{q_4\}$$

  - Detecting $cs173$:

$$\delta(q_4, \Sigma - \{c\}) = \{q_4\}, \quad \delta(q_4, c) = \{p_0\}$$

$$\delta(p_0, \Sigma - \{s\}) = \{q_4\}, \quad \delta(p_0, s) = \{p_1\}$$
$$\delta(p_1, \Sigma - \{1\}) = \{q_4\}, \quad \delta(p_1, 1) = \{p_2\}$$
$$\delta(p_2, \Sigma - \{7\}) = \{q_4\}, \quad \delta(p_2, 7) = \{p_3\}$$
$$\delta(p_3, \Sigma - \{3\}) = \{q_4\}, \quad \delta(p_3, 3) = \{p_4\}$$
$$\delta(p_4, \Sigma) = \{p_4\}$$

- Start state: s
- Accepting states: $A = \{s, q_0, q_1, q_2, q_3, p_0, p_1, p_2, p_3\}$

This NFA consists of:

- States q_0, q_1, q_2, q_3, q_4 tracking the occurrence of $cs374$.

- States p_0, p_1, p_2, p_3, p_4 tracking the occurrence of $cs173$.

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Homework 3 Problem 2

Solution:

(a) Let $M = (Q, \delta, s, A)$ be the DFA that accepts L, and we construct a new DFA $M' = (Q', \delta', s', A')$ that accepts L_{shifted} as follows.

M' reads the input string w and simulates M running on input shift(w).

Intuitively, M' ignore the first symbol as long as it is 0 or 1 , and simulates M running on the rest of the string.

- The state (q, before) means (the simulation of) M is in state q and M' has not read first symbol.

- The state (q, after) means (the simulation of) M is in state q and M' has read first symbol.

$$Q' = Q \times \{\text{before, after}\}$$
$$s' = (s, \text{before})$$
$$A' = A \times \{\text{after}\}$$

$$\delta'((q, \text{before}), 0) = \delta'((q, \text{before}), 1) = (q, \text{after})$$
$$\delta'((q, \text{after}), 0) = (\delta(q, 0), \text{after})$$
$$\delta'((q, \text{after}), 1) = (\delta(q, 1), \text{after})$$

Since we successfully constructed a DFA M' that recognizes L_{shifted}, we conclude that L_{shifted} is regular.

(b) Let $M = (Q, \delta, s, A)$ be the DFA that accepts L, and we construct a new DFA $M' = (Q', \delta', s', A')$ that accepts L_{shifted} as follows.

M' reads the input string shift(w) and simulates M running on input w.

Intuitively, M' save the first symbol in buffer b, and simulates M running on the rest of the string.

State (q,b):

- The state (q, 1) means (the simulation of) M is in state q and M' gets 1 from the first symbol.

- The state (q, 0) means (the simulation of) M is in state q and M' gets 0 from the first symbol.

- The state (q, ϵ) means (the simulation of) M is in state q and M' has not get any value from the first symbol.

$$Q' = Q \times \{0, 1, \epsilon\}$$
$$s' = (s, \epsilon)$$

$$A' = \{ (q,b) \in Q \times \{0,1\} \mid \delta(q,b) \in A \} \cup \{ (q,\epsilon) \in Q \times \{\epsilon\} \mid q \in A \}.$$

$$\delta'((q,\epsilon),1) = (q,1)$$
$$\delta'((q,\epsilon),0) = (q,0)$$

$$\delta'((q,1),0) = (\delta(q,0),1)$$
$$\delta'((q,1),1) = (\delta(q,1),1)$$
$$\delta'((q,0),0) = (\delta(q,0),0)$$
$$\delta'((q,0),1) = (\delta(q,1),0)$$

Since we successfully constructed a DFA M' that recognizes $L_{\text{unshifted}}$, we conclude that $L_{\text{unshifted}}$ is regular.

(c) Let $M = (Q, \delta, s, A)$ be the DFA that accepts L, and we construct a new NFA $M' = (Q', \delta', s', A')$ that accepts L_{deleted} as follows.

M' reads the input string deleteOnes(w) and simulates M running on input w.

M' is a NFA that can nondeterministically skips $1s$ while processing $0s$ based on orignial DFA M.

$$Q' = Q$$
$$s' = s$$
$$A' = A$$

$$\delta'(q,1) = \{q, \delta(q,1)\}$$
$$\delta'(q,0) = \{\delta(q,0)\}$$

Since we successfully constructed a NFA M' that recognizes L_{deleted}, we conclude that L_{deleted} is regular.

(d) Let $M = (Q, \delta, s, A)$ be the DFA that accepts L, and we construct a new NFA $M' = (Q', \delta', s', A')$ that accepts $L_{\text{undeleted}}$ as follows.

M' reads the input string w and simulates M running on input deleteOnes(w).

M' is a NFA that nondeterministically removes 1 and processes $0s$ according to the original DFA M.

$$Q' = Q$$
$$s' = s$$
$$A' = A$$

$$\delta'(q,1) = \{q\}$$
$$\delta'(q,0) = \{\delta(q,0)\}$$

Since we successfully constructed a NFA M' that recognizes $L_{\text{undeleted}}$, we conclude that $L_{\text{undeleted}}$ is regular.

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