CSE 321 - Homework 2

Due date: 15/11/2022, 23:59

1. ~~***20 pts.*** Solve the following recurrence relations by using Master Theorem and give Θ bound for each of them. If any of the relations cannot be solved by using Master Theorem, state that this is indeed the case together with the explanation of the reason.~~

   (a) $\cancel{T(n) = 2 \cdot T(\frac{n}{4}) + \sqrt{nlogn}}$
   (b) $\cancel{T(n) = 9 \cdot T(\frac{n}{3}) + 5n^2}$
   (c) $\cancel{T(n) = \frac{1}{2} \cdot T(\frac{n}{2}) + n}$
   (d) $\cancel{T(n) = 5 \cdot T(\frac{n}{2}) + logn}$
   (e) $\cancel{T(n) = 4^n \cdot T(\frac{n}{5}) + 1}$
   (f) $\cancel{T(n) = 7 \cdot T(\frac{n}{4}) + nlogn}$
   (g) $\cancel{T(n) = 2 \cdot T(\frac{n}{3}) + \frac{1}{n}}$
   (h) $\cancel{T(n) = \frac{2}{3} \cdot T(\frac{n}{3}) + n^5}$

2. ~~***10 pts.*** Apply the insertion sort algorithm to the following array in ascending order. Explain every step in detail. What is the reasoning behind each operation? What is the updated version of the array?~~

   $\cancel{A = \{3, 6, 2, 1, 4, 5\}}$

3. ***20 pts.*** Consider an array and a linked list, both with n elements. Answer the following questions for both data structures.

   (a) Analyze the worst-case time complexity of the following operations. Explain your answer in detail.

      i. ~~Accessing the first element.~~
      ii. ~~Accessing the last element.~~
      iii. ~~Accessing any element in the middle.~~
      iv. ~~Adding a new element at the beginning.~~
      v. ~~Adding a new element at the end.~~
      vi. ~~Adding a new element in the middle.~~
      vii. Deleting the first element.
      viii. Deleting the last element.
      ix. ~~Deleting any element in the middle.~~

   (b) ~~Analyze the space requirements.~~

4. ~~***15 pts.*** Construct an algorithm that converts a given binary tree with size n to a binary search tree (BST). Make sure you preserve the structure of the tree, i.e. you should not add or delete a node. Write down the pseudo-code of the algorithm, explain your reasoning, and analyze the best-case, worst-case, and average-case time complexities.~~

5. ~~***15 pts.*** Consider an integer array $A = \{a_0, a_1, ..., a_n\}$ and an integer x. You are asked to find a pair (a_i, a_j), if any, within this array such that $|a_i - a_j| = x$. Design an algorithm with $O(n)$ time complexity to solve the problem. Write down the pseudo-code of the algorithm, and explain your reasoning in detail.~~

6. ~~***20 pts.*** For each of the statements below, indicate true or false with the explanation of the reason (explain the reason for each of them, not only for false ones).~~

   ~~(a) Shape of a BST (full, balanced, etc.) depends on the insertion order.~~

   ~~(b) The time complexity of accessing an element of a BST might be linear in some cases.~~

   ~~(c) Finding an array's maximum or minimum element can be done in constant time.~~

   ~~(d) The worst-case time complexity of binary search on a linked list is $O(log(n))$ where n is the length of the list.~~

   ~~(e) Worst-case time complexity of the insertion sort algorithm is $O(n)$ if the given array is reversely sorted.~~