CS224d
Deep NLP

Lecture 7:
Recurrent Neural Networks

Richard Socher

richard@metamind.io

Overview

- Traditional language models
- RNNs
- RNN language models
- Important training problems and tricks
  - Vanishing and exploding gradient problems
- Bidirectional RNNs
- RNNs for other sequence tasks

Language Models

A language model computes a probability for a sequence of words: $P(w_1, \ldots, w_T)$

- Useful for machine translation
  - Word ordering:
    p(the cat is small) > p(small the is cat)
  - Word choice:
    p(walking home after school) > p(walking house after school)

Traditional Language Models

- Probability is usually conditioned on window of n previous words
- An incorrect but necessary Markov assumption!

$$P(w_1, \ldots, w_m) = \prod_{i=1}^{m} P(w_i \mid w_1, \ldots, w_{i-1}) \approx \prod_{i=1}^{m} P(w_i \mid w_{i-(n-1)}, \ldots, w_{i-1})$$

- To estimate probabilities, compute for unigrams and bigrams (conditioning on one/two previous word(s):

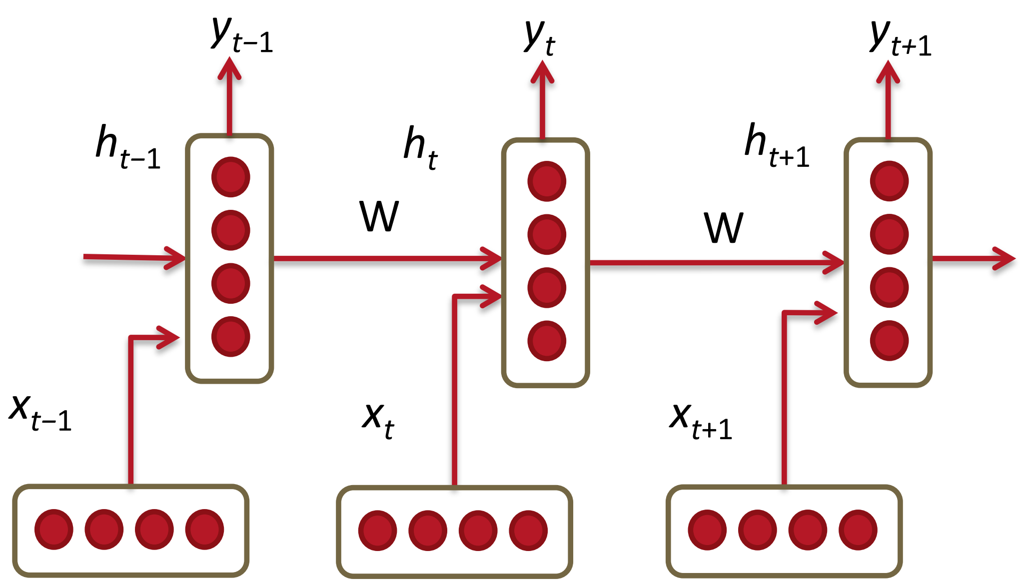
$$p(w_2|w_1) = \frac{\text{count}(w_1, w_2)}{\text{count}(w_1)} \qquad p(w_3|w_1, w_2) = \frac{\text{count}(w_1, w_2, w_3)}{\text{count}(w_1, w_2)}$$

Traditional Language Models

- Performance improves with keeping around higher n-grams counts and doing smoothing and so-called backoff (e.g. if 4-gram not found, try 3-gram, etc)
- There are A LOT of n-grams!
  → Gigantic RAM requirements!
- Recent state of the art: *Scalable Modified Kneser-Ney Language Model Estimation* by Heafield et al.:
  "Using one machine with 140 GB RAM for 2.8 days, we built an unpruned model on 126 billion tokens"

Recurrent Neural Networks!

- RNNs tie the weights at each time step
- Condition the neural network on all previous words
- RAM requirement only scales with number of words



Recurrent Neural Network language model

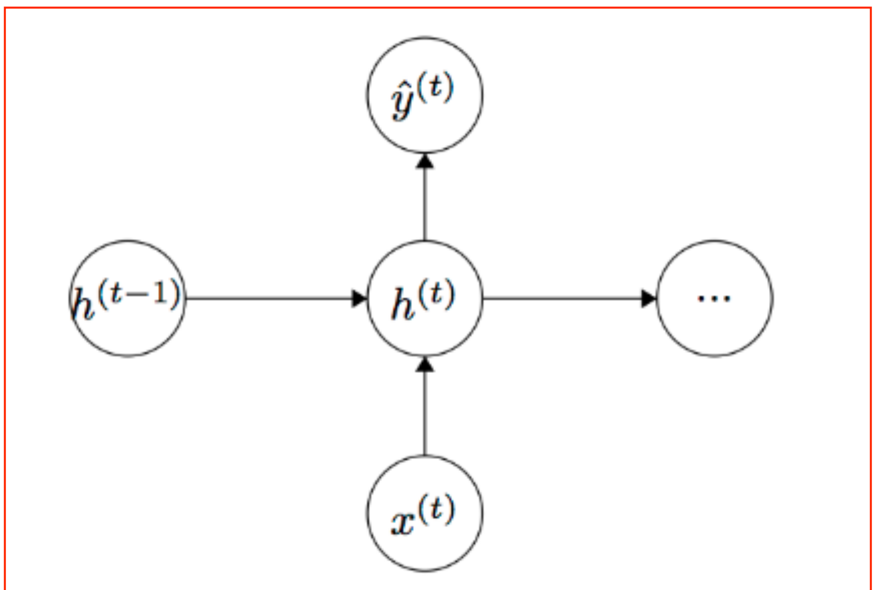
Given list of word **vectors:** $x_1, \ldots, x_{t-1}, x_t, x_{t+1}, \ldots, x_T$

At a single time step:

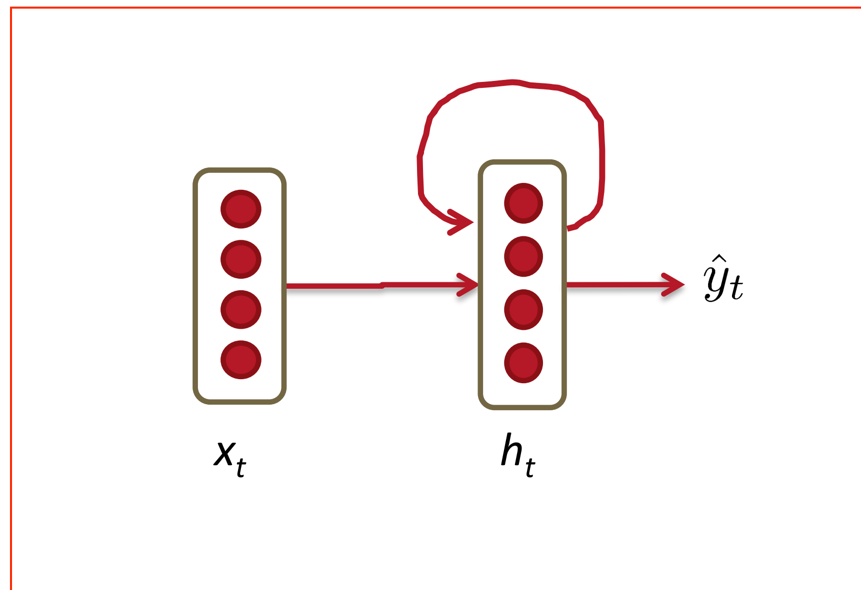
$$h_t = \sigma\left(W^{(hh)} h_{t-1} + W^{(hx)} x_{[t]}\right)$$

$$\hat{y}_t = \mathrm{softmax}\left(W^{(S)} h_t\right)$$

$$\hat{P}(x_{t+1} = v_j \mid x_t, \ldots, x_1) = \hat{y}_{t,j}$$



$\leftrightarrow$



Recurrent Neural Network language model

Main idea: we use the same set of W weights at all time steps!

Everything else is the same:

$$
\begin{aligned}
h_t &= \sigma\left(W^{(hh)}h_{t-1} + W^{(hx)}x_{[t]}\right) \\
\hat{y}_t &= \mathrm{softmax}\left(W^{(S)}h_t\right) \\
\hat{P}(x_{t+1} = v_j \mid x_t, \ldots, x_1) &= \hat{y}_{t,j}
\end{aligned}
$$

$h_0 \in \mathbb{R}^{D_h}$ is some initialization vector for the hidden layer at time step 0

$x_{[t]}$ is the column vector of L at index [t] at time step t

$W^{(hh)} \in \mathbb{R}^{D_h \times D_h}$ $\quad W^{(hx)} \in \mathbb{R}^{D_h \times d}$ $\quad W^{(S)} \in \mathbb{R}^{|V| \times D_h}$

Recurrent Neural Network language model

$\hat{y} \in \mathbb{R}^{|V|}$ is a probability distribution over the vocabulary

Same cross entropy loss function but predicting words instead of classes

$$J^{(t)}(\theta) = -\sum_{j=1}^{|V|} y_{t,j} \log \hat{y}_{t,j}$$

Recurrent Neural Network language model

Evaluation could just be negative of average log probability over dataset of size (number of words) T:

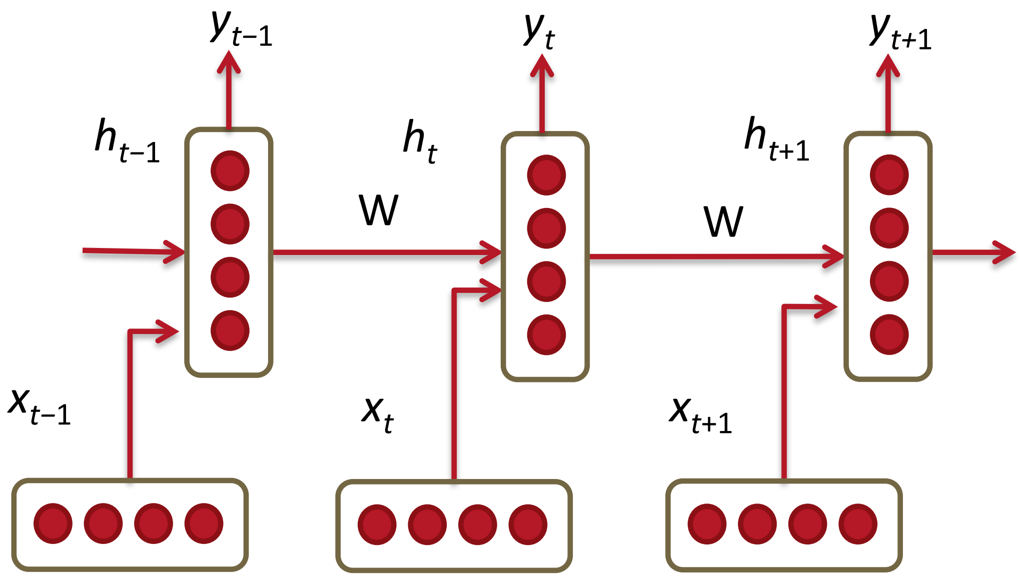
$$J = -\frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{|V|} y_{t,j} \log \hat{y}_{t,j}$$

But more common: Perplexity: 2^J

Lower is better!

Training RNNs is hard

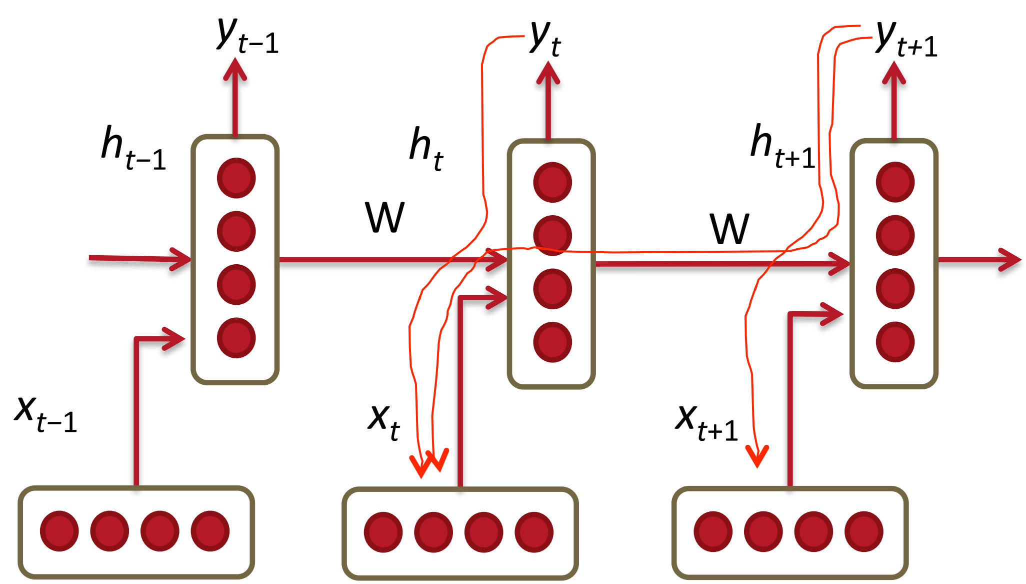
- Multiply the same matrix at each time step during forward prop



- Ideally inputs from many time steps ago can modify output *y*
- Take $\frac{\partial E_2}{\partial W}$ for an example RNN with 2 time steps! Insightful!

The vanishing/exploding gradient problem

- Multiply the same matrix at each time step during backprop



The vanishing gradient problem - Details

- Similar but simpler RNN formulation:

$$\begin{aligned} h_t &= W f(h_{t-1}) + W^{(hx)} x_{[t]} \\ \hat{y}_t &= W^{(S)} f(h_t) \end{aligned}$$

- Total error is the sum of each error at time steps t

$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

- Hardcore chain rule application:

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W}$$

The vanishing gradient problem - Details

- Similar to backprop but less efficient formulation
- Useful for analysis we'll look at:

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \boxed{\frac{\partial h_t}{\partial h_k}} \frac{\partial h_k}{\partial W}$$

- Remember: $h_t = W f(h_{t-1}) + W^{(hx)} x_{[t]}$
- More chain rule, remember:

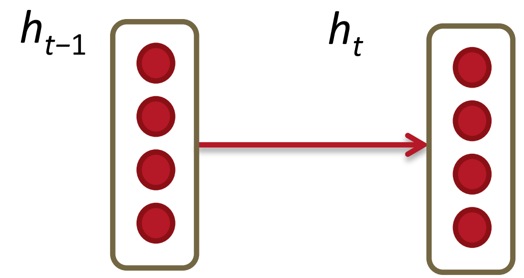
$$\frac{\partial h_t}{\partial h_k} = \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}}$$

- Each partial is a Jacobian:

$$\frac{d\mathbf{f}}{d\mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial x_1} & \cdots & \frac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

The vanishing gradient problem - Details

- From previous slide: $\frac{\partial h_t}{\partial h_k} = \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}}$



- Remember: $h_t = W f(h_{t-1}) + W^{(hx)} x_{[t]}$

- To compute Jacobian, derive each element of matrix: $\frac{\partial h_{j,m}}{\partial h_{j-1,n}}$

$$\frac{\partial h_t}{\partial h_k} = \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}} = \prod_{j=k+1}^{t} W^T \mathrm{diag}[f'(h_{j-1})]$$

- Where: $\mathrm{diag}(z) = \begin{pmatrix} z_1 & & & & \\ & z_2 & & 0 & \\ & & \ddots & & \\ & 0 & & z_{n-1} & \\ & & & & z_n \end{pmatrix}$

Check at home that you understand the diag matrix formulation

The vanishing gradient problem - Details

- Analyzing the norms of the Jacobians, yields:

$$\left\| \frac{\partial h_j}{\partial h_{j-1}} \right\| \leq \|W^T\| \|\text{diag}[f'(h_{j-1})]\| \leq \beta_W \beta_h$$

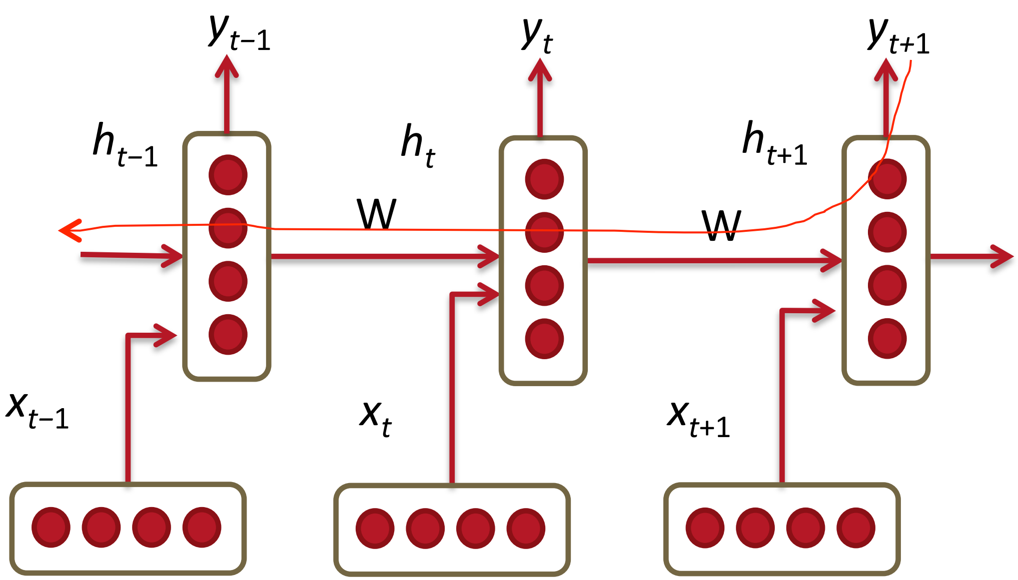
- Where we defined β's as upper bounds of the norms
- The gradient is a product of Jacobian matrices, each associated with a step in the forward computation.

$$\left\| \frac{\partial h_t}{\partial h_k} \right\| = \left\| \prod_{j=k+1}^{t} \frac{\partial h_j}{\partial h_{j-1}} \right\| \leq (\beta_W \beta_h)^{t-k}$$

- This can become very small or very large quickly [Bengio et al 1994], and the locality assumption of gradient descent breaks down. → **Vanishing or exploding gradient**

Why is the vanishing gradient a problem?

- The error at a time step ideally can tell a previous time step from many steps away to change during backprop



The vanishing gradient problem for language models

- In the case of language modeling or question answering words from time steps far away are not taken into consideration when training to predict the next word

- Example:

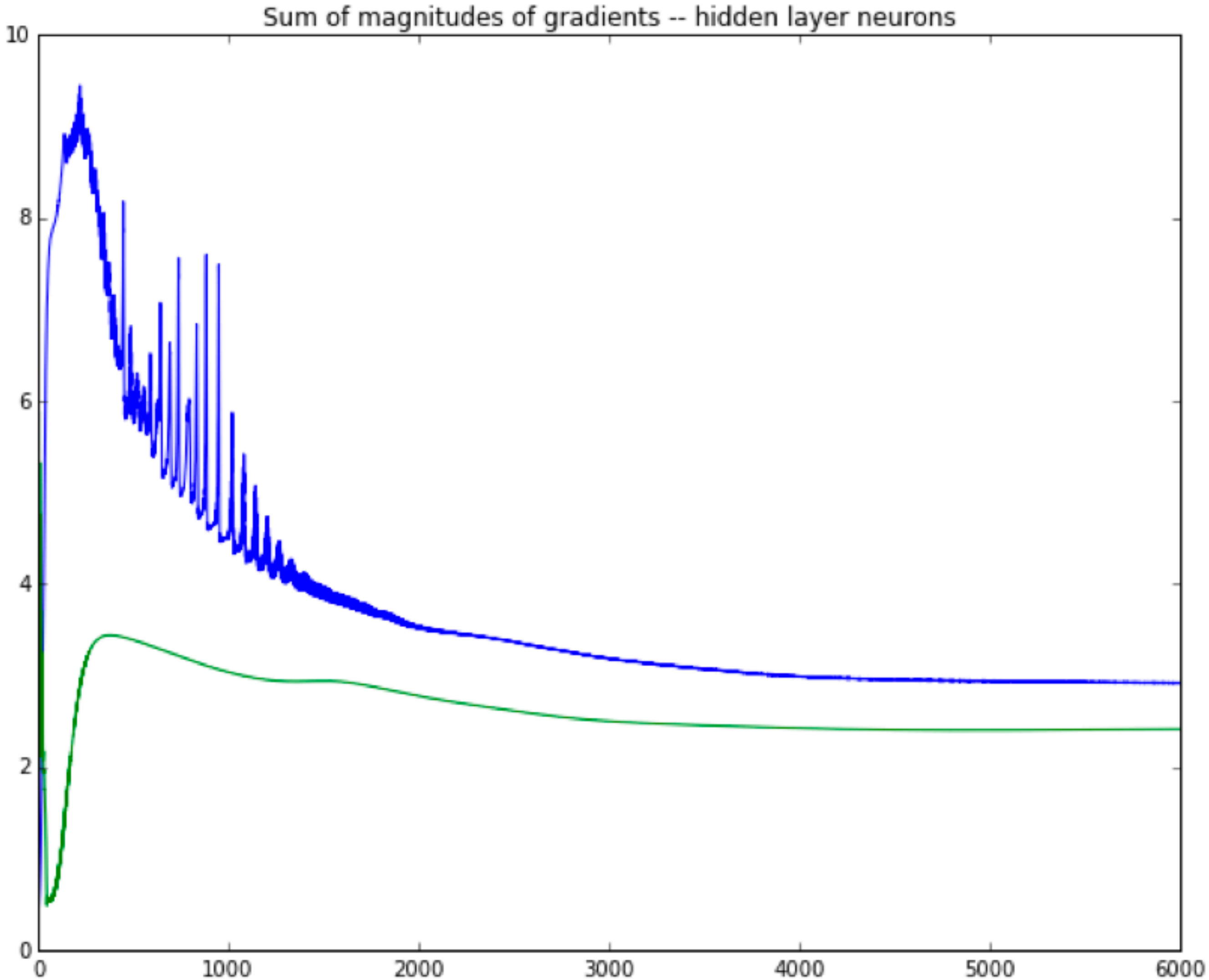
  Jane walked into the room. John walked in too. It was late in the day. Jane said hi to ____

IPython Notebook with vanishing gradient example

- Example of simple and clean NNet implementation
- Comparison of sigmoid and ReLu units
- A little bit of vanishing gradient

In [21]:
```
plt.plot(np.array(relu_array[:6000]),color='blue')
plt.plot(np.array(sigm_array[:6000]),color='green')
plt.title('Sum of magnitudes of gradients -- hidden layer neurons')
```

Out[21]:
```
<matplotlib.text.Text at 0x10a331310>
```



Trick for exploding gradient: clipping trick

- The solution first introduced by Mikolov is to clip gradients to a maximum value.

Algorithm 1 Pseudo-code for norm clipping the gradients whenever they explode

---

$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$
if $\|\hat{\mathbf{g}}\| \geq threshold$ **then**
  $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|}\hat{\mathbf{g}}$
end if

- Makes a big difference in RNNs.

Gradient clipping intuition

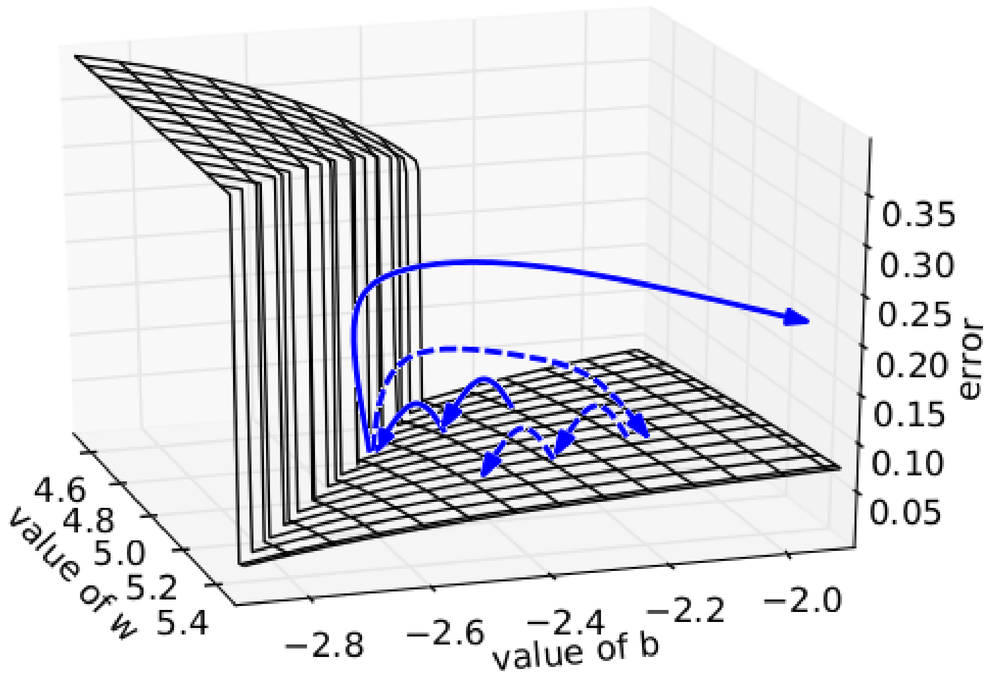


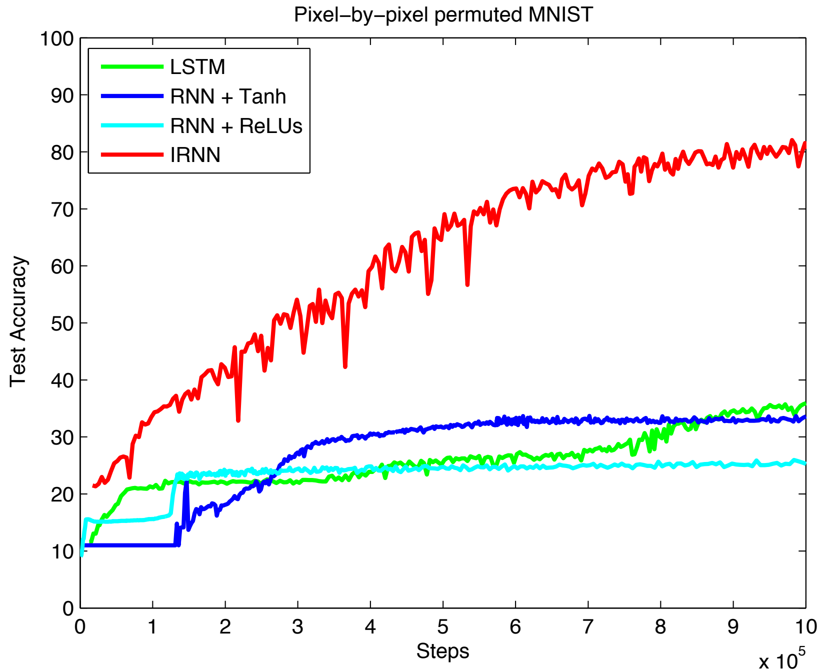
Figure from paper:
On the difficulty of training Recurrent Neural Networks, Pascanu et al. 2013

- Error surface of a single hidden unit RNN,
- High curvature walls
- Solid lines: standard gradient descent trajectories
- Dashed lines gradients rescaled to fixed size

For vanishing gradients: Initialization + ReLus!

- Initialize $W^{(*)}$'s to identity matrix I
  and
  f(z) $= \mathrm{rect}(z) = \max(z, 0)$

- → Huge difference!



- Initialization idea first introduced in *Parsing with Compositional Vector Grammars*, Socher et al. 2013

- New experiments with recurrent neural nets 2 weeks ago (!) in *A Simple Way to Initialize Recurrent Networks of Rectified Linear* Units, Le et al. 2015

Perplexity Results

KN5 = Count-based language model with Kneser-Ney smoothing & 5-grams

Table 2. *Comparison of different neural network architectures on Penn Corpus (1M words) and Switchboard (4M words).*

	Penn Corpus		Switchboard	
Model	NN	NN+KN	NN	NN+KN
KN5 (baseline)	-	141	-	92.9
feedforward NN	141	118	85.1	77.5
RNN trained by BP	137	113	81.3	75.4
RNN trained by BPTT	123	106	77.5	72.5

Table from paper *Extensions of recurrent neural network language model* by Mikolov et al 2011

Problem: Softmax is huge and slow

Trick: Class-based word prediction

$$p(w_t|\text{history}) = p(c_t|\text{history})p(w_t|c_t)$$

$$= p(c_t|h_t)p(w_t|c_t)$$

The more classes, the better perplexity but also worse speed:

Table 3. *Perplexities on Penn corpus with factorization of the output layer by the class model. All models have the same basic configuration (200 hidden units and BPTT=5). The Full model is a baseline and does not use classes, but the whole 10K vocabulary.*

Classes	RNN	RNN+KN5	Min/epoch	Sec/test
30	134	112	12.8	8.8
50	136	114	9.8	6.7
100	136	114	9.1	5.6
200	136	113	9.5	6.0
400	134	112	10.9	8.1
1000	131	111	16.1	15.7
2000	128	109	25.3	28.7
4000	127	108	44.4	57.8
6000	127	109	70	96.5
8000	124	107	107	148
Full	123	106	154	212

Sequence modeling for other tasks

- Classify
  - NER
  - the sentiment of each word in its context
  - opinionated expressions

- Example application and slides from paper *Opinion Mining with Deep Recurrent Nets* by Irsoy and Cardie 2014

Opinion Mining with Deep Recurrent Nets

Goal: Classify each word as

direct subjective expressions (DSEs) and
expressive subjective expressions (ESEs).

DSE: Explicit mentions of private states or speech events expressing private states

ESE: Expressions that indicate sentiment, emotion, etc. without explicitly conveying them.

Example Annotation

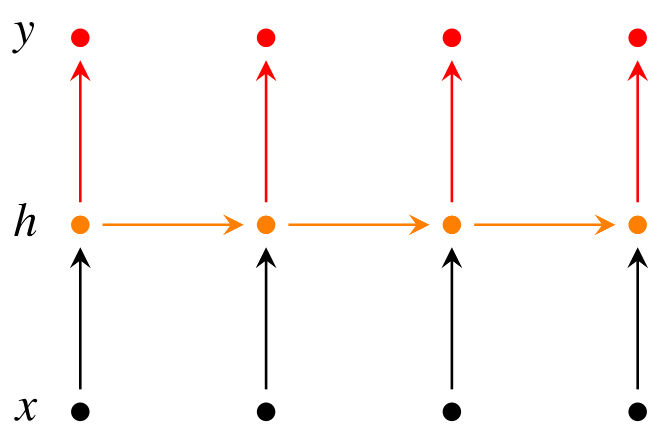
In BIO notation (tags either begin-of-entity (B_*X*) or continuation-of-entity (I_*X*)):
The committee, [as usual]$_{\text{ESE}}$, [has refused to make any statements]$_{\text{DSE}}$.

The	committee	,	as	usual	,	has
O	O	O	B_ESE	I_ESE	O	B_DSE

refused	to	make	any	statements	.
I_DSE	I_DSE	I_DSE	I_DSE	I_DSE	O

Approach: Recurrent Neural Network

- Notation from paper (so you get used to different ones)

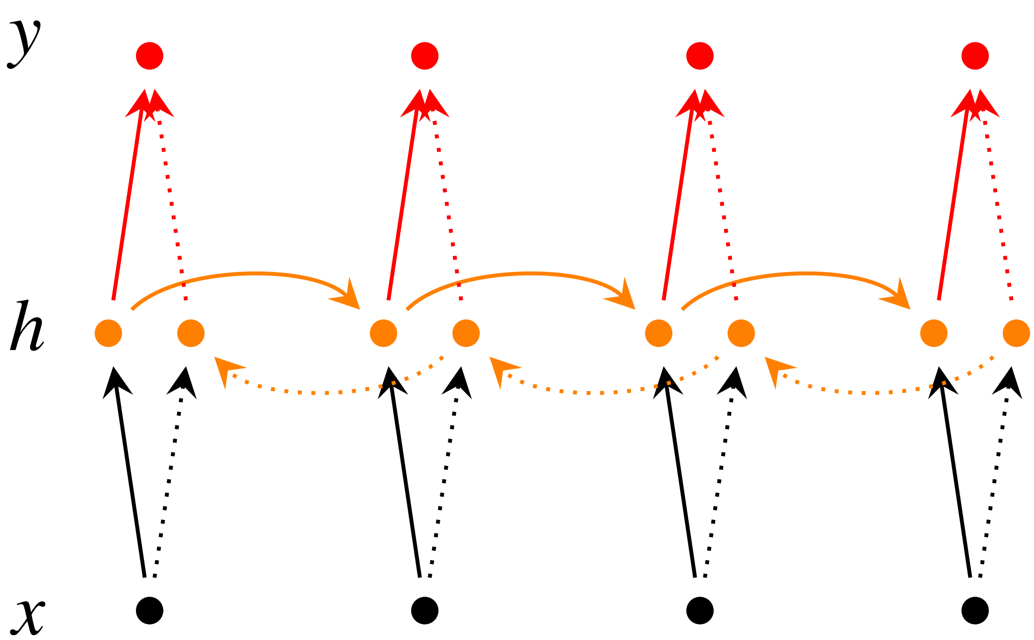


$$h_t = f(Wx_t + Vh_{t-1} + b)$$
$$y_t = g(Uh_t + c)$$

- *x* represents a token (word) as a vector.
- *y* represents the output label (B, I or O) – g = softmax !
- *h* is the memory, computed from the past memory and current word. It summarizes the sentence up to that time.

Bidirectional RNNs

Problem: For classification you want to incorporate information from words both preceding and following



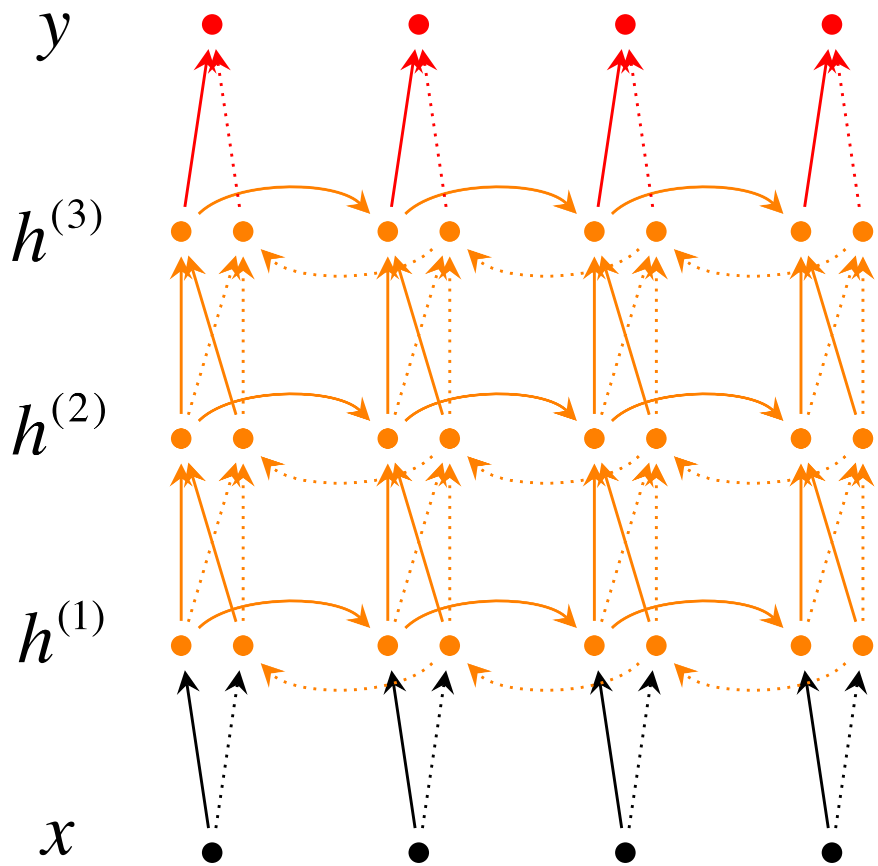
$$\overrightarrow{h}_t = f(\overrightarrow{W} x_t + \overrightarrow{V}\overrightarrow{h}_{t-1} + \overrightarrow{b})$$

$$\overleftarrow{h}_t = f(\overleftarrow{W} x_t + \overleftarrow{V}\overleftarrow{h}_{t+1} + \overleftarrow{b})$$

$$y_t = g(U[\overrightarrow{h}_t;\overleftarrow{h}_t]+c)$$

$h = [\overrightarrow{h};\overleftarrow{h}]$ now represents (summarizes) the past and future around a single token.

Deep Bidirectional RNNs



$$\overrightarrow{h}_t^{(i)} = f(\overrightarrow{W}^{(i)} h_t^{(i-1)} + \overrightarrow{V}^{(i)} \overrightarrow{h}_{t-1}^{(i)} + \overrightarrow{b}^{(i)})$$

$$\overleftarrow{h}_t^{(i)} = f(\overleftarrow{W}^{(i)} h_t^{(i-1)} + \overleftarrow{V}^{(i)} \overleftarrow{h}_{t+1}^{(i)} + \overleftarrow{b}^{(i)})$$

$$y_t = g(U[\overrightarrow{h}_t^{(L)}; \overleftarrow{h}_t^{(L)}] + c)$$

Each memory layer passes an intermediate sequential representation to the next.

Data

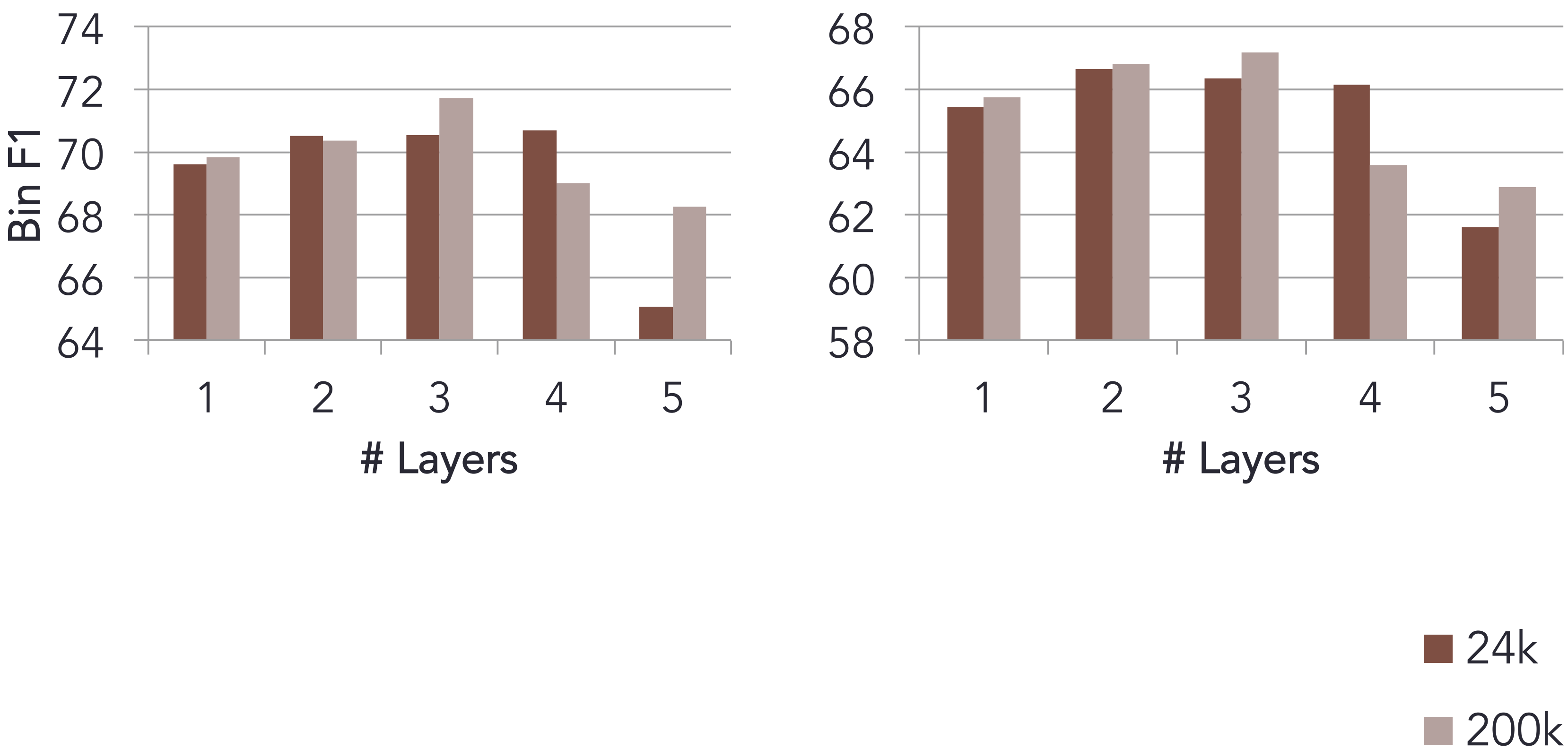
- MPQA 1.2 corpus (Wiebe et al., 2005)
- consists of 535 news articles (11,111 sentences)
- manually labeled with DSE and ESEs at the phrase level
- Evaluation: F1

$$\text{precision} = \frac{tp}{tp + fp}$$

$$\text{recall} = \frac{tp}{tp + fn}$$

$$F1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

Evaluation



74
72
70
68
66
64
Bin F1
1
2
3
4
5
Layers
68
66
64
62
60
58
1
2
3
4
5
Layers
24k
200k