SA104X – Degree Project in Engineering Physics, First Level 15.0 Credits

Project 2 : Autonomous Helicopter Landing on a Mobile Platform

Supervisors : Xiaoming Hu, Johan Markdahl
hu@kth.se, markdahl@kth.se

Problem statement

This project concerns modeling of helicopter dynamics, design of a stabilizing feedback control law and simulation of the resulting system of differential equations. The project has been divided into two parts, each of which is to be carried out by a group of two students. Group 1 is required to find a feedback control law that stabilize the dynamics governing the helicopter attitude, *i.e.* to align the helicopter yaw, pitch and roll angles with given reference values. Group 2 is required to find a feedback control law that stabilize the dynamics governing the translational motion of the helicopter. Both groups are advised to choose their models bearing their respective control design aim in mind.

Planning

Any degree project is expected to be carried with a great degree of independence on the part of the students. The supervisors will be available as counselors to the students, with the possibility of meeting every week for an hour or so of discussion and planning. The students will themselves plan their work, find and read relevant background material, design a control law for the model, simulate it and write a detailed report in a word-processor of their choice.

To begin with, two groups should be formed. Also, a time-line for the project needs to be constructed. There are seven weeks of full-time studies to distribute on modeling, control design, simulation, report writing and presentation over a 20-week semester [1]. Some time will be required for finding, evaluating and reading background material.

Modeling

The dynamics of helicopter flight are well documented in the literature. Find a suitable *state-space model*, either dynamic or kinematic, describing the motion of the helicopter in a reliable textbook or paper of your choice.

The model should be a system of differential equations on the form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ are the *states* of the model, $\boldsymbol{u} \in \mathbb{R}^m$ the *input/control signals* and the derivative of $\boldsymbol{x}$ is taken with respect to time, $\boldsymbol{x} = \boldsymbol{x}(t)$. A model of few states is preferable to one of many, and a so-called *control-affine* model of the form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}. \tag{2}$$

is preferable (if possible) to a more non-linear model since control design of these type of systems has been thoroughly studied.

The text [2] is a recent reference on control of autonomous helicopters. The entire book can be read online if you first go to the KTH library web page,

`http://www.lib.kth.se/main/`,

then search for "Springer" among databases, log in with your KTH library account and finally search for the book ISBN at the Springer web page.

Control design

In the control design part, we will design $u = \mu(\boldsymbol{x})$ to make sure that the solution $\boldsymbol{x}(t)$ of

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \mu(\boldsymbol{x}))$$

converges to a reference value $\boldsymbol{r}(t)$ that may or may not be time-dependent for all $x(0) \in \Omega \subset \mathbb{R}^n$, where $\Omega \neq \emptyset$ is the so-called *region of attraction* – the set of initial-points from which convergence is guaranteed.

It might be the case that our knowledge of the system is limited to measurements of a few quantities $\boldsymbol{y} \in \mathbb{R}^k$ (*output signals*) that are related to the states by

$$\boldsymbol{y} = h(\boldsymbol{x}).$$

For example, a cruise control designed to control the speed of a vehicle to some reference value may be limited to use measurements of engine speed. In such cases, an output feedback control law may be specified as $u = \mu(\boldsymbol{y})$.

For a non-linear system like (1) on control-affine form (2) (which the model is likely to be), there are various techniques to be used depending on the more detailed form of the model, see for example [3, Chapter 14].

In this project, we will focus on linear feedback control laws, $\mu(\boldsymbol{x}) = K\boldsymbol{x}$ where $K \in \mathbb{R}^{n\times n}$ is a constant matrix. The matrix K of the linear feedback law should be such that it stabilizes a *linearization* of the nonlinear dynamics (1) locally around some operating point x_0, u_0. The linearization is done by means of a Taylor expansion, see for example [4, Chapter 8].

For a *linear system*,

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, the *equilibrium point* $\boldsymbol{x} = \mathbf{0}$ is *stabilized* by setting $\mu(\boldsymbol{x}) = K\boldsymbol{x}$, where the matrix $K \in \mathbb{R}^{m\times n}$ is chosen so that the eigenvalues of $A+BK$ are strictly negative. This feedback steers the system

$$\dot{\boldsymbol{x}} = (A + BK)\boldsymbol{x}$$

to $\mathbf{0}$ as time goes to infinity regardless of $\boldsymbol{x}(0)$, *i.e.* $\Omega = \mathbb{R}^n$. A nice introduction to linear systems is [5], the lecture notes for the Mathematical Systems Theory course SF2832.

For a linear output given by $\boldsymbol{y} = C\boldsymbol{x}$, where $C \in \mathbb{R}^{k\times n}$, a linear output feedback law $\mu(\boldsymbol{x}) = K\boldsymbol{y} = KC\boldsymbol{x}$ stabilizes the system (3) if the eigenvalues of $A + BKC$ are strictly negative. In the case of $k = n$, $C = I$, output feedback is reduced to state feedback.

After having found a stabilizing control for the linearized system, it remains to investigate the stability properties of the system

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, KC\boldsymbol{x})$$

in a neighborhood of x_0.

Simulation

The system equations should be solved numerically using a MatLab inbuilt solver for various parameter values/initial states. Such simulations may *e.g.* serve to estimate the region of attraction Ω if this cannot be done analytically, *i.e.* determine a set of point in which the linearization around x_0 is a sufficiently good model of the nonlinear system.

Simulation may also be done in Simulink, if so desired. Simulink is a useful tool for developing signal processing applications. The user interacts with Simulink by picking blocks from a library, placing them on a canvas and routing signals between them in a point-and-click fashion. Simulink is developed by The MathWorks and therefore highly integrated with MatLab which can be used as a data pre- and postprocessor.

A good introduction to SIMULINK can be found at

`http://www.me.cmu.edu/ctms/`

Literature

Finding the literature needed to complete this project is part of the requirements. Textbooks are often preferable to papers when looking for basic level information, while papers may sometimes be needed to find in-depth information. Below is a list of search engines that you may find useful while looking for information.

Books

- `http://libris.kb.se/`
- `http://books.google.com/`

Papers

- `http://www.scopus.com/home.url` (only at KTH)
- `http://scholar.google.com/`

Master's Theses

- `http://www.uppsatser.se/`
- `http://liu.diva-portal.org/smash/search.jsf`

References

[1] Course web pages,
`http://www.kth.se/sci/gru/kurser/kansli/SA104X/VT10-1`
`http://www.math.kth.se/optsyst/grundutbildning/kandexjobbVT11/`

[2] Raptis I, Valavanis K. *Linear and Nonlinear Control of Small-Scale Unmanned Helicopters*. Springer, 1st edition, 2011. ISBN 978-94-007-0022-2.

[3] HK Khalil. *Nonlinear systems*. Prentice Hall, Upper Saddle river, 3rd edition, 2002. ISBN 0-13-067389-7.

[4] Glad T, Ljung, L. *Reglerteknik*. Studentlitteratur, 3rd edition, 2006. ISBN 91-44-04308-2.

[5] Lindquist A, Sand J. *An Introduction to Mathematical Systems Theory* Department of Mathematics, KTH.