Week 11 & Week 12: Finite Element Analysis (FEA)

Accelerate

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1 What's different

This week you won't be given a set set idea. I want you to look around you. Look at the objects that make up your everyday life. How do we know they won't break? I want you to try to model something around you in your day to day. Whether it's your house's roof, or a telephone pole. I want you to model it using FEA.

2 Intro to FEA

This is going to be short. So bear with me.

The process goes something like.

1. Model the geometry of the problem.
2. Create a mesh of the problem. That is split into elements of a finite size.
3. Design Boundary Conditions and Initial Conditions.
4. Think about what you're going to calculate, and how you're going to calculate it for each distinct element.
5. Run it computationally, to calculate for all the elements.
6. Visualise the results.

The core idea here is. Split the problem up, and solve it smaller to get a solve for the big picture. It uses Divide and conquer :yay: !

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Version January 16, 2026

3 The Fundamentals

The main things you'll need to calculate for each element are.

3.1 Stress

Stress measures the internal forces within a material.
Stress is defined:

$$\sigma = \frac{F}{A}$$

It has units of Nm^{-2}
It has three types:

- **Tensile stress**: pulling apart
- **Compressive stress**: pushing together
- **Shear stress**: sliding forces parallel to the surface

3.2 Strain

Strain measures how much a material deforms relative to its original size.
It is defined as:

$$\epsilon = \frac{\Delta x}{x} = \frac{x - x_0}{x_0}$$

And has no units.
where x_0 is the original length and x is the deformed length.

3.3 Material Properties & Constitutive Relations

Now we know stress and strain, but how are they related? That depends on the material.

3.3.1 Young's Modulus

Young's Modulus (E) tells us how stiff a material is. It relates stress to strain in the linear elastic region:

$$E = \frac{\sigma}{\epsilon}$$

Or rearranged (If the material is following Hooke's Law):

$$\sigma = E\epsilon$$

A higher Young's modulus generally means the material is stiffer.

3.3.2 Poisson's Ratio

When you stretch something in one direction, it usually gets thinner in the perpendicular directions. Poisson's ratio (ν) quantifies this effect:

$$\nu = -\frac{\epsilon_{transverse}}{\epsilon_{axial}}$$

For most materials, ν ranges from 0 to 0.5. Rubber is close to 0.5 (nearly incompressible - squeeze it and it bulges out), while cork is close to 0 (doesn't change shape in other directions when compressed).

3.4 Moments

A moment (or bending moment) is what happens when a force tries to rotate something.

The moment is calculated as:

$$M = F \times d$$

where F is the force and d is the perpendicular distance from the force's line of action, to the pivot point,

In FEA, moments are crucial because bending creates stress distributions across elements. The classic beam bending equation is:

$$\sigma = \frac{My}{I}$$

where:

- M is the bending moment
- y is the distance from the neutral axis (center of the beam)
- I is the second moment of area (a geometric property)

This tells us that stress is highest at the furthest points from the center. That's why I-beams are shaped the way they are - material far from the center is most effective at resisting bending!

3.5 Why This All Matters for FEA

In FEA, each element needs these properties to calculate how it responds to forces and moments. The solver uses:

- Stress and strain definitions to measure deformation
- Material properties (E and ν) to predict behavior through constitutive equations
- Moment calculations to handle bending and rotation

4 Conclusion

So go on, use these ideas and make your own FEA model!