Week 7 & Week 8: Electrostatics Electromagnetic Waves

Accelerate

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ELECTROSTATIC EXAMPLE GITHUB

1 Introduction

Electromagnetic fields are everywhere. They're fundamental to modern technology, from the headlights on your car, to the WiFi you have at home. While simple cases have easy mathematical solutions, real-world situations need computer simulations. In this I'll introduce the basics of electromagnetic field simulation.

2 From Point Charges to Fields

2.1 The Basics

Two charges exert forces on each other. Coulomb's law says the force between charges q_1 and q_2 separated by distance r is:

$$F = k\frac{q_1 q_2}{r^2} \tag{1}$$

where $k = 9 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$. Positive charges repel, opposite charges attract.

2.2 The Electric Field Concept

Instead of thinking about forces between charges, we can think about fields. A charge q creates an electric field $\vec{E}$ everywhere in space:

$$\vec{E} = k\frac{q}{r^2}\hat{r} \tag{2}$$

This field exists whether or not there's another charge to feel it. Any charge q_{test} placed in this field feels a force: $\vec{F} = q_{\text{test}}\vec{E}$.

Think of the field like a map showing what force a test charge would feel at each location.

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2.3 Adding Magnetic Fields

Moving charges (currents) create magnetic fields $\vec{B}$. A wire carrying current I creates a circular magnetic field around it. The force on a charge moving with velocity $\vec{v}$ in a magnetic field is:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{3}$$

This is why magnets can deflect charged particles.

2.4 From Static to Dynamic

So far, we've talked about static charges and steady currents. But what if charges accelerate? What if currents change? This is where things get interesting, where EM takes center stage. Maxwell realized that changing fields create new fields, leading to waves that propagate through space.

2.5 Maxwell's Equations

Maxwell used these ideas about electricity and magnetism to create four equations.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{Gauss's Law} \tag{4}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday's Law} \tag{5}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{Ampère-Maxwell Law} \tag{6}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{Gauss's Law for Magnetism} \tag{7}$$

Here, ρ is charge density (charge per volume), $\vec{J}$ is current density (current per area), and ϵ_0, μ_0 are constants that determine light speed: $c = 1/\sqrt{\mu_0 \epsilon_0} \approx 3 \times 10^8$ m/s.

What we really need to get from this is that changing magnetic fields create electric fields (equation 5) and changing electric fields create magnetic fields (equation 6).

2.6 Electromagnetic Waves

Due to this, when a particle moves back and forth, it has changing electric and magnetic fields that propagate away as waves:

$$\vec{E}(z,t) = \vec{E}_0 \sin(kz - \omega t) \tag{8}$$

where $k = 2\pi/\lambda$ (wavelength) and $\omega = 2\pi f$ (frequency). The wave travels at $c = \lambda f$.

3 Simulating Fields with Computers

Maxwell's equations describe how fields evolve, but solving them by hand is only possible for simple cases. For almost everything we need computational methods. The Finite Difference Time Domain (FDTD) method is the simplest approach.

3.1 The FDTD Idea

The basic concept:

1. Divide space into a grid
2. Store $\vec{E}$ and $\vec{B}$ at each point
3. Update fields step-by-step using Maxwell's equations

3.2 The Update Rules

In 1D, we alternate between updating E and H (we use $H = B/\mu_0$ for simplicity):

$$E_x^{\text{new}} = E_x^{\text{old}} + \frac{\Delta t}{\epsilon \Delta x}\left[H_y^{\text{right}} - H_y^{\text{left}}\right] \tag{9}$$

$$H_y^{\text{new}} = H_y^{\text{old}} - \frac{\Delta t}{\mu \Delta x}\left[E_x^{\text{right}} - E_x^{\text{left}}\right] \tag{10}$$

Translation: E changes based on how H varies in space, and vice versa. This is Maxwell's equations in discrete form.

3.3 Stability Condition

Choose your time step carefully:

$$\Delta t \leq \frac{\Delta x}{c} \tag{11}$$

If you violate this, your simulation explodes! Use $\Delta t = 0.5\Delta x/c$ to be safe.

3.4 Boundary Conditions

Every simulation needs boundaries:

- **Metal walls:** Set electric field parallel to surface to zero
- **Absorbing boundaries:** Use special conditions to prevent reflections (simulates infinite space)

3.5 Implementation Tips

- **Grid size:** Start small (e.g., 200 points). Bigger grids are more computationally expensive.
- **Adding waves:** Create a source at one point: $E_x[\text{source}] += A\sin(2\pi f t)$
- **Visualization:** Plot E_x vs. position to see the wave propagate
- **Materials:** Change ϵ in different regions to simulate glass, air, etc.

3.6 Basic Algorithm

1. **Setup:**
   - Create arrays: `Ex[200]`, `Hy[200]`
   - Set $\Delta x = 0.01$ m, $\Delta t = 0.5\Delta x / c$
2. **Time loop:** Repeat many times:
   - Update all H from neighboring E values
   - Update all E from neighboring H values
   - Add source: `Ex[100] += sin(2*pi*f*t)`
   - Save snapshot for animation
3. **Visualize:** Plot E_x vs. position to see the wave move!

3.7 Testing Your Code

How can we check it works?

- **Wave speed:** Measure how long a pulse takes to cross your grid. Should be $c = 3 \times 10^8$ m/s.
- **Energy:** Total energy should stay constant (if no losses). If it grows or shrinks, check your time step, and method.

4 What's Next?

Once you have a basic Electrostatic simulator, try:

4.1 Electromagnetic Wave Simulation

Build a simulation that simulates electromagnetic waves.

4.2 Multiple Materials

Extend it, setting different ϵ values in different regions. Watch waves reflect and refract at boundaries, see the difference between glass, water and other materials in real time.

4.3 Pulses and Interference

Send two pulses from opposite directions. Simulate the superposition of these waves.

4.4 2D and 3D

Extend to higher dimensions. Now you can simulate antennas, waveguides, even optical devices. The algorithm is the same, just more neighbors to consider.

5 Conclusion

Electrostatics are an easy start to EM simulation.

Electromagnetic simulation brings invisible fields to life. Maxwell's equations describe everything from radio to light, and FDTD makes simulating that easy.

Make your electrostatic simulation, then start with 1D EM waves, watch them propagate. Once you understand the basics, extend it to 2D, 3D, and real-world applications.